

**Fundamentals of Combustion**  
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**Lecture - 13**

**First Law and Second Law of Thermodynamics Applied to Combustion - Part 05**  
**Chemical Equilibrium (Contd.)**

(Refer Slide Time: 00:14)

### Gibbs Free Energy Change

Using the partial molal Gibbs function of individual species, Gibbs free energy of a mixture of ideal gases may be expressed as,


$$G_{mix} = \sum N_i \bar{g}_i(T, p) = \sum N_i \left\{ \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{p_i}{p_0} \right) \right\}$$


$H = \sum n_i h_i$

Here,  $N_i$  is the number of moles of  $i^{th}$  species. For fixed  $T$  and  $p$ , the equilibrium condition is given as  $dG_{mix} = 0$ . This leads to,

$$\sum_i dN_i \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{p_i}{p_0} \right) \right) + \sum_i N_i d \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{p_i}{p_0} \right) \right) = 0$$

Since pressure is constant, the second term in the above equation is zero.





Ok now, once you know the Gibbs function or Gibbs free energy for a particular species  $i$  then the Gibbs free energy of a mixture of ideal gases can be calculated by just same as  $H, \sum n_i H_i$ . Same thing I am using here  $\sum n_i g_i$  but calculated at a particular temperature and pressure.

So, that you can now expand this term here and write.  $\sum n_i [g_i(T, p_0) + R_u T \times \ln(p_i/p_0)]$ . So, this is the Gibbs function at a particular temperature calculated at atmospheric pressure.

Again, we have tables for this and this even you can calculate knowing the mole fraction of the particular species. In fact, if you want to determine mole fraction, this term is which is going to be useful for us.

Now, this is the equation

$$G_{mix} = \sum N_i \bar{g}_i(T, p) = \sum N_i \left\{ \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{p_i}{p_0} \right) \right\}$$

Now, the condition for equilibrium at a given temperature and pressure is  $dG_{\text{mix}} = 0$ . So, you have to just differentiate this by chain rule.

$$\sum_i dN_i \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{P_i}{P_0} \right) \right) + \sum_i N_i d \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{P_i}{P_0} \right) \right) = 0$$

So,  $dN_i g$  plus  $N_i dg$ . Now, you can see that the second term here goes to 0 because at a given temperature  $g$  has a constant value. Similarly, pressure is constant; partial pressure is depend on the total pressure that is constant. When you sum all the partial pressure you get the total pressure and total pressure is held constant. So, this second term goes to 0. That means,  $dG_{\text{mix}}$  will be just equal the first term.

(Refer Slide Time: 02:24)

### Equilibrium Constant


The condition for equilibrium is written as,

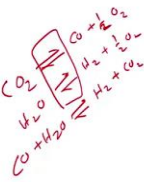
$$dG_{\text{mix}} = \sum_i dN_i \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{P_i}{P_0} \right) \right) = 0$$


Consider a molecular (elementary, reversible) reaction taking place under a constant pressure process,  $aA + bB \leftrightarrow cC + dD$ . Here, A and B are reactants, C and D are products, for the forward reaction, and  $a - d$  are the stoichiometric coefficients.

The change in the number of moles of a species is directly proportional to the corresponding stoichiometric coefficient, and therefore,  $dN_A = -k_a$ ;  $dN_B = -k_b$ ;  $dN_C = +k_c$  and  $dN_D = +k_d$ , where  $k$  is a proportionality constant.

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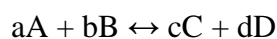


$$dG_{\text{mix}} = \sum_i dN_i \left( \bar{g}_i(T, p_0) + R_u T \ln \left( \frac{P_i}{P_0} \right) \right) = 0$$

So, this is the condition for equilibrium.

Now, this has to be implemented for any reaction. So, for example, when I said equilibrium reactions, I consider  $\text{CO}_2 \rightarrow \text{CO} + \frac{1}{2} \text{O}_2$ . Similarly,  $\text{H}_2\text{O} \rightarrow \text{H}_2 + \frac{1}{2} \text{O}_2$ . These are all equilibrium reaction. I have to apply this condition to the equilibrium reaction to get the equilibrium products.

