Wheeled Mobile Robots
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Lecture - 09
Holonomic and Non-Holonomic Mobile Robots

Welcome back to Wheeled Mobile Robot course. So, this is a lecture 9, you know like lecture 8 we were talking about generalize wheel model along with the example. So, and I gave a brief introduction there itself we would be classifying the mobile robot that too like wheeled mobile robot into different category.

So, that is what we are trying to cover in this lecture 9. Where we would be talking about you can say wheel configuration based on the wheel configuration, what would be the nature of the total mobile base based on that we would be classifying and we would be seeing little further, how this classification can happen and what would be the you can say seriousness on this.

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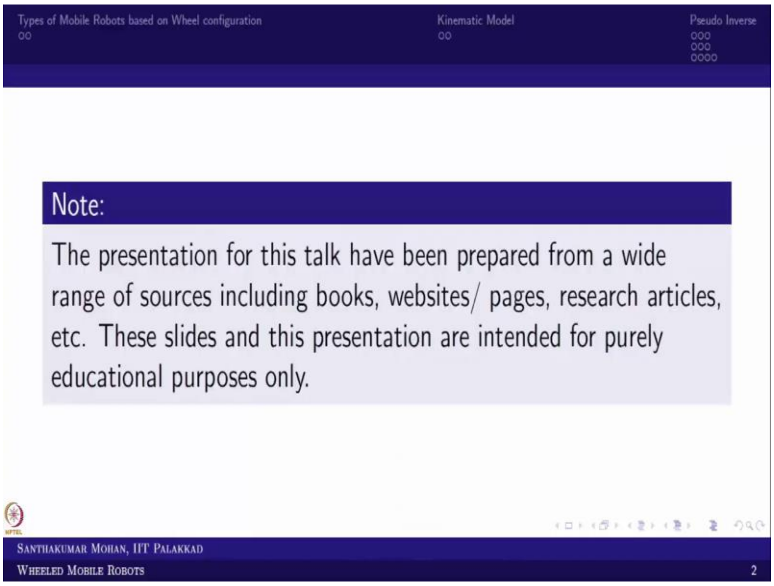

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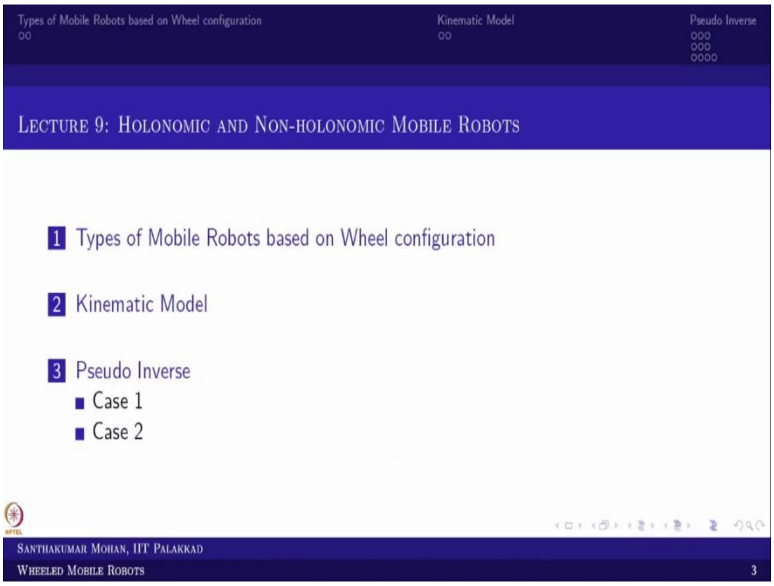

So, if that is the case what we will actually like see here. So, in this particular lecture we would be seeing about.

So, types of mobile robot based on the wheel configuration and we would see the kinematic model and in that one particular aspect what we have seen that the W which is nothing but the wheel you can say configuration matrix or input configuration matrix. That matrix how it would actually like play for this particular what you call holonomic or non-holonomic.

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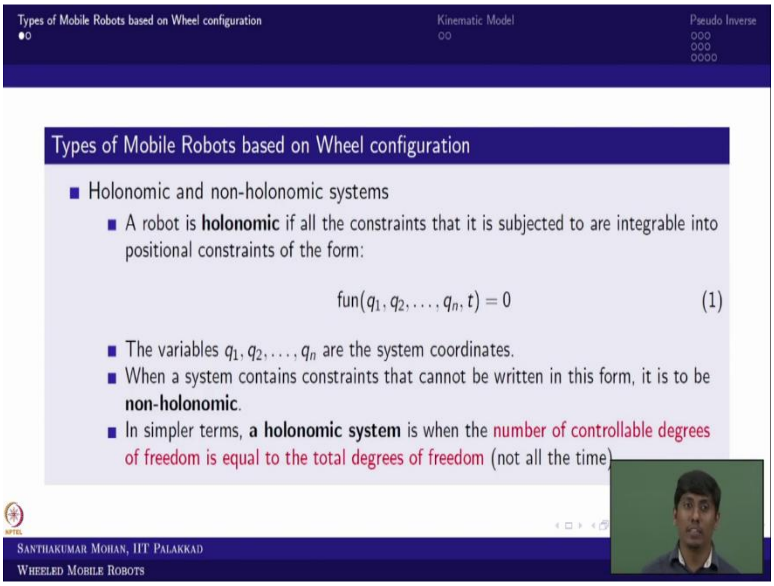

