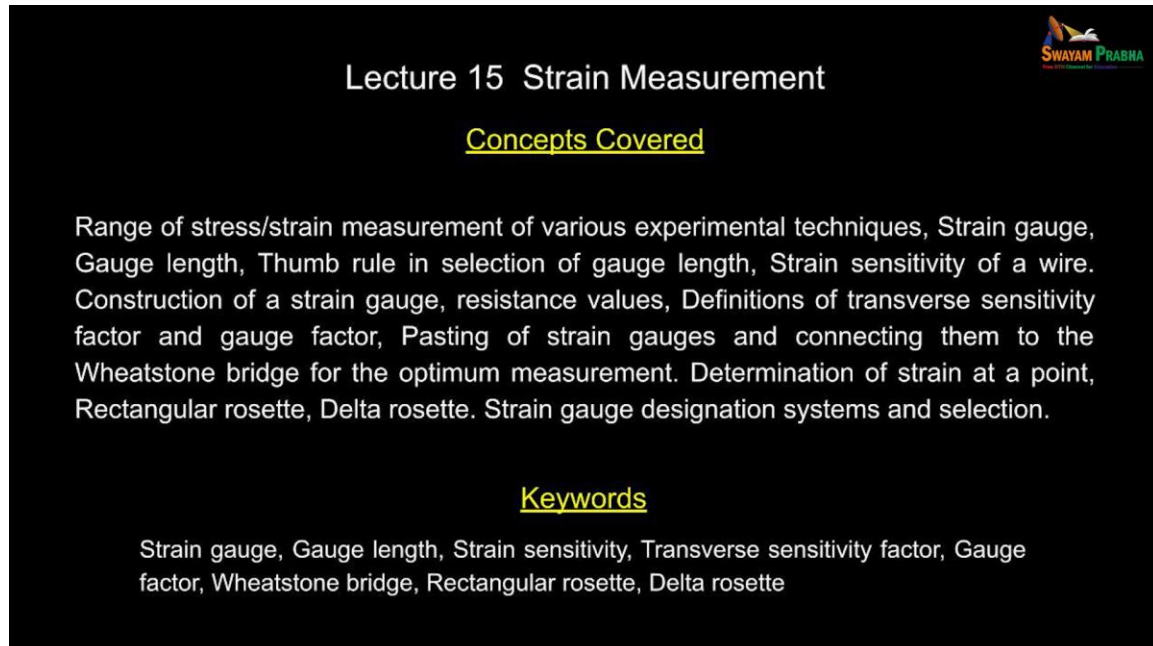



**Strength of Materials**  
**Prof. K. Ramesh**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Lecture - 15**  
**Strain Measurement**

(Refer Slide Time: 00:20)





**Lecture 15 Strain Measurement**

Concepts Covered

Range of stress/strain measurement of various experimental techniques, Strain gauge, Gauge length, Thumb rule in selection of gauge length, Strain sensitivity of a wire. Construction of a strain gauge, resistance values, Definitions of transverse sensitivity factor and gauge factor, Pasting of strain gauges and connecting them to the Wheatstone bridge for the optimum measurement. Determination of strain at a point, Rectangular rosette, Delta rosette. Strain gauge designation systems and selection.

Keywords

Strain gauge, Gauge length, Strain sensitivity, Transverse sensitivity factor, Gauge factor, Wheatstone bridge, Rectangular rosette, Delta rosette

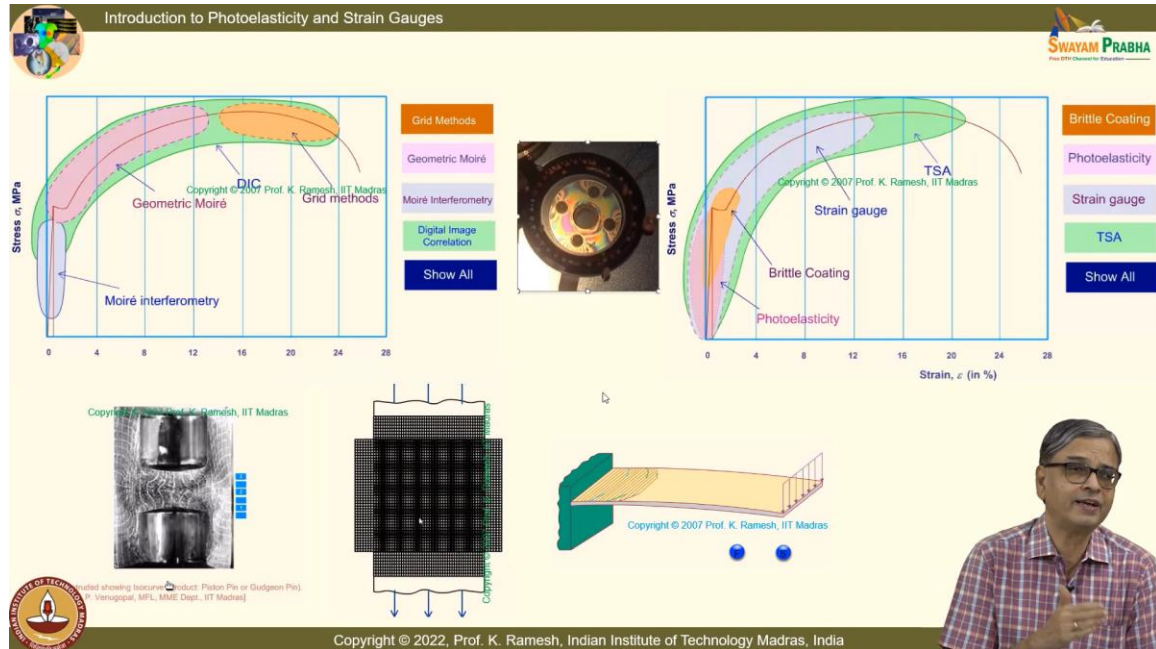
See, in the last class, we have learnt what is strain at a point. When you say strain at a point, you recognize that you have to refer to state of strain at a point. And when you look at in two dimensions, you have three components, fine? And we have also looked at, when we talk about infinitesimal strain, the quantity was very very small. We are talking about in terms of micro strain and we said it is  $10^{-6}$  mm/mm. It is a very small quantity. So, you should recognize, when we want to make strain measurement, you should be sensitive that we are going to measure very very small quantity.

