Strength of Materials
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Lecture - 14
State of Strain

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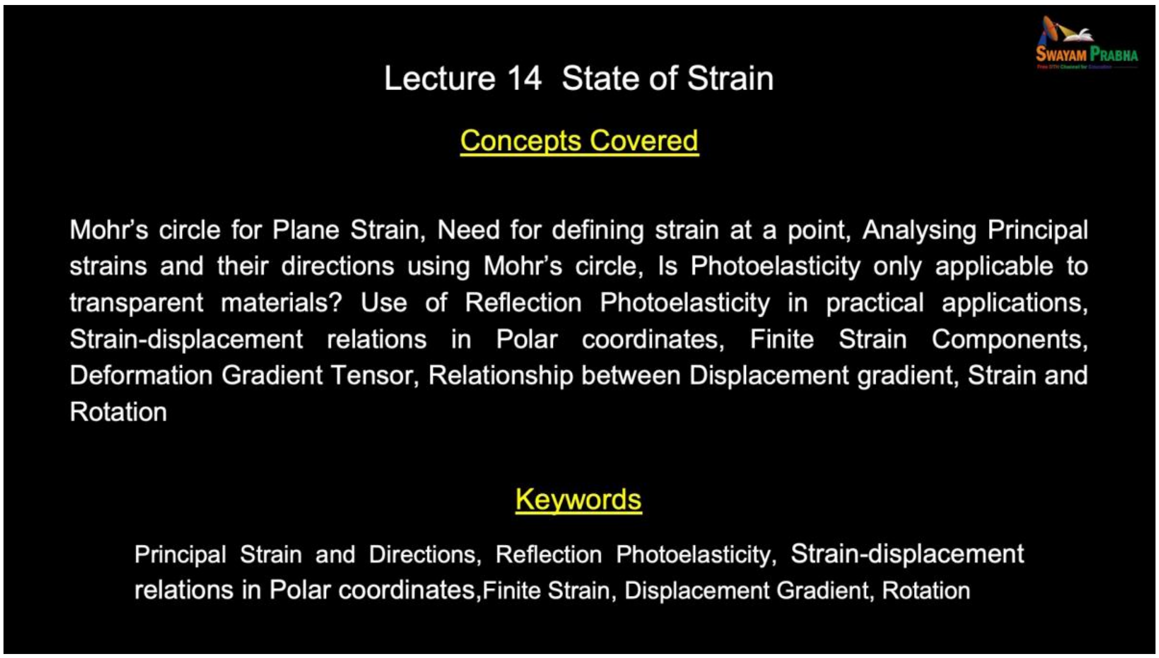

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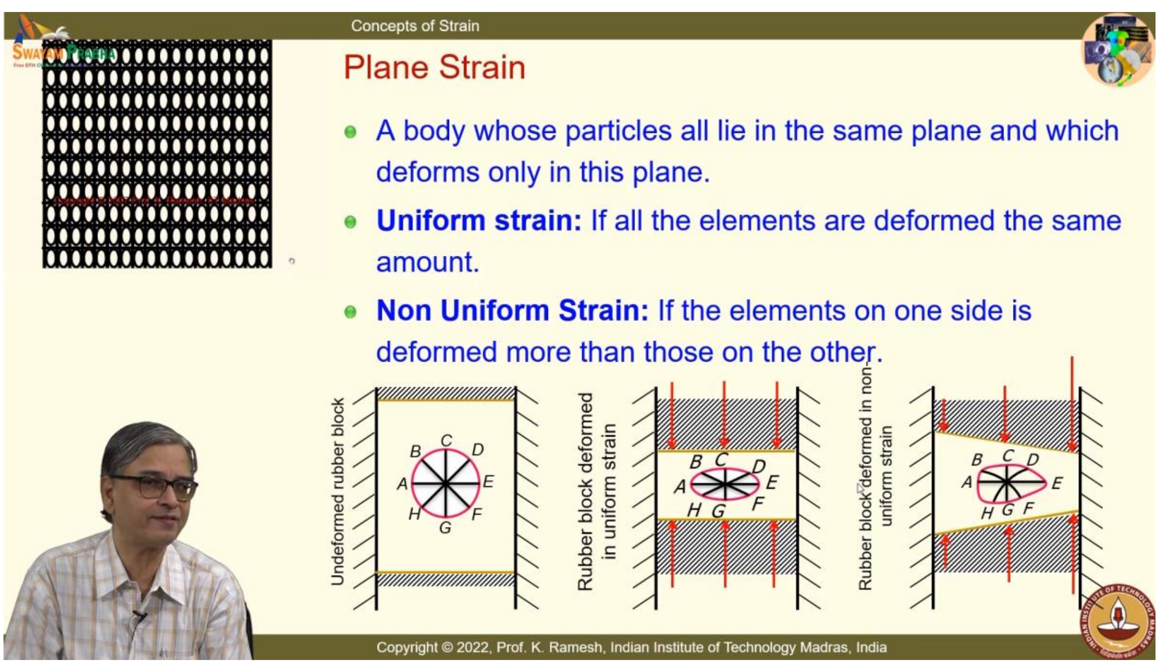

Let us continue our discussion on concepts related to strain. See, both in the case of concepts related to stress as well as strain, I said they took material of different cross section and when they pulled it, they could have this material response as a single graph, when you plot on the *x*-axis quantity called strain and you plot stress on the vertical axis. So, we had a simple definition of change in length divided by original length as strain, force divided by cross sectional area is stress. From then on we developed that these are all not scalar quantities, they are indeed tensorial quantities. Stress is a tensor of rank 2, strain is also a tensor of rank 2. So, whatever the discussion we have had on stress, you can have a parallel set of discussion on strain.

You know, we were confining our attention to plane strain, a simplistic definition is given here. A body whose particles all lie in the same plane and which deforms only in this plane. And what you see here is actually a stretching of a member, where you have the circles drawn and what you find is, when I apply this load, I have also have a lateral strain. I am taking a planar example, so I also have a lateral strain.

So, by default when you apply the loads, you will have multi-axial strain. On the other hand, if I have to have strain in only one direction, I have to have external constraints, please never forget that, both are illustrated in this slide. Here I take a rubber block and compress it, when I compress it what I do? It will bulge out, I prevent that bulging out by these constraints. And when I have a uniform load applied, a circle deforms into an ellipse and you also find these lines get rotated. And this also gives, suppose I reduce this circle to a point, in the limit, you find that depending on the direction, the value of strain also changes like what we have seen in the case of stress.

In this case of stress, we looked at all the possible infinite planes passing to the point of interest, in the case of strain all the possible directions, that is the difference. And suppose I have a non-uniform force applied, you can have a very distorted shape and you know, you need to have the mathematics to evaluate the strain quantities. We have seen, you have axial strain as well as shear strain and we also saw that there is also rigid body rotation. We have determined the expressions in strength of materials very comfortably.

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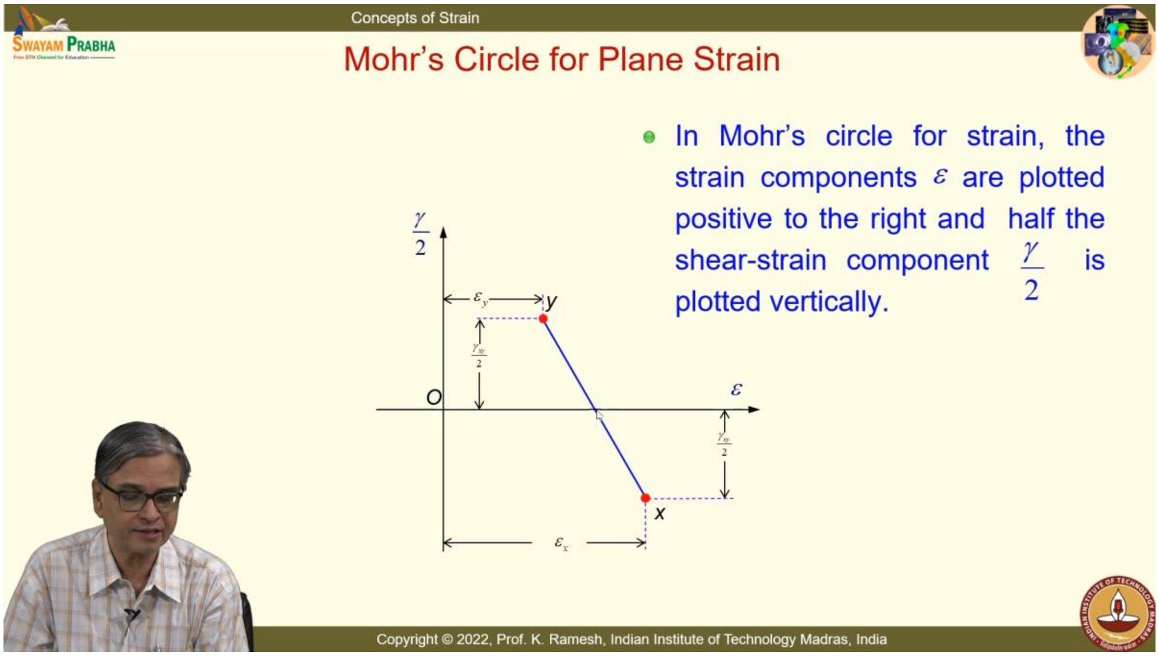

And when you develop these quantities, you also need to know, if I change the frame of reference, how do I express or get these quantities? That is what you do in stress transformation or strain transformation.

Same thing if you do it for all the possible directions, you have a Mohr's circle. And I have said historically people developed shear strain as γ, but tensorially it is only $\gamma/2$ behaves. So, when like what we have done in the case of a stress, here also you plot what is the axial quantity, here it is axial deformation on the *x*-axis. And on the *y*-axis, you plotted shear stress there, here you plot shear strain, if we express it as γ, it is $\gamma/2$. If we express it as ε_{xy}, which is already known as $\gamma/2$, you plot ε_{xy} here.

And what is the purpose of this Mohr's circle of strain? Whatever I see in the real plane, I should be able to correlate. So, I need to follow a convention. The convention is same as what you do it for stress. So, you plot the positive shear strain for the *x* direction downwards and you have no difficulty in plotting what is ε_x, that you will naturally put on the positive axis, there is no difficulty there. But you have to follow a convention, if I have a positive shear strain in the *x* direction, you plot it downwards.

And for the *y* direction, you can follow simple logic, what you have been using it for plotting any graph, when it is positive shear and positive axial strain, the point *y* is located here. So, once you have located the direction *x* and *y*, simply join them, it cuts this axis at point *C*, with this as center, you draw the Mohr's circle. So, very similar to what we have seen in the case of stress analysis. This is called Mohr's circle of strain. And all the properties, we saw principal stresses, here we will have principal strains.