So, now I already talked about holonomic. So, we will just talk about, how we can classify the mobile robot that too like wheeled mobile robot based on the kinematic or wheel configuration. If that is the case I mentioned that it can be classified two broad way, so one is holonomic, the other one is non holonomic; the name itself says that holonomic. Holonomic means it is having all the features right that is what we are trying to see in (Refer Time: 01:43).

So, in that sense what we are trying to see? A robot is holonomic if all the constraints that it is subjected to or integrable into a positional constraint of the form. For example, if I say that it is in a planner so, I can write 3 constraint equation, all 3 constraint equation should be integrable form.

So, that is what we call holonomic. It is actually like quite tricky right to understand. So, that is why we are actually like going to define this holonomic and non holonomic based on the wheel configuration matrix.

But, before going to see that then what is non holonomic? So, if anything any concern which cannot be written in this you call integral integrable form then what you call that is non holonomic. One simple example you see in the last class, what we have seen as a differential wheel drive where u and r only would be the component when you write it. So, the v component was not coming. In the sense the v equation if I write that would be 0.

So, then somebody can say that is a it is integrable form because v = 0 if I integrate that would be giving as a constant. Yes I do agree, but that is not the real integrable form right because v = 0 the constant can come anything when I integrate, but I need proper you call integrable form that if I able to write all the constraint equation then I call that is holonomic.

So, if that is the case what one can easily feel it? If the number of you can say controllable inputs is equal to number of degrees of freedom then you can say holonomic yeah certain extent it is right but not for all the time. For example, you would have seen the skid steering right. Why it is called skid steering? I did not told there.

So, you can see that this robot if you have actually like simple differential wheel only 2 that actually like a actual like you what you can feel it; it is just it would rotate and as well as it is actually like manual. But, now you have 4 wheel and 4 wheel independently you control. What happened?

You can see that it would be going very fast and as soon as it is actually like sliding in the way in the real time that is why it is actually call skid steering. It is not a steering system, but you can actually like apply based on that the rotational can happen by skidding so, that is what the idea.

So, in that sense what the number of controllable inputs for that skid steering robot? There are 4 right, but that is not holonomic system. Why? The v is still in non integrable form.

So, that is why I said certain extent you can say that number of controllable degrees of freedom is equal to the total degrees of freedom in the sense the number of input is equal to the total degrees of freedom then you can actually like say it is holonomic system. So, it is actually like you can see the overall picture right.

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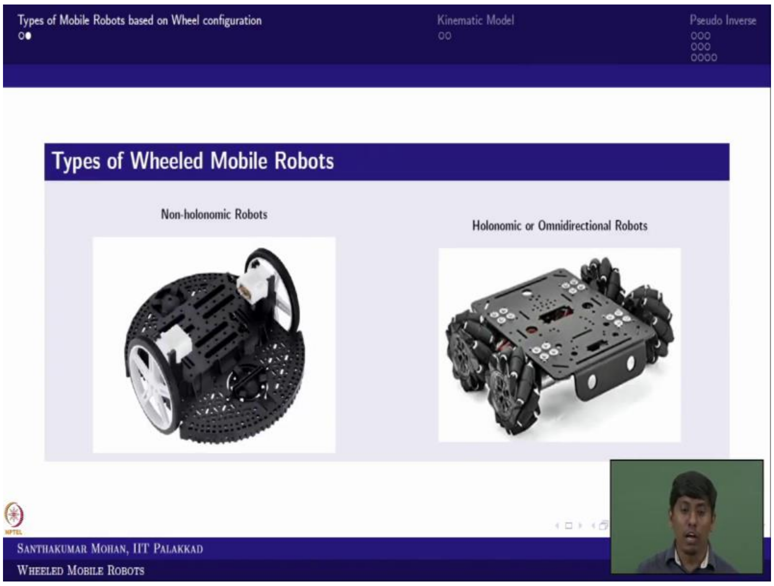

So, now I am just giving a small, you can see pictorial view. You can see this is a non holonomic. It is a differential wheel there are 2 power wheel and 2 casters are there. But you can see that based on the configuration it is a fixed wheel, the lateral direction is going to in a non integrable form. So, that is what we are actually saying this is non holonomic.

Whereas, you would take here because of the passive roller which is attached in this mecanum wheel, so what it is giving? It is actually like giving all 3 constraint can be written in a integrable form so, that is what we call holonomic. And most of our robotic community people call it is omni directional or omni directional. What that mean?