Now, let us consider elementary reactions which are reversible in nature. So, let us consider an arbitrary elementary reaction written like this capital A, B, C, D represent the species and small a, b, c, d represent the stoichiometric coefficients.



So,  $aA + bB$  reversibly giving small  $cC + dD$ , that is the general equation we are trying to consider here.

We can also have a bi-molecular reaction like this. Now, you get any general equation written like this and try to apply this condition and see how to evaluate this.

So, before going to this evaluation, we should understand what is the change in number of moles  $dN_i$ . So, the change in the number of moles of a species is directly proportional to the corresponding stoichiometric coefficient.

$$dN_A = -ka; dN_B = -kb;$$

So, for the species A, if a, small a is the stoichiometric coefficient, then the change in the number of moles of the species a will be proportional to stoichiometric coefficient. So, I add a constant of proportionality k. I also put a minus sign here because that is the reactant.

So, we definitely know that if I consider forward reaction here, then the reactants are consumed. So,  $dN_A$  and  $dN_B$  should be negative. So, I put a negative sign and the change in the number of moles is proportional to the corresponding stoichiometric coefficients a, b etcetera here. So, I put proportionality constant k.

$$dN_C = +kc \text{ and } dN_D = +kd$$

Coming to the products, the change in number of products will be correspondingly proportional to their coefficients. So,  $dN_C$  will be proportional to c,  $dN_D$  will be proportional to small d and I put a plus sign because they are produced the forward reaction and the constant of proportionality is k.

So, you can now replace  $dN$ 's by ka, kb, etcetera for the particular reaction here with the sign.

When I consider the reverse reaction then  $dN_A$  will be positive,  $dN_B$  will be positive. On the other hand,  $dN_C$  and  $dN_D$  will be negative. So, the kb value etcetera what we are going to calculate next will be dependent on which direction you consider this reaction.

(Refer Slide Time: 06:09)

**Equilibrium Constant** ✓  $dG_{mix} = 0$



Therefore, for the elementary reaction,  $aA + bB \leftrightarrow cC + dD$ , the condition for equilibrium is written as,

$$-a \left[ \bar{g}_A(T, p_0) + R_u T \ln \left( \frac{p_A}{p_0} \right) \right] - b \left[ \bar{g}_B(T, p_0) + R_u T \ln \left( \frac{p_B}{p_0} \right) \right] + c \left[ \bar{g}_C(T, p_0) + R_u T \ln \left( \frac{p_C}{p_0} \right) \right] + d \left[ \bar{g}_D(T, p_0) + R_u T \ln \left( \frac{p_D}{p_0} \right) \right] = 0$$

By grouping the partial molal Gibbs function on one side, the condition for equilibrium may be written as,

$$- [c \times \bar{g}_C(T, p_0) + d \times \bar{g}_D(T, p_0) - a \times \bar{g}_A(T, p_0) - b \times \bar{g}_B(T, p_0)] = R_u T \ln \left[ \frac{(p_C / p_0)^c (p_D / p_0)^d}{(p_A / p_0)^a (p_B / p_0)^b} \right]$$

Dr. V. Raghavan, IIT Madras 28

$$a \left[ \bar{g}_A(T, p_0) + R_u T \ln \left( \frac{p_A}{p_0} \right) \right] + b \left[ \bar{g}_B(T, p_0) + R_u T \ln \left( \frac{p_B}{p_0} \right) \right] = c \left[ \bar{g}_C(T, p_0) + R_u T \ln \left( \frac{p_C}{p_0} \right) \right] + d \left[ \bar{g}_D(T, p_0) + R_u T \ln \left( \frac{p_D}{p_0} \right) \right]$$

Now, let us apply this rule and calculate  $dG_{mix}$  for the reaction. When I make it equal to 0 then I get this. So,  $k$  cancels out,  $a \times g_A(T, p_0) + R_u T \times \ln(p_A/p_0) + b \times g_B(T, p_0) + R_u T \times \ln(p_B/p_0)$  that is equal to the  $c$  and  $d$ 's correct.