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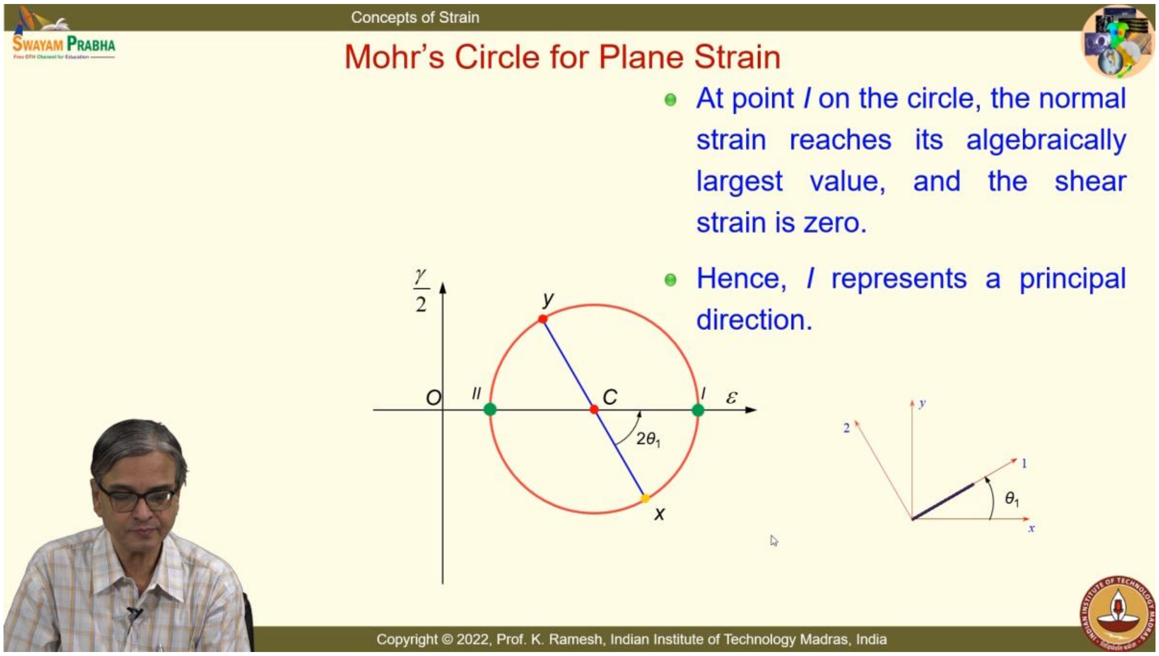

So, what you have here is, I have the point *I*, where you have the maximum normal strain, there is no shear strain on this point. Like in the case of principal planes, there are no shear stresses, in the principal directions, there are no shear strain. And I have the point located and in Mohr's circle, we will always have twice the angle. So, that is also illustrated here. And I have this as, if you move from this to this, you get it and this is angle $2\theta_1$.

And physical plane, it is θ_1. So, whatever the discussion we have developed in the case of stress, here again you repeat the same, but you should recognize on the *y*-axis, I plot $\gamma/2$ that is the only difference. And I have principal strain direction 1 and principal strain direction 2. Like principal stresses, you also arrange them algebraically. So, that is what is listed here.

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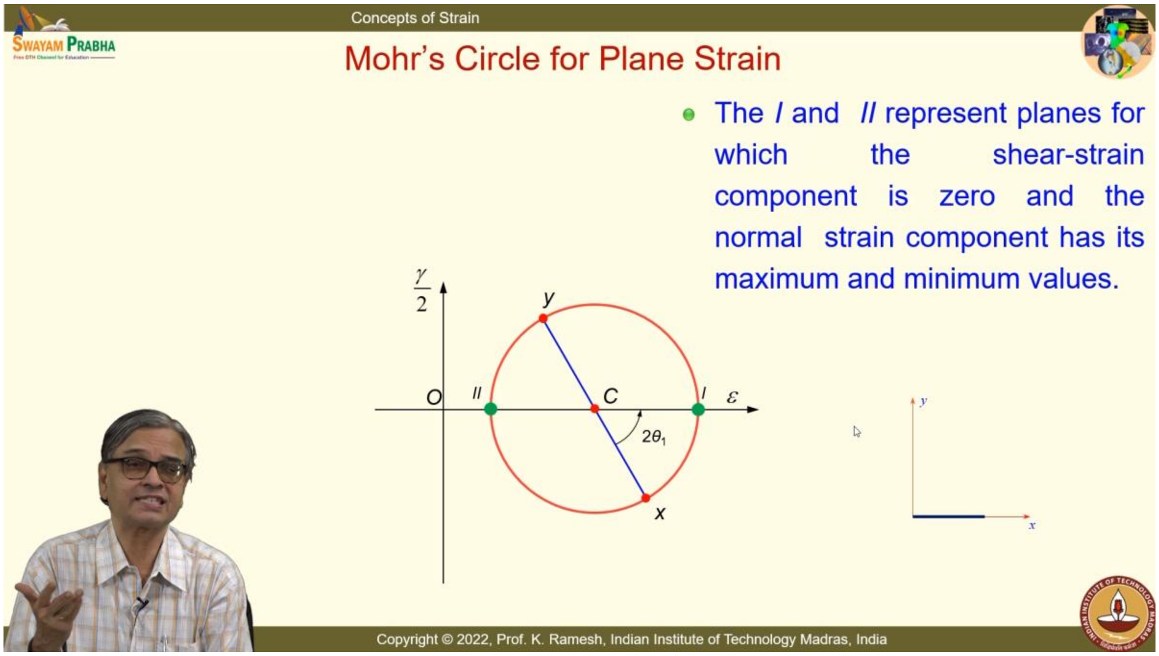

Then like we had seen, maximum shear stress plane, we will also have a maximum shear strain, fine. And they can be located easily, because you know very well how to locate them.

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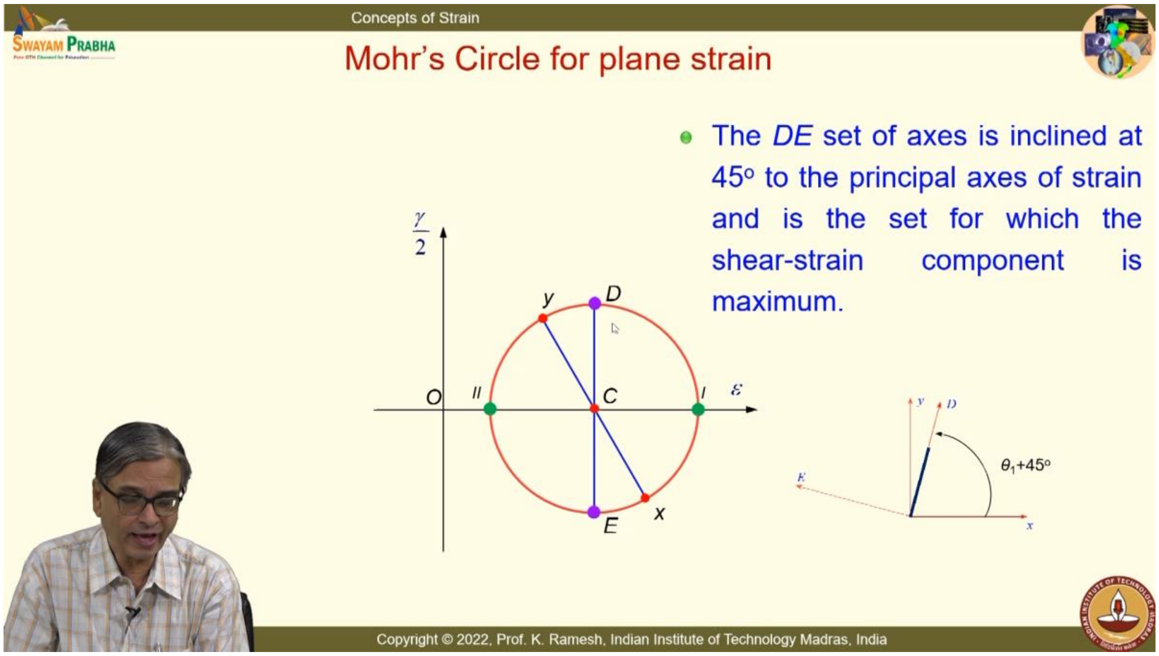

I have this on the circle labeled as *D* and *E*. And like we have discussed, whether it is a local maximum or a global maximum. That discussion what we have done for stress is also valid here.

To make your diagram simple, I take both the strains as positive, so that this axis is not interfering with the diagram. And we have seen in the case of your pressure vessel, you have a biaxial stress, one is $\frac{\text{Pr}}{t}$, another is $\frac{\text{Pr}}{2t}$, both are positive. So, such situations are very common. And what you have here is, the location of the maximum shear strain direction is fixed with respect to the principal direction. This is at an angle 45° in the physical plane, in the Mohr circle it is at 90°.

And you have in general, on the maximum shear strain direction, you also have some value of normal strain. Normal strain is not absent. Suppose I go to a pure shear stress state or strain state, then I will have maximum shear strain, there is no normal strain. That means, the entire shifted to this place. Now, it is away from the origin, so I have some value of normal strain as well.

So, you have this as $\theta_1 + 45$, that is what you see in the physical plane and in the Mohr circle it is twice the angle.

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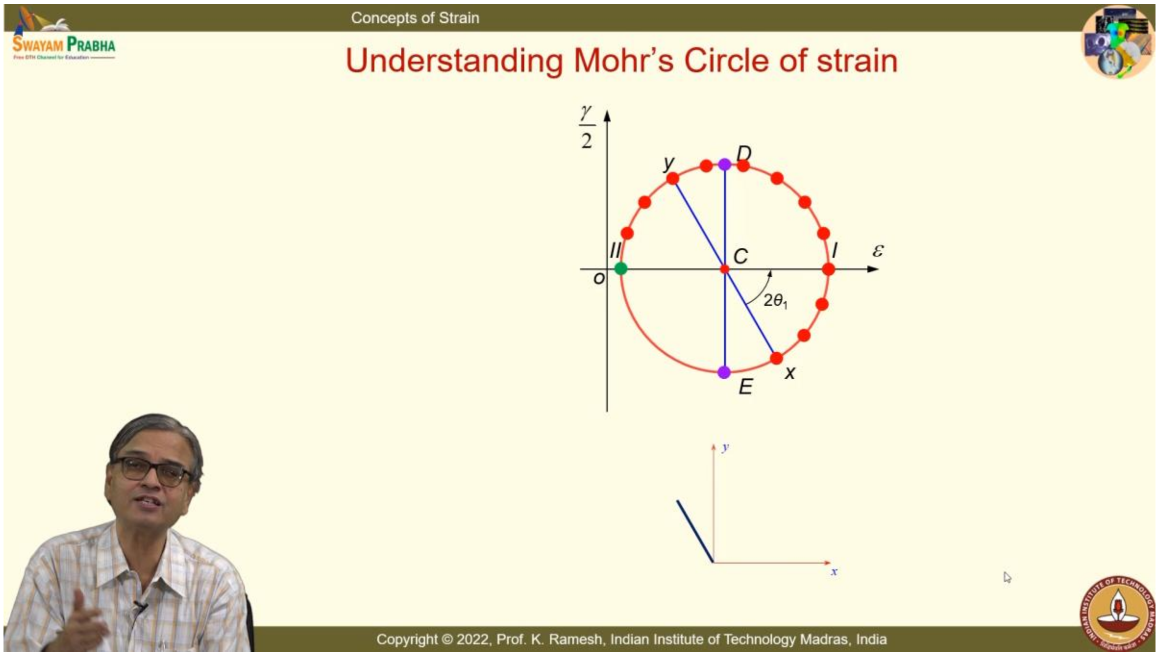

And similarly, what we have seen in the case of stress, we can also appreciate when you have a circle, I have infinite points forming the boundary of the circle. So, it gives you all the complete information of what is the value of strain on all these directions. You know, we also solved a numerical problem in the case of stress, where we understood state of stress at that point is unique, which can be expressed by various components. On similar lines, state of strain is also unique at that point.

I can express it as components in the *x* and *y* directions. I can express as components in the principal directions *I* and *II*. And if I can also express as components along the shear directions, maximum shear direction, here it is the local maximum. And if you take any point that represents strain in a particular direction. See, I have just shown the direction, I have not taken effort to show the change in the length, which I was able to do easily in the case of stress, but here it is too difficult to do.

