Fundamentals of Combustion

Prof. V. Raghavan
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 50
Turbulent Flames – Part 2
Turbulent length scales and turbulent stresses

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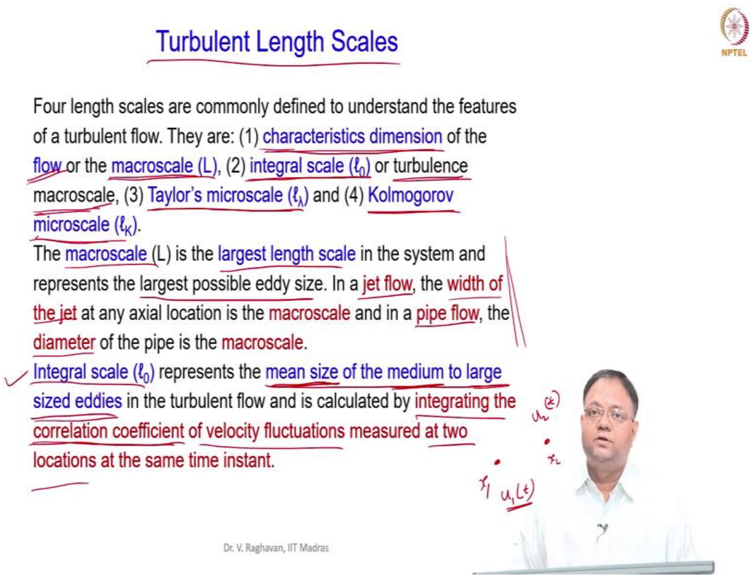

So, next is the turbulent length scales. There are four length scales which are commonly defined and used. The 1st one is called the characteristic dimension of the flow itself. So, that is the biggest time this length scale what we can have.

So, the dimension of the flow; please understand dimension of the flow itself. For example, in a boundary layer, the boundary layer thickness can be the characteristic dimension we can use turbulent boundary layer thickness Δt.

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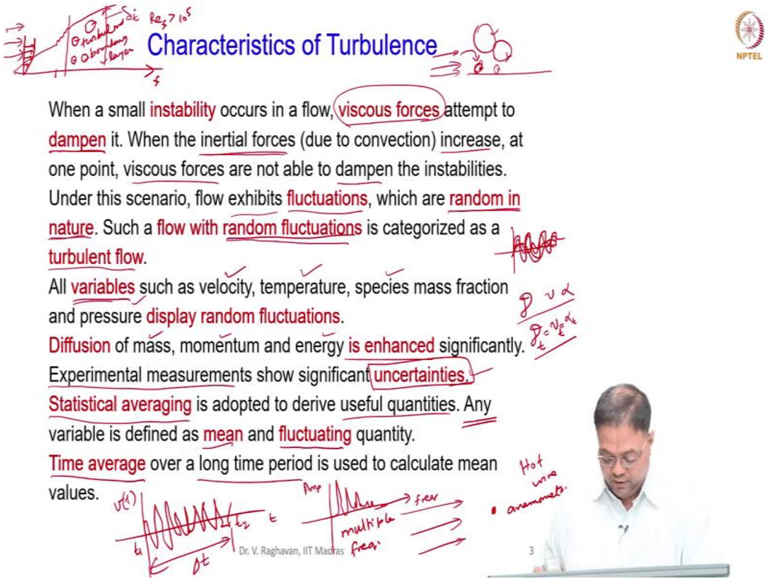

So, we go back here, this is the turbulent boundary layer thickness Δt that will be a characteristic dimension for us. So, that will be the 1st one which is the characteristic dimension of the flow or we just call it a macroscale, capital L, macroscale. So, that will be the characteristic dimension of the particular flow. So, it may be internal flow, external flow anything. So, for the category, dimension associated with that is called the macroscale.

The next three are turbulent scales which will basically depend on the turbulence intensity and other things. So, first we define what is called the largest turbulent scale which is called integral scale l suffix 0, integral scale.