So, the products,  $c$  is the coefficient of the species  $C$  that into  $g_C(T, p_0) + R_u T \times \ln(p_C/p_0)$ . Similarly, you can do this for  $D$ . So, this is the equation I get when I subject this equation to  $dG_{mix}$  equal to 0.

Now, you now try to group the Gibbs functions in one side and the partial pressures on the other side.

$$- [c \times \bar{g}_C(T, p_0) + d \times \bar{g}_D(T, p_0) - a \times \bar{g}_A(T, p_0) - b \times \bar{g}_B(T, p_0)] = R_u T \ln \left[ \frac{(p_C / p_0)^c (p_D / p_0)^d}{(p_A / p_0)^a (p_B / p_0)^b} \right]$$

So,  $-c \times g_C(T, p_0) + d \times g_D(T, p_0) - a \times g_A(T, p_0) - b \times g_B(T, p_0)$  will be equal to now group this log terms  $R_u T$  times logarithm of the product partial pressure of product raised to the stoichiometric coefficient.

So,  $(p_C/p_0)^c \times (p_D/p_0)^d$  divided by the reactants again say same products product of the reactant partial pressures  $(p_A/p_0)^a \times (p_B/p_0)^b$  raised to the stoichiometric coefficient. So,

this is the equation what we get and I am grouping the g's on one side, partial pressure terms on the other side and get this equation.

Now, I started from this equation, applied  $dG_{\text{mix}} = 0$ , I got this equation and I grouped the g's in the left-hand side and the partial pressures in the right-hand side I get this equation.

$$\Delta G(T, p_0) = c \times \bar{g}_C(T, p_0) + d \times \bar{g}_D(T, p_0) - a \times \bar{g}_A(T, p_0) - b \times \bar{g}_B(T, p_0)$$

(Refer Slide Time: 08:46)

### Equilibrium Constant

The expression in the left hand side is defined as the standard state Gibbs function change  $\Delta G(T, p_0)$ , given by, = f(T) only

$$\Delta G(T, p_0) = c \times \bar{g}_C(T, p_0) + d \times \bar{g}_D(T, p_0) - a \times \bar{g}_A(T, p_0) - b \times \bar{g}_B(T, p_0)$$


Equilibrium constant based on partial pressures,  $K_p$ , is defined as,


$$K_p = \frac{(p_C/p_0)^c (p_D/p_0)^d}{(p_A/p_0)^a (p_B/p_0)^b}$$

Finally, the condition for equilibrium for constant pressure process for a general system can be written in any one of the forms as:

$$\Delta G(T, p_0) = -R_u T \ln K_p \quad \text{K}_p = f(T) \text{ only}$$

$$K_p = \exp\left[\frac{-\Delta G(T, p_0)}{R_u T}\right] \quad K_p = \exp\left[\frac{-\Delta H(T)}{R_u T}\right] \exp\left[\frac{-\Delta S(T, p_0)}{R_u}\right]$$





Dr. V. Raghavan, IIT Madras 29

Now, I define the left-hand side as standard state Gibbs function change  $\Delta G(T, p_0)$ . You can see in the Turns book, this is represented by this in a compact notation  $\Delta G^0$  that is for standard atmospheric pressure and the subscript T represent any temperature.

So, I have just written  $\Delta G(T, p_0)$ , you can also write like this  $\Delta G_T^0$  that will be equal to the products. Gibbs free energy of the products  $c \times \bar{g}_C(T, p_0) + d \times \bar{g}_D(T, p_0)$ , then minus again add the reactant sides.

So, all the product Gibbs free energy multiplied by stoichiometric coefficients should be added and that should be subtracted to this reactants Gibbs free energy, the sum of reactants.

So, this is the Gibbs function change, standard state Gibbs function change calculated at any temperature, but only at atmospheric pressure. So, that means,  $\Delta G$  is a function of temperature only.