So, what is indicated here is, when I go one full circle of 360°, I will traverse 0 to 180 on the physical plane. So, every point denotes what is the strain in that particular direction. In general, it can have a normal strain component as well as a shear strain component. In particular cases, you may have only normal strain and when you have a pure shear strain state, you may have only shear strain. Otherwise, when you have maximum shear strain like this, you will always have a normal strain component.

So, every point denotes what happens along that direction. And if you draw a line and find out what is the angle from this, one half of that is what is the direction in the physical plane. So, all the understanding that we have developed in the case of stress, you can also extrapolate in the case of strain. See, while we discussed stress, we also developed photoelasticity. We had developed a stress optic law and then we showed that you can get beautiful fringe patterns and we were using only models that were transparent.

Now, the question arises, suppose I do not have a transparent model, can I still take advantage of the use of photoelasticity? If you ask the question, the answer is yes. What you do is, you take a specimen and then coat the specimen with a photoelastic coating. That would be transparent, but what you do is, you coat it with a reflective backing and then put the material. So, this main member experiences strain. The strain is faithfully transmitted to the coating. Because of the strain, the coating will get deformed and because the coating is birefringent, all the concepts that we discussed in the case of stress analysis, you can have a parallel one like you have a stress optic law, you can also have a strain optic law.

And this is what you see me 15 years back, you know, I pulled out my old video and this shows a reflection polariscope.

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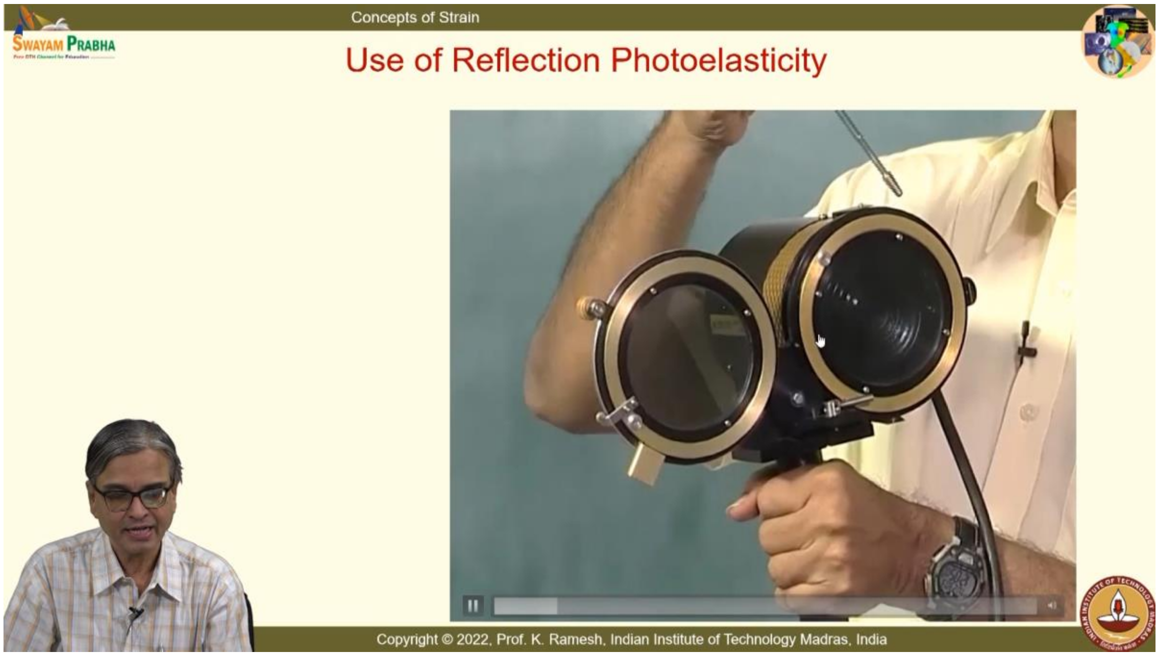

It is very compact, you have a light source, you have a polarizer, you have a first quarter wave plate, you have the second quarter wave plate and you have the analyzer, which you had seen a normal polariscope should have this.

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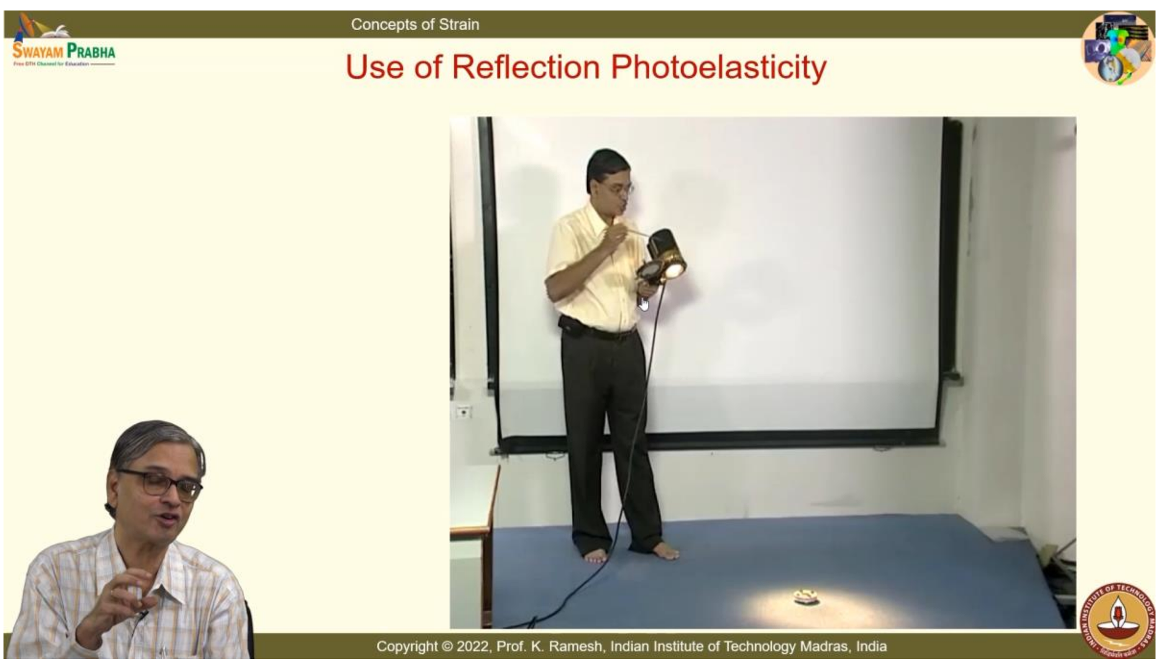

So, what you do is, the light comes out of this, it impinges on the model, you analyze the reflected light, fine. And in the case of normal photoelasticity, you know, you did not have that angle problem.

Here you have to keep the equipment away from the specimen, so that you minimize the angle of deviation. You want to limit it with within 4° and what you have here is, you have a flange coupling, it is tightened with 4 bolts. And this is, the light is impinged on this and the reflected light will be viewed through the analyzer and quarter wave plate.

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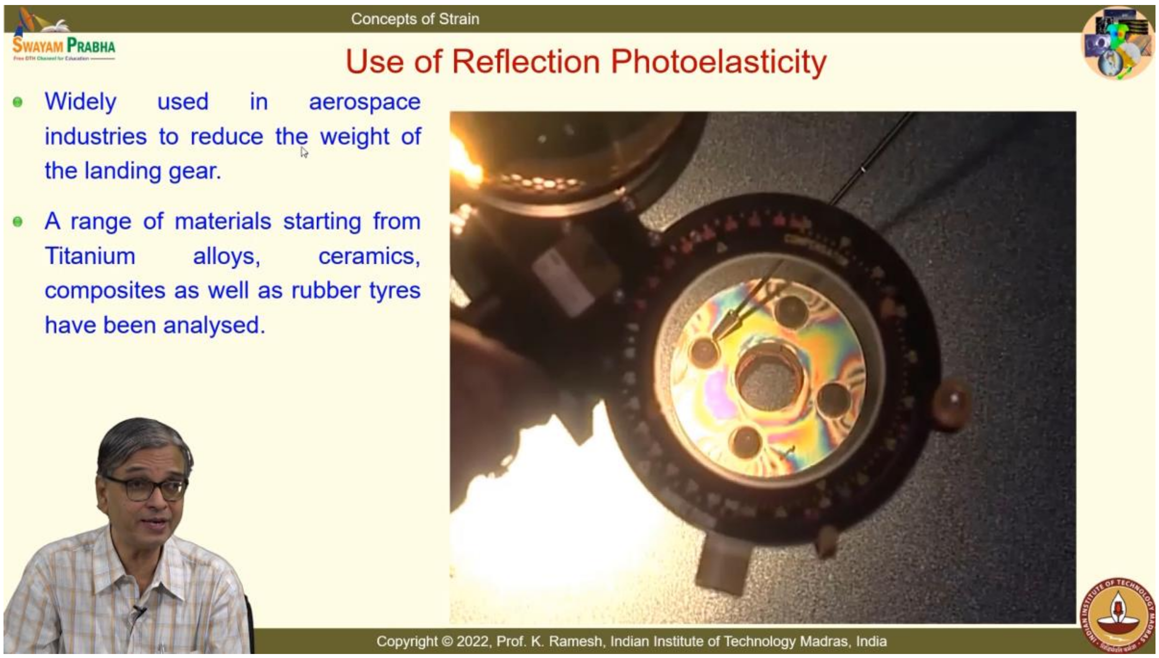

And for you to see, instead of me looking at it, we will have an electronic camera port and you will see beautiful fringe patterns, ok. And this only explains how the light gets reflected on this and you see this beautiful fringe pattern, do you see them? See, normally if you are given a spanner, what do you go and do? You simply go and tighten it to your maximum, that is not the way engineering is done.

And this is a very complex problem. See, you would have seen flange couplings in many many applications, you will have big pipelines, you will have a flange and it will be transmitting fluid at a high pressure. In those applications, what people normally do is, they have what is known as a torque wrench and they will measure what is the torque to be applied. And when I have number of bolts like this, here I have only 4. If you have a huge flange, there is a particular sequence in which you have to tighten the bolts.

The sequence of tightening influences the performance, you should not do over tightening. And you see here, the problem should have been symmetric because of non-uniform tightening, the fringes are not symmetric. And it is very difficult to model analytically, please understand. The kind of stresses that are introduced, happen because of the assembly. Assembly stresses are very difficult to analyze analytically.

Whether you want to go and do a numerical analysis, there again modeling assembly stresses is extremely difficult. The only recourse is a technique like photoelasticity or any other experimental technique. Because here you actually work on a prototype, you do not work on a model. So, from analytical perspective, it is a very complex problem. And you also find this equipment is very compact, you can hold it in a hand like you have a stethoscope, like the doctor tests you and then finds out whether your heart is beating properly.

So, I can take this and then find out what happens to the structure. And if you look at, this is extensively used in aerospace industries and one of the applications there is when you have a landing gear, if I save 1 kilogram of weight on the landing gear, I save enormous amount of fuel in the lifetime of the aircraft. And this is a structural optimization. Even today when they go from A 360 to A 380, it is done only experimentally because the problem is extremely complex, fine. And they actually have chemical engineers making a three-dimensional model of this landing gear and then do a transmission as well as reflection photoelastic analysis.

And whatever the loading rig that they developed for A 360, A 380, they will use it for the model study for A 380 because the capacity is increasing, you also need larger loading frames. So, it is a very big job and it is also used for a range of materials. You have aluminum titanium alloys, ceramics, composites. If you look at composite from analytical analysis, it is extremely difficult because it is anisotropic. It can be orthotropic or it can even be anisotropic whereas, the coating is isotropic in nature.