And you know, you have used a scale for the measuring length, you have also used a vernier caliper, you have also used a screw gauge; you must have done it. And scale, you know, you can measure 0.5 mm reasonably well. When you go to a vernier caliper, you can go down to 0.01 mm. When you go to screw gauge, you can go down to 0.001 mm. But you also find the range with which these measurement methodologies can be used, they also keep reducing. So, in any measurement system, you have a resolution.

If it is of very high resolution, the range is also limited. Is the idea clear? And you have to

be sensitized that we are attempting to measure the quantity *strain* which is extremely small.

(Refer Slide Time: 02:17)



And you know, in the Gudgeon pin, we have put square grid and then we have been able to see, after deformation, beautifully the deformed lines; and that comes under grid methods. Then you go to geometric Moiré, what they have done is, if I view the square grid with a master grating, that is what you have here, you have the master grating here. I have these black and white fringe patterns are seen which is by mechanical interference, which has the ability to magnify the deformation.

And the fineness of the technique depends on how many lines per millimeter that you draw. So, you have variation. I have a geometric Moiré that is what is shown here. And when you say geometric Moiré, this graph brings out, on the  $x$ -axis, the level of strain. The idea is, like in your measurement scenario, when you use a scale, when you use a vernier caliper, when you use a screw gauge, your resolution keeps improving and the range is also getting smaller.

So, when we go for any experimental measurement, they are applicable only for a particular range. Grid is a very crude way of finding out the strain. When you have very large strain, the grid is capable of showing good distortion. So, you are in a position to have an idea of what way the body is strained, ok? The next step is, you go to geometric Moiré. I view the square grid with another square grid which is called a master grating.

Because of mechanical interference of these two grids, you have improved the accuracy that comes under the category of geometric Moiré. So, this can operate somewhere around 2% strain to somewhere around 13% or 14% strain. And theoretically, you can have up to 2000 lines per millimeter. It is very very fine. And that comes under the category of Moiré interferometry.

And this Moiré interferometry can start from something like 0.05% strain and then it can go up to just before yielding. And you have a very new technique which is called digital image correlation which works from near yielding to near breakage of the specimen for a very large range. And we have also looked at photoelastic coatings. And in the reflection scenario, you have the coating bonded onto a specimen.

So, the strain from the prototype or the specimen is transferred to the birefringent coating. So, it is strain induced and you get beautiful fringe patterns. So, you have photoelasticity which is in this range, complete elastic range from almost close to zero strain till yielding. Then you have brittle coating. We have said that we will spray a brittle material. And you know it is very interesting. It is a brittle material. So, when the normal stress reaches a critical value, you have cracks formed. So, in this technique, cracks are information. Normally you are worried about the crack developed in any structure. Here, crack is used as a measurement tool to find out what is the level of strain introduced. And it is used in conjunction with strain gauges. That is what we are going to discuss in detail. So, the idea is you can minimize the number of strain gauges if you know what is the direction of the principal strain at that point of interest. So, the brittle coating requires some level of strain. It cannot work below  $500 \mu\epsilon$ . From  $500 \mu\epsilon$ , little after yielding it can work. See, I have shown the stress-strain curve for mild steel just for illustration. The focus here is, when I go for measurement, there are multiple measurement techniques. Many of these techniques operate in a particular range.

You should know what is the range of strain that it can use. And the advantage of strain gauges, it can work from almost close to zero strain to somewhere around 12% to 13% of strain. And you also have another very interesting technique called thermoelastic stress analysis, which works from zero strain to about 20% strain. It is a very very expensive technique. The physical principle is when the material is stressed, the temperature at the point changes.

So, you need infrared cameras to find out the temperature. It is a very expensive technique. So, this gives you an idea. You have multiple techniques. Many of them directly measure displacement.

From the displacement, you find the derivatives and find out the strain. On the other

hand, the strain gauge has an advantage that you have an electrical system that directly gives you the strain quantity.

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Introduction to Photoelasticity and Strain Gauges

GLOBAL STRESS ANALYSIS

Strain Gauges 8

SWAYAM PRABHA

### Gauge Length – Definition

- The gauge length of a strain gauge is the active or strain-sensitive length of the grid.
- The gauge measures the axial strain in the direction of the gauge length.
- The end-loops and solder tabs are considered insensitive to strain because of their relatively large cross-sectional area and low electrical resistance.

Gauge length ranges from 0.2 mm to 100 mm.

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And many times, you know, people say strain gauge gives me strain, which is a misnomer. See, what you will have to keep in mind is, I have a strain gauge which is blown-up here. I have what is known as a gauge length.

I want you to draw a sketch of this. And you have tabs where you connect it to lead wires. These are connected to the equipment for measurement. A strain gauge can give only a component of strain along the gauge length. So, that is what is summarized here.

The gauge measures the axial strain in the direction of the gauge length. Do not say strain gauge gives you strain. It gives you only a component of strain and it can basically measure only axial strain, ok? It has a finite size. It has a finite size and you have a grid. And you have the end loops and solder tabs are considered insensitive mainly because I make them big.

You know from your simple high school physics that when the area of cross-section is larger and larger, the resistivity is small, ok? So, you have a strain gauge where this element, we will also see in a short while how this is constructed. And when you say strain at a point, you may think that I want to measure at a point. We have gauge length that ranges from 0.2 mm to 100 mm.

So, you cannot have a point. The minimum least length is 0.2 mm. But many of the common strain gauges readily available have a gauge length of 3 mm. If I go for 0.2 mm, it is going to be very expensive and even the measurement technique has to be very sophisticated.

So, the common strain gauge's gauge lengths are in the range of 3 mm. And you use 100 mm length on heterogeneous materials because you would like to have an average of the strain over a length. So, you have to select what is the gauge length that is appropriate for your given application. And strain gauge selection is also a very important issue. And I want to caution, strain gauge is a widely used and abused technique.

People abuse it because they do not have a formal introduction to what a strain gauge is, what are the sensitivities because the electrical principle is very simple and straightforward. When you stretch a conductor, whatever the stress introduced influences the change in the resistance. But the resistance change also can occur because of change in temperature. So, in all strain gauge measurements, you must account and segregate any effect due to temperature.

(Refer Slide Time: 11:42)

Introduction to Photoelasticity and Strain Gauges

Strain Gauges

SWAYAM PRABHA

Error in measurement ....contd

Strain

Deviation

peak strain

Average strain

$\epsilon_a < \epsilon_p$  - Not desirable.

Failure of the component will be predicted wrongly.

Axial distance

Representative strain distribution in the vicinity of a stress concentration

★ Demonstrates the error in strain indicated by a gauge which is too long with respect to the zone of peak strain.