So, it can actually like maneuver longitudinal, lateral and as well as rotate freely that is what you call omni directional. So, you can actually like resemble what is omni directional wheel. The wheel can actually like go in longitudinal and as well as it can slide in lateral. The same way this particular robot which is having all 3 you can say motion possible that we call omni directional robot.

So, now you got a clarity right what is non holonomic, what is holonomic right. So, now, we will move forward why the wheel generalized model is required and why the W matrix is coming into a picture.

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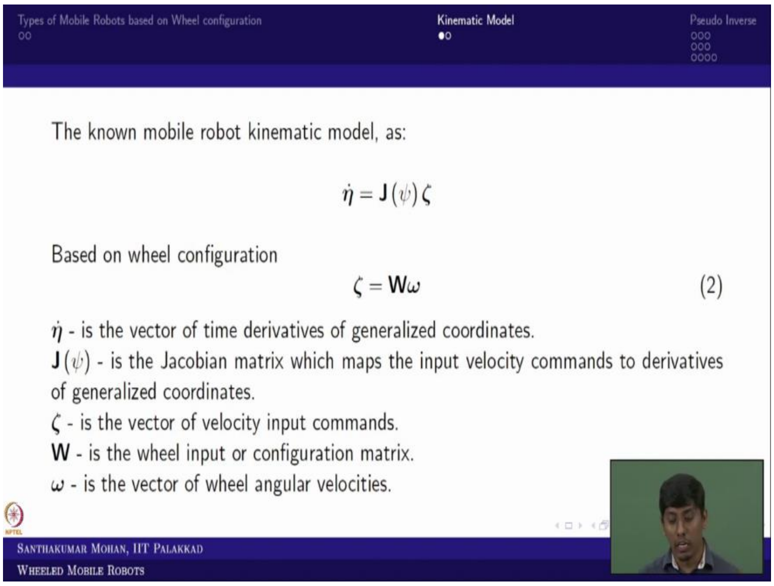

So, now we will recall. So, the known mobile robot kinematic model is nothing, but $\dot{\eta} = J(\Psi)\xi$, but what we have written based on the generalized you can say kinematic model of the wheel? So, what we brought? The $\xi = W \times \omega$, where W what we call? Wheel input or configuration matrix right.

So, now can I actually like bring this non integrable instead of writing non integrable you can say a constrained equation, can I take this W as the nature of W. Can I classify non holonomic or holonomic? Yes you can do. So, for example, if you take a land based system, so what would be the W rank? So, it would be actually like the left hand side which is ξ.

So, this ξ is actually like would be having 3 × 1 whereas, the W can have maximum 3 × n. The n can be less than 3 or more than 3, but what one can see? The maximum possible rank for the W is 3 and how many motions are required? 3. So, if you have actually like a rank of 3 of W then you can call that is a holonomic robot. If the rank is actually less than 3, what one can see? At least one or more motions are not possible.

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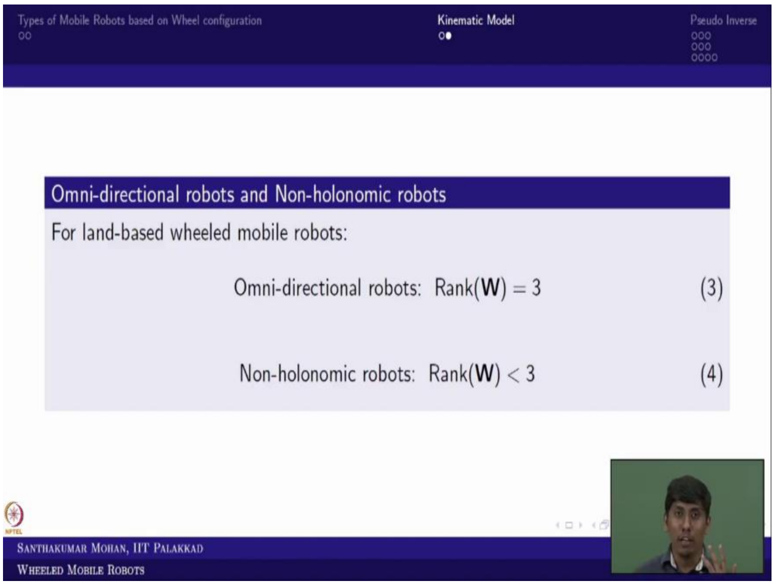

So, that is what we are actually trying to say. So, the omni directional and non holonomic I am classifying. Based on the nature of the W what I say that omni directional robot is if the rank of W is 3. If the rank is less than 3 then I call non holonomic.

So, now, you got a very clarity picture right, but further end is actually coming into a picture. What the further end? If you are going further inverse differential kinematics. What inverse differential kinematic? The ξ you can write in the form of $\dot{\eta}$.

So, what you do? $J^{-1} \times \dot{\eta}$, but now you assume that that I can obtain because the J^{-1} always exist because the J^{-1} is actually like possible because the J is a simple orthogonal matrix and the Ψ is actually like even if you put 0 or 90 that would be no longer singular. So, in the sense you can see like you can always get ξ based on the $\dot{\eta}$ desired.