And you see, you all use a cycle. So, you treat your cycle tire without any respect because it is after all a cycle tire. Same is not applicable when you go in an aircraft. Imagine while landing, the entire weight is taken care of by your tires. That is what we said when we looked at that pressure vessel with two helical coils, I said your tire is also very complex. You have coils in the helical direction and it is a very complex to analyze.

Particularly, when you go to aircraft tires, people have analyzed stability of those tires in those applications only by a technique like photoelasticity. You develop a photoelastic coating, apply it appropriately. It has to bear the entire dynamic load while landing. So, tire burst is a very severe problem and you rarely come across that kind of accidents these days because these are all tested and you need a via media to do the testing. So, I have a variation that helps you to work on prototypes, fine.

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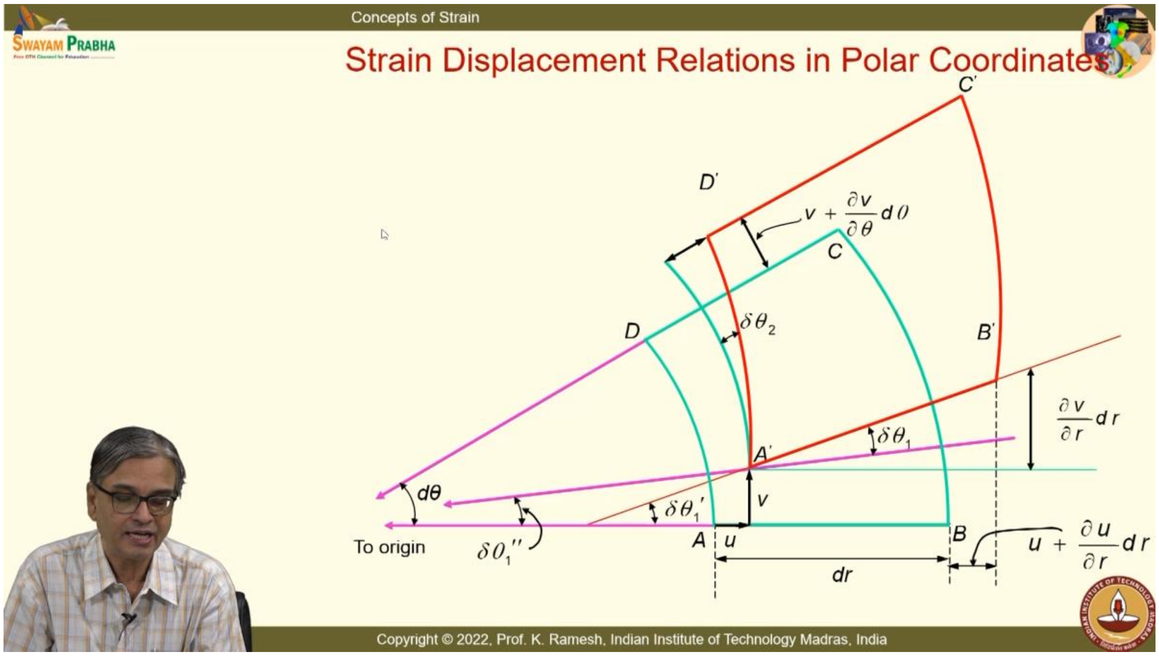

We will also have to learn what happens in polar coordinates. See, the animation is very good. I would like you to devote a very nice page in your notebook. Draw the diagram very big. If you draw the diagram systematically, you know how to write the quantities, fine.

So, I have first drawn this line. It goes to origin. I take a small element *ABCD*. I have shown the segment *AD*. So, I have a b which is a distance *dr*.

I am going to go slow. So, which will help you to draw as I am drawing, fine. I have the segment *BC*. These are all exaggerated pictures. You should keep in mind. So, I have taken a simple element in the radial and circumferential direction.

I have these two lines. So, I have an element *ABCD*. Under the given action of loads, this element gets distorted and let me say that I have a displacement field *u* and *v* for us to write it very comfortably. So, the point *A* gets shifted, *B* gets shifted, *C* gets shifted and *D* gets shifted and you carefully draw how do I locate these various lines. So, I take this angle as $d\theta$. So, I have an element *abcd* and I have a displacement at *A*, I have a radial component which is denoted as *u*. I have a circumferential component which is denoted as *v*.

In the limit, you can say that this is tangential to this, fine. And I have point *A* shifted to A'. And let me see how I locate point B'. I have moved by a distance *dr*.

So, *u* is a function of both *r* and θ, fine. *v* is also a function of both *r* and θ. So, I should write the incremental quantities from Taylor's approximation properly, ok. So, I have, let me first complete this distorted element. I have this arc becomes like this, distorted like this and this is no longer purely circumferential like this.

It is distorted because of both axial and shear strain. Now, we have to locate what is the coordinates of B'. And for me to do that later I have to get the shear strain. So, I draw this line, I recognize this angle as $\delta\theta_1$ because this is, you know, again radial, fine. So, if I have to find out original rectangle, so I should also draw one more line here which is circumferential.

So, this is orthogonal. If I draw a tangent here and this line, they are perpendicular. So, I have to measure anything related to shear strain as deviation from this line and deviation from this line. Is the idea clear? Shear strain, we have said it is the original rectangle becomes what? And it is $\delta\theta_2$. But I will also have to recognize, if I have to calculate $\delta\theta_1$, I should recognize that I can get this angle comfortably.

I join this and then extend it. This I can express it. This I will put it as $\delta\theta_1^{'}$. I will also have another line drawn. What we have already drawn is this and this is $\delta\theta_1^{''}$. I need these angles for me to find out what is $\delta\theta_1$. For me to get $\delta\theta_1$, I have to have $\delta\theta_1^{'} - \delta\theta_1^{''}$.

That is what you get this as $\delta\theta_1$. So, I have the point B' located. So, from this how do I write this variation? This is the variation in the radial direction. I have here it as *u* and I have shifted by the distance *dr* and what is this distance? Can you guess and write down? Write down what is the way I can do that? Then check what I have written. Can I write this as $u + \frac{\partial u}{\partial r} dr$? Because I have moved by distance *dr*, this *A* has become A', *B* has become B'. So, *B* will have *u* plus variation of *u* because of the shift in distance *dr*.

And I also have, I can comfortably write this vertical distance. I have put a horizontal line here. I can write this distance easily. Can you find out what is this distance? Because I have come to *v* and then at a distance *dr*, I am writing what has happened to *v*. Can you say what is this value here? See *u* and *v* are functions of *r* as well as θ, fine.

But here, I have moved only by distance *dr*. So, it will be a function of *r*. So, I will have this as $\frac{\partial v}{\partial r} dr$, fine. See I am not writing from this bottom. If I write it from this bottom, it will be $v + \frac{\partial v}{\partial r} dr$. I have written this directly here. So, do not get confused. I write

$u+\dfrac{\partial u}{\partial r}dr$ here, suddenly I drop v. I am denoting it different quantity here. And if I know this, I can find out what is this angle. I can find out what is this angle. $\delta\theta_1^{'}$ I can easily find out, fine. So, once you have understood this, same logic will extend it for other positions. So, this will become, it is because of θ variation. So, $v+\dfrac{\partial v}{\partial \theta}d\theta$ and this you will have this as $\dfrac{\partial u}{\partial \theta}d\theta$. So, once you have understood how to draw the distorted element and label them systematically, finding out the strain components from this diagram is simple and straightforward, fine.

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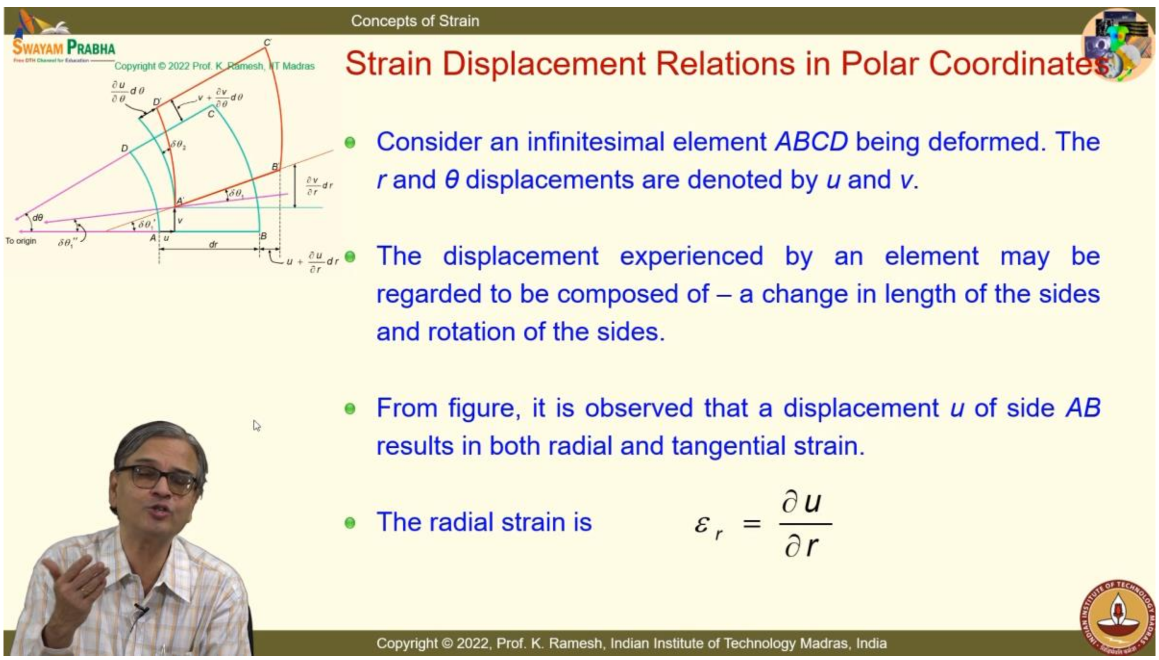

So, what I am going to do is to find out the normal and shear strain. Here again, I have the animation. Just watch the animation, how the diagram is drawn, how the quantities are denoted. Even if you have missed some idea, you get clarity from the way the diagram is drawn. So, we have to consider an infinitesimal element *ABCD*.

And there is peculiarity in the case of polar coordinates. See, when the radius changes, the circumference also changes, they are interrelated, fine. So, results in both radial and tangential strain. If u change, because of *u*, it will have a component on the radial direction as well as tangential direction. Now, let us look at what is the radial strain.

Radial strain is very straightforward. I have *u*, it has gone to $u+\frac{\partial u}{\partial r}dr$. So, I simply have change in length divided by original length as ε_r equal to $\frac{\partial u}{\partial r}$. There is no difficulty at all in writing radial strain, which was also like ε_x as $\frac{\partial u}{\partial x}$, fine. There is similarity between what you had seen in Cartesian coordinates and what you are looking at in polar coordinates. And, we have to look at what is the strain due to the tangential strain due to displacement *u*, because when the radius elongates by a distance *u*, you also have a circumferential distance changed, ok.

