Strength of Materials
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Lecture - 17
Stress Strain Relations

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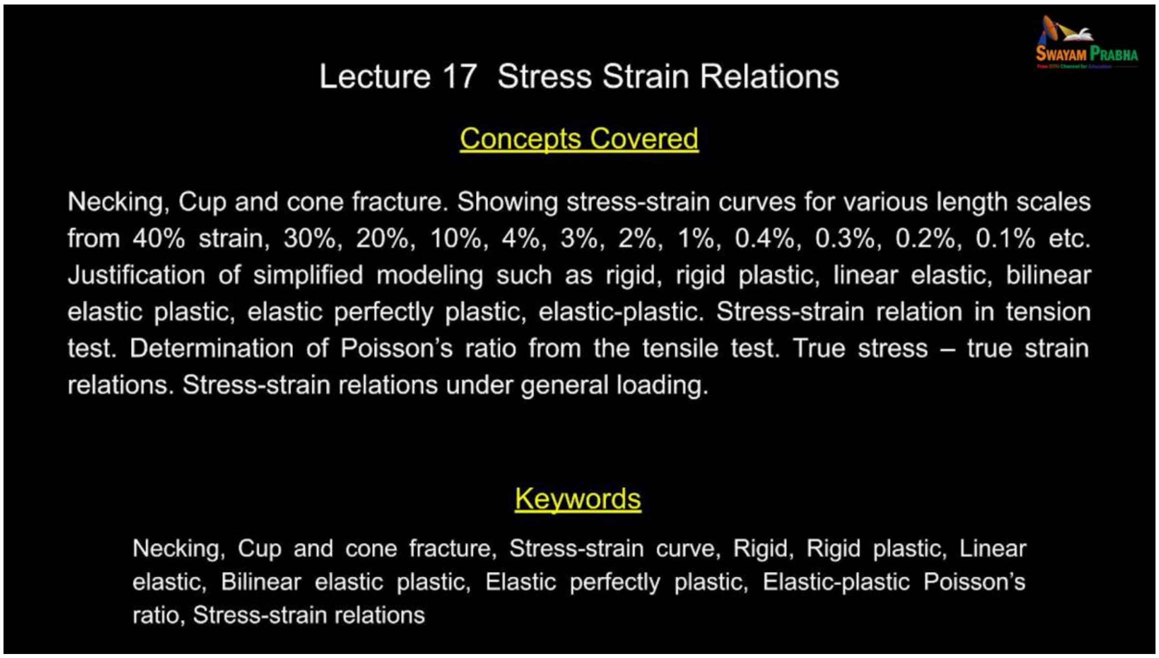

See, one of the important aspects of learning strength of materials is to select an appropriate material and also the appropriate cross-section for a given application. And I said, you have to characterize the material and one of the important questions the founders of solid mechanics had was - how to characterize a material, how many parameters that you need to characterize a material. These are two important questions. And we looked at a simple tension test in the last class and we would like to use whatever the information we get in tension test, in a meaningful way in strength of materials, because that is the simplest test possible and you try to get the maximum out of the test. So, from the test you also go and graduate to formulate the constitutive relations. What is the relationship between stress and strain? In a simplistic manner, it is a stress-strain relationship. In a very bombastic terminology, you are talking about a constitutive relationship. How does the material behave under the application of different types of loading? So, that is what we are aiming at.

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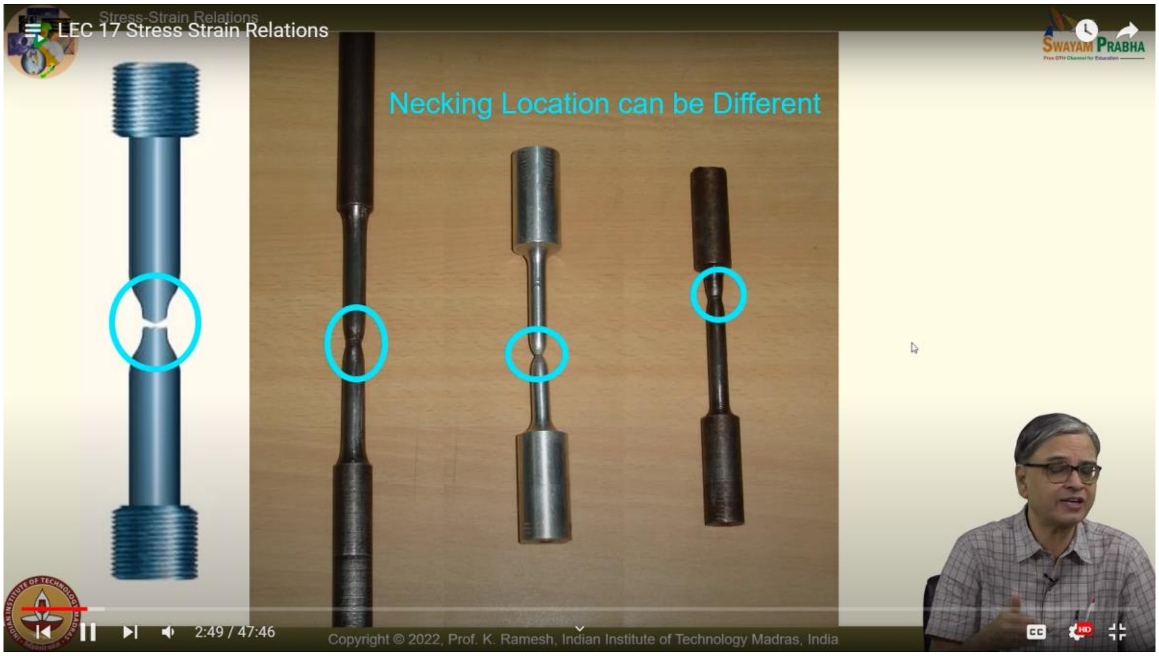

