Fundamentals of Combustion

Prof. V. Raghavan

Department of Mechanical Engineering

Indian Institute of Technology, Madras

Lecture - 51

Turbulent Flames - Part 3

Axisymmetric turbulent jet

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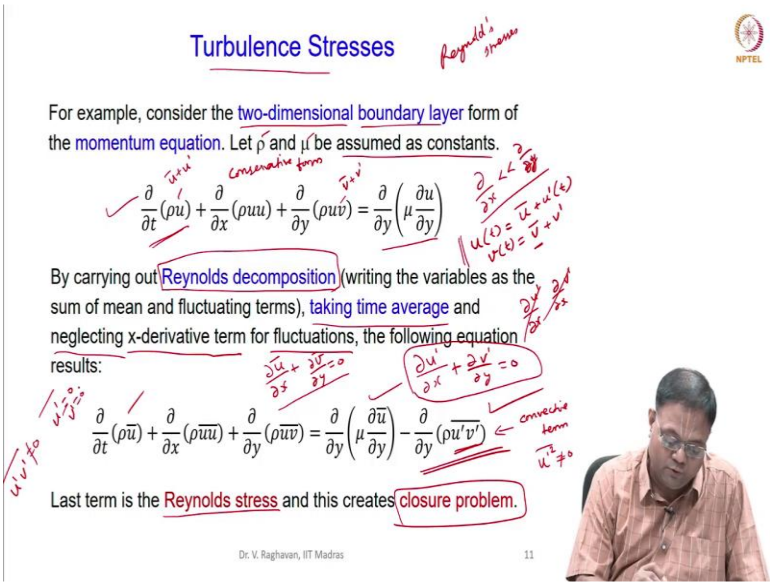

So, we will continue with what is called Turbulent Stresses or Reynolds Stresses. So, this is the extra terms which are going to be coming because of the fluctuation terms. So, to illustrate this let us consider a two-dimensional boundary layer equation. Here you can see that the derivative in the x direction is not prominent as the derivative in the y direction.

So, the diffusion term basically does not have the x derivative. And since the turbulence is time dependent, so we have included a time dependent term also here. So, ∂(ρu)/∂t, the x momentum equation equal to ∂/∂x with the convective term ∂(ρuu)/∂x and the ∂(ρu)/∂y and this is written in conservative form. So, writing like this is conservative form and you can see this diffusion term in the right hand side.

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho uv) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

So, this is what we are taking into account now. And let us do the Reynolds decomposition where you know that the u is at any time instant t is written as the time average value plus the fluctuating value again which is a function of time. Similarly, u v also you do the same. So, once you do this one more assumption which is made is the properties ρ and μ are assumed to be constants here. If it is not, then that will also fluctuate and that also should be added like mean and the fluctuating quantity.

So, just to illustrate the stress terms let us consider ρ and μ to be constants and Reynolds decomposition we will do like this. Writing flow velocity as the mean value plus the fluctuating value, that is the Reynolds decomposition. And time average you know how to take this. You have to take considering a longer, sufficiently long time period and average of fluctuations.

So, once you substitute this in the equation, for example, for u substitute $\bar{u}$ + u'. Similarly, for v you can write here $\bar{v}$ + v'. Then what you do? You have to take time average of the entire equation. Now, if you take time average and since we are assuming this, that is the x derivatives is much less than the y derivative, the x derivative term of the fluctuations.

So, you get derivatives like ∂u'/∂x, ∂v'/∂x etcetera those things we will neglect. So, neglecting the x derivatives of the fluctuations we will get this equation. So, you can see that this equation if you take the time derivative, we have the mean value of the time derivative. So, you can say $\bar{u'}$ = 0. So, you can see that.

$\partial \bar{\rho}\bar{u}$ /∂t will come. Similarly, the convective derivatives basically remain the same with the overhead bar indicating that they are mean values. However, the extra term which has come here is this, ∂/∂y of because I have already neglected the x derivative of the fluctuations. But keeping y derivative of fluctuations you see ∂(ρu'v')/∂y is there in this.

So, that actually comes in from the convective term. So, this term comes in from the convective term. But it is convenient actually to write it as a diffusion term. So, we have put this in the right hand side along with the stress term. So, along with the stress term we have put this in and this particular term is now called Reynolds stress.

So, similarly, if you do it for x momentum and y momentum everything, you will get additional terms. These additional terms will create closure problems. See for example, continuity equation can be written as this. In general, if it is laminar this term will not be there, the last term will not be there.