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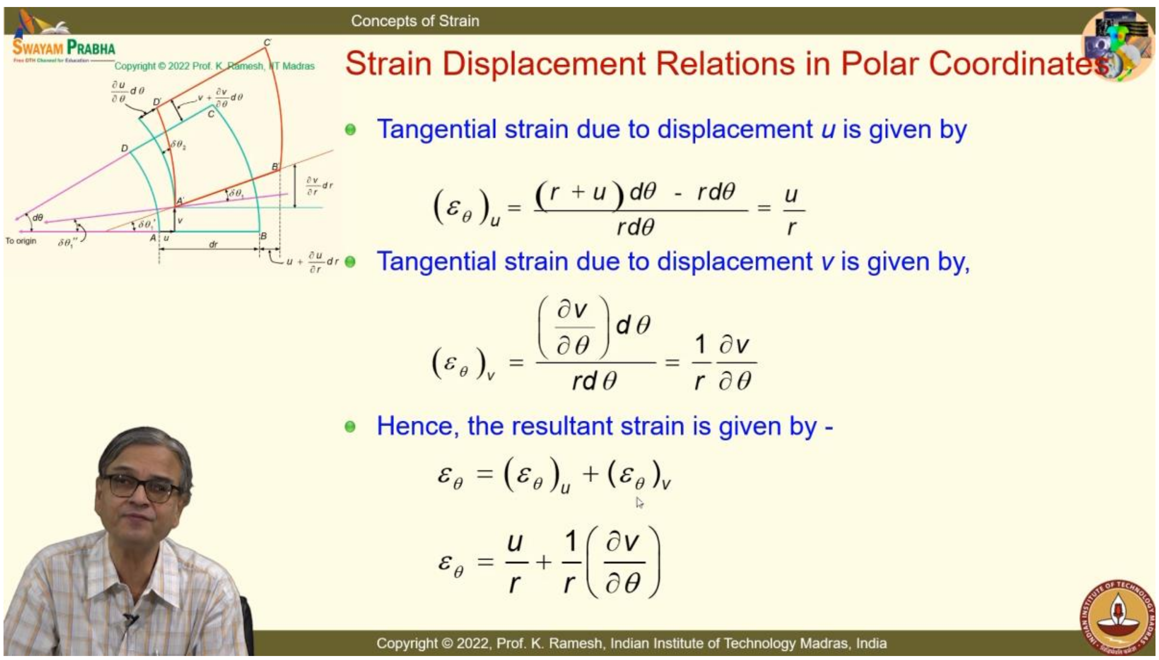

So, you will have this as $r+ud\theta$ is a circumferential change. Original one was $rd\theta$. So, change in length becomes *u*/*r*, which is something very peculiar, which happens only in polar coordinates, fine. I have one component because of what happens to *u*. I will also have another component in the tangential direction because of what happens to *v*, ok. So, I will write this as

$$(\varepsilon_\theta)_v = \frac{\left(\frac{\partial v}{\partial \theta}\right)d\theta}{rd\theta} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

So, this is subtle and you have to know what is physically happening in polar coordinates. You have every likelihood you may ignore this, you may account for this, but you may ignore this because any radial change there will also be a connected circumferential change. And when I say ε_θ, I am talking about what is the change in the circumference. So, it has a component because of the change in the circumference, v is what is the change in the circumference length, u is what is the change in the radial length.

So, this also will have a component. So, the resultant strain is

$$\varepsilon_\theta = (\varepsilon_\theta)_u + (\varepsilon_\theta)_v$$

$$\varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\left(\frac{\partial v}{\partial \theta}\right)$$

Is the idea clear? And for you to find out the shear strain, I have already elaborated how do you get the angles and things like that, but nevertheless we will have advantage of the animation. We will see the animation. You just look at the angles, look at how it is drawn, in what sequence these quantities are labeled, all that contributes to your basic understanding.

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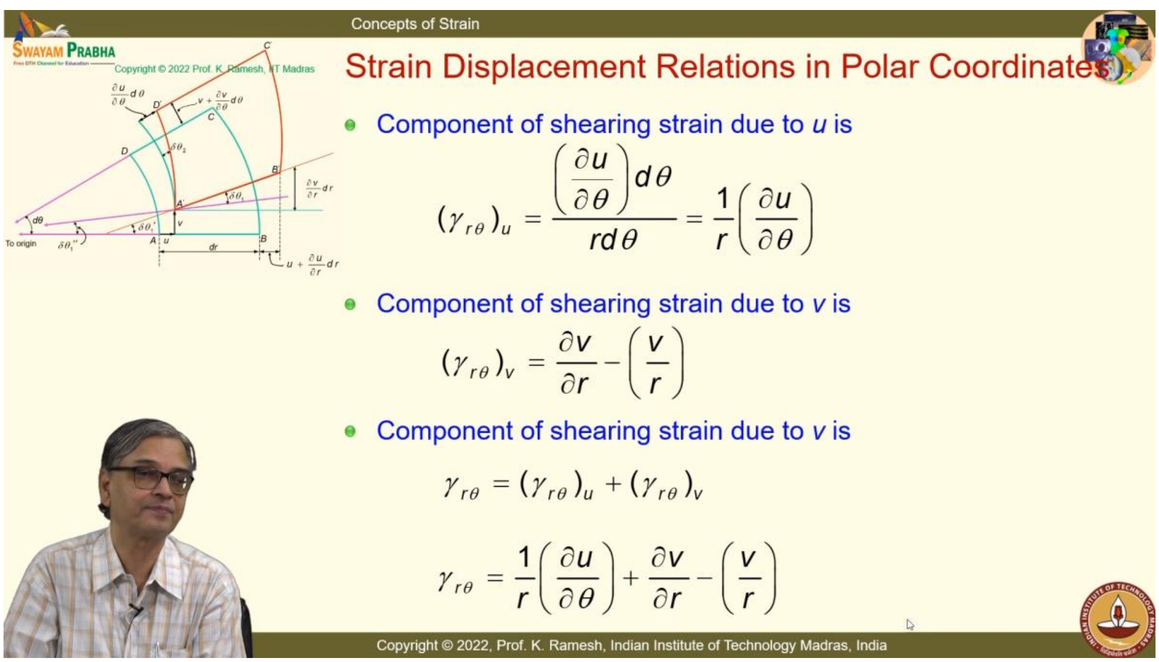

So, you look at this and component of shearing strain due to u, we have already drawn what is the reference here. I have a radial line, I have a circumferential line. Is the idea clear? Radial line is drawn straight, circumferential line is drawn as curved because I have concentric circles and then radial lines. In the limit they will all look like straight lines, but this is an exaggerated picture and when we do that we have to show that as circumferential line. So, I have $(\gamma_{r\theta})_u$ the contribution from *u*, I have this as $\frac{\partial u}{\partial \theta}$, see, this you are able to see. I am first writing out $\frac{\delta\theta_2}{rd\theta}$ that is the original length, original length is this, original length is this, that is this is with angle $d\theta$. So, I have this as $rd\theta$. So, I have this as $\frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right)$ and what is the component due to the displacement *v*? I have $\frac{\partial v}{\partial r}$, see you will have this $\delta\theta_1^{'}$. So, this is $\frac{\frac{\partial v}{\partial r}dr}{dr}$, I will get this as $\frac{\partial v}{\partial r} - \frac{v}{r}$, see you look at here $\delta\theta_1^{''}$, this is *v* and this distance is *r*, we neglect this *r* + *u*.

So, I have this as, I have this as *v*/*r*. So, the shear strain is addition of these two. So, that I finally, get this as

$$\gamma_{r\theta} = \frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right) + \frac{\partial v}{\partial r} - \left(\frac{v}{r}\right)$$

So, what you find is, when you want to find out the strain quantities in polar coordinates, it is not completely similar to what we have seen in Cartesian coordinates, you have additional terms, you should not ignore the additional terms.

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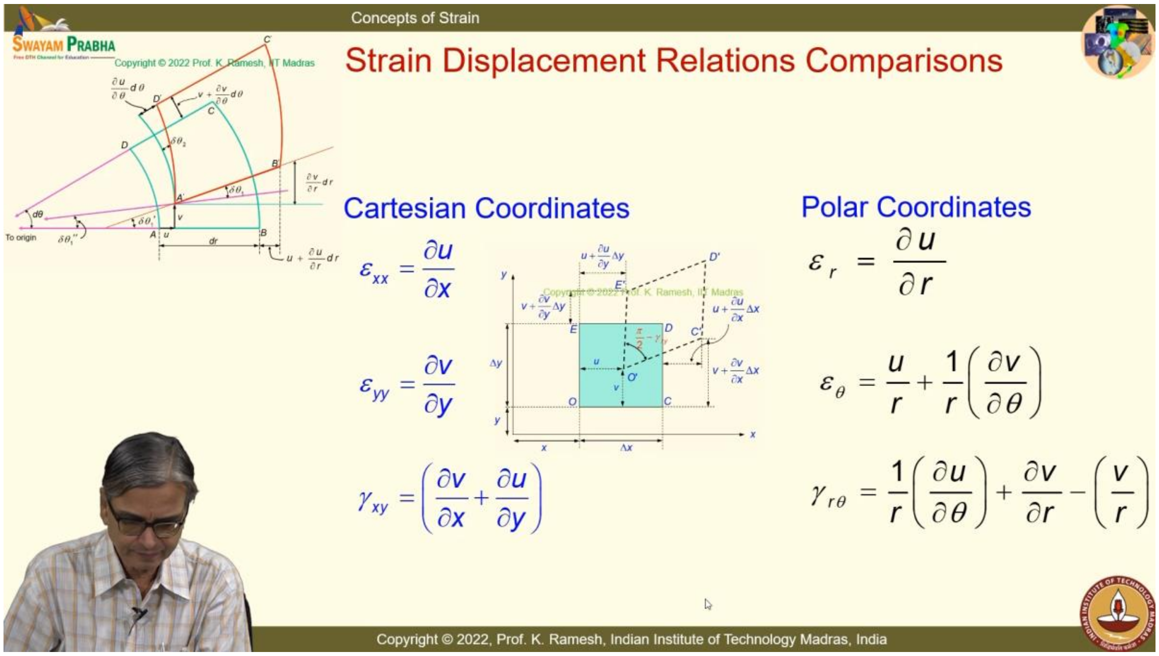

Now, let us look at a comparison because there is also a need that you need to remember them mnemonically, fine. I will try to give you some hints about how you can remember some of these quantities. So, you have the animation for the Cartesian, you have the animation for the polar coordinates and you have ε_{xx} as $\frac{\partial u}{\partial x}$ and ε_{rr}, this is what I said, we will use this symbols interchangeably because *xx*, I can also write it as ε_x, ε_{rr} I can also write it as ε_r. They are very similar $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial r}$, no difficulty at all. When I come to ε_{yy}, it is like $\frac{\partial v}{\partial y}$, I do have the term $\frac{\partial v}{\partial \theta}$, but in addition I have a term *u*/*r* and whenever I have a $d\theta$ at the bottom, I will always have a multiplier like *r* because $rd\theta$ is the distance. So, that is one way of remembering. Here I have ε_{yy} as $\frac{\partial v}{\partial y}$. So, similarly ε_θ will be $\frac{\partial v}{\partial \theta}$ because I have theta l/1/*r*, that you can write comfortably, but you have to remember that you will also have a nuisance value from *u*/*r*, ok. Now, when I go to shear strain, you look at here, I have $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ opposite.

So, I will also have similar terms here, I have $\frac{\partial v}{\partial r}$, no problem. I have $\frac{\partial u}{\partial \theta}$, but when I have θ term, I will have one multiplier *r*. So, I have $\frac{1}{r}\left(\frac{\partial u}{\partial \theta}\right)$. So, this basic structure you can write from Cartesian without any difficulty, but you will have in the case of shear strain and this is again engineering definition of shear strain. I have this as -*v*/*r* for engine for this shear strain, for the tangential strain I have *u*/*r*, that you will have to remember or learn that you can draw that sketch and then write it. So, that cannot be avoided, ok. See, with these definitions of strain, we can comfortably do strength of materials, but you are learning this course in 2022 and you will also have to be aware what are all the other strain measures. And I am going to have one simple development, which is not that complicated. We will have a look at it.