Now, in the right-hand side, I have already grouped the partial pressures like this in the natural logarithm term. Now, that I define as equilibrium constant based on partial

pressures. This partial pressure is the ratio of product of the partial pressures of products raised to the stoichiometric coefficient divided by corresponding atmospheric pressure. Similarly, that divided by the product of the reactant partial pressure ratios raised to corresponding stoichiometric proportions. This ratio is called the equilibrium constant at partial pressure  $K_p$ .

$$K_p = \left[ \frac{(p_C / p_0)^c (p_D / p_0)^d}{(p_A / p_0)^a (p_B / p_0)^b} \right]$$

So, now I can write the previous equation like this  $\Delta G(T, p_0) = -R_u T \ln(K_p)$ .

So, you please understand that to calculate  $K_p$ , I need this. So, temperature is known already. Since temperature is known,  $\Delta G$  can be calculated because  $\Delta G$  is a function of temperature only. It is done at atmospheric pressure.

Then  $K_p$  can be calculated. So,  $K_p$  is also a function of temperature only.

$$K_p = \exp \left[ \frac{-\Delta G(T, p_0)}{R_u T} \right]$$

Now, how the pressure effects come, because of the definition of  $K_p$  pressure effect comes here. So, partial pressures are involved, so the partial pressures of products they are multiplied together with the ratio with atmospheric pressure raised to corresponding stoichiometric proportions and that divided by the product of the partial pressure ratio of the reactants raised to corresponding stoichiometric proportions.

So, this definition. The partial pressure itself  $p_C$ ,  $p_D$ ,  $p_A$ ,  $p_B$  will change based upon the total pressure. So, the pressure dependence comes by the definition of  $K_p$ .  $K_p$  itself is calculated as a function of temperature and you can substitute the value of  $K_p$  in this to get the product composition.

So, in terms of H and S,  $K_p$  can be written like this. If you do not want to use G, you can also use H and S and both are same.

$$K_p = \exp \left[ \frac{-\Delta H(T)}{R_u T} \right] \exp \left[ \frac{-\Delta S(T, p_0)}{R_u} \right]$$

(Refer Slide Time: 13:06)

### Equilibrium Composition



It is clear that the standard Gibbs function change is a function of temperature only. Therefore,  $K_p$  is also a function of temperature only.

By estimating the partial molal Gibbs function of the species considered, the Gibbs function change may be estimated and from that  $K_p$  may be determined.

Then definition of  $K_p$  in terms of partial pressures is used. The ratio  $p/p_0$  is called activity coefficient. If the combustion takes place at a pressure  $p$ , then using the mole fraction of  $i^{\text{th}}$  species,  $X_i$ , the activity coefficient is written as,  $X_i p/p_0$ .

$$K_p = \exp\left[\frac{-\Delta G(T, p_0)}{R_p T}\right]$$

$$K_p = \frac{X_C^c X_D^d}{X_A^a X_B^b} \left(\frac{p}{p^0}\right)^{c+d-a-b}$$

$$X_C = \frac{n_C}{n_p}$$

*Handwritten notes:  $p_C = X_C p$ ;  $p_D = X_D p$*

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30



Now, to summarise you know that the Gibbs function change is a function of temperature only. So,  $K_p$  is also a function of temperature only. Now, after calculating  $K_p$  like this as a function of temperature you can use this equation. So, what I have done here is instead of  $p_C$  I am putting  $X_C \times p$ .

Similarly,  $p_D$  I am writing  $X_D \times p$  and substitute here. This equation here when you substitute your  $p_C$ ,  $X_C \times p$  similarly for  $p_B$ ,  $X_B \times p$  etcetera you can write in terms of mole fractions and the total pressure  $p$ .

$$K_p = \frac{X_C^c X_D^d}{X_A^a X_B^b} \left(\frac{p}{p^0}\right)^{c+d-a-b}$$

So, after evaluating  $K_p$  at a given temperature with Gibbs function change, standard Gibbs function change, then you can substitute this to calculate or to generate expressions. We cannot calculate everything so, if there is only one unknown, this one equation can be used to do that. Or you have to generate equations in order to solve for the mole fractions.