So, we can solve continuity equation and x momentum equation to solve the variable u and v. But the thing is now the additional terms have come, so number of unknowns are higher than number of equations, so this is called closure problem. So, that is what we get.

So, one more thing what we should understand here is when you do this for continuity equation you get the average the continuity is obeyed by the mean value as well as the fluctuation value. So, even the fluctuation of v that will obey the continuity. So, continuity equation is actually used for understanding the relationship between u and v, basically this u and v. So, you have to understand that $\left(\overline{u}'\overline{v}'\right) \neq 0$.

Similarly, we have already seen that $\overline{u}'^2 \neq 0$. So, $\overline{u}' = 0$, $\overline{u}' = 0$, etcetera. So, individually they are 0, but when you take average, the product of that that will not be equal to 0. So, that is the contribution by this term here, and that is going to be the extra term which is called Reynolds stress. Please understand that this term comes actually by the convective equation, convection terms, but it is now clubbed with the stress term and it is called Reynolds stress.

Since the additional terms appear you get into what is called the closure problem. So, somehow you have to go about solving this. When you do not do the Reynolds decomposition we have to go to direct numerical modeling, as I told earlier, we have to go to extremely smaller grids, extremely smaller time steps and bigger problems cannot be done, very small problems can be attempted with DNS.

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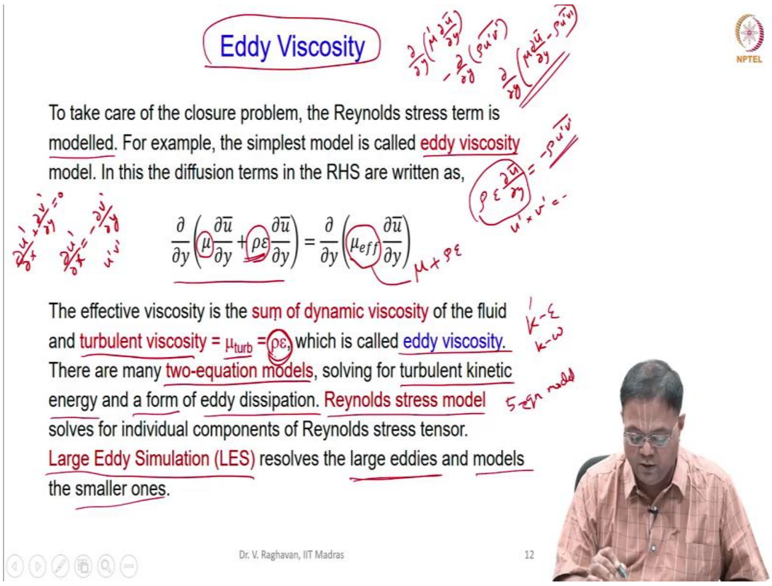

Now, eddy viscosity. The simplest concept what we have is to just take out the closure problem what we try to do here is the Reynolds stress term is modeled; that means, some change we make to the Reynolds stress term. For example, if you go back the term in the right hand side can be written like this. So, these two terms I am writing here. So, $\partial(\mu\partial\bar{u}/\partial y)/\partial y$ - $\partial(\rho\overline{u'v'})/\partial y$.

$$\frac{\partial}{\partial t}(\rho\bar{u})+\frac{\partial}{\partial x}(\rho\overline{uu})+\frac{\partial}{\partial y}(\rho\overline{uv})$$
$$=\frac{\partial}{\partial y}\left(\mu\frac{\partial\bar{u}}{\partial y}\right)-\frac{\partial}{\partial y}(\rho\overline{u'v'})$$

So, this now I can write as $\partial(\mu\partial\bar{u}/\partial y)/\partial y$ - $\partial(\rho\overline{u'v'})/\partial y$. So, this term is what is written here, you can see this. So, now, $\partial(-\rho\overline{u'v'})//\partial y$ is now written as plus $\rho\varepsilon\partial\bar{u}/\partial y$. So, what we are trying to see here is see this mean term we have viscous stress here this is the shear stress here.

So, the viscous stress, this is the molecular viscosity, dynamic viscosity, molecular dynamic viscosity that contributes to the first term here. The second term is due to the turbulent fluctuation. So, we introduce a viscosity called Eddy viscosity which is nothing, but $\rho\varepsilon$.