Since this is the maximum possible length scale within the turbulence. So, we say this is called turbulence macroscale. So, when you consolidate all the eddies together and see what will be the maximum dimension in the domain so, that will be the macroscale or the characteristic dimension of the flow that is the first scale.

Second one is the integral scale which is also called turbulent macroscale. Turbulence macro scale is nothing but the large size turbulent eddy that will be represented by this. Then comes the microscale which is called Taylor's micro scale l suffix λ, Taylor's microscale and the smallest scale is called Kolmogorov microscale l suffix k.

So, these are the four things and out of which the 2, 3, 4 are the representative of the eddy sizes in the turbulent flow. So, these are the four length scales, macroscale which is the characteristic dimension of the flow, then integral scale which is the largest eddy size, in this it is called turbulent macroscale, then the Taylor's microscale which represents the medium like a level lesser than the integral scale basically, then the smallest length scale is called the Kolmogorov scale microscale l_K.

So, as I told you the macroscale is nothing, but the largest length scale in the system or the control volume. For example, if you take a jet flow, the width of the jet half width, for example, can be taken as the macroscale or the characteristic dimension and for the pipe flow, the diameter of the pipe itself so, the maximum possible eddy size will be this. So, this is the flow thing, the jet spreads to its width. So, the maximum eddy what we can anticipate here is the width. So, after that there is no jet at all. So, the eddy cannot be larger than that.

So, the macroscale normally represents the largest possible eddy size. Since I say macro, it does not talk about the particular flow field like eddy size and so on. But anyway, this is a general thing.

So, for example, we can use this. Even this length scales can be used for laminar flows also. For example, diameter of the pipe, we have used to define the Reynolds number in a pipe flow. Similarly, the jet width, half width of jet etcetera we can use as a length scale for the jet. So, this we can also use in the laminar flow that is a macro scale.

2^{nd} scale as I told you 2^{nd}, 3^{rd} and 4^{th} are specifically to the turbulent eddy size and this integral scale which is also called the turbulence macroscale represents the mean size of the medium to large size eddy, mean size, average size of medium to large size eddy. Now, please understand that once you go into the turbulence, we cannot just take a particular size of eddy.

So, here, we have definite the macroscale or the flow, the characteristic dimension of the flow we know that the jet width will be like this based upon a Reynolds number and the pipe diameter is this, fixed everything, but in this when you go into turbulence, we have to do some statistics.

So, the statistic what you do for the integral scale is nothing but the mean size. So, you have to take some average of what the medium to the large eddies which are present in the flow. When you take the average or mean values of the medium to large size eddies, then I get that scale.

Now, how will you do this, how will you calculate the eddy size? So, that is done actually by using what is called correlation coefficient. See correlation is nothing, but when I do a measurement, see I can do a measurement using a hot anemometer at a particular location, then I get see for example, I measure the velocity, one of the common velocity say u with this. So, I get u as such a function of time. So, I now fix the location say x_1 is the location; x_1 is the location in which I put the probe, hot anemometer and do the measurement. Soon I get a time, with time how u varies.

So, I can do this for another location x_2, and I get say this is a $u_1(t)$ and this may be $u_2(t)$, I get this. Now, I can see what will be the relationship between these two so, that is called correlation.

When I do, for example, $u_1 \times u_2$ etcetera whether it is cancelling out or it is enhancing so, that will give the correlation. So, the correlation is nothing but when I do measurement in two locations as a function of time at the same instant how these two correlates. So, that correlation is what will tell you the size of eddies. So, if the points are within a particular eddy, it will correlate better. If the points are not within the eddy, it will not correlate.

So, based upon the correlation, I will decide the size. So, when I do this measurement at different x_1, x_2 etcetera, I will know what is the medium size eddy, what is the large size eddy etcetera.

Once I know this correlation, I integrate from the medium to large size. So, actually I do for some particular distance I take and put the probes and take the measurements and try to correlate till which distance I get the correlation that will decide the mean size. So, that if I take, that will correlate the medium to large size eddies. So, that scale is called integral time scale.