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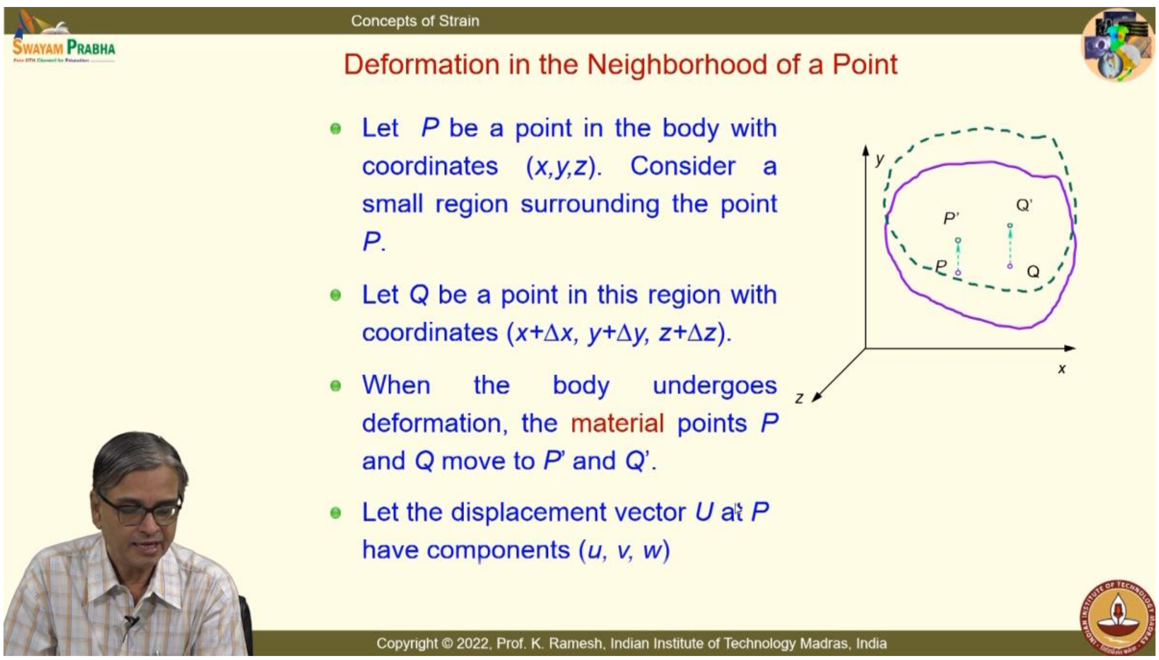

So, I have a reference frame *x*, *y*, *z*. I have the object and let us consider a point *P* with coordinates *x*, *y*, *z*. Consider a small region surrounding that point, I take a point *Q*, which has the coordinates $x+\Delta x$, $y+\Delta y$, $z+\Delta z$. And now I have a deformed structure, the points have displacements, *P* has gone to P' and then *Q* has gone to Q'. And I denote this as material points, you know there are material coordinates and spatial coordinates. And if we go to higher studies of strain, because in infinitesimal strain, we work on the undeformed configuration. When we said we are working on small deformation, there is no distinction between deformed and undeformed.

I can live in the comfort of undeformed configuration. So, we never worried about it, but the moment you come for large deformation, there are many many definitions. I will not get into all of them and you understand that these are all material points, these are all material coordinates. And let their displacement vector be *u*, *v* and *w*, fine. And what we do is, we want to find out the change in length of the linear element.

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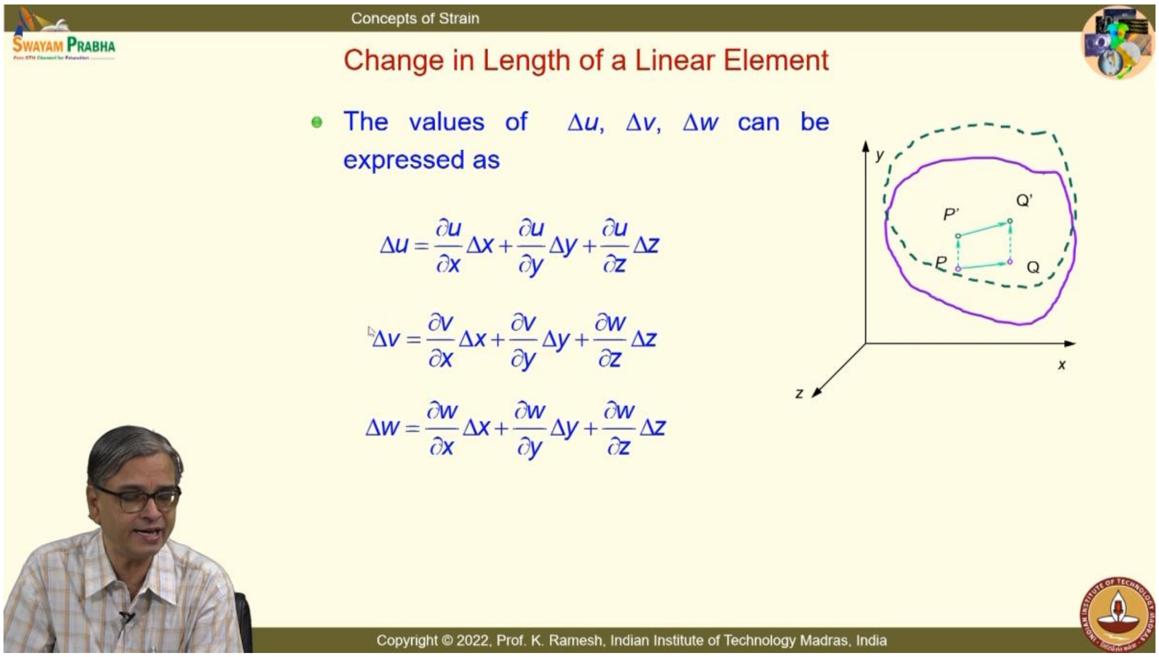

So, I need to have Δu , Δv and Δw. And we are looking at a three-dimensional space. So, *u* is a function of *x*, *y*, *z*, *v* is a function of *x*, *y*, *z*, *w* is also a function of *x*, *y*, *z*. So, when I have Δu , I can write from Taylor's approximation as

$$\Delta u = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

On similar lines, I can also write Δv as

$$\Delta v = \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z$$

And you have Δw as

$$\Delta w = \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z$$

For this, you have a mathematical background, no difficulty at all. If you write one expression, even if you do not have time to write the other expression now, you can fill it up later.

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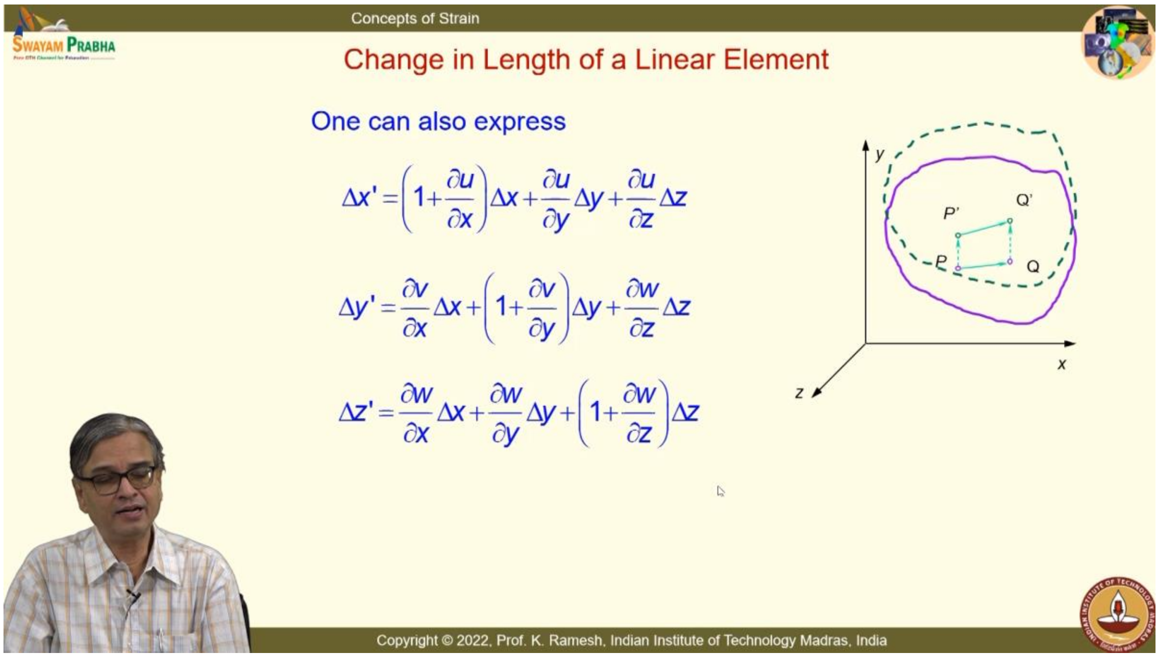

And one can also express Δx as become $\Delta x + \Delta u$. So, when I know what is Δu, I can write

$$\Delta x' = \left(1+\frac{\partial u}{\partial x}\right)\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

$$\Delta y' = \frac{\partial v}{\partial x}\Delta x + \left(1+\frac{\partial v}{\partial y}\right)\Delta y + \frac{\partial w}{\partial z}\Delta z$$

$$\Delta z' = \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \left(1+\frac{\partial w}{\partial z}\right)\Delta z$$

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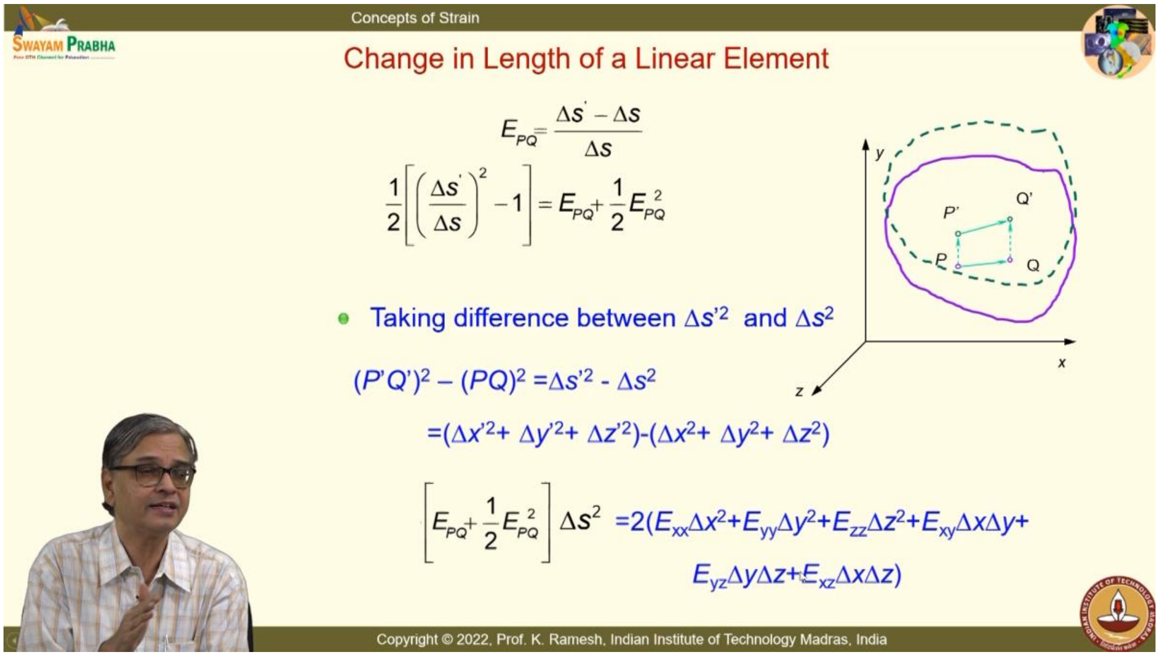

See, our interest is only to find out a quantity like this, when you are talking about P', Q', ok, we are only looking at what is the strain in the direction PQ. I have to find out what is $\frac{\Delta s' - \Delta s}{\Delta s}$. That is the definition of strain, change in length divided by the original length. But instead of computing it from this, it is done by using the squares,

$$\frac{1}{2}\left[\left(\frac{\Delta s'}{\Delta s}\right)^2 - 1\right]$$

If you look at, if you expand this definition and substitute and do the simplification, you can write this in terms

$$E_{PQ} + \frac{1}{2}E_{PQ}^2$$

Is the idea clear? My interest is to find out $\Delta s' - \Delta s$, but I will compute $(\Delta s')^2 - (\Delta s)^2$. We have defined what is Δx, we have defined what is $\Delta x'$. So, I can find out what is the difference. So, the taking the difference between $(\Delta s')^2 - (\Delta s)^2$, I get this as $(P'Q')^2 - (PQ)^2$

$$=(\Delta x'^2+\Delta y'^2+\Delta z'^2)-(\Delta x^2+\Delta y^2+\Delta z^2)$$

$$\left[E_{PQ}+\frac{1}{2}E_{PQ}^2\right]\Delta s^2=2(E_{xx}\Delta x^2+E_{yy}\Delta y^2+E_{zz}\Delta z^2+E_{xy}\Delta x\Delta y+E_{yz}\Delta y\Delta z+E_{xz}\Delta x\Delta z)$$

where the quantities E_{xx}, E_{yy}, which we will see later are the finite strain components. And on the left-hand side, you have this. When I have an infinitesimal strain, if E_{PQ} is small, $\left(E_{PQ}\right)^2$ is small, much smaller than this.