And we have looked at a very important aspect when we conducted the tension test, we also saw material separation. That is also important, because once you understand that every component may have inherent defects, these defects can grow during service and you can have fracture; such failures have happened.

And we have also seen that what you have as necking; in the simulation, I have shown it exactly at the center. And when I take a circular rod and then follow the ASTM standards, I have got for one specimen exactly at the center. This is of aluminum and this is slightly shifted from the center. I have also picked up another specimen which is way off. You know, necking is a very very material dependent; at that point in time, what kind of defects are there in that specimen, it is precipitated by that. So, it can happen anywhere in the gauge length, fine? And mathematically, it is very difficult to analyze what happens during this instability.

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And from our understanding, what is important is how does the failure take place? You have a cup and a cone fracture; this is how it is referred to. So, what happens is, initially you have a crack develops at the center. Because of the brittle failure, it propagates and then finally you have this as shear lips.

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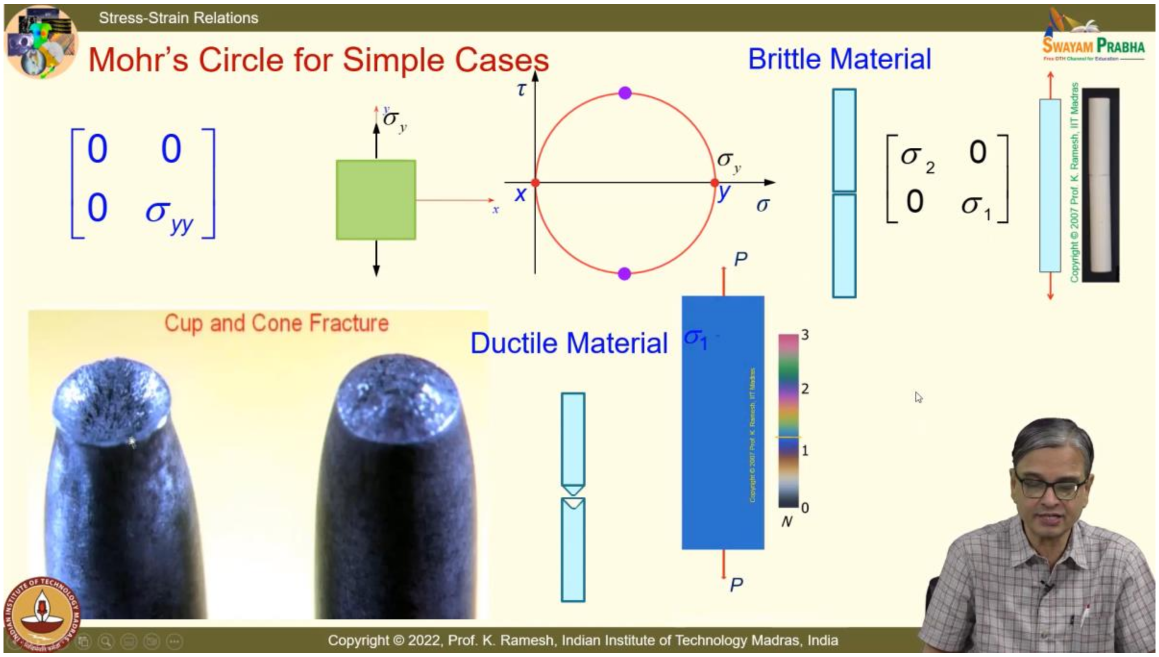

And we have also looked at, from Mohr's circle for a uniaxial loading, we have discussed why does a chalk fail like this. We have also seen epoxy which behaves like in a brittle manner.

In an actual video, I showed that when you pull it, it was getting separated horizontally, fine? So, that reinforces a reason that brittle materials fail by maximum normal stress. That can be a good model for your failure theory. And the moment you come to ductile materials, we have seen that it is the shear that precipitates this failure. And when you have this axis, you know the shear points are marked on these two and this is oriented at 90° in the case of Mohr's circle. In physical plane, it is about 45°.

So, that explains why I have a cup and cone fracture when I have a ductile material. And we have also looked at one more way to make the specimen fail. We have looked at, if I apply the load repeatedly, then also after some number of cycles depending on the material and depending on the load, you find failure has occurred. And with your hand, you cannot apply a very high load; you can easily understand. So, when I do a repeated loading, I am forced to operate at lower loads in actual service.