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And when I say I have a strain gauge which has a finite length and if I paste it on an area where the strain varies in a fashion like this, I have a peak strain here. Because I have a gauge length, it would only measure an average strain over this length. The average strain would be like this. So, if my average strain is smaller than peak strain, then this is not good selection of the gauge length for the given application. Is the idea clear? So, you

will underestimate the actual strain coming on the prototype at that point.

See, you have looked at photoelasticity. When I had a model made of photoelastic material, I get information on all points in the structure. Such techniques are called whole-field techniques. See, you need to know what happens in the neighborhood point. So, one of the challenges in strain gauge is, though it directly gives you component of strain, it is a point-by-point technique. You can get the information only where you have pasted the strain gauge.

And when you paste the strain gauge, you should have selected the gauge length appropriate to the problem under consideration. If the stress state is uniform, no issue. If I want to paste it on a cantilever where the stress is varying linearly as a function of distance, average will equate to what is the strain at the point of interest. But when I have a plate with a hole, that is what we are going to see in the next slide.

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Introduction to Photoelasticity and Strain Gauges

MENTAL STRESS ANALYSIS

Strain Gauges 10

SWAYAM PRABHA

### Thumb rule in selection of gauge length

- As a rule of thumb, when practicable, the gage length should be no greater than 0.1 times the radius of a hole, fillet, or notch, or the corresponding dimension of any other stress raiser at which the strain measurement is to be made.

Crack

Gauge Location

Gauge Location

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So, what is the thumb rule in selection of gauge length? Suppose, I want to put a strain gauge on the fillet, that is what is shown here. See, I have a tensile root fillet where you have also got a crack developed in service. And if you want to see in the compressive fillet, I have a fillet radius. And you will study in your machine elements course, if you want to reduce the stress concentration, one of the ways you try to reduce the concentration is by providing a generous fillet wherever possible. So, the idea is, you should have the gauge length not greater than 0.1 times the radius of a hole or the fillet. These are all thumb rules, you know, you can only satisfy to the extent possible.

See, other famous problem is your plate with a hole where you have stress concentration developed. And if I want to find out, these are all the places where you have maximum stress. And if I want to do this, I should have a selection of the gauge length depending on the size of the hole. And ideally, I should bond it on the inner surface, on the circumferential direction, then make a measurement. Only then that measurement will give you the peak stress or strain correctly. So, gauge length is a very important factor that you will have to select.

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Introduction to Photoelasticity and Strain Gauges

MENTAL STRESS ANALYSIS

Strain Gauges 03

SWAYAM PRABHA

### Strain Sensitivity of a Wire

- Resistance of a conductor can be written as

$$R = \frac{\rho L}{A} \quad \longrightarrow (1)$$

- Where  $\rho$  is specific resistance,  $L$  is length of the conductor and  $A$  is the cross sectional area of the conductor.
- Differentiating Eq. (1) and dividing it by  $R$  gives

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \longrightarrow (2)$$

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And you know, if you look at the physics, it is only high school physics, there is nothing great in this. And you all know the resistance of a conductor is  $R = \frac{\rho L}{A}$ . Now, I take simple wire, I want to find out how it is sensitive to the strain, fine? Now, I differentiate this and also divide by  $R$ , I get an expression

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

And you all know; strain is nothing but change in length divided by the original length. Why do you have to study the strain sensitivity of a wire? We have to understand, why do you make a strain gauge as a grid? Why you not have a simple speck of material to give you strain? That is one issue. Another issue is, see, when I want to paste a strain gauge in an object, *a priori* I do not know what is the level of strain. Is the idea clear? It can go from elastic region to plastic region. When I want to interpret the data from the strain gauge, whatever the calibration constant that I use, it should be same for elastic as well as

plastic ranges, otherwise I cannot make a measurement.

So, when I take a conductor and look at it as a function of the specific resistance as well as the geometry, I get a knowledge what should be my calibration constant, in what way I should tweak it so that I do not have a difficulty to move from elastic to plastic strain, fine?

(Refer Slide Time: 16:58)

The slide is titled "Strain Sensitivity of a Wire" and is part of a presentation on "Introduction to Photoelasticity and Strain Gauges". It contains the following text and equations:

- If the wire diameter is  $D$ , the change in area can be calculated as
$$A = \frac{\pi}{4} D^2$$

$$\frac{dA}{A} = 2 \frac{dD}{D}$$
- From the definition of Poisson's ratio one can write,
$$\frac{dD}{D} = -\nu \frac{dL}{L}$$

Hence,

$$\frac{dA}{A} = -2\nu \frac{dL}{L}$$

The slide also features a video inset of Prof. K. Ramesh on the right side, a navigation bar at the bottom, and logos for IIT Madras and SWAYAM PRABHA.

Now, we will simplify it and we will also have to talk about what we have looked at in the simple rubber experiment. You know, we have looked at when I stretch the rubber, there was a contraction in the lateral direction. This happens at a fixed ratio and that ratio is credited to Poisson. We will use that, ok? So, we have the cross-section, we have taken a circular cross-section.

So, the area is  $\frac{\pi}{4} D^2$ . So,  $\frac{dA}{A}$ , I can write it as  $2 \frac{dD}{D}$ . Now, I replace it in terms of the axial strain, which we have looked at in the simple experiment and this ratio is known as Poisson's ratio. So, I can replace what is  $\frac{dD}{D}$  in my expression in terms of this strain  $\frac{dL}{L}$ . These are all high school physics, you have learnt it, you know all this differentiation.

I would like you to write down these expressions. These are very simple to use. And there is also a limiting value of Poisson's ratio. We will see that in the next class. It will go to 0.5 when the material becomes plastic, ok?



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The slide is titled "Strain Sensitivity of a Wire" and is part of a video lecture on "Introduction to Photoelasticity and Strain Gauges". It includes the following content:

- Strain sensitivity of the conductor  $S_A$  is defined as
- Equation (4): 
$$S_A = \frac{dR/R}{dL/L} \rightarrow (4)$$
- In terms of specific resistance and Poisson's ratio of the strain gauge material.
- Equation (5): 
$$S_A = \frac{d\rho/\rho}{dL/L} + 1 + 2\nu \rightarrow (5)$$
- Strain sensitivity approaches 2 when the gauge experiences plastic deformation.

The slide also features a navigation bar at the bottom with icons for back, forward, and search, and a small portrait of Prof. K. Ramesh on the right side.