So, it is got by integrating the correlation coefficient of velocity fluctuations at two locations at the same time instant. So, I get the time data and at the particular time instant, what is the correlation between this? If the data is within a same eddy, it will correlate better. So, with that I can see these two points are within the same eddy.

So, I know the dimension of the eddy now. Similarly, I say if the there is no correlation between one point to another point, then these two points are not within the same eddy and so on. So, some type of meaningful derivations can be done by these measurements. But please understand again we have to do correlations with some statistical procedure. So, that is what we try to do this and get the value of the integral scale.

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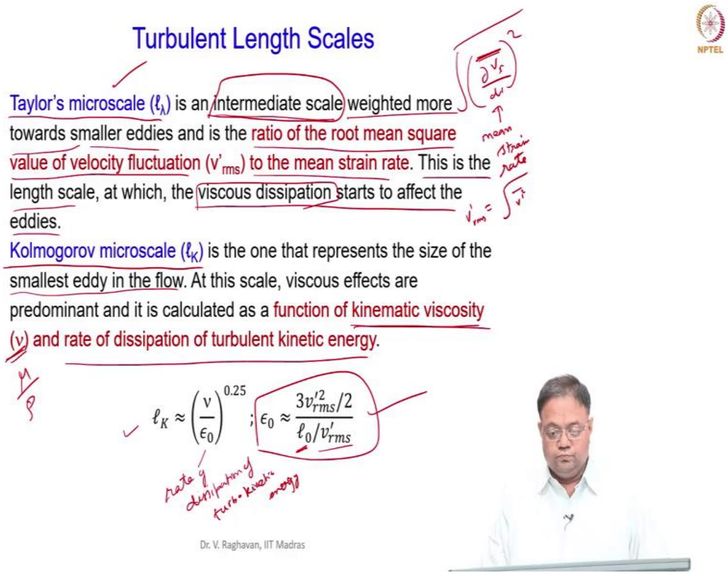

Then comes the microscale which is called Taylor's microscale l_λ. It is an intermediate scale. Now, the largest flow field scale is macroscale and the next is integral scale which is the turbulence macroscale which is actually the average of the size of eddies present in medium to the large size.

Then comes the microscale which is an intermediate scale and it is weighted more towards the smaller eddies. So, we have medium to large, now, medium to smallest eddies we need to cover. So, small to intermediate, in that range.

So, it will be weighted more towards smaller and that means it will represent the average of the smaller eddies. So, you can say average of the smaller eddies which is actually defined as the ratio of root mean square value of the velocity fluctuation $\dot{v}_{rms}$ and the ratio. So, this to the mean strain rate.

So, mean strain rate we can say $\sqrt{\left(\frac{dv_x}{dx}\right)^2}$ under square root something that. So, this may be the mean strain rate. Strain rate is nothing, but the derivative of velocity, but whenever we take derivative, normally we take a mean square and take a square root. So, that will be the this. So, that positive value you can take.

So, this may be called the mean strain rate, mean means time average, mean strain rate. Now, $\dot{v}_{rms}$ will be nothing but $\sqrt{\bar{\dot{v}}^2}$. So, this if you take a ratio of this $\dot{v}_{rms}$ to this strain rate, mean strain rate, you get the Taylor's microscale. So, again please see that when I define the Taylor's microscale, the turbulent length scales like the integral scale, l_0 or l_λ

or l_K we will use only the turbulent quantities like $\dot{v}_{rms}$ or correlations between two fluctuating quantities and so on.

So, this is the length scale at which the viscous dissipation starts to affect the eddies. Viscous dissipation will start to affect the eddies, that means that there is a energy dissipation from the smaller eddies normally, bigger eddies will take the energy from the free flow, the main flow and it will shut the eddies to the smaller and smaller eddies. The smallest eddy will dissipate it, the smaller eddies also will start to dissipate it so, that is the thing. So, the viscous dissipation starts to affect the eddies at this scale.