See, these are all justifications why we want to confine our attention to small deformation, whether it is only a bombastic assumption or does it really help in solving our day-to-day problem. If it helps in solving a day-to-day problem, it is an excellent simplification. See, engineers have to simplify the problem. You are supposed to deliver a solution for a given requirement. It is not that you complicate the problem and then revel in your mathematics, fine? That is also needed.

But the focus of an engineer is to provide solution. And I have also been re-emphasizing that you should not only look at what happens at selected points, you should also get the field information. Now, you should be able to appreciate when you use any of the whole field techniques, you get the field variation of the parameter. That is also very important.

I told you in the case of a Concorde rudder. And if you look at many of the aircrafts, see, Concorde is not in service now. It was the first supersonic aircraft and in its entire life of service, there were not many failures, fine? That was mainly because judiciously combining a whole field technique and a point-by-point technique to solve the practical problem. So, you should have the basic understanding and we have also discussed that it gives you σ_1-σ_2 and we have also learnt how to label σ_1 and σ_2. It is because algebraically larger one should be labeled as σ_1. So, when I have σ_{yy}, σ_{yy} is σ_1. And we have also said that in photoelasticity, you get only contours of σ_1-σ_2. See, all the experimental techniques exploit a particular physical principle.

It cannot give you all the information. It can give you only a particular kind of information based on the physics that is being exploited. With a strain gauge, you can measure only strain. You cannot get stress out of it. You have to use stress-strain relation and find out stresses.

On the other hand, if you go to digital image correlation, you can only measure displacement. From the measurements, go to the strain displacement relation, find out strain and constitutive relations, you find out stress. So, this you will have to appreciate.

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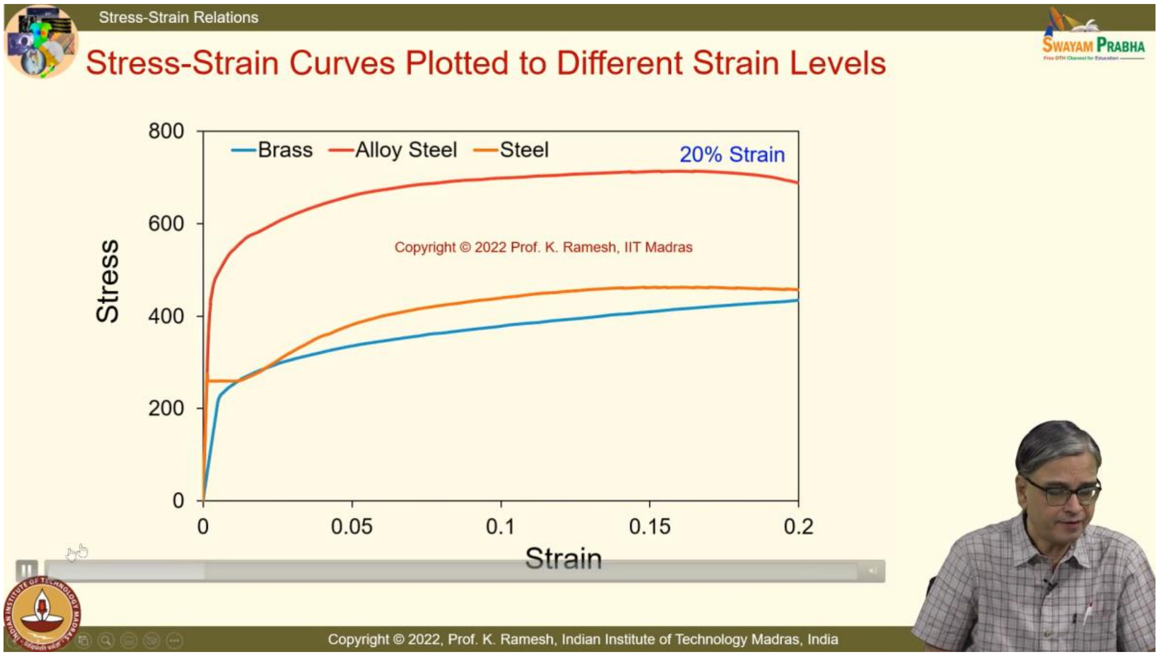

And we have also looked at a very important graph and it is also plotted in a very nice fashion. I would like you to sketch. See, this plot is done initially you saw it for 40% strain.

You know the same data is used in which the horizontal axis is plotted differently. Because once you collect data, what does the data say? How do you interpret the data? Why do you make an approximation that I can live in a small deformation? Is it justifiable? And what are the other properties? See, we have also looked at, you know, in this graph, it is little clearer that I have this mild steel and I have this alloy steel. Both of them share the initial slope as identical. It is a very very important information. See, the desire of a material scientist is to develop a material which has all the good qualities.