So, if you write this $\partial(\rho\bar{u})/\partial y$ I say this is equal to $-\rho\overline{u'v'}$. So, basically $-\overline{u'v'}$ is a positive term. So, $u'v'$ when you multiply that is actually a negative term. So, that can be illustrated by the continuity equation here. So, as I told you the continuity is obeyed by the fluctuation terms also. Now, if you write this, you can understand that $\partial u'/\partial x$ can be written as $-\partial v'/\partial y$.

So, let us say if in the x direction the fluctuation increases u' increases then; that means, that in the negative y direction v' will increase. So, this term u'v' will be negative. So, negative of negative will be positive, so this term you substitute for this and put it here. So, this is called eddy viscosity or we can say turbulent viscosity, eddy viscosity or turbulent viscosity which is nothing but $\rho\varepsilon$.

Now, the effective viscosity μ_{eff} which is contributed by the molecular viscosity plus the eddy viscosity. So, why we concentrate on this? Because we can see that the diffusivity is one which is much enhanced, even though you have to keep in mind always that this term is generated by the convective term but the effect of turbulence is basically to increase the diffusion. Molecular diffusion is slow, but this basically increases the diffusion mixing.

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial\overline{u}}{\partial y}+\rho\varepsilon\frac{\partial\overline{u}}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial\overline{u}}{\partial y}\right)$$

So, you can see that the when you add this the μ increases. So, μ/ρ basically, so μ increases then you can see that the diffusion term will be stronger and this is what is contributed by the turbulence. So, these fluctuations basically increase the mixing; so, the diffusion strength. And so, when you have effective viscosity that will be equal to the molecular viscosity plus the eddy viscosity or the turbulent viscosity; so, this turbulent viscosity or eddy viscosity is the same.

Now, this is the simplest model where you can say that the effective viscosity will be say several 100 times more than the normal molecular viscosity. So, that is the effect. See when the intensity increases the mixing will increase. So, that is what the contribution of turbulence. So, this is a simple model. It is called the zero-equation model, we do not need any additional equation for this.

There are several other models Prandtl's, mixing length base models and so on. In fact, to evaluate this we need the Prandtl's mixing length. Next, to improve this we go for what is called two equation models. I am not going to go to details, but I am just mentioning this. In the two-equation model we have a equation to be solved for turbulent kinetic energy. So, that is the first equation we solve. And the second equation is for solving what is called epsilon equation.

So, we have what is called k-epsilon equation, k-omega equation and so on where you solve for the k (the turbulent kinetic energy) and epsilon omega are some forms of eddy dissipation. So, when you solve this then these stress terms are modeled using this. So, we have to handle the closure problem.

For example, if I write this in the simplest way $-\rho\overline{u'v'}$ is written as $\rho\varepsilon\partial\overline{u}/\partial y$ then you can see that I call for some effective viscosity here and I try to solve this. So, that is the call here. Similarly, higher order models, this is two equation then Reynolds stress model that will solve for all the terms appearing like this. In 3D we will have 5 or 6 terms, so for all these we will model this.

Now, we will have a separate equation to solve. So, that is solved by the Reynolds stress model, so 5 equation model. Then, we go to what is called large eddy simulation which resolve for larger eddies, resolves larger eddies, but you do not go for resolving smaller eddies, that is modeled. So, the smaller eddies are modelled and resolve those larger eddies.

In the k-epsilon, k-omega models whatever be the size of the eddy you just model it. Here in the large eddy simulation the large eddies are modeled. So, you go for finer grids to resolve this large eddies. However, the smaller ones are modeled. So, if you take any turbulence book you will understand more about the turbulence modeling.

And it is not so easy to understand all this. For our basic course we need to understand that the effect of turbulence is going to affect the flame structure, both in premix as well as the non-premixed flames, turbulent regime premixed and turbulent regime non-premixed flames. So, we need to understand how we are going to understand the effect of this mixing and the eddies interaction with the reaction zones and so on.

So, that is what our thing is. Now, you have to understand the extra terms which are going to come and how you are going to model it.