So, mole fraction can be written in terms of the number of moles of a particular species see for example, mole fraction of C can be  $n_C/n_T$  or  $n_p$ ;  $n_p$  I will say that is the number of moles of the product  $n_p$ .  $n_C$  is the number of moles of the C so,  $X_C$  will be this.

When you substitute, you can also substitute this equation in terms of number of moles. So, straight away we can write the equation in terms of number of moles  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  what we saw earlier. So, once we do that, we can calculate.

The procedure of calculating the equilibrium product is first you find the Gibbs free energy of individual species then calculate the standard Gibbs function change at a given

temperature at the reference pressure of one atmosphere so,  $\Delta G$  is got, then you calculate the value of  $K_p$  using this expression then substitute this for the given equation and find the number of moles or form an equation with the number of moles involved.

(Refer Slide Time: 15:25)

### Equilibrium Composition


Consider the example of  $\text{CO}_2$  dissociation, the mixture has  $(1 - \alpha)$  kmol of  $\text{CO}_2$ ,  $\alpha$  kmol of  $\text{CO}$  and  $\alpha/2$  kmol of  $\text{O}_2$ . At a given temperature, standard Gibbs function change and equilibrium constant are calculated as,

$$\Delta G(T, p_0) = \bar{g}_{\text{CO}}(T, p_0) + 0.5 \times \bar{g}_{\text{O}_2}(T, p_0) - \bar{g}_{\text{CO}_2}(T, p_0)$$

$$K_p = \exp\left[\frac{-\Delta G(T, p_0)}{R_u T}\right]$$

In terms of partial pressures,  $K_p$  is expressed as,

$$K_p = \frac{X_{\text{CO}} X_{\text{O}_2}^{0.5}}{X_{\text{CO}_2}} \left(\frac{p}{p^0}\right)^{0.5} = \frac{\left(\frac{\alpha}{1 + \alpha/2}\right) \left(\frac{\alpha/2}{1 + \alpha/2}\right)^{0.5}}{\left(\frac{1 - \alpha}{1 + \alpha/2}\right)} \left(\frac{p}{p^0}\right)^{0.5}$$



*Handwritten notes:*

- $\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2} \text{O}_2$
- $K_p = \exp\left(\frac{-\Delta G}{R_u T}\right)$
- $K_p = \left(\frac{p_{\text{CO}}}{p_0}\right) \left(\frac{p_{\text{O}_2}}{p_0}\right)^{0.5} / \left(\frac{p_{\text{CO}_2}}{p_0}\right)$
- $p_{\text{CO}} = X_{\text{CO}} p$
- $p_{\text{O}_2} = X_{\text{O}_2} p$
- $p_{\text{CO}_2} = X_{\text{CO}_2} p$
- Total moles =  $1 - \alpha + \alpha + \frac{\alpha}{2} = 1 + \frac{\alpha}{2}$
- $X_{\text{CO}} = \frac{\alpha}{1 + \alpha/2}$
- $X_{\text{O}_2} = \frac{\alpha/2}{1 + \alpha/2}$
- $X_{\text{CO}_2} = \frac{1 - \alpha}{1 + \alpha/2}$

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31

So, so, let us just take the equation what we considered in which we have some  $\text{CO}_2$  dissociation taking place the equation is  $\text{CO}_2 \rightarrow \text{CO} + \frac{1}{2} \text{O}_2$ . Now, in this case, we know that the mixture has  $1 - \alpha$  kilomole of  $\text{CO}_2$ ,  $\alpha$  kilomoles of  $\text{CO}$  and  $\alpha/2$  kilomoles of  $\text{O}_2$ .

At a given temperature, now we can calculate  $\Delta G$  at atmospheric pressure. First of all, you have to group the products so, these were the products. So,  $g_{\text{CO}}$  at a particular temperature, then  $0.5$  times  $g_{\text{O}_2}$  at a particular temperature calculate them add up then calculate reactant  $g_{\text{CO}_2}$  at the same temperature then subtract that to this.