You will never have; in any of the physical systems, some aspects are good, some aspects are bad. And what you have also seen is, you know, if I have to, ok, I will go through this graph, then I will come back. Then we will see, if I increase my yield strength, you will find the maximum strain it can take will become slightly smaller. It is not that I want to have best of all the worlds, fine? So, there is always a trade-off. And one of the issues that material scientists want to do is, if I have to use minimum material, I should have very high strength.

I should also have very high strength, I should also have very high toughness, very high ductility. These are all the questions people ask. And you know, you see here, this is about 550 MPa or so, if you will have 0.2% strain and then find out the yield strength, it may be about 550 MPa, whereas your mild steel is about 275 MPa or 300 MPa. So, by alloying the steel, you are in a position to increase the yield strength; that is fine. And people also do heat treatment. People do what is known as quenching. They heat it and then put it in water, fine? That improves the hardness. You know, our ancient people have understood this. When they were making their weapons, when they were making their swords, they were making the material processing technology very well developed, they could get very high strength swords made.

And India is credited for that. It is all done in the, you know, not in a technical manner; you have a nice workshop where you do it. All done in huts; hutments where they have developed this technology. There are also names for it. People also have looked at the evolution of steel in India and people have understood.

See, experiential learning has always been there. By performing set of experiments, by varying certain things, people have understood how to process the material. But analysis always comes later, ok? And what is done here is, you have the same data, which is the advantage of your multimedia type of knowledge delivery.

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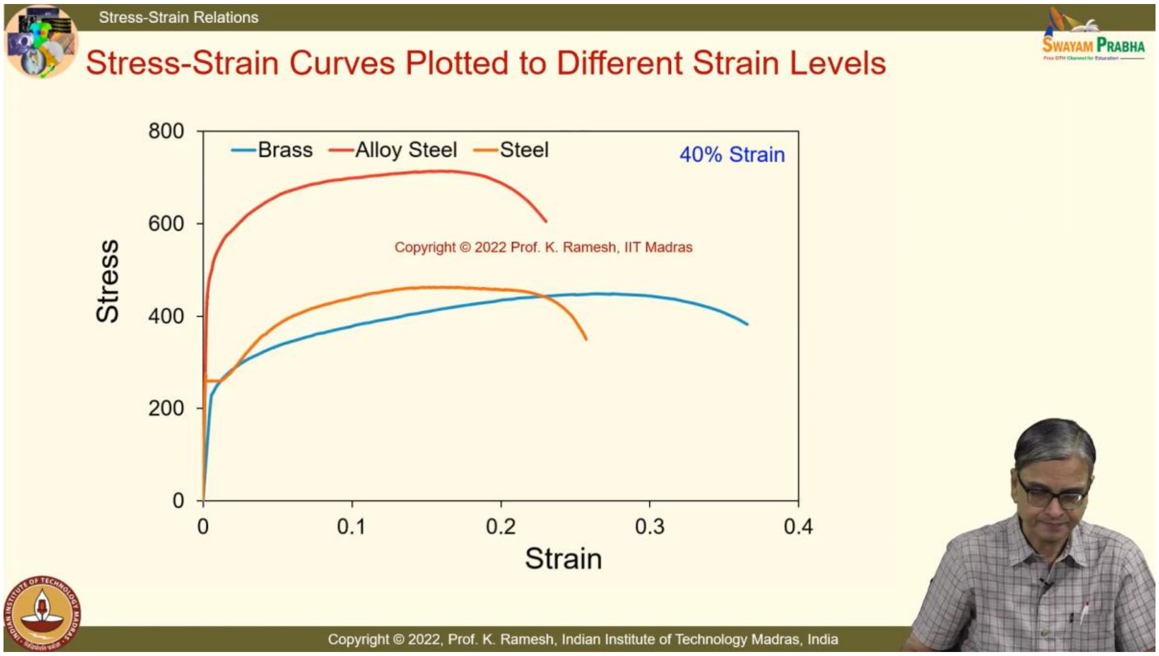

See, what I said was, when I have a mild steel, it fractures at a larger strain. When I have an alloy steel, I have improved my yield strength, but this has shifted to the left.

So, you always have a trade-off. When you want to increase the yield strength, you may have to lose something else. But people also have addressed this. One of the challenges for material scientist is to develop exotic materials. See, right now you have alloy steel, which is giving you 550 MPa.

People have developed in defense application, where the yield strength is in the order of GPa. People may not reveal what is the composition, what is the processing technology, you understand? So, these are all trade secrets, ok? People have also achieved some of those alloys. And here you are unable to see that these two slopes merge. Now, we will look at the change.