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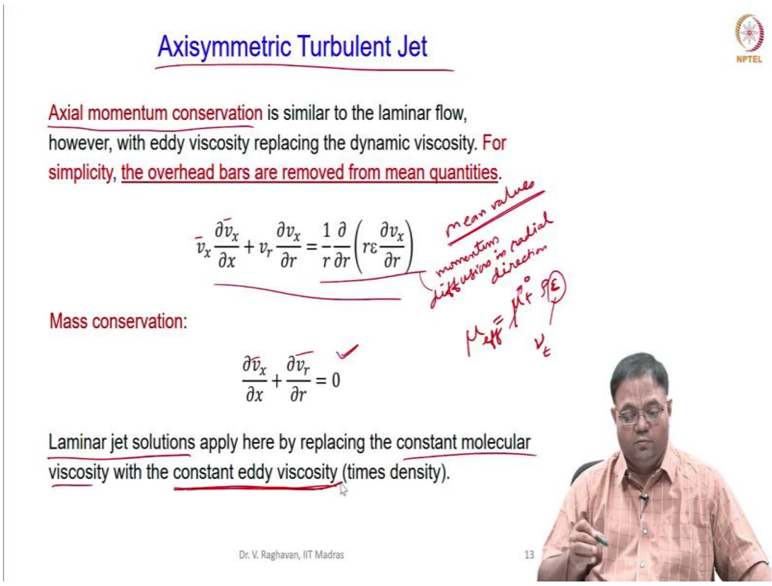

Now, let us do a simple analysis of an axisymmetric turbulent jet. We have already seen the axisymmetric laminar jet analysis. So, now, let us try to see how we can apply this eddy viscosity, what we have introduced here to solve for the turbulent jet.

So, axial moment momentum conservation equation is written here. So, v_x, please understand that we have to put bars here, but I have removed the overhead bars for simplicity. So, all are mean terms.

So, you can see that the axial momentum conservation $v_x\ \partial v_x/\partial x$ plus the radial velocity $v_r \partial v_x/\partial r = 1/r\ \partial(r\varepsilon\partial v_x/\partial r)/\partial\ r$. So, this is the radial diffusion in the radial direction.

$$v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon \frac{\partial v_x}{\partial r}\right)$$

So, this is the equation we have and of course, the mass conservation. Again, please see that all are mean quantities, mean values only. So, here since we are writing like this, you can see there are similarity between this equation and the previous equation. So, the laminar jet solution is applicable here.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} = 0$$

One more thing is please understand that we have seen that mu effective will be equal to $\mu + \rho\varepsilon$. So, here this is very small, so; we have neglected this also. So, the dynamic viscosity also is neglected, so we have just substituted μ for $\rho\times\varepsilon$ and when I divide this equation by ρ. I do not have any ρ here.

So, this is what we are going to substitute for the μ now, there is $\mu = \rho\varepsilon$.. So, this will be equivalent to ν turbulent. So, that is a turbulent kinematic viscosity for turbulent. Now, this again you can see the continuity equation.

So, you see the similarity between the laminar counterpart. So, we can replace the molecular viscosity in the laminar solutions by this eddy viscosity and we can go for prescribing solution do you understand. So, when I write the equation for the turbulent jet I have to replace by the above concept I have to replace the μ by μ_{eff} effective. So, μ_{eff} is nothing, but the dynamic viscosity, molecular level plus the eddy viscosity.

Now, since the dynamic viscosity is much smaller than the eddy viscosity we neglect this and write only the μ effective as $\rho\varepsilon$. And by dividing this equation by ρ I get the epsilon here.

So, $v_x\partial v_x/\partial x$, please understand these are all mean quantities + $v_r\partial v_x/\partial r$ will be equal to 1 by r by dou r of r epsilon dou v_x by dou r. So, this is the equation I get. So, we can see this is similar to the laminar jet axial momentum conservation. And the continuity equation also is the same when you just see the mean quantities are put in here. So, they are the same. So, the solution of laminar jets apply here, but only thing is we have to replace the μ by $\rho\varepsilon$.

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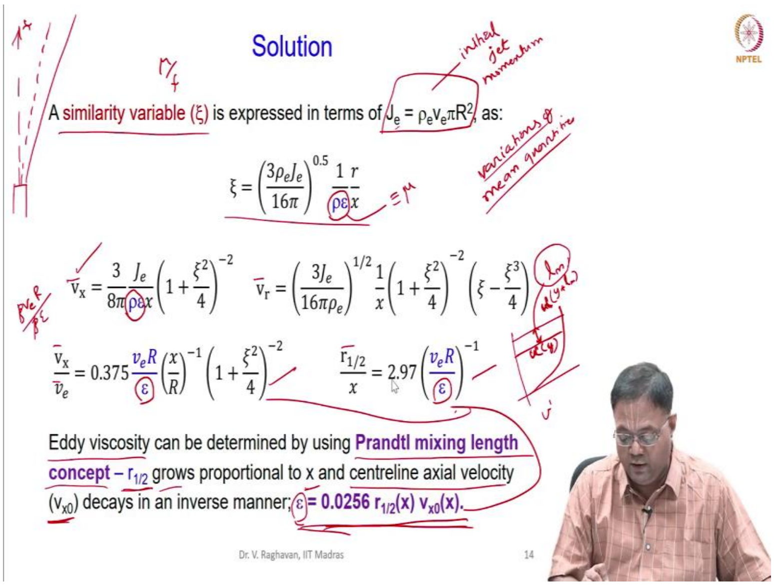