But what we are actually looking at? We are looking at the point of wheel configuration where the ω can I obtain. So, for that what we will take. The W inverse into ξ would be giving as ω. So, now, the W^{-1} is exist or not your big question right, why? The W can be a rectangular.

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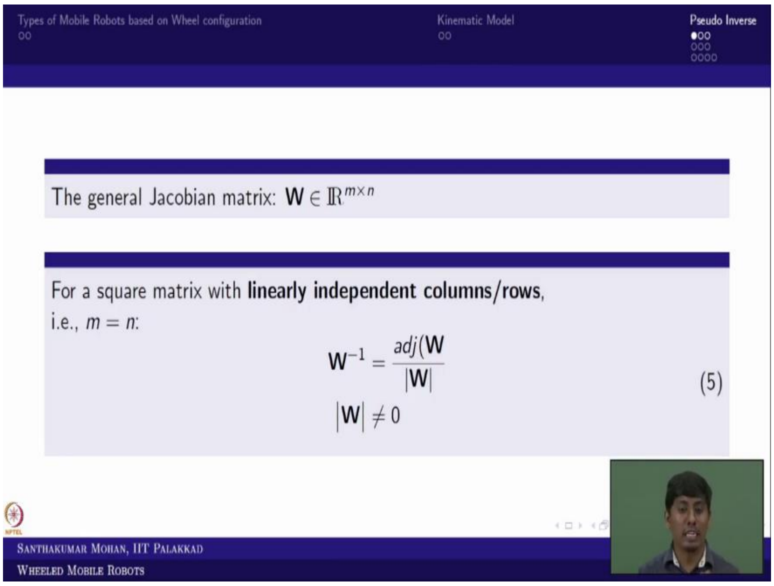

So, if it is a rectangular, so what one can see? So, the inverse is exist or not? So, if it is actually like a W I write in a; you can say real form where it is actually like a real of m × n, in the sense m × n as a matrix. So, what one can see? If this m = n we are easily find it right because we assume that m = n in the sense it is a square matrix, in addition to that.

So, this is having independent columns or rows, so what one can see? The singularity is no longer exist, in the sense W inverse is exist. Why it is? So, the determinant of W is nonzero. So, in that sense what you can find? The W inverse you can find it with the help of $\frac{adj(W)}{|W|}$ which is nothing but a determinant.

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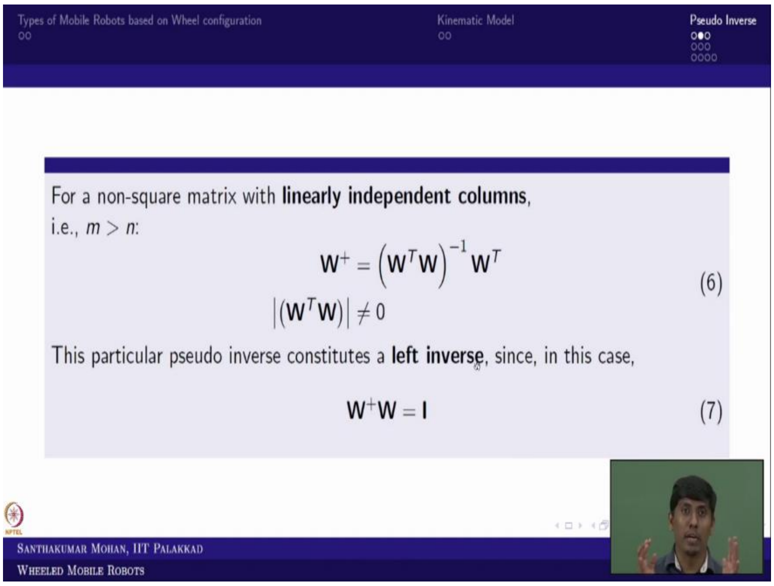

So, now, the $|W|$ is exist then you can do it right. So, now, this is actually like one case, but now imagine this is a rectangular where m > n. So, what that mean? So, your number of; you call angular velocity in the sense number of powered wheel or number of controllable inputs are less than the total number of system states, what that mean? So, you have 3 are the states, but your number of wheels powered wheels are actually less than that.

You take a differential wheel so where that is having 2. So, in the sense 3 < 2 right or you can say 3 > 2. So, in the sense of what one can see? If it is a non square matrix, so, if you have actually like independent columns; in the sense, here what would be the independent columns? So, it would be 3 × 2 right. So, in the sense if you have independent column of 2 then what would be the maximum rank? So, n.

So, in that sense of what one can see, can I actually like find the inverse? That is what whole idea right. Whole idea is actually like the ξ is given. Can I find the ω which is what you call angular velocity vector of the wheels? So, can I find it? So, for that we are trying to use a pseudo inverse which is nothing but Moore Penrose we are actually like using it.

So, what we can call? We can call a left inverse can be applied in this case. So, we will see detail about the pseudo inverse in the further slide, but now you are calling left inverse, why it is called left inverse? So, this $W^{+} = \left(W^{T}W\right)^{-1}W^{T}$ that would not give identity matrix.