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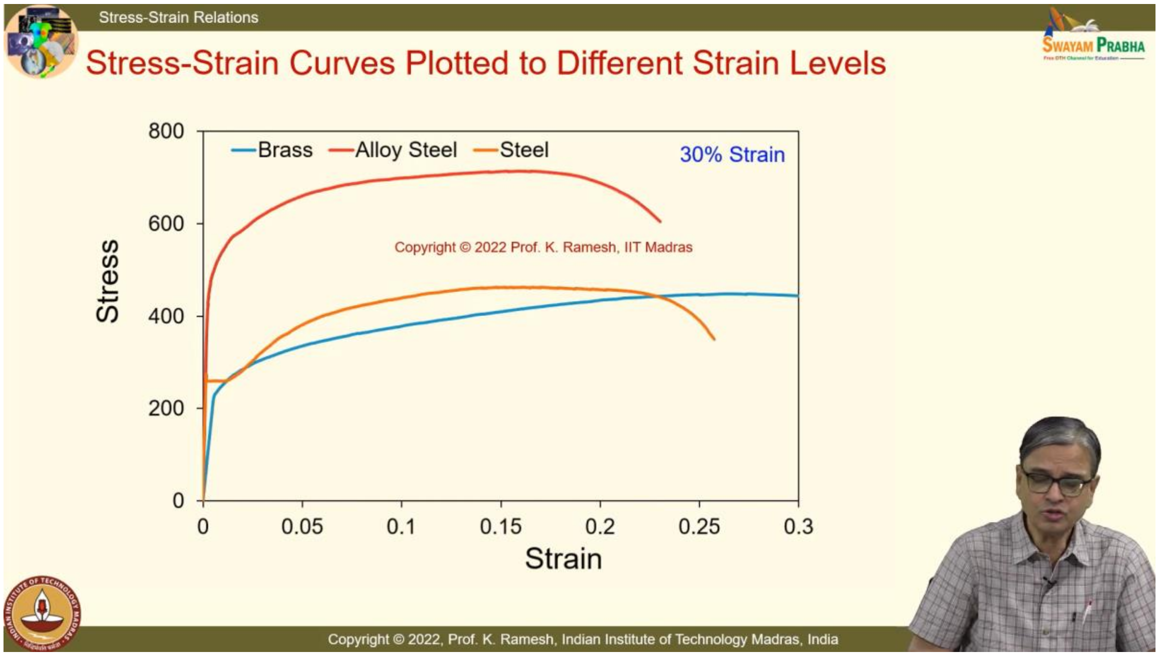

This is the 30% strain. So, you should see the *y*-axis is not changed, only the *x*-axis is changed, ok? You know, I have also said that strain can be mentioned in terms of percentage or a number or also as micro-strain. You must be comfortable switching between these representations.

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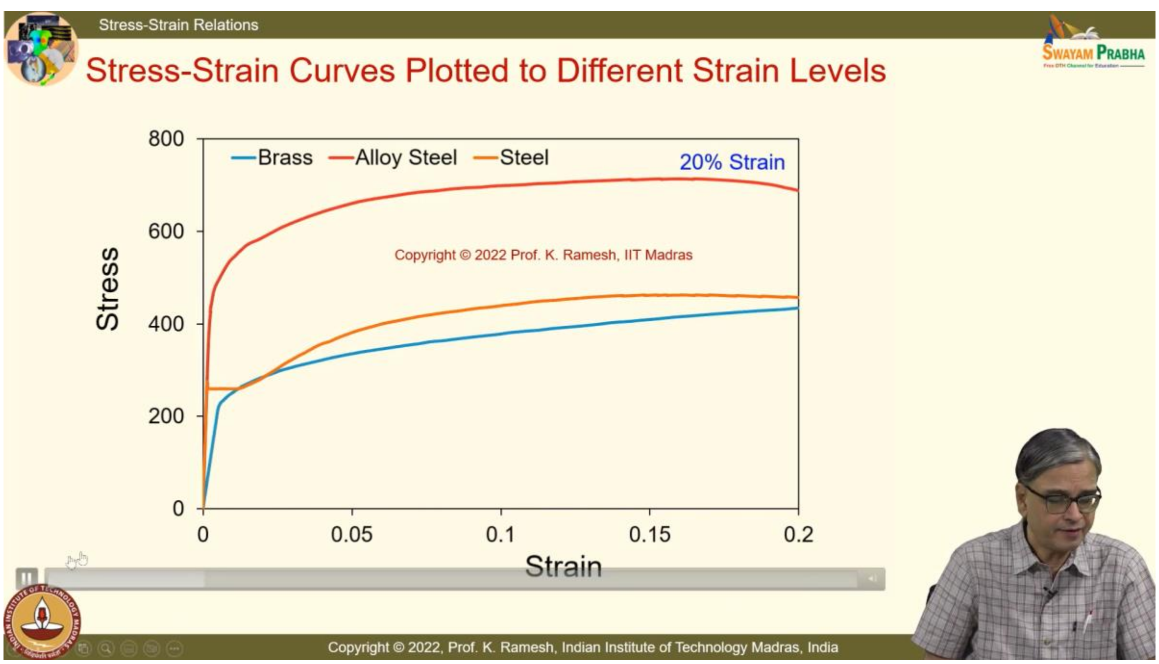

Then I have this as 20% strain.