Now, the smallest length scale is called Kolmogorov scale l_K. So, that represents the size of the smallest eddy in the flow. So, how will you calculate it? Basically, it is calculated as a function of kinematic viscosity. Because the viscous dissipation is the largest in this.

So, viscous affects are predominant in this particular scale. So, we use the kinematic viscosity ν, that is μ/ρ, dynamic viscosity by density and the rate of dissipation of turbulent kinetic energy. So, if I use that, then I get the Kolmogorov scale.

So, you can see the ν/ε_0 this is the rate of dissipation of turbulent kinetic energy. So, this can be written as the dissipation of a turbulent kinetic energy, rate of dissipation of turbulent kinetic energy can be written as 3/2 $(v_{rms})^2$ divided by the integral scale divided by $\dot{v}_{rms}$. So, this if we substitute here, you get the l_K.

So, these are the four time scales, the last three are exactly the representative of the size of eddies. For example, the integral scale is representative of the mean size of the medium to large eddies, then the Taylor microscale is the average size of the smaller eddies and Kolmogorov scale is the size of the smallest eddy.

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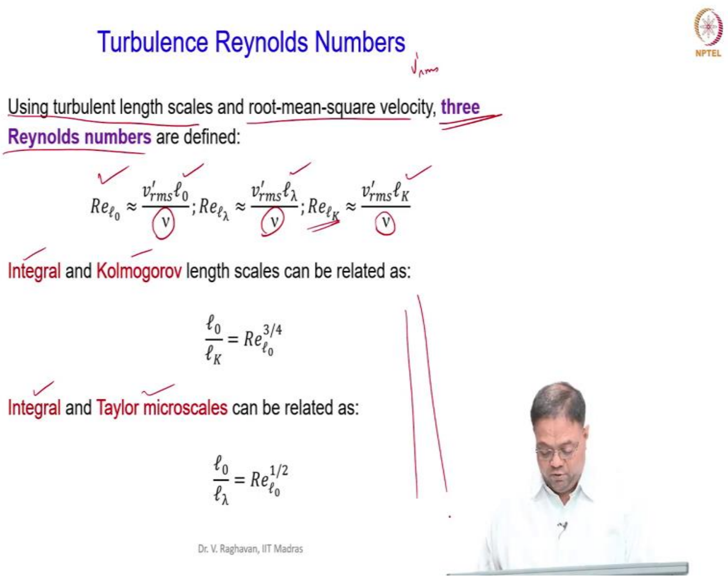

So, based upon the length scales, using the turbulent length scales and the root mean square value of velocity so, $\dot{v}_{rms}$, we get three Reynolds numbers defined. So, Re_{l0} = $\dot{v}_{rms}$ l_0/ν.

So, we actually take the ν value which is molecular kinematic viscosity. So, that we use it and define the Reynolds numbers. So, Re_{l0} is the Reynold number defined based upon the integral scale. $Re_{l\lambda}$ depend upon the Taylor's microscale and Kolmogorov scales Re_{lK} represent the Reynolds number that is defined based upon the Kolmogorov scale.

Now, with Reynolds number, we can relate the length scales. So, for example, the integral length scale l_0 by Kolmogorov length scale l_K is nothing but, or you can prove this Re_{l0} which is defined here that is the integral scale based Re value to the power of 3 by 4. Similarly, the integral scale and the Taylor microscale l_λ can be related. l_0 by l_λ can be related using the Re defined based upon the l_0 and to the power of half; 1 by 2.

So, these are very important thing and also, we will see that based upon the length scales, we will be able to characterize the turbulence of a turbulent mixing premixed flames.