So, when I substitute this in the expression, I get this as the strain sensitivity  $S_A$  is defined as

$$S_A = \frac{dR/R}{dL/L}.$$

When I substitute for  $dR/R$  in terms of the specific resistance and the other quantities which we have evaluated, I get strain sensitivity of the wire. It is not the strain gauge. Strain gauge is made out of a grid. Strain sensitivity of a wire equal to

$$S_A = \frac{d\rho/\rho}{dL/L} + 1 + 2\nu$$

So, when the material becomes plastic, then value of  $\nu$  goes to 0.5 and the first term goes to zero. So, this goes very close to 2. So, if I have the calibration constant when I have to find out, from the resistance change, how to get the strain values. If I have that closer to 2, then I can comfortably go from elastic range to plastic range. So, that knowledge you get from this and people also have developed. See, I said it is very difficult to get a speck of material for me to make the strain measurement.

If I not get speck of material, at least I should have the shortest length possible. What is the way that you can have the shortest length to have a highest resistance? One way is you develop alloys which have high specific resistance. Do not use copper. They use copper-nickel alloys which are called *Constantan* and *Advance* which have very high

specific resistance.

That is one way of doing it. What is the other way of doing it? You know, resistance is  $\frac{\rho L}{A}$ . So, the other way of doing it is, reduce the cross-section to the least possible. With all those calculations, people have found even if I want to have 100  $\Omega$  as the base, as a resistance, I need about 100 mm length of the wire to do it. Because if I have higher and higher resistances, any small change I can make the measurement. See, one of the challenges when they discovered strain gauge was how to measure micro volts, because I am going to have micro strain.

So, the electrical change also will be in very small quantity. So, people needed amplification circuits for them to have measurement of these very small voltages. You have to recognize, that is  $10^{-6}$ . I do not know how many of you have gone with your mother to go and buy gold.

Has anyone gone? We have girls here. At least boys have not gone. At least girls should have gone. Have you noticed? Buying gold is one aspect. Have you noticed how he weighs it in the shop? He has a balance. He closes that balance. He does not even want an air current and you are measuring gold in terms of grams. Is the idea clear? And here you are going to make a measurement which is  $10^{-6}$ . We have not measured anything like that in our life earlier. So, you should be very sensitive to that, ok?

How many of you are from north India here? Raise your hands. You all use your heaters in your winter. You have electrical coil. Am I right or not? Have you seen it? At least you would have seen it and it generates lot of heat and provides you warmth. Now, I have a very small tiny strain gauge. I am going to pass current through this. This also acts like a micro heater,  $I^2R$  loss is there.

Because I am going to measure  $10^{-6}$ , you cannot feel the heat. In a domestic heater, it is 1000 watts. This will be very small wattage, ok? 0.04, 0.06 W or something like that, but that heat generation also affects your electrical response. So, you have to be very sensitive that you are measuring extremely small quantity. Never forget that.

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Introduction to Photoelasticity and Strain Gauges

Strain Gauges 7

## Gauge Construction

....contd

- Obviously one cannot measure strain at a point using a long wire!
- Hence the gauge is formed by folded grids etched on metal foil.

Standard Resistances

120 and 350  $\Omega$

500, 1000 and 3000  $\Omega$  are for special purpose gauges.

Copper Coated Tails  
Grid  
Backing - Encapsulation

Courtesy: Vishay Micro-Measurements

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So, what is a gauge construction? So, you have to make strain measurement using a long wire and you have a photo grid which is made very large. So, they take pains to make the strain gauge grid like this. And these are all done in metal foil. And what you have is, you have a straight-line portion. We have already said that we are going to measure strain along the gauge length, fine? So, I ensure that my design is also such.

I have sensitivity high along the axial direction and wherever I have loops, I have made the loops very thick so that it has comparatively lower resistance. And I have this for tabs and very carefully you make the strain gauge, made of a very thin metal foil. Even though it is made of metal, you cannot go and take it on your hand. You have to take it through a tweezer. I will show the strain gauge towards the end of the class, ok? You have to, otherwise it will become, it will be broken.

It will get broken and when you connect it, it will not show any reading, ok? And it is like a basic resistor. How many of you have played with your electronic circuits? Have you constructed any electronic circuits in your school or for your fun? You go and buy resistance in a shop, electronic shop. You get resistance of all values. Strain gauge is also a resistor, but it is not available in all values.

It is available only in specific values. And I have said even making 120  $\Omega$  is very difficult. Now, you have 350  $\Omega$ , 500  $\Omega$ , 1000  $\Omega$  and 3000  $\Omega$ . Please write down these numbers. Numbers are very important. See, in engineering, certain numbers, certain

appreciation, when I say that calibration constant, what I am looking for should be close to 2, never forget that.

And when you go and look at a strain gauge, the basic one we said that even if I want to have 100  $\Omega$ , I need a long wire and the standard strain gauge, the least resistance that is available is 120  $\Omega$ . And it is a technologically influenced technique. As the technology has evolved and you have finer and finer measurements possible in strain gauge. With the technology right now, you can confidently measure 1  $\mu\epsilon$ . No other strain measurement can give you this accuracy.

But if you want to get this accuracy, you have to select the strain gauge properly. You should be sensitive to what is the heat generation, how the heat is dissipated. Now, we are in a world where we are bombarded with composites because we want to have weight reduction. What is the difference between a composite and a metallic structure? Because I have been talking about heat, can you tell me? It is made of plastic.

You have studied in your physics, thermal conductivity. Am I right? How do you expect a plastic to behave in compared to metal? Metal will dissipate heat fast; plastic is not going to dissipate heat. So, when I going to work with composites, please use a strain gauge which is of 1000  $\Omega$ . Why? What is the loss when a current goes in a conductor?  $I^2R$ . If the voltage is fixed constant and if the resistance is very high, you reduce the current flow.

So, you reduce the heat generation. You reduce the heat generation because you know you are going to work on a material which is not going to dissipate away the heat as efficiently as like a metal. So, many times, many laboratories what they do is, they make the strain measurement with the available strain gauge without paying attention to which material they are working on. So, you have to select, like I said that you have to select the gauge length, you should also select the resistance of the strain gauge.

It is not trivial, it is very very important. Having said this, now I have a grid. Obviously, since you have studied what is state of strain at a point and you have also looked at from a photoelastic experiment, how the strain field is, in the form of fringes. When I am going to have a finite grid like this, it is going to be sensitive to the axial strain, transverse strain as well as shear strain at the point.