So, that is what you call left inverse so that is what we are actually like taking it. So, now, you can see that the pseudo inverse using you can say T.

So, in the sense $W^T \times W = I$ can invert it. So, that is what we are doing it here, we will see in detail how this $W^T \times W$ has come. So, similarly if it is actually like the other way around non square but it is actually like where m is actually like < n.

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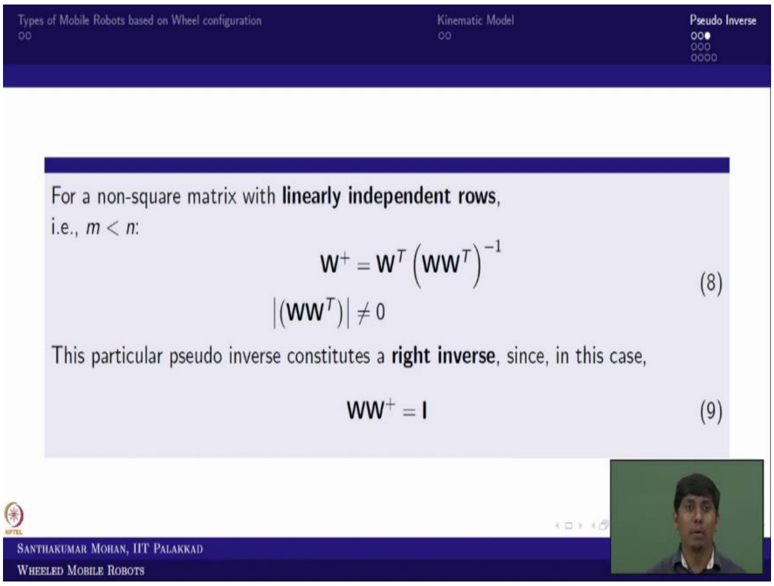

For example, you take a mecanum wheel 4 mecanum wheel drive so where n would be you can say ω_1 to ω_4 where n = 4, but you have $\begin{bmatrix} u \\ r \\ v \end{bmatrix}$ in the sense 3. So, 3 is actually less than 4 in the sense what you can see the non square is exists. Here, what would be the maximum rank you can expect?

So, 3, in the sense so, m. What you can actually call it? If it is having a linear independent rows so what would be the pseudo inverse possible that is what we are trying to use it. So, earlier we have taken left inverse, so; obviously, what we call here is right inverse so that is what we are applying it.

So, here W = what you call that would be coming with you call right inverse. So, now, W + can be actually like post multiply with W then only you would get identity matrix so this is what we call right inverse. Now, there would be one question will come in your mind.

So, how you are saying that this W, W^T, whole inverse, earlier case $(W^T W)^{-1}$. What is the logic? So obviously, I am trying to address although it is not exact part of this course, but I thought I can address little bit on this.

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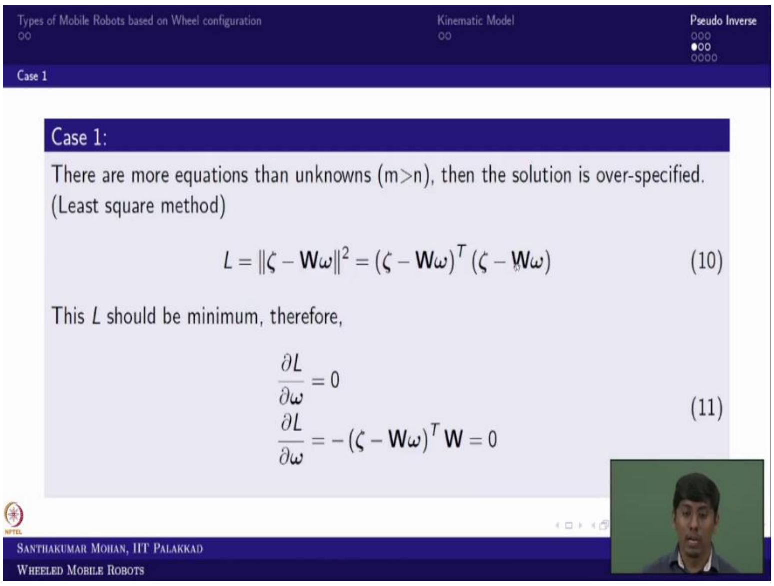

In the sense the case 1, so, you know like a simultaneous equations right. So, now, you bring in that way as a mathematician you can bring it. So, there are actually like more equations than unknowns, what; that means? If you have actual like m = 3 and n =2. So, what you will get? The n is unknown which are actually 2, but you have actual 3 equation to be written. So, what that mean?

So, your equation is actually like over specified because you have only 2 unknown, these 2 unknown you have to you can say put it in the 3 equation in such a way that this would be qualify. In mathematics this is no solution exist, but here what we are trying to find out, since no solution exist if you leave it then the differential will drive or you call tricycle and all would not work right. Then we have to see what we can do it? So, this particular situation what we call over specified.