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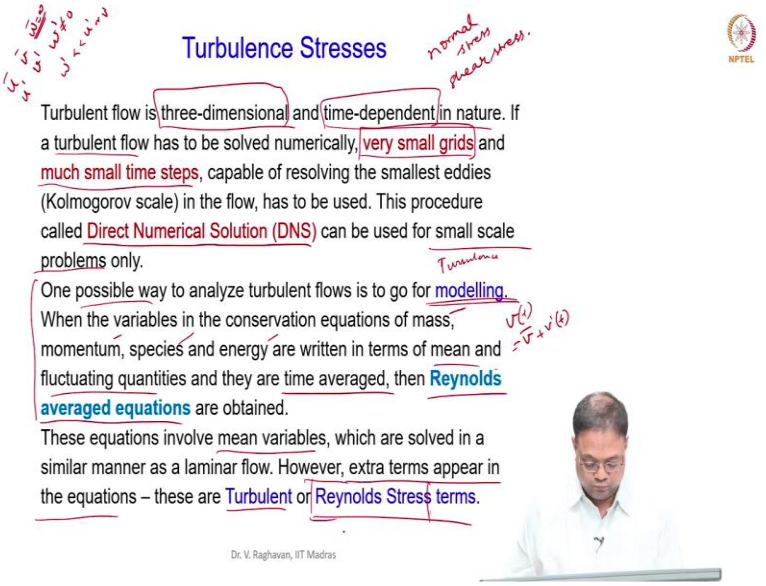

Now, turbulent stresses. So, you know the stress terms normally, we know in flow field there are normal stress and the shear stress, there are stress terms. But if you see turbulence actually causes some stress terms, but it is not going to cause a stress term, but we can have a stress term defined by analysing a Reynolds averaged equations. So, we will come to that here.

So, first of all, another important characteristic of turbulent flow is it is three-dimensional in nature and it is also time dependent in nature because chaotic fluctuations are present.

So, that will surely cause fluctuations which cannot be taken as a steady state oscillation. So, they are going to be time dependent, then they are three-dimension. Even though the predominant flow is in say two-dimension x and y only, predominant flow takes place, but there will always be oscillation in the third direction. Oscillations cannot be prevented.

So, for example, in the three-dimensional thing, u and v are present in two-dimensional flow field, but the oscillations in u that is u' oscillations in v which is v' and the oscillation in w or the exit direction which is w' will be present. So, here, w may be 0; w may be 0, but the fluctuation may not be 0.

So, that is the characteristics. You can see that the third direction, there is no mean velocity; I see these are the mean velocities. The mean velocity in x, y direction may be non-zero, but the mean velocity in the y, z direction, the mutual perpendicular direction can be 0.

However, the fluctuating quantity w’ may not be 0. So, that actually leaves the turbulent flow as a three-dimensional flow. But anyway, we can neglect that. So, anyway we can do, but this I have to understand. So, the fluctuation may not be too large when compared to u’ and v’. So, if w’ is much less than say u’ or v’, then we can neglect w’, but w’ cannot be mentioned as 0.

So, that is the main point here, w’ is not equal to 0 however, w’ may be much smaller than u’ and v’ so that we can neglect that. We can say $\overline{w}$ that is the mean value of w = 0 that is possible, but w’ can be negligible, not 0. That we have to understand.

Also, I told you the chaotic oscillations will introduce the time dependent nature of the turbulent flow. So, that means, if I want to solve the turbulent flow, first of all, I have to go for a 3D modelling, then smallest eddy, very small eddies are present and the large eddies are also present.

So, if you want to resolve large eddy, you can go for bigger mesh, but if the smaller eddy you want to resolve, that size itself will be say 0.1 mm, I am just giving an example so, now, what happens in that case? If you want to resolve that at least your mesh size would be less than that.

So, at least say 5 times less than that or 3 times less than that and so on to actually capture that particular eddy, the fluctuations due to that eddy. So, this means that very small grids are required to resolve the smallest size. Similarly, time scale. So, the smallest eddies oscillation, largest eddy oscillation etcetera will have different time scales, several frequency oscillations are there so, multiple frequencies are present. So, the time step; the time steps also will be multiple. So, we have to use the smallest time step to resolve the multiple time step aspect here.

So, time steps are to be small, then we can use, we have to use smaller grids to resolve the Kolmogorov scale and so on. So, this leaves that as to when you want to resolve it without any worry about the fluctuating components, then we have to use what is called direct numerical solution DNS. So, this direct numerical solution basically is not easy to apply for any other scale. So, this can be used only for small scale problems that is it. If you increase the scale of a problem, this will not work. So, that is very important.