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Introduction to Photoelasticity and Strain Gauges

Strain Gauges 16

SWAYAM PRABHA

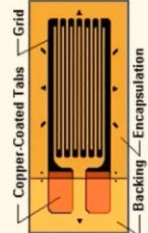
### Transverse Sensitivity Factor, Gauge Factor – Definitions

- In general  $S_s$  is small and can be neglected

$$\frac{dR}{R} = S_a (\varepsilon_a + K_t \varepsilon_t) \quad \rightarrow (2)$$

$$K_t = \frac{S_t}{S_a} \quad K_t \text{ is the transverse sensitivity factor}$$

The response of the gauges subjected to  $\varepsilon_a$ ,  $\varepsilon_t$  and  $\gamma_{at}$  is given by

$$\frac{dR}{R} = S_a \varepsilon_a + S_t \varepsilon_t + S_s \gamma_{at} \quad \rightarrow (1)$$


Courtesy: Vishay Micro-Measurements

- Strain gauge manufacturer supplies a gauge factor  $S_g$  which is experimentally determined for an uniaxial stress field, for that particular batch.

$$\frac{dR}{R} = S_g \varepsilon_a \quad \rightarrow (3)$$

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Let us look at what way we have handled this, ok? So, in this language it is put;  $\varepsilon_a$  is axial strain that means strain along the gauge length;  $\varepsilon_t$  is the strain perpendicular to this, transverse to the gauge length and  $\gamma_{at}$  is the shear strain in the plane of  $a-t$ .

So, I can write this  $dR/R$  with three factors. See, when I was talking about a conductor, I put capital  $A$ . When I am talking about a strain gauge, the axial response I put it as  $S_a$ . So, it is sensitive to  $\varepsilon_a$ , it is sensitive to  $\varepsilon_t$  as well as sensitive to shear strain. But because the way I construct the gauge, I can neglect the effect of shear strain, but I cannot neglect the effect of transverse strain in critical applications, ok? So, what I have is, this expression is simplified to

$$\frac{dR}{R} = S_a (\varepsilon_a + K_t \varepsilon_t)$$

And  $K_t$ , you call that as a transverse sensitivity factor, which is the ratio of  $S_t/S_a$ . That is the ratio of the transverse sensitivity to axial sensitivity. We saw what was there for a simple wire. From the wire, we have graduated to a grid and we have seen grid is unavoidable. And a common gauge length what you have is only 3 mm, that is what is available cheap.

Even the cheap strain gauge will cost you 10 dollars. So, it is very very expensive. It is 10 dollars to 1000 dollars or 3000 dollars, one strain gauge. So, it is a very expensive measurement technique, but very accurate. And what does the manufacturer give you?

Manufacturer generates strain gauges in a batch and selects few of them, conducts a test and gives you what is known as gauge factor  $S_g$ , which is defined as

$$\frac{dR}{R} = S_g \varepsilon_a .$$

So, from your electrical measurement, you will be in a position to get  $dR/R$ . From the manufacturer supplied value, you have the gauge factor  $S_g$ . And in IIT, in your strength of materials lab, you have an experiment using strain gauge to measure the strain in the case of a cantilever and also for a torque measurement in the case of a twisting of a shaft. So, if you recall, you would have the gauge factor around 2. I have already given a reason why it should be around 2. And it is so fortunate the manufacturer simplifies your exercise. He has done a calibration in such a manner by using this expression, you directly get the axial strain.

And if you have to incorporate transverse sensitivity factor, you have methodologies developed. People also use such sophisticated improvements on the basic measurement available as a software.

(Refer Slide Time: 31:46)

Introduction to Photoelasticity and Strain Gauges

TAL STRESS ANALYSIS

Strain Gauges 16

SWAYAM PRABHA

Wheatstone Bridge

$$V_{AB} = \frac{R_1}{R_1 + R_2} V$$

$$V_{AD} = \frac{R_4}{R_3 + R_4} V$$

Two-wire quarter-bridge circuit

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How do I measure this small quantity? This is again you have done it in your high school physics. So, draw the Wheatstone bridge. And we exploit the Wheatstone bridge to the hilt in strain measurement.

See, we have already seen, I want you to draw the sketch, make the sketch. We have to

find out because we are going to get one very important expression that will help us to amplify my signal with minimal effort. I have also said, you have load cells. You know even a vegetable vendor these days an electronic balance. Am I right? How do these electronic balances operate? They all have a strain sensing element, which is basically a transducer, where you have strain gauge pasted on an appropriate spring element.

And when I use it as a transducer, my focus is to measure the weight accurately. So, in order to get the signal properly, I would like to amplify the signal. There is a difference between strain measurement and a transducer application. In a strain measurement scenario, whether the strain is small, my strain gauge should be able to give me the value. In a transducer application, in order to improve the accuracy, I would like to maximize the signal. You will learn from the construction of the Wheatstone bridge, ok? So, I can write

$$V_{AB} = \frac{R_1}{R_1 + R_2} V \quad \text{and} \quad V_{AD} = \frac{R_4}{R_3 + R_4} V$$

(Refer Slide Time: 33:33)

Introduction to Photoelasticity and Strain Gauges

CENTRAL STRESS ANALYSIS

Strain Gauges 1B

SWAYAM PRABHA

### Wheatstone Bridge

....contd

$$E = V_{BD} = V_{AB} - V_{AD} = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V$$

- The bridge is initially balanced ( $E = 0$ ) when  $R_1 R_3 = R_2 R_4$
- For an initially balanced bridge the output voltage  $\Delta E$  for an incremental change in the resistances can be shown to be

$$\Delta E = V \frac{R_1 R_2}{(R_1 + R_2)^2} \left( \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad (1)$$

Valid for strains up to 5% only.

- Opposite arms add and adjacent arms subtract.

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And I get this voltage drop across the points  $DB$  as given as

$$E = V_{BD} = V_{AB} - V_{AD} = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V$$

And when bridge is initially balanced, I have  $R_1 R_3 = R_2 R_4$ . So, one thumb rule is you make these resistances  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  equal, to start with. And you make the null

balance measurement, which is very very accurate. And you have to keep in mind that we are measuring extremely small changes, which we are not accustomed to, fine? So, you have to understand every aspect is important. And when I write this final expression, I have a very interesting relationship, which tells me how I should use the arms of the bridge effectively. The change in voltage is related to  $V \frac{R_1 R_2}{(R_1 + R_2)^2}$ . This is more or less a constant for a configuration, if I have selected the voltage.