So, in the mathematics we will take one of the tool what we call least square method. So, what we are trying to take? I am taking one of the parameter what I call L. So, L is actually like length of the vector, what that vector here? So, $\xi = W \times \omega$. So, now, I am bringing it that as $\xi - W \times \omega = 0$, but that is not possible in this case. Why? Because there are only 2 unknowns. So, then what I can do? This length can I reduce to 0 or minimum?

So, in that sense what I can bring as you call objective function L as a length of the vector I am taking as you can say two norm of this $||\xi - W \times \omega||$. So, this I can write in an inner product style in the sense $(\xi - W \times \omega)^T \times \xi - W \times \omega$. So, this is what you can write in an inner product form.

So, now what I can do it? So, this L, if I actually like take differentiation with respect to what we wanted? We wanted in a ω form right. If I actually take $\frac{\partial L}{\partial \omega} = 0$, what I will get? So, I will actually I get something useful right. So, what that can be either minimum or maximum. If I take $\frac{\delta^2 L}{\delta \omega^2}$, if that is actually like you can say positive sign then you can call as a minima right.

So, now bringing that into a case, I am trying to bring actually like $\frac{\partial L}{\partial \omega} = 0$. So, now, I am actually like differentiating this equation partially differentiating with respect to ω. So, then what I will get? So the equation would be giving because this is actually like as similar. So, I am keeping this as T whole term as single and this I am actually like differentiating.

So, in the sense it would be - W right. So, in the sense I can rewrite this in this form. So, now, I am not done. So, what I am not done? So, I did not get still the equation right.

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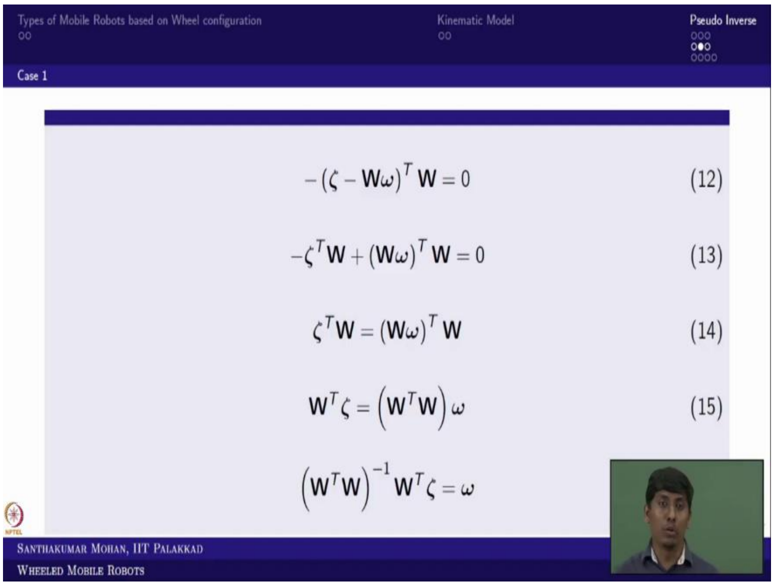

So, what I can do it? I am taking this equation further; I am actually like taking the T throughout in the sense $\xi^T - (W \times \omega)^T$ where the - sign also I am taking into account and multiplying with a W. So, what I will get? $-\zeta^T \mathbf{W} + (\mathbf{W}\omega)^T \mathbf{W} = 0$.

So, now, you can see that something we have got it right. What we got it? So, I can rewrite this equation in this form. So, now, you can easily see right. So, what you can see? So, if I take the T both side, so, what I will get? So, I will get ω sorry, $W^T \times \xi$ the other side it would be in the form of ω where $W^T \times W$. What you do not know? ω, you assume that ξ is known.

So, then what you can see? You take the W^T W which is nothing but a square matrix. We assume that the determinant of this W^T W is not nonzero then you can take inverse. So, that is what we have taken now you can see that what happened? The left inverse right. So, if it is actually like a over specified, what you obtained? You obtained the left inverse.

Now, W if you I have given as a rectangular where m is actually greater than n you can apply this right. So, this is what the Moore Penrose have given, this is what we call Moore Penrose pseudo inverse. So, this is what we have finally, written in the normal form.

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So, now, only one condition is W^T W determinant supposed to be nonzero right, this is the condition where W + I am writing as a left inverse.