Now, you get  $\Delta G$ , when  $\Delta G$  is known calculate  $K_p$ .

$$K_p = \frac{X_{\text{CO}} X_{\text{O}_2}^{0.5}}{X_{\text{CO}_2}} \left(\frac{p}{p^0}\right)^{0.5} = \frac{\left(\frac{\alpha}{1 + \alpha/2}\right) \left(\frac{\alpha/2}{1 + \alpha/2}\right)^{0.5}}{\left(\frac{1 - \alpha}{1 + \alpha/2}\right)} \left(\frac{p}{p^0}\right)^{0.5}$$

So, we can write in a compact form like this. So, this is the expression you make.

Then, use the definition of  $K_p$ .  $K_p$  for this equation is written as partial pressure of product first of all so, partial pressure of  $(p_{\text{CO}}/p_0)^1$  and  $(p_{\text{O}_2}/p_0)^{0.5} / (p_{\text{CO}_2}/p_0)^1$ .

Now, if the pressure of the reaction is  $p$  and is equal to the total pressure, then  $p_{\text{CO}}$  will be equal to  $X_{\text{CO}} \times p$ . Similarly, you can write the  $X_{\text{O}_2} \times p$  as  $p_{\text{O}_2}$  and  $p_{\text{CO}_2}$  will be equal to  $X_{\text{CO}_2} \times p$ . So, substitute this here, you get this equation.




Then, what is mole fraction? Total number of moles =  $1 - \alpha + \alpha + \alpha/2 = 1 + \alpha/2$ . So,  $X_{CO}$  will be equal to  $\alpha / (1 + \alpha/2)$ .

Similarly, you can substitute for other things and you could write this in terms of  $\alpha$ . So,  $X_{O_2}$  is  $\alpha/2 / (1 + \alpha/2)^{0.5}$ . Similarly, number of moles of  $CO_2$  is  $1 - \alpha / (1 + \alpha/2)$ .

If you write, then you get a non-linear equation in  $\alpha$ . So,  $K_p$  is known, right-hand side  $\alpha$  in a non-linear is there. The total pressure is known, so, the pressure ratio can be calculated.

(Refer Slide Time: 18:49)




### Equilibrium Composition

A non-linear equation results. After solving this equation and considering the physically possible roots, the value of  $\alpha$  is got.

With respect to the effect of pressure, it has been observed that when pressure increases at constant temperature, the backward reaction will be favoured.

Dissociation normally occurs at high temperatures at atmospheric pressure.

The adiabatic flame temperature calculated considering the equilibrium products of combustion is called equilibrium flame temperature. This has to be determined iteratively.



Dr. V. Raghavan, IIT Madras

32

A non-linear equation results. If you are solving that, you get the value of  $\alpha$ . So, it is the only one unknown, so, one equation is enough. But non-linear equation will give multiple roots. You have to choose physically possible roots that is all, that will be the value of  $\alpha$ . We know that the Gibbs function change is a function of temperature only.

So,  $K_p$  is a function of temperature only. The temperature dependence on the products of equilibrium comes due to that. The pressure dependence comes due to the mole fraction and this pressure term here. So, with that we take into account both the effects of pressure and temperature and calculate the equilibrium value of a particular species.

Now, some observations. When pressure increases at a constant temperature, the backward reaction is favoured. Dissociation normally occurs at high temperatures at atmospheric pressure. When pressure increases, then there is a decrease in the dissociation.



Now, adiabatic flame temperature when you calculate by calculating the equilibrium products of combustion, then that value is close to the measured value. So, such a

temperature is called equilibrium flame temperature. Again, as I told you it has to be determined iteratively. So, this is the completion of this particular topic  
(Refer Slide Time: 20:22)

### Course Contents

- (1) Fuels and their properties
- (2) Review of basic thermodynamics of ideal gas mixtures
- (3) Stoichiometry
- (4) First and Second Laws of Thermodynamics applied to combustion;  
Heat, temperature and composition of products in equilibrium ✓
- (5) Mass transfer basics
- (6) Fundamentals of combustion kinetics
- (7) Governing equations for a reacting flow
- (8) General characteristics of combustion flame and detonation
- (9) Laminar flame propagation-Flammability limits and quenching of laminar flames-Ignition-Flame stabilization
- (10) Gas jets and combustion of gaseous fuel jets
- (11) Turbulent premixed and non-premixed flames
- (12) Droplet evaporation and combustion
- (13) Combustion of a carbon particle.