I have labeled this as in clockwise direction  $R_1, R_2, R_3, R_4$ . Even this is very important. And when I look at this expression, I get this as  $\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}$ . So, I would use this change in sign appropriately in my design of the spring element. And the way to understand this, when I have labeled this like this, opposite arms add, ok? Because I have  $\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}$ . Adjacent arms  $R_2$  and  $R_4$ , you have their subtraction. So, I have also connected two strain gauges in a Wheatstone bridge and make the measurement. This should have one sign of strain; this should have opposite to that. Only then, it will add. If they are of the same sign, you will get only zero as the answer. I can also have another configuration. I can have all four arms connected as strain gauge. Either you can have a single strain gauge, two strain gauges or only four. You do not have anything with three. You cannot make any measurement with three.

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The slide contains several diagrams illustrating the use of strain gauges in a Wheatstone bridge:

- Top Left:** A cantilever beam of length  $L$  and thickness  $t$  fixed to a wall. A load  $P$  is applied at the free end. Two strain gauges, labeled 1 and 2, are attached to the top and bottom surfaces respectively.
- Top Middle:** A 3D perspective of a rectangular strain gauge element with a central diamond-shaped stress element. The stress element shows normal stress  $\sigma$  and shear stress  $\tau$  on its faces.
- Top Right:** A Wheatstone bridge circuit diagram with four resistors  $R_1, R_2, R_3, R_4$  and a central voltmeter  $V$ . A source voltage  $\Delta E$  is applied across the bridge.
- Middle Left:** A circular stress element with principal stresses  $\sigma_1$  and  $\sigma_2$  acting at  $45^\circ$  to the horizontal. Strain gauges 1, 2, 3, and 4 are oriented along these principal stress directions.
- Middle Right:** A square stress element with normal stress  $\sigma$  and shear stress  $\tau$ . Strain gauges 1, 2, 3, and 4 are oriented along the  $x$  and  $y$  axes.
- Bottom Left:** A square element showing shear strain  $\gamma$  and the corresponding shear stress  $\tau_{xy}$ .
- Bottom Middle:** A circular stress element with principal stresses  $\sigma_1$  and  $\sigma_2$  acting at  $45^\circ$  to the horizontal. Strain gauges 1, 2, 3, and 4 are oriented along these principal stress directions.
- Bottom Right:** A circular stress element with principal stresses  $\sigma_1$  and  $\sigma_2$  acting at  $45^\circ$  to the horizontal. Strain gauges 1, 2, 3, and 4 are oriented along these principal stress directions.

Equation:  $\epsilon_1 = \epsilon_3 = -\epsilon_2 = -\epsilon_4$

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I have a very interesting application. I have a cantilever beam and then I apply the load here and then I put a strain gauge on the top. I put the strain gauge at the bottom. Even though we have not formally studied what is the variation in a beam, I have already shown it schematically. Do you recall what was the strain variation or stress variation? It is maximum at the top fiber and least at the bottom fiber of opposite sign, ok? So, I have pasted one strain gauge on top, one strain gauge at the bottom. When I connect them in the Wheatstone bridge like this, I amplify the measurement of whatever the signal I get, I amplify.

So, my measurement of force becomes more and more accurate. If I suppress noise, amplify signal, my instrumentation system will give me reliable values of the force. That is my interest. Suppose I have a system where I have a cantilever, I have put two strain gauges and I have connected like this. What do you anticipate? Tell me, you have looked at, just now looked at what way the Wheatstone bridge behaves.

And you know I have put strain gauge 1 here and strain gauge 2 here. Both are in the top surface; both are experiencing same signature. You will be coming and telling that this is a very strong beam, nothing happens to the beam is what you will conclude. Whereas, you have done a mistake in your measurement. Because any load I put, I will get only zero. Is the idea clear? See, in a sense, you also learn application of your knowledge of stress and strain in this measurement also.

I have taken a shaft under torsion. I have shown, I have taken a shaft and I have twisted it

anticlockwise. Can you sketch what is the state of stress? Suppose I take a point here, ok, you tell me what way it is twisted. You have in front of you, I put a point in front of me, I put it anticlockwise. Can you sketch the state of strain? It is a pure shear strain. Can you sketch the direction of strain and then check with my diagram? First thing is you should know how to find out what is the load applied on the system.

So, you take a square element, draw the strain acting on that or stress acting on that, whichever way you can do it. Make a mistake, no problem. And I have already told you a strain gauge can measure only axial strain. The question here is how do I use my strain gauge to measure what happens in torsion? What way do I have to align my strain gauge when I want to make the measurement? That is also very important. In strain gauge instrumentation, all these aspects are important. What should be the size of the gauge length? What should be the resistance? And what way I should align my strain gauge? Have you figured out the stress what is happening on the element? I have this as in the negative direction because I am putting it anticlockwise.

I am putting a point in front of me, ok? When I put a point in front of me, it will appear one way. When I put on the other side, it will appear the other way. This is what I am looking at, ok, which you can visualize. Once you have drawn this, you have already studied Mohr's circle. And when we are dealing with an isotropic material, in an isotropic material, whether I have stress state and strain state, they are identical.

Your principal stress direction will coincide with principal strain direction. With that knowledge, can you visualize this as combination of axial strain? A pure shear stress state is introduced. Can you go to Mohr's circle? Can you draw the Mohr's circle for this? Draw the Mohr's circle for this and find out at what angle you have the principal stresses.

It is an application of what we have studied earlier. It is nothing new. Please draw the Mohr's circle. I will draw the Mohr's circle for you now, ok? Before the Mohr's circle, I have drawn this, ok. Let us see the Mohr's circle. I have the Mohr's circle drawn like this.

And I have these two are the points where you have the maximum normal stress. They are the principal stress direction. They are also the principal strain direction in the case of an isotropic material. And this is located at  $45^\circ$ . So, I have it as clockwise.

So, when I rotate clockwise, I have this as maximum stress which is tension, which is also the value of  $\tau$ . So, I have tension. So, that is why your element is stretched like this and compressed the other way. And if you look at from a diagram like this, I can write an equivalent stress state in this fashion. Is the idea clear? That is important for you to recognize how do I use my strain gauge to find out what happens when the object is

undergoing a pure shear stress state.