So, anyway some assumptions and some simplifications have to be made and if it is not a DNS, how will you proceed with this? So, that is what we have to see. So, the possible way to proceed without doing a DNS is called modelling, turbulence modelling.

So, how will you model the turbulence flow? So, this means we do not resolve the smallest eddy size or the time step which is required to capture the oscillation and so on.

So, what we try to do here is when the variables in the conservation equations of mass, momentum, species and energy are written in terms of mean and fluctuating quantities. So, we have already defined it.

Let us say for example, v at a particular time instant will be equal to $\bar{v}$ + v'(t). Now, you take the equation, the governing equation and substitute for each variable mean quantity plus fluctuating quantity. So, that we try to do, and now when I split the variable into mean and fluctuating quantity, then I do a time average and this type of process is called Reynolds averaging, Reynolds averaging and what we end up is with the Reynolds averaged governing equations.

So, Reynolds averaging is very important. So, that will give you mean quantity based on equation and the fluctuating quantity terms will be appearing there correct. So, mean quantity is like a laminar flow solution. So, due to the turbulence, the fluctuating quantities will give rise to additional terms.

So, when I do the Reynolds averaging, I get Reynolds average governing equations which have mean; mean variables which are solved in the similar manner as laminar flow, that has a same meaning as that of the variable what you use in the laminar flow.

However, extra terms come in because of the turbulent oscillations or the fluctuation terms. So, these extra terms which appear in the governing equations representing the fluctuating components is called a turbulent stress or stress terms, Reynolds stress. So, this is very important. So, additional terms appear, those terms are called Reynolds stresses or the turbulent stresses.

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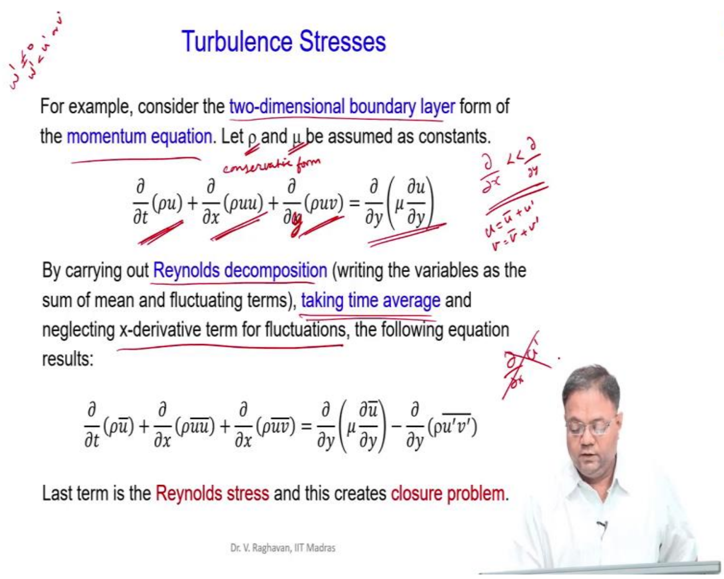

So, let us illustrate that with a small example. Let us consider here boundary layer type equation that means, when I invoke this, the gradient in the flow direction is not as high as the gradient in the transverse direction the y. So, if the flow is in x direction, then we can invoke the boundary type of equation.

For example, we can say that $\partial(\mu\partial u/\partial y)/\partial x$ is equal to 0. So, this is the equation I take. Now, please understand that as I mentioned previously the turbulent flow is actually three-dimensional in nature, but why I am using the two-dimensional here? Even though w' is not equal to 0, but w' is less than u' or v'. So, in that case, I neglect the fluctuation component of the z direction. So, I do not need to use z here, the w or z derivatives of that in this. So, when I write the momentum equation in the two-dimensional boundary layer form, I get this equation.