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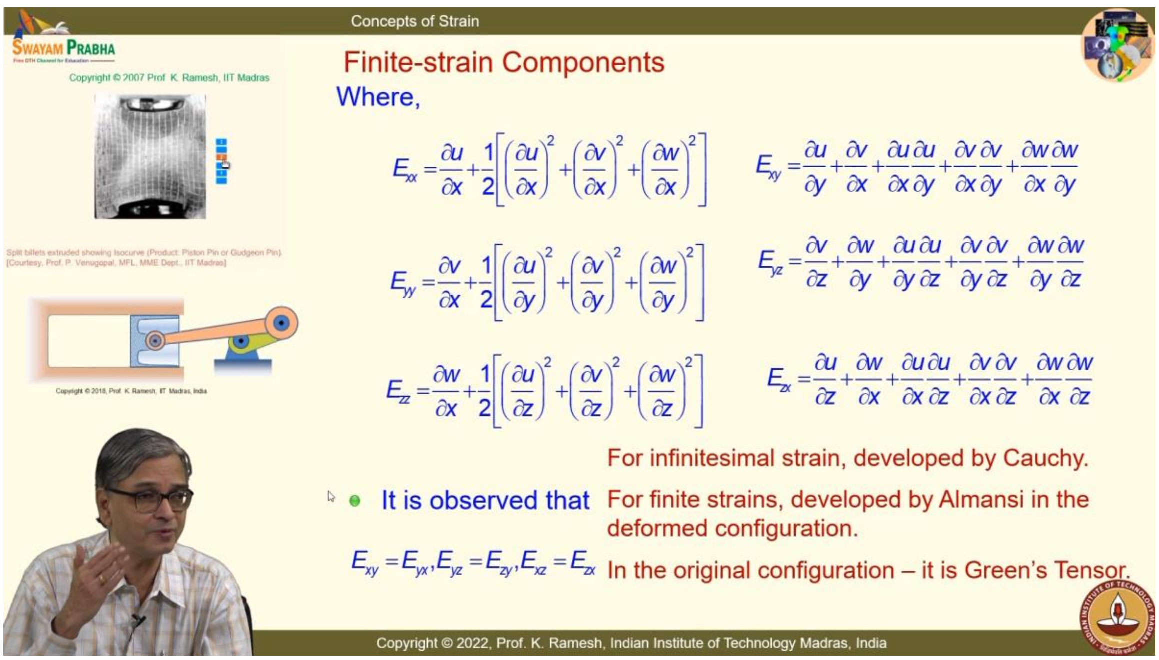

So, we will knock off this term and we will also knock off the higher order terms on the right-hand side. That is what you have done in infinitesimal strain, fine. And we have already seen what is the expression for $\Delta x'$. When I substitute this and simplify, I can get these expressions in this form.

$$E_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right] \qquad E_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

$$E_{yy}=\frac{\partial v}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \qquad E_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}+\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}+\frac{\partial v}{\partial y}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial w}{\partial z}$$

$$E_{zz}=\frac{\partial w}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right] \qquad E_{zx}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial z}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial z}$$

Do you find any similarity between these expressions and your infinitesimal strain components? Suppose I look at E_{xx},

$$E_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]$$

And you have this repeated cyclically. And when I go to the shear component E_{xy}, I have this as

$$E_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

This is identical to what you have seen in your strength of material infinitesimal strain. This we have seen this as shear strain, ok.

Like what we have seen in strength of materials, you have equality of cross strains also available. E_{xy} becomes E_{yx} and then E_{yz} becomes Ez_y, E_{xz} become E_{zx}. And multiple people have developed these kind of expressions in various coordinate systems. So, for infinitesimal strain itself Cauchy has developed it. And for finite strains, it was developed by Almansi in the deformed configuration.

In the original configuration, it is Green's tensor. There are multiple names. Fine, that if you get into that literature, you will get idea of it by, but my interest is to tell you, you know, we have looked at the simple rubber. Suppose you imagine that this is the original length, without any difficulty I can double the length, without any difficulty I can triple the length, nothing happens to it. When I release the hand, it comes back to its elastic condition. You cannot have analysis of this unless you have finite strain, ok, in the case of a rubber. And we have also seen how a Gudgeon pin is formed. So, in metal forming, it will be of use and people also develop incremental plasticity theory and so on and so forth. In metal forming, even the level of strain is much less compared to what you see in the case of a rubber. And we have already seen how a Gudgeon pin is formed.

And I said that this is used in the case of IC engine. This is the pin where you have. And initially you have drawn a square lines, horizontal and vertical forming a square. And when you deform, you could see very clearly with your naked eye, you are able to see the deformed lines. So, these are all very high strains. I said strain is of the order of 20 to 22 %.

Whereas, in the case of infinitesimal strain, I said you should limit within 0.1%. If you say what is 0.1%, it is like 1 millimeter extension of a distance of, if I have original length is 1000 mm, if it extends by 1 mm, you call that as 0.1%. So, you talk of very small strain in components that are working. You have a bearing, you have a shaft which is embedded into that, you need to have this work comfortably during the service, you cannot have large deformation.

You can also argue that they were behaving almost rigid, but they do deform. So, that was comfortable to analyze with infinitesimal strain. But when I come to a problem like this, I need to have finite strain quantities. And the literature says that these are not approximations, these are exact expressions for finite strain, that is what the literature says. And you know bioengineering is becoming order of the day. I have shown you the rubber like recoverable elastic strain and bioengineering is very very important, that is the many IITs are also starting a medical school.

You have to analyze what happens in the case of a heart. See if you go to human beings right from your blood, it is not Newtonian mechanics, blood is a non-Newtonian fluid. So, our body is much more complex than what we analyze as engineers fine. So, when I go to bioengineering, people work with very soft material, people have developed. This is again from photoelasticity, you have soft material which is mimicking the human skin.

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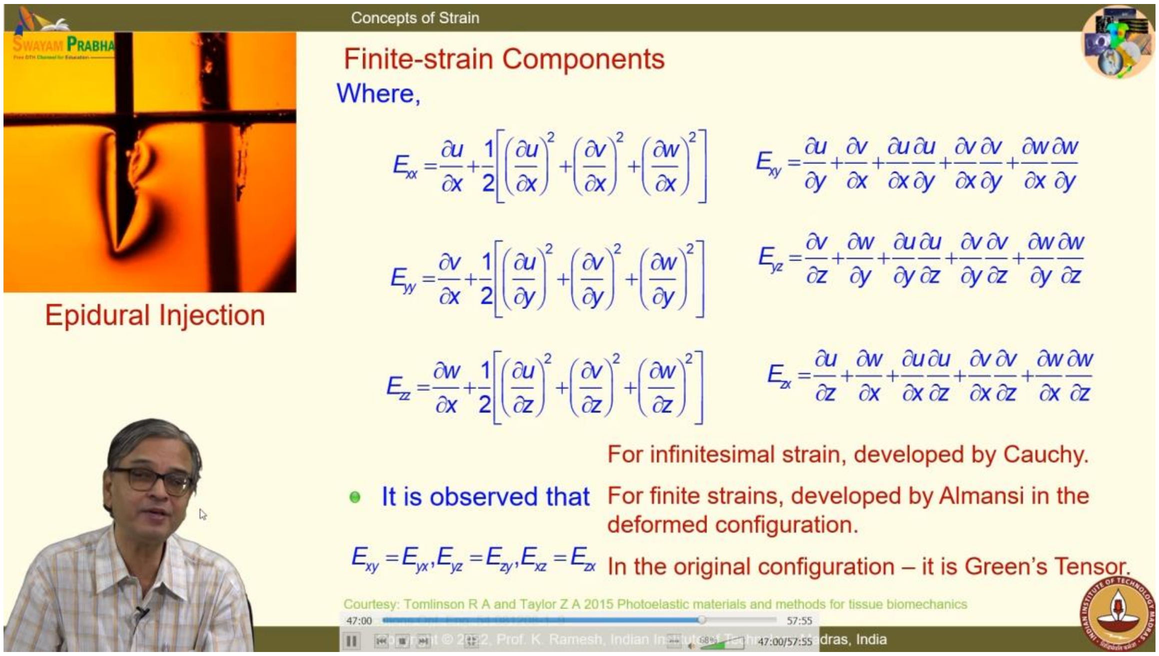

And I am not sure how many of you have heard of, if they have to perform an operation, they put an epidural injection. It is not like your normal injection where you take your antibiotics. It is a very long injection that they have to penetrate to your spine and then they will put it. And here the question is what should be the shape of the tip? And as you penetrate, you know the advanced studies also show the resistance is also different. And people have developed, you have I do not know whether you know there is a new school of thinking, you have what is known as haptics, where they develop the force resistance.

So, you have simulation modules that are developed to train the doctors. If they do not put the injection properly, the person can become paralytic for the rest of the life. And here you deal with soft material, large deformation. So, current problems require your exposure to finite strain. You cannot stay away from it. Even though in this course, we will confine ourselves to infinitesimal strain, that is good enough for you to handle the next level course on machine design.

If you look at and step out of IIT and if you want to do any engineering, there is very great scope in medical diagnostics. And people want to apply more and more mechanics to understand the heart, understand the kidneys. So, all of them are soft material and they have much more complex problem than what you can think of.