So, the solution again we go for the same similarity variable which is dependent on r/x and initial jet momentum is same.

$$\xi = \left(\frac{3\rho_e J_e}{16\pi}\right)^{0.5} \frac{1}{\rho\varepsilon}\frac{r}{x}$$

$\rho_e v_e$ that is exit velocity, the density at the exit of the nozzle into the area of cross section of the nozzle or the port which is πR^2. So, this is the initial jet momentum. So, similarity variable is already written. But please see that the μ term which was used there, so this is the μ term which was used in the laminar that is now substituted by the eddy viscosity.

$$v_x = \frac{3}{8\pi}\frac{J_e}{\rho\varepsilon x}\left(1+\frac{\xi^2}{4}\right)^{-2}$$

So, everywhere else I do the same. Similarly, we got the solution for the axial velocity v_x. That is now replaced, now the same equation is written, but the μ part is now replaced by this. Similarly, wherever I have indicated this you can see that here also the Reynolds number was used basically. So, $v_e R$ by mu, you can see that this is now replaced here.

$$v_r = \left(\frac{3J_e}{16\pi\rho_e}\right)^{1/2} \frac{1}{x}\left(1+\frac{\xi^2}{4}\right)^{-2}\left(\xi - \frac{\xi^3}{4}\right)$$

So, actually this should be $\rho v_e R$ divided by this, so ρ cancels, so we get $v_e R/\varepsilon$. So, you can see this. So, the Reynolds number is now written like this. Similarly, here also the Reynolds number was there. So, $2.97 Re^{-1}$. Now, Reynolds number is replaced by this because ρ and ρ cancels.

So, we can see that the solution what we got for and please understand these are the variations of the mean quantities only. Basically, we are interested only in the mean quantities, so the mean quantity variation, so all are mean quantities. Please understand everything is mean quantity here.

$$\frac{v_x}{v_e} = 0.375 \frac{v_e R}{\varepsilon} \left(\frac{x}{R}\right)^{-1} \left(1 + \frac{\xi^2}{4}\right)^{-2}$$

So, the mean quantity variation is what? What we are interested in. So, that is got by just applying the laminar solution; however, replacing the dynamic viscosity, molecular level dynamic viscosity by the eddy viscosity ρε. So, that is the way followed. So, in this context it is very simple. However, the only problem is how do you evaluate this value ε. So, the eddy viscosity can be determined by Prandtl mixing length, as I told you Prandtl mixing length concept. So, actually Prandtl what he told was, if you take a velocity variation like this. Prandtl postulated a length called mixing length.

So, this is say l_m, mixing length and for example, in this the velocity will be u' and here it is u', say at a particular y and u' at $y + l_m$.

So, some packet of fluid these are all going to flow in this direction, so I need not put u' here, it says u. So, these are the layers which are going to flow parallel to the mean quantities, but the fluctuation quantities are going to be there.

So, for example, if there is a v' fluctuation then that will contribute to some packets coming across the transverse direction. So, that Prandtl assume that due to these y directional fluctuations, the packet can travel at most a length what is called the mixing length.

So, with that assumption basically the epsilon can be evaluated. So, for this particular context the jet spread, we can say that the $r_{1/2}$ that is the jet half width, grows proportional to x we have seen this.

So, this is the jet exit and this is jet axis and a jet spreads like this. So, if you take $r_{1/2}$, so we can say $r_{1/2}$ also comes something like this here. So, $r_{1/2}$ increases with the x, grows proportional to the x direction. So, this is the x direction, so and $r_{1/2}$ grows increases as x is increased.