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If you see in this particular section, we have actually dealt with this. We have seen how to calculate heat, temperature, adiabatic flame temperature iteratively. So, heat calculations require enthalpy calculations of individual species absolute enthalpy then calculate the reactant enthalpy and the product enthalpy, apply first law. In a constant volume process heat of reaction is  $U_P - U_R$  and in a constant pressure process it is  $H_P - H_R$ .

The flame temperature you calculate based upon some iterative procedure, for which you need composition of the products. The adiabatic flame temperature is the maximum temperature attainable, when the product enthalpy matches the reactant enthalpy for a constant pressure system or the product internal energy matches the reactant internal energy for a constant volume system.

Composition of the products in equilibrium is determined using second law, a variable defined using second law is entropy. So, based on entropy, two free energies have been defined one is Helmholtz free energy, another one is Gibbs free energy used in constant volume and constant pressure processes, respectively.

So, using that, we calculate the Gibbs function change and from that, we calculate the equilibrium constant which is defined as the ratio of partial pressures of product mixture and the reactant mixture and from that you calculate the equilibrium composition. So, this finishes this topic.

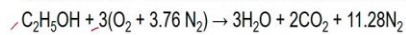
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### Worked Example 1

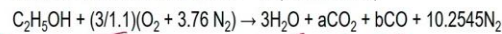


(1) Ethanol ( $C_2H_5OH$ ) is burnt in a furnace with equivalence ratio of 1.1. If no hydrogen or oxygen is found in the exhaust, determine the product composition per kmol-fuel. For the constant pressure process, determine the standard heat of combustion per kg of fuel. Take standard heat of formation of ethanol as  $-235310$  kJ/kmol.

**Solution:** Stoichiometric ethanol combustion is written as,



Actual combustion reaction at  $\Phi = 1.1$ , is written as,



$$\text{C balance: } 2 = a + b$$

$$\text{O balance: } 1/2 + 3/1.1 = a + b/2 + 3/2$$

$$\text{Solving, } a = 1.4546; b = 0.5454$$

$$\text{Products per kmol-fuel: } 3H_2O + 1.4546CO_2 + 0.5454CO + 10.2545N_2$$

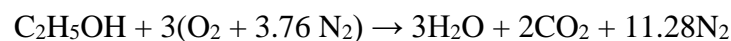
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Now, we go for worked examples. This worked example is corresponding to the heat, temperature and equilibrium calculation. Let us do some simple example first.

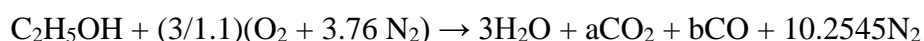
Let us consider the first problem, ethanol,  $C_2H_5OH$  is burnt in a furnace with a equivalence ratio of 1.1 and exhaust gas analysis specifies that there is no hydrogen or oxygen left in the exhaust gas.

Now, for a constant pressure process, you have to determine the standard heat of combustion per kg of fuel. Now, the standard heat of formation of ethanol is given here that you can use that is the problem.



So, ethanol stoichiometric combustion is written here  $C_2H_5OH + 3(O_2 + 3.76 N_2)$  giving  $3H_2O + 2CO_2 + 11.28$  kilomoles of  $N_2$ . Now, actual combustion occurs at an equivalence ratio of 1. So, you rewrite the equation for the actual combustion by replacing 3 by  $3 / 1.1$  and it is given that no hydrogen or oxygen is left. We have already seen this in the previous worked example.