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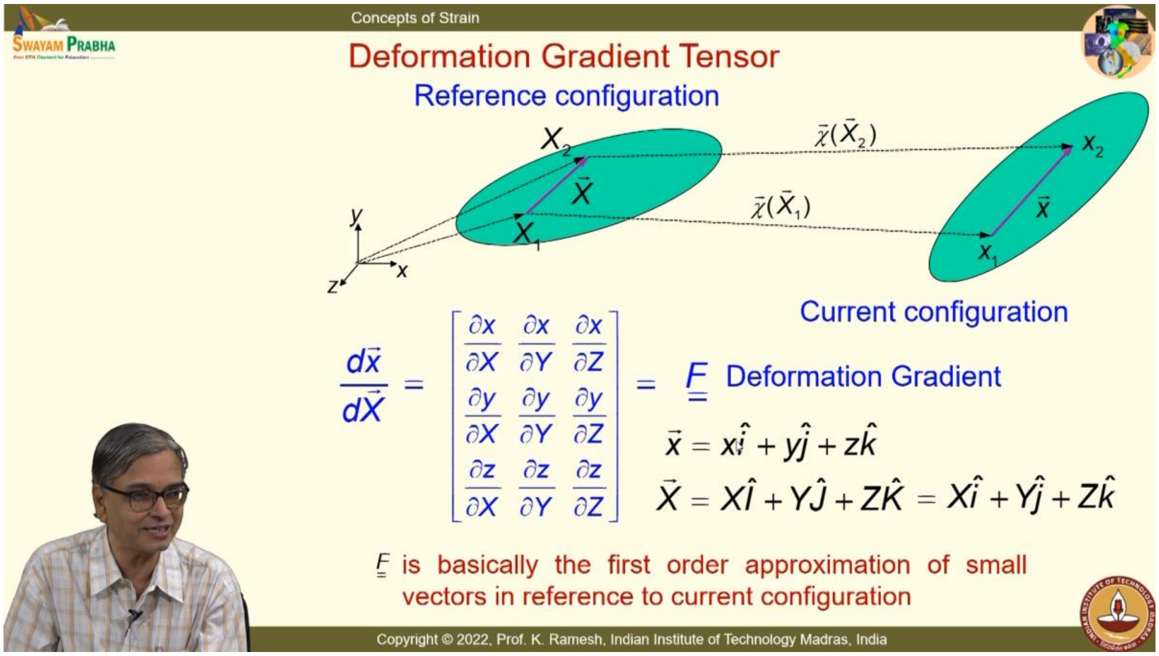

And you know I will also give you a flavor from continuum mechanics literature, how do you label, how do they label things. They you have, we have seen displacement, there it is a deformation gradient tensor. And I have a vector, this is material coordinates. So, I have this as X and you also have the vector notation is given. And this gets transformed to the current configuration. So, they have, there is a reference configuration and there is a current configuration that is given by the symbol C and you have all small letters used for this current configuration. And you have a displacement field and you have what is known as dx, this is a vector field dx dx divided by dX.

And you call this matrix as a deformation gradient tensor. And the tensorial quantities are illustrated with two horizontal bars below the quantity that says it is a tensor of rank 2. See, you recall when I developed concept of stress vector, it was so inconvenient to write the vectorial notation. I said you please understand whenever I have T_n, if there are no subscripts put you understand that as a vector, it is easy for you to write. But the moment you step into continuum mechanics literature, it is flooded with all these bars and you also have these lines below to indicate that this is a tensor and this is a vector, here this is a vector and this is a unit vector all those symbolism is put, fine.

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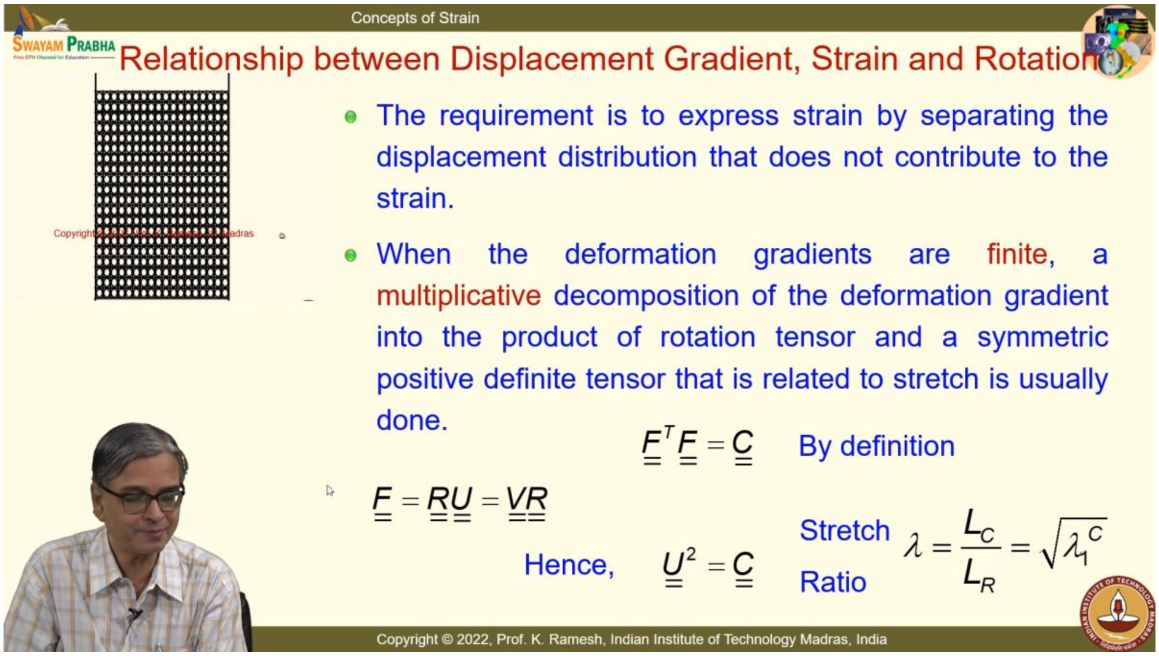

And what you have is the requirement is to segregate the rotation and deformation. And what people have found is you can split the deformation gradient when they are finite as a multiplicative decomposition of a rotation and a stretch, ok. This is how the literature goes. In the case of infinitesimal strain, we saw strain matrix strain tensor plus rotation tensor gives you displacement matrix, fine. Here you write the deformation gradient tensor either as a product of R into U, R is a rotation matrix. And when this is on the right side, or left side they call it as right-side tensor, left side tensor all those symbolism that they use will not get into them, but at least you understand those words are used.

And you have by definition C is given as F^TF and if I write F^TF and F is defined as RU, I can relate what is the relationship between U and C, I get U^2 equal to C. And then if I have, you know I said when you have strain looked and your principals strain direction, you see that as a stretch, ok, only axial elongation. So, I can also define the deformation in terms of what is known as a stretch ratio, where L_C is the length in the current configuration and L_R is the length in the reference configuration that is the language that is used and this ratio is called stretch, ok. And this stretch can be related to Eigen value of this matrix, this tensor C that is what you have this is λ_1^C. So, there are interrelated quantities and you can also imagine suppose I have $L_C = L_R$, I get the stretch ratio is 1 when there is no deformation, which is not convenient for you to write the constitutive relations, you want to have this goes to 0 that is why strain concept was developed from a mathematical perspective.

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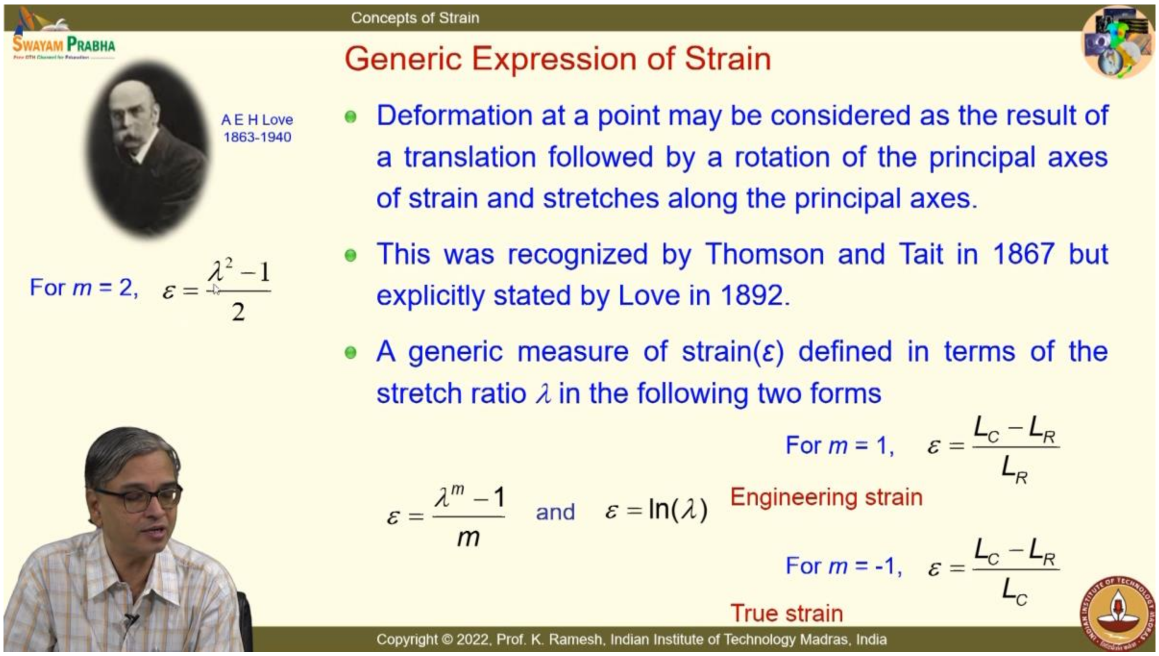

And you also have a generic expression of strain and deformation at a point may be considered as a result of a translation followed by a rotation of the principal axis of strain and stretches along the principal axis. See, when we took a state of stress at a point, I said you can have multiple avatars of state of stress, it can be expressed in different ways. Similar thing happens in strain also, strain also can be expressed as a matrix with all non-zero elements. If I refer it with respect to principal direction, I will have only diagonal element, I will have 0. When I express this as diagonal elements, you can recognize that these are nothing but stretches, it could be stretches or it could be contraction either of the two, fine.

And this was recognized by Thomson and Tait in 1867, explicitly stated by Love, he was one of the pioneer in writing the first book on theory of elasticity, but the symbolism is very difficult to decipher in the modern notation. And there is also a generic measure of strain defined in terms of the stress ratio in the following two forms basically, but this could be expressed in a much simpler way for different values of *m*. So, you have strain is defined as a $\frac{\lambda^m - 1}{m}$ as well as $\varepsilon = \ln(\lambda)$. See this is a natural logarithm, this is all used in plasticity theory, incremental plasticity or people deal with soft materials, they express strain like this. From our course point of view, if I put *m* equal to 1, I get this as $\frac{L_C - L_R}{L_R}$, this is nothing but changing length divided by original length.

This is what we defined as strain when we started strength of materials, what is the definition of normal strain. And there is also, which is also known as engineering strain. There is also another form which we will see when we look at stress strain relation, I can also have *m* is -1, when I put -1, I have this as $\frac{L_C - L_R}{L_C}$. That means, this is referred with respect to the current configuration. And when I do that, there is a specific name, you know people want to coin new names, they call this as true strain as if the other one is untrue, it is not like that.

Fine, we will see stress strain relations, we will talk about strain hardening and we will see how the graph changes when I use this expression. These two we will be using in strength of materials as well. And it is also interesting to see, when I have *m* equal to 2, you have this $\varepsilon = \frac{\lambda^2 - 1}{2}$, this is nothing but your finite strain. So, my idea is to give you a bird's eye view of what is the current type of thinking in advanced literature. It is necessary, you may not have a background to fully understand or develop it mathematically, but you should know such quantities do exist.

So, this brings a curtain riser on what we have learnt on concepts related to strain. Once you have understood state of stress at a point, state of strain at a point is easy to grasp, appreciate all these quantities. Like we have invariance in strain stress tensor, we also have invariance in strain tensor. You should also know how to use the strain tensor invariance properly. And we have also seen definition of strain in Cartesian coordinates as well as in polar coordinates. And I said in polar coordinates, how to remember you can derive from appearance of Cartesian components, but certain components you need to have, understand how it is originated. Thank you.