However, the centerline velocity, axial velocity, this is v_{x0} means v_x at R = 0. So, centerline velocity at the axis decays. So, you have seen after the potential core. Anyway, this solution is not applicable near to the jet exit. So, far field only it is applicable.

We have seen that Reynolds will also $0.375Re_j$, if that is the location then from that location onwards this solution will be valid. So, in that region, you can see that the decay of axial velocity happens, the mass fraction axial velocity etcetera happens in that.

$$\frac{r_{1/2}}{x} = 2.97\left(\frac{v_e R}{\varepsilon}\right)^{-1}$$

So, based upon this the ε can be written in terms of constant times, $r_{1/2}$ of the particular x and v_{x0} that is the central line velocity at the particular x. So, this we can write. Now, using this solution we can write the equations for $r_{1/2}$ and v_{x0} and try to get the values in terms of v_e and d radius. So, that is what we are going to do.

So, please understand that as per the Prandtl mixing length concept, a mixing length is defined where the velocity gradients are used to get the epsilon values. So, using that concept we can derive this basically to get eε value for this jet spread as a constant time $r_{1/2}$ at a particular x and v the centreline velocity at the x.

$$\varepsilon = 0.0256\, r_{1/2}(x)\, v_{x0}(x).$$

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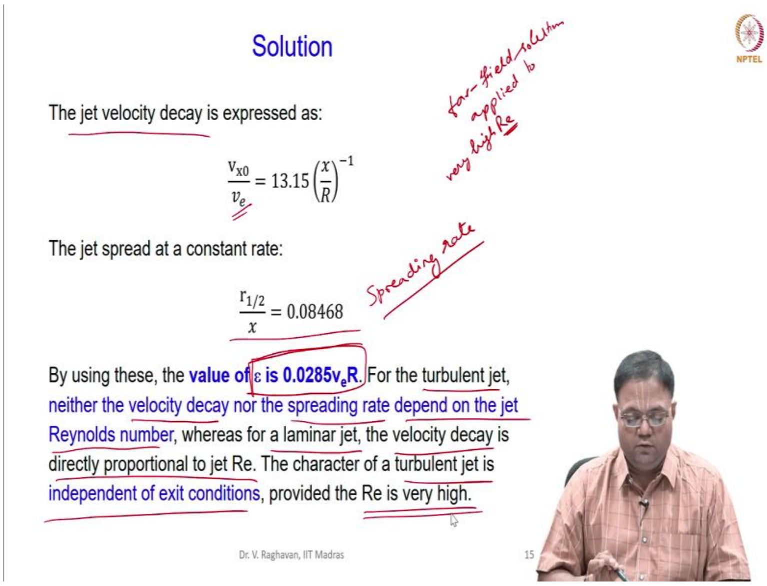

So, now, so let us try to write this. The velocity decay you can see that v_x/v_e normalized by the jet exit velocity, $v_{x0}/v_e = 13.15(x/R)^{-1}$. Similarly, jet spread rate is a constant. When you just put here, the values you can see that is a constant here $r_{1/2}/x$, when you substitute this epsilon value in these equations.

So, this equation and this equation you substitute the epsilon values, then you get these two. So, from that you can see that the epsilon value will be. That is why when we postulated the problem, we said that we will use the constant eddy viscosity for this solution of the turbulent jet. So, anyway a lot of simplifications have to be put in when you derive a theoretical solution for a complex problem.

So, that is what we are trying to do here. So, when you substitute this in the v_x/v_e equation and set R = 0. Similarly, $r_{1/2}/x$ and substitute this you get these equations and finally, you get the epsilon value as a constant $0.285(v_eR)$. So, we can understand that for a turbulent jet neither the velocity decay nor the spreading rate, depend upon the jet Reynolds number.

That is actually not for just transition regime or anything, it is actually in the strong turbulent regime. So, for example, the Reynolds number should be very high. When this is very high we can get the solutions. So, these are all far field solutions applied to very high Reynolds number. So, that we have to keep in mind.

So, in the laminar jet case the velocity decay was directly proportional to the jet Re, but here you can see that the velocity decay is not. So, that means, that the turbulent jet is independent of the nozzle exit conditions. So, exit condition means nozzle exit conditions.

So, that is very important to understand here. So, the eddies which are going to be present after the potential core region basically that is going to contribute to the characteristics of the turbulent jet. So, we can see that the spreading rate is a constant.

Once you get past a particular Reynolds number, for all the Reynolds number this characteristic will be intact. So, that is the main thing you have to understand from this solution.