So, it tells me that I have to align my strain gauge this way. So, when I see the side view, I will put one strain gauge on the top. I will put the other strain gauge; we have put this as 1, 2, 3, 4, ok? This is experiencing, I have this direction, ok? This is, if I say, if I say this as 1, I will say this as 3, whichever way, ok? So, I put this on this surface, I put this on the other side, ok? And this is oriented at  $45^\circ$ . So, similarly I put two other strain gauges that measures the compression. So, I can measure only axial strain, fine? So, we do not give up and you know from stress analysis, what is the stress state there. From your Mohr's circle, you have an understanding it can be visualized as combination of tension and compression. And then when you do this, you also recognize  $\varepsilon_1$  and  $\varepsilon_3$  are positive,  $\varepsilon_2$  and  $\varepsilon_4$  are negative and connect them appropriately in your Wheatstone bridge.

Do not connect it wrongly. I will put this as 1, 2, 3, 4. So, they all add up. So, when I do this, I get four times the signal. So, if my interest is to find out torque, I said that you have to use a torque wrench when you want to tighten the bolt. So, when I want to have a torque wrench, I should have a system to measure the torque and give you that as a reading. So, there are many instances where you have to find out the torque. So, it is a transducer application. Your intention is to find out the torque applied. So, you design a mechanical element, spring element, in which you paste more than one strain gauge and connect them appropriately so that you amplify the signal. So, I can make a torque wrench using this as a strain sensing element and I can amplify my signal.

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The slide is titled "Introduction to Photoelasticity and Strain Gauges" and "Strain Gauges 23". The main heading is "Determination of Strain at a Point". It contains the following bullet points:

- A single strain gauge can give only the strain component along its gauge length.
- In many applications, one wants to measure in-plane strain on a free surface.
- In such a case, three components of strain are to be determined.
- This is done by measuring strain along three directions at the point of interest.

The diagram shows a 2D Cartesian coordinate system with x and y axes. Three strain gauges, labeled A, B, and C, are shown as blue rectangles. Gauge A is oriented at an angle  $\theta_a$  from the x-axis. Gauge B is oriented at an angle  $\theta_b$  from the x-axis. Gauge C is oriented at an angle  $\theta_c$  from the x-axis. The origin is labeled O. The axes are labeled x and y, and a third axis c is also indicated.

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And the moment I say from stress analysis point of view, I want to find out strain at a point. When you say strain at a point, you are looking on a plane surface, ok? That is, you have a free surface on which you want to measure, you have three quantities  $\varepsilon_x$ ,  $\varepsilon_y$  and  $\gamma_{xy}$ .

So, when I want to measure three quantities, I need minimum of three strain gauges. You get my point? I have a strain gauge 1 oriented at  $\theta_a$ , strain gauge 2 oriented at  $\theta_b$ , strain gauge 3 oriented at  $\theta_c$ . I told you, transformation laws are very very important. You cannot afford to forget it. You should be able to write them very comfortably, fine? You find an application in the case of strain gauges. So, I can go and find out what happens.

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Introduction to Photoelasticity and Strain Gauges

Strain Gauges 26

Determination of Strain at a Point ....contd

- Based on strain transformation equations, it is possible to get the components of strain tensor from these measurements.

$$\varepsilon_j = \varepsilon_{xx} \cos^2 \theta_j + \varepsilon_{yy} \sin^2 \theta_j + \gamma_{xy} \sin \theta_j \cos \theta_j$$

- Pre-aligned strain gauges mounted on a single carrier are available from strain gauge manufacturers – these are known as strain gauge rosettes.
- The conversion of strain data to stress on planes normal to the free surface at a point of interest requires knowledge of the elastic constants  $E$  and  $\nu$  and the strain in the three directions.

Transformation matrix:

$$\begin{bmatrix} x' & y' \\ x & y \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

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And thankfully the strain gauge manufacturers put three strain gauges in a backing and supplies you and this is called as a *rosette*. Because aligning it on field is extremely difficult. You have pre-aligned strain gauges.

So, whenever somebody says strain gauge gives you strain, do not say that. It gives you only component of strain. A strain rosette which has three elements in it can give you state of strain in a two-dimensional situation, ok? And you know the reason why I have put this is, you know you should have written this transformation law. This is nothing but  $x'x'$ , ok?  $\varepsilon_j$  what is indicated here is nothing but  $x'x'$ . That is given as

$$\varepsilon_j = \varepsilon_{xx} \cos^2 \theta_j + \varepsilon_{yy} \sin^2 \theta_j + \gamma_{xy} \sin \theta_j \cos \theta_j$$

You have to use this and then extract the value of strain at the point of interest. See, you

have a physical difficulty. I have to have three strain gauges and I have to find out what is strain at a point.

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Introduction to Photoelasticity and Strain Gauges

Strain Gauges 26

SWAYAM PRABHA

### Three element rectangular rosette

$$\varepsilon_j = \varepsilon_{xx} \cos^2 \theta_j + \varepsilon_{yy} \sin^2 \theta_j + \gamma_{xy} \sin \theta_j \cos \theta_j$$

$$\theta_A = 0^\circ; \theta_B = 45^\circ; \theta_C = 90^\circ;$$

$$\varepsilon_A = \varepsilon_{xx} \quad \varepsilon_C = \varepsilon_{yy}$$

$$\varepsilon_B = \varepsilon_{xx} \cos^2 45 + \varepsilon_{yy} \sin^2 45 + \gamma_{xy} \sin 45 \cos 45$$

$$= \frac{1}{2}(\varepsilon_A + \varepsilon_C + \gamma_{xy})$$

$$\varepsilon_{xx} = \varepsilon_A$$

$$\varepsilon_{yy} = \varepsilon_C$$

$$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C$$

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Let us see how they have constructed the rosettes. You have a very famous rosette called a rectangular rosette. I have strain gauges mounted like this. So, you deliberately make an approximation as if these strain gauges give strain at the point  $O$ . So, there are engineering approximations involved in strain measurement. And you may make the strain gauge as small as possible. You also have strain gauge where they are stacked one over the other so that the center of the strain gauge forms at this point.

But you amplify heat generation three times. So, you should have very positive way of taking away the heat. So, your measurement should be sensitive to that. You may not have the luxury. So, you have this. And when I have a strain gauge like this, this is called as a rectangular rosette and you have these angles as fixed as  $0^\circ$ ,  $45^\circ$  and  $90^\circ$ .