So, hydrogen combust fully to  $H_2O$  because it is highly reactive. However, the remaining oxygen is shared to form  $CO_2$  and  $CO$ . Now, by incurring C and O balances, you get two equations involving a and b by solving you get a equal to 1.4546 and b equal to 0.5454 and the products are this



$$\text{C balance: } 2 = a + b$$

$$\text{O balance: } 1/2 + 3/1.1 = a + b/2 + 3/2$$

Now we can write the equations. So, product composition is known, so, we can find the standard heat of combustion.

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### Worked Example 1

The reaction is written as,

$$\text{C}_2\text{H}_5\text{OH} + (3/1.1)(\text{O}_2 + 3.76 \text{N}_2) \rightarrow 3 \text{H}_2\text{O} + 1.4546 \text{CO}_2 + 0.5454 \text{CO} + 10.2545 \text{N}_2$$

$\Delta H_c = -\Delta H_R = -(H_P - H_R)$

Standard heat of combustion,  $\Delta H_c = H_R - H_P$ , is evaluated at 298 K. Only heat of formations of species are involved and taken from tables.


$H_R = h_{f,\text{C}_2\text{H}_5\text{OH}} = -235310$  kJ/kmol, since standard heats of formation of oxygen and nitrogen are zero.


$H_P = 3h_{f,\text{H}_2\text{O}} + 1.4546h_{f,\text{CO}_2} + 0.5454h_{f,\text{CO}} + 0$   
 $= 3(-241845) + 1.4546(-393546) + 0.5454(-110541) + R_{f,N_2}$

$\Delta H_c = 1122966$  kJ/kmol-fuel ✓

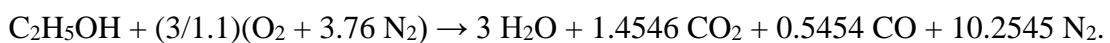
Per kg of fuel,  $\Delta H_c = 1122966/M_{\text{C}_2\text{H}_5\text{OH}} = 24412.3$  kJ/kg.

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Now, the entire equation is written here 1 kilomole of ethanol + 3 / 1.1 × air 4.76 kilomoles of air giving 3 kilomoles of water vapour, 1.4546 kilomoles of CO<sub>2</sub> 0.5454 kilo moles of CO and 10.2545 kilomoles of nitrogen.



Now, standard heat of combustion  $\Delta H_c$  is nothing but  $-\Delta H_R$  which is nothing but  $-(H_P - H_R)$ , which is written as  $H_R - H_P$ . So, this is the standard heat of combustion.

Now, when I say standard, this has to be evaluated at 298 K, all the enthalpies have to be evaluated at 298 K. So, only heat of formation of species are involved in this.

For example,  $H_R$  the reactant mixture contains ethanol, O<sub>2</sub> and N<sub>2</sub>; however, since O<sub>2</sub> and N<sub>2</sub> are basic elements heat of formation of them are 0. The  $H_R$  is contributed only by the heat of formation of ethanol which is given as -235310 kJ/kmol.

So, that is the enthalpy of reaction. Then, what is enthalpy of products? Enthalpy of product is nothing, but  $\sum n_i h_i$ .

$$\begin{aligned} H_P &= 3h_{f,\text{H}_2\text{O}} + 1.4546h_{f,\text{CO}_2} + 0.5454h_{f,\text{CO}} + 0 \\ &= 3(-241845) + 1.4546(-393546) + 0.5454(-110541) \end{aligned}$$

Now, since we are talking about 298 K, this is  $n_i \times h_{fi}$ , heat of formation that is it. so, now that is what we are trying to do here 3 times the enthalpy of formation of H<sub>2</sub>O and 1.4546 times the enthalpy of formation of CO<sub>2</sub> + 0.5454 times enthalpy of formation of CO + 0, that is 0. There is no enthalpy of formation of nitrogen because it is a naturally occurring species.

So, when you do  $H_R - H_P$ , you will get the positive  $\Delta H_C$  value because say exothermic reaction. So, 1122966 kJ/kmol of fuel so, that is this. If you want per kg of fuel, then divide by the molecular weight of fuel you will get 24412.3 kJ/kg.

So, this is a simple example where you try to find the calorific value or heat of combustion at standard state when there is a slightly rich mixture of ethanol and air reacting.