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho uv) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

Now, I assume the density and the viscosity to be constant here. So, this is another assumption I make. So, to just illustrate the Reynolds averaging. Now, I take, but I am including time derivative term, it may be steady, then you can neglect this, the convection terms, these two are the convection terms.

So, time derivative of the x momentum $\rho\times u$ per unit volume plus the convective term here this is y so, $\partial(\rho uu)/\partial x + \partial(\rho uv)/\partial y$ that will be the convective terms and written in conservative form that will be equal to the diffusion term which is $\partial(\mu\partial u/\partial y)/\partial y$. So, this is the momentum equation which I take.

Now, what I do is ρ I take as constant so, I do not need write ρ here. So, $u = \bar{u} + u'$ similarly, $v = \bar{v} + v'$ when you do this, substitute here so, I substitute this, $u = \bar{u} + u'$, $v = \bar{v} + v'$ and now I take what is called the time averaging. So, now, take time average to this equation. So, if I writing in a Reynolds decomposition methodology where I write a variable as this mean value plus the fluctuating value, I complete this equation and take the time averaging of the entire equation.

Now, please understand that in this case, there will be some x derivative of the fluctuations which is due to this term, the second convective term, second term basically. Now, I neglect the x derivative of the terms for fluctuations. So, for example, $\partial/\partial x$ of say u' etcetera is neglected. So, that I do not use. So, when I use the time average and neglect the x derivative of the fluctuations, then I get this equation.

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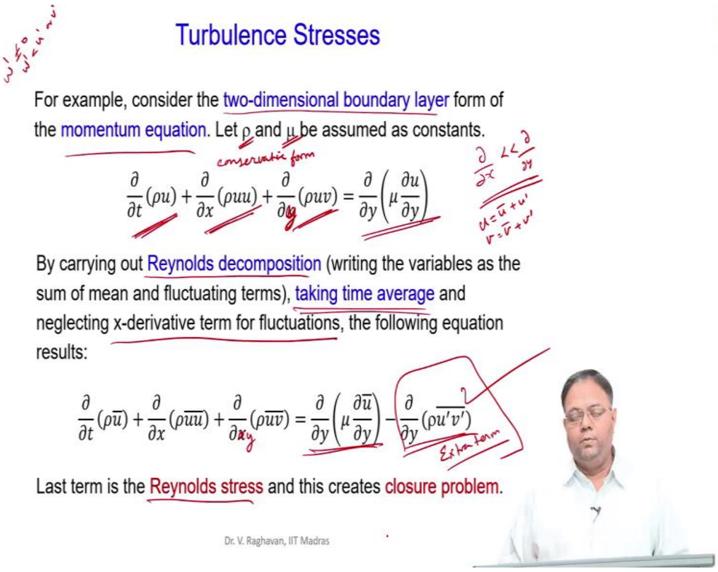

So, you can see that the overhead bar represents the mean value. So, instead of ρu, I write $\rho\bar{u}$. So, time derivative of $\rho\bar{u}$ + ∂(ρuu)/∂x, the same term ρuu comes here and please understand that there will be additional term which is going to go to the other side basically, then ρuv so, here also this y so, ∂(ρuv)/∂y. So, here also we get the $\rho\overline{uv}$. So, that will be equal to the momentum term that is ∂(μ∂u/∂y)/∂y and the extra term, this is the extra term what we get.

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho u v) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial}{\partial t}(\rho\bar{u}) + \frac{\partial}{\partial x}(\rho\overline{uu}) + \frac{\partial}{\partial y}(\rho\overline{uv}) = \frac{\partial}{\partial y}\left(\mu\frac{\partial\bar{u}}{\partial y}\right) - \frac{\partial}{\partial y}(\rho\overline{u'v'})$$

When you compare these two equations, you can see these terms on the left-hand side are now common and the right-hand side is also common, but I have put an overhead bar here to indicate that I have substituted the mean values here.

But I see that I get an extra term. So, this term is called the Reynolds stress term under that this creates what is called a closure problem. Because I have extra term u'v' so, how will you resolve this u'v'? That is the closure problem what we have.