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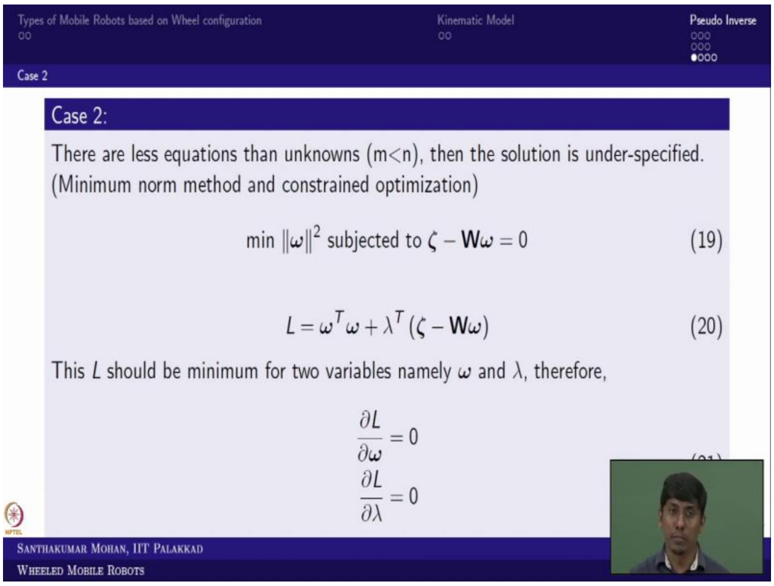

Now, we will come to the 2nd case. What would be the 2nd case? So, there would be less equations than unknowns where, m < n. So, you take the same case what we have discussed earlier; you take the mecanum wheel there are 4 and there are number of you can say steer are 3. So, 3 is actually like less than 4. So, now what we are actually trying to see the situation? It is under specify. Why it is under specify? So, you have actually like more unknown and less equation.

So, now, the unknowns you can actually like manipulate in number of way. If you talk in a mathematic form this would have multiple solution theoretically it is having infinite number of solution. So, then you can see this is not at all specified right in fact, it is not at all specified. So, that is why we call the solution is under specified. So; obviously, you can see there are several solution which will give ξ - W×ω = 0. Then what we wanted? We wanted which solution will give minimum length.

So, that is why we bring into as a minimum norm method. So, what then? So, you have to take the length that length would be fulfilling one constraint. So, earlier what we did? Simple optimization, but here we are doing a constrained optimization. What that mean? So, minimum is what we are interested, minimum ω in the sense length of the ω should be smaller.

So, for that the length we are trying as a norm, so here 2 norm. So, minimum of you can say $\|\omega\|$, subjected to what? So, the ξ - W×ω = 0 that is what whole idea right. So, if that is the case what we can bring it? We can bring it these 2 in a single optimization equation where Lagrangian multiplier will come because there is a constraint equation right.

So, I can write the earlier equation in a different form; earlier equation is straightforward, but now it is having a constraint. So, in the sense $L = \omega^T \omega + \lambda^T (\zeta - \mathbf{W}\omega)$. So, now, what you can see? This can be minimum in two instant, where actually like it can be minimum based on λ based on ω.

So, in the sense what we can see? We can actually differentiate partially differentiate this $\frac{\partial L}{\partial \omega} = 0$ $\frac{\partial L}{\partial \lambda} = 0$. So, both supposed to be 0; in the sense 2 unknowns, 2 equation you will get you can find it right.

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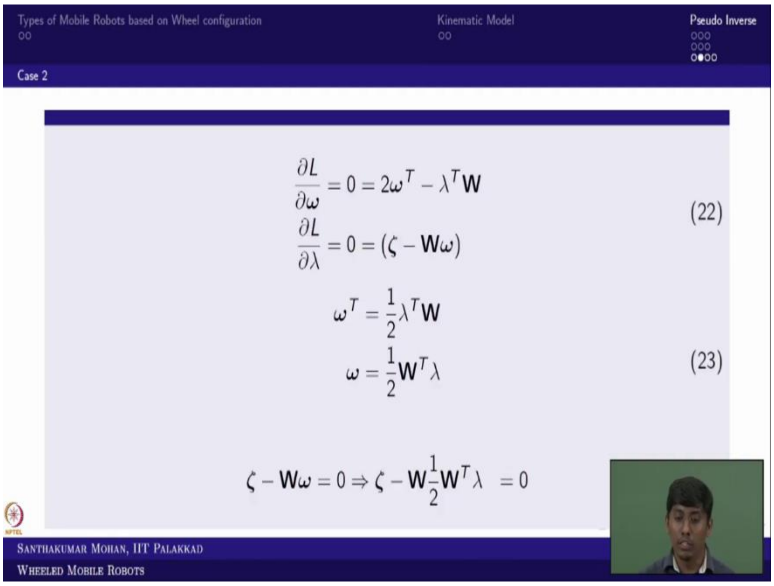

So, I am taking $\frac{\partial L}{\partial \omega} = 0$. So, you know this is the equation, I am differentiating with respect to ω I got this equation right. Similarly, if I take a differentiation with respect to λ this is $\frac{\partial L}{\partial \lambda} = 0$ what the equation. Now, you have 2 equations 2 unknowns. So, we will try to bring 1 first unknown with another unknown as a relation.

So, the ω^T I am writing into a λ form, since it is ω T I am taking T throughout. What I will get? $\omega^T = \frac{1}{2}\lambda^T W$, $\omega = \frac{1}{2}W^T\lambda$, when I see the second equation ok, so what I can actually like find it? So, $\xi - W\times\omega = 0$ this is not giving any usefulness; because this is what we started right.