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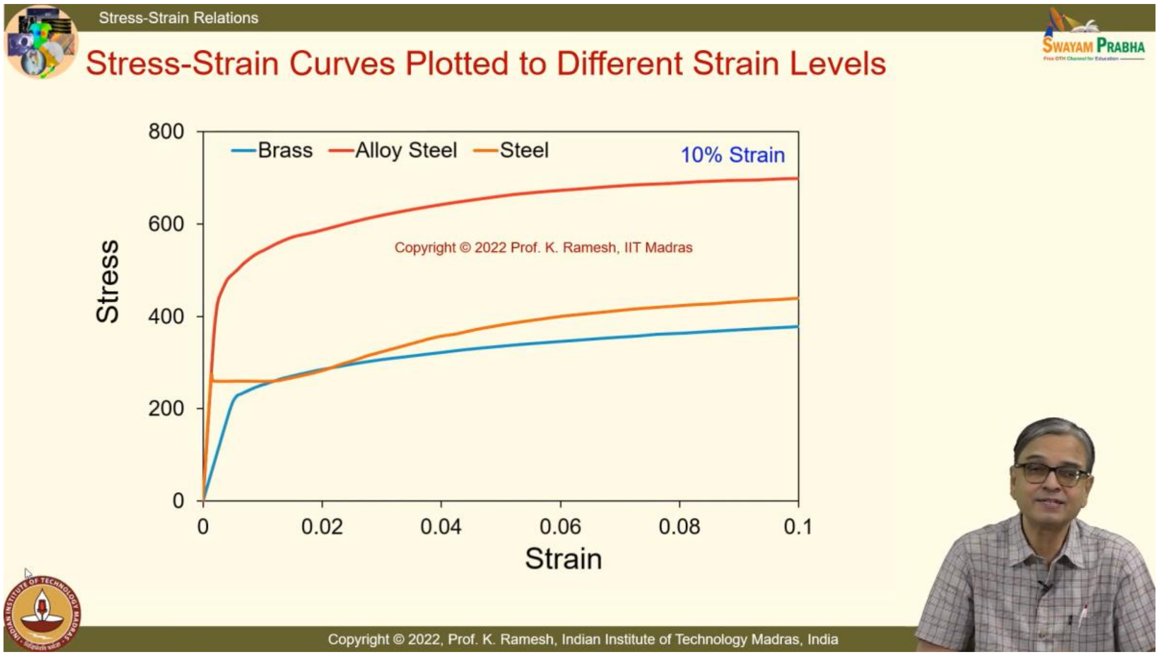

And I see the same graph at 10% strain. I would like you to draw the 10% strain. Likewise, we will have few steps I want you to draw it and keep it for your reference.

This explains many of the simplistic models that we want to adopt to characterize a material. For example, if you look at here, for the mild steel, I have a straight portion. If you ignore this small kink, it becomes horizontal. Don't you see that? That was not very clear when we looked at 40% strain. So, I have a straight line followed by a horizontal line.

That kind of an approximation is also made. All that comes from reprocessing the data that you have recorded. That is also very important. See, once you collect data, you should also know how to use it effectively. So, what is done here is, I have changed the percentage of strain and then I have tried to bring out whatever the simplified material model, you have a justification inherent in the data collected.

It is for you to see. See, there are optical illusions people throw before you and then say what do you see and then they characterize what way you think. It can be fun. It can also be psychologists using that as a methodology to find out how your brain functions. So, many times you see what you want to see. Is the idea clear? So, when you get the data, you must get the maximum out of it and this is plotted for different percentages.

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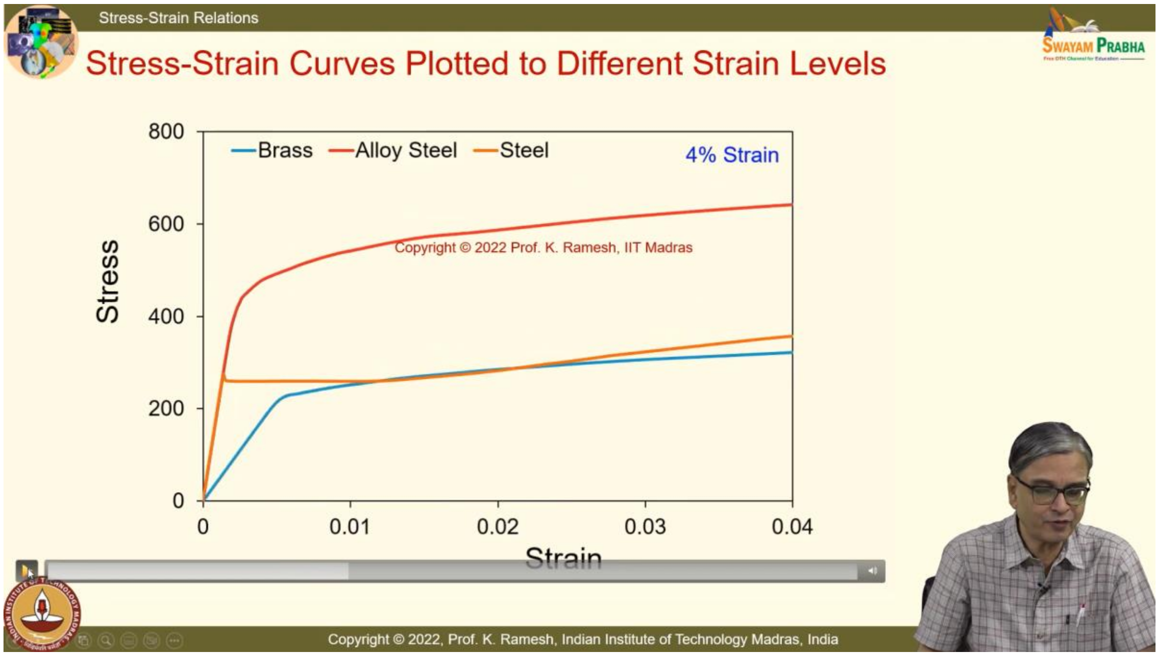