You can apply the transformation law and tell me what is  $\varepsilon_A$  in terms of  $\varepsilon_{xx}$ ,  $\varepsilon_{yy}$  and  $\gamma_{xy}$ . So, the idea is to find out  $\varepsilon_{xx}$ ,  $\varepsilon_{yy}$  and  $\gamma_{xy}$  from the measurements of strain along direction  $A$ . When I put the strain gauge as  $A$ , I can also say that strain along direction  $A$ , strain along direction  $B$ , strain along direction  $C$ . And when you simplify, I get  $\varepsilon_{xx}$  identical to  $\varepsilon_A$ ,  $\varepsilon_{yy}$  is identical to  $\varepsilon_C$  because I have aligned it along the reference axis  $x$  and  $y$ .

And  $\gamma_{xy}$  comes as  $2\varepsilon_B - \varepsilon_A - \varepsilon_C$ . It is very simple and straightforward. There is nothing

new. It is all high school physics. But what you have to keep in mind is, technologically strain gauge measurement is very very sensitive. You need high precision strain gauges and how do you bond them? How to bond a strain gauge on a given surface on a prototype itself is a very important training as a strain gauge measurement specialist, you have to undergo.

And there are certificates given by British Society of Strain Measurement. You have to renew that certificate every 3 years. So, it is not a simple job. It is not that anybody can walk in and take a strain gauge available, put it on any substrate and then make the measurement. That is why it is, I say it is abused. If you know the technique, you should select the strain gauge, follow appropriate procedures to bond and then make the measurement.

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Introduction to Photoelasticity and Strain Gauges

EXPERIMENTAL STRESS ANALYSIS

Strain Gauges 26

SWAYAM PRABHA

Delta rosette

$\theta_A = 0^\circ; \theta_B = 120^\circ; \theta_C = 240^\circ;$

$\epsilon_A =$

$\epsilon_B =$

$\epsilon_C =$

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I also have another variation which I am not going to spend much time. You have a variation like this. You can work it out, ok?

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Introduction to Photoelasticity and Strain Gauges

## Strain Gauge Designation Systems and Selection

A simple procedure for selection of strain gauges for routine and conventional stress analysis is as follows.

1. Type of Strain Gauge

```

    graph TD
      A{Biaxial stress field} -- No --> B[Single element strain gauge]
      A -- Yes --> C{Principal stress direction known}
      C -- No --> D[Delta or Rectangular Rosette]
      C -- Yes --> E[T-Rosette]
  
```

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Now, you know if I go to strain gauge manufacturers, they have an elaborate list of strain gauges available, ok? There is a designation system. I want you to write one of this, ok? Just write this. Because you should know all these parameters need to be selected for a given application. I should know what should be the foil alloy.

I should know what is the backing; on what substrate this strain gauge is put, ok? And there is also something called self-temperature compensation number. You know some alloys are made. If there are small changes in temperature, this itself can annul that. I can have just one strain gauge connected to the Wheatstone bridge for a short range of temperatures, it can cancel the temperature effects. And what is the gauge length? Because the gauge length is very important, you cannot say I will always use 0.2 mm. It is so expensive and very difficult to do, ok? And what is the way I should have the grid because it should help me to provide the access. And what is the resistance? I said in a composite, please use 1000  $\Omega$ , at least 500  $\Omega$ , never use 120  $\Omega$ . And there are many many options available, ok? And these are the two people, one is the Vishay micro-measurements, another is HBM. They are the people who are dominating the strain gauge supply.

And they have a different system, ok? So, you should know if you are using, there are 1000s of configurations available. So, you have to select appropriately the strain gauge suitable for your application and then use it. And you know, I would also like to show, you know it is a very very precious. I said it is not cheap, it is available in a very nice packet like this. You have a small strain sensing element. I cannot go and take it with my

fingers. I have to use tweezers, ok? And then go and take it out. I must handle it very delicately. If I handle it roughly, the strain gauge is gone.

Once I paste the strain gauge, I cannot reuse it. Is the idea clear? Even though it is made of metal, it is a very thin film to have very high resistance. So, I should handle it very very carefully. It is very expensive, ok? Then, there are also from measurement perspective, what all that you require. Whether I need a single element strain gauge, whether I need a rosette of two or three, all that I have to look at, whether I am working in a biaxial stress field, whether the principal stress directions are known. See, you know, we have looked at what is brittle coating.

In the brittle coating, when I have a crack pattern, I said tangential to the crack and perpendicular to the crack, you know the principal strain direction. When a principal direction is known, I can paste just one strain gauge for me to make the strain measurement or two strain gauges. See, at a point, I need three strain gauges. If I need three strain gauges, I need three instruments to measure the strain. Is the idea clear? Three Wheatstone bridges are required.

If I reduce one component of strain not to be measured, I can go to brittle coating and then align it along the principal directions. It is enough, I put only two strain gauges. I can use a  $T$ -rosette. So, I reduce my measurement requirement. And in a generic situation, I should use a delta rectangular rosette because I need three components of strain. So, whether I use a single element strain gauge, two element strain gauge or three element strain gauge also a decision you have to make, because when you go to a point-by-point technique, how many points I make the measurement at, is going to decide my design of the component once I get the measurement.

I have a limitation in the capacity of how many points I can handle or how many channels I can handle. If I have three strain gauges, it occupies three channels. If it is two strain gauges, it occupies two channels of my measurement system. So, if I have a 1000 channel system, if I want to increase my number of data points, you should not say I will simply use, I have enough money to spend, I will simply use rosettes at every point. Then you cannot make best out of your experimental measurement. Some places, you may have used single or two elements so that I can make large number of measurements. So, this is the difference between a whole-field technique and a point-by-point technique.

And you know the rudder of a Concorde aircraft was failing very repeatedly. They had done a finite element analysis, they have also pasted a strain gauge, still the design was not improved. Only when they pasted a photoelastic coating and made the measurement, they found they pasted the strain gauge slightly off from the peak strain what they had actually got in the structure. So, that failure was prevented only with the help of a whole-



field technique.

So, with this, you have a bird's eye view of what a strain gauge technique is. A strain gauge can give you state of strain at a point provided you use a rosette. And strain gauge has to be selected appropriately for a given application. Always keep in mind, you have to look at the thermal effects, you should know how to cancel it, you can cancel it electrically by having two strain gauges on adjacent arms, one experiencing actual strain, other experiencing the ambient temperature. Or you use self-temperature compensated gauges and select an appropriate gauge for a given material. Do not simply use what is the strain gauge available at the eleventh hour you plan and then try to make measurement and finally blame the technique that the strain gauge technique is not giving you correct results. It is a very sophisticated technique, you have to be sensitive from the first stage of the experiment and handle the technology, listen to all the manufacturer's recommendation, paste the strain gauge and make the measurement. Thank you.