But what usefulness is giving? This particular equation 23, you substitute into this equation. So, what you will get? So, you will get this right. So, now, you can see that this particular equation is having only one unknown which is nothing but λ.

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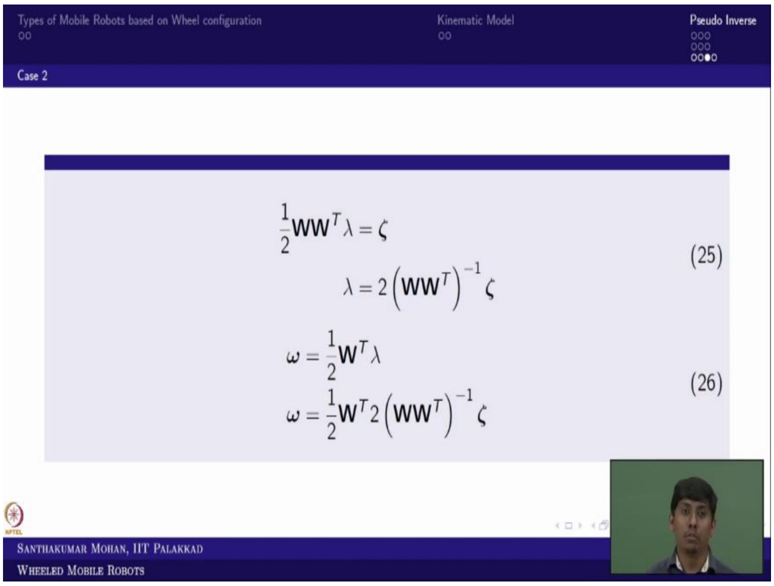

So, I am rewriting this equation in this you can say simplified form where this half I am bringing in to one side and you can see this is what the left hand side and right hand side. So, now, what is unknown to us? The ξ is known based on the inverse differential kinematics, the λ is unknown. So, you can actually take that. What we have done? We take the W into W T inverse. So, now, this is what the idea.

Now, what is λ? So, λ I can write in the form of what you call ω. And if you recall the $\omega = \frac{1}{2}W^T\lambda$. So, now, I already found the λ right; I substitute that into this equation. So what I will get? ω; so now you can see that the 2 and 2 goes out.

So, what you will get? So, the right inverse, right where $(W\times W^T)^{-1}$ is multiply with pre you can say post multiply in the case with W^T that is what you call right inverse so you got it right.

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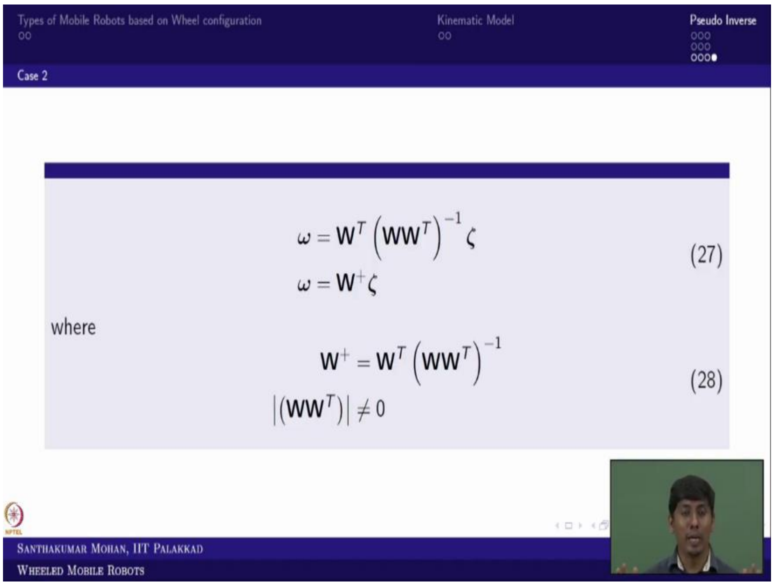

So, what is the basis of this pseudo inverse have come that you have clarified here. So, now, you can see that this W + for both the cases we have calculated, only one condition is the square matrix what you are obtaining $W\times W^T$ or $W^T\times W$ that determinant supposed to be nonzero ok. So, this is a condition, so now, you can see that we are almost ready to take off in a mobile robot more close to the real time. What more close to real time?

So, for what we have simulated is only a bar rectangular base as the vehicle and we are tried to simulate. But now what we got it? We got the wheel configuration and we obtained the W matrix and then we can actually like do the more you can say realistic simulation.

So, that is what we are going to cover in the next class. So, where the next class would be focused on more about what we have derived in the lecture you call 8 and what we have derived now in the lecture 9 these fused and we would be doing more realistic simulation which is more close to the what you call reality.

So, in that sense lecture 9 what we have covered? So, what is actually like the non holonomic and holonomic and now we know like what to be done in the further end ok.

So, now lecture 10 we would be seeing more on the kinematic simulation in the MATLAB base, with that I am actually like saying thank you and see you at lecture 10, bye.