Now, it is also plotted for 4% strain. Here again, you are able to see, I have a straight-line portion and a horizontal portion. This is a very celebrated approximation people do. When the moment they want to enter into plasticity, they do not want to analyze plasticity in all its complexities. People would like to have elastic and plastic, perfectly plastic.

That is the way they look at it. Because you have a graph which has horizontal and because the graph is increasing here, they have to come out and distinguish what is happening. That is why I coined that as strain hardening. You know, terminologies also come like that.

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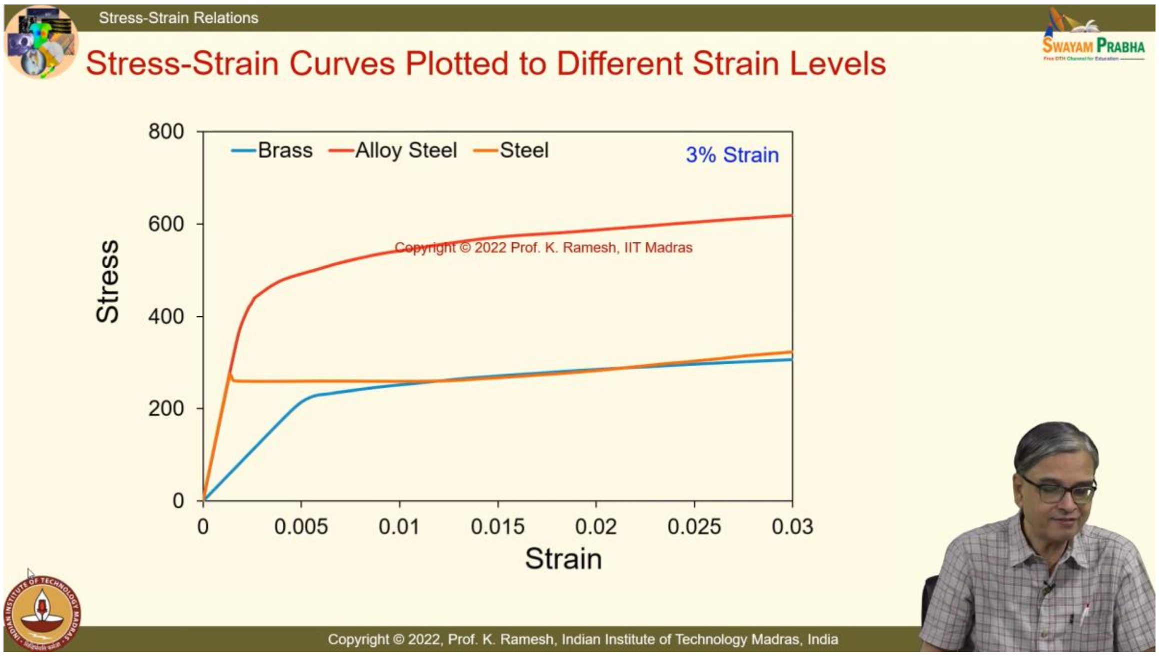

Then I have this as 3% strain, ok?

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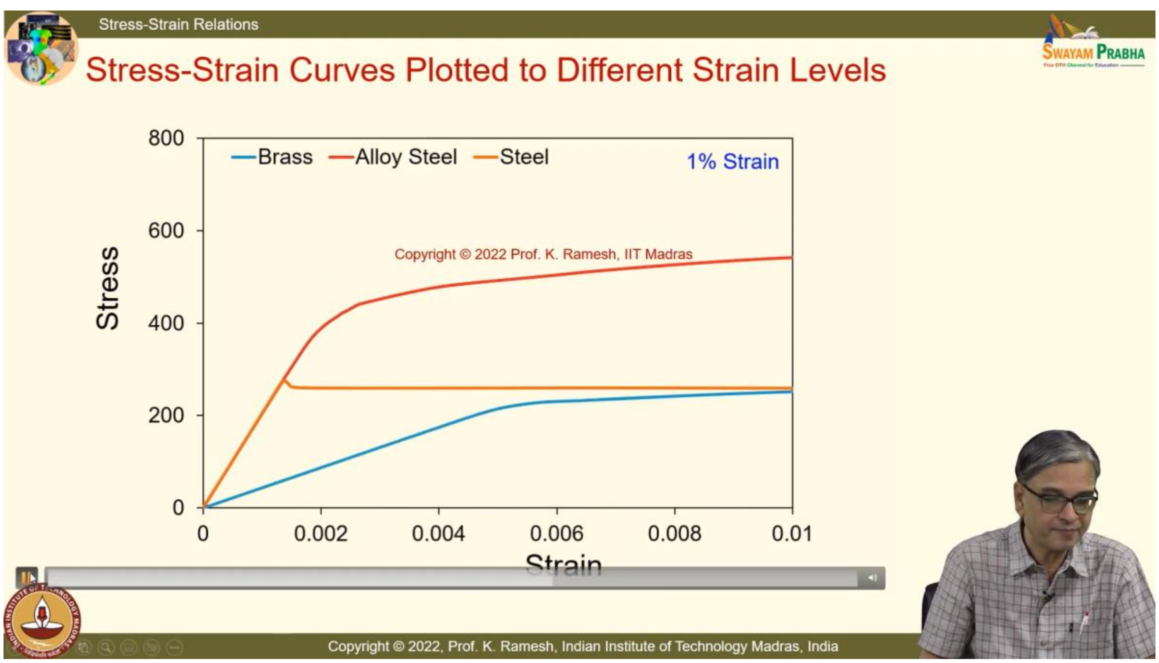

Then I have 1% strain. See, 1%, if you go and invest money in your bank, 1% is very small.

If you go to Japan, you have to pay money to keep the money in the bank. Do you know that? Only in India, you have very high interest rates and normally when you say 1% from the monetary perspective, it is very small. But from material perspective, if you say 1% strain, it is very very high strain. You do not want to go anywhere near 1% strain. Why? Because we have seen, when I have a stress strain graph which is not having a clear demarcation of the yield strength, I have to draw a line parallel to this initial slope from 0.002 and then find out what is the point it touches and find out what is the corresponding value as yield strength. So, what you have to appreciate is, at 0.2% strain, the material yields. It is a very very important knowledge. Never forget this.

So, we operate at a very very small level of deformation. I can say it as 0.2% strain or I can call it as 0.002. I can also say 2000 $\mu\varepsilon$. So, whichever way it is mentioned, you should be able to appreciate this.

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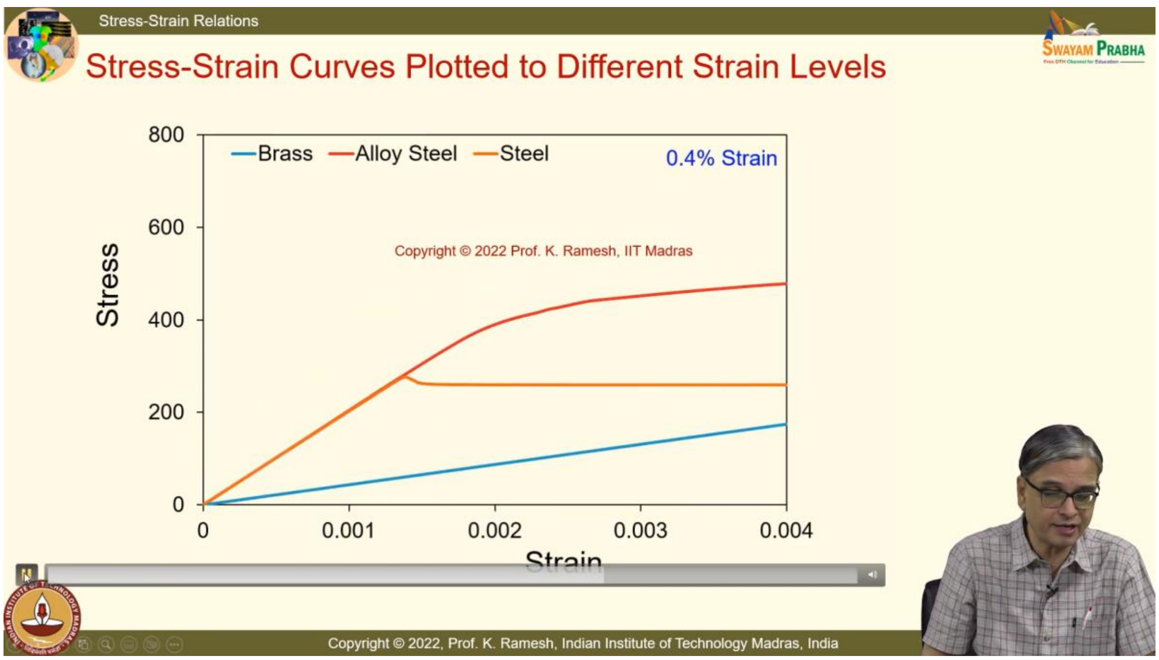

And we have also drawn it for 0.4% strain. So, you can understand it is just a linear graph. It is becoming very very prominent and particularly for a brass, you see for the entire range, it is still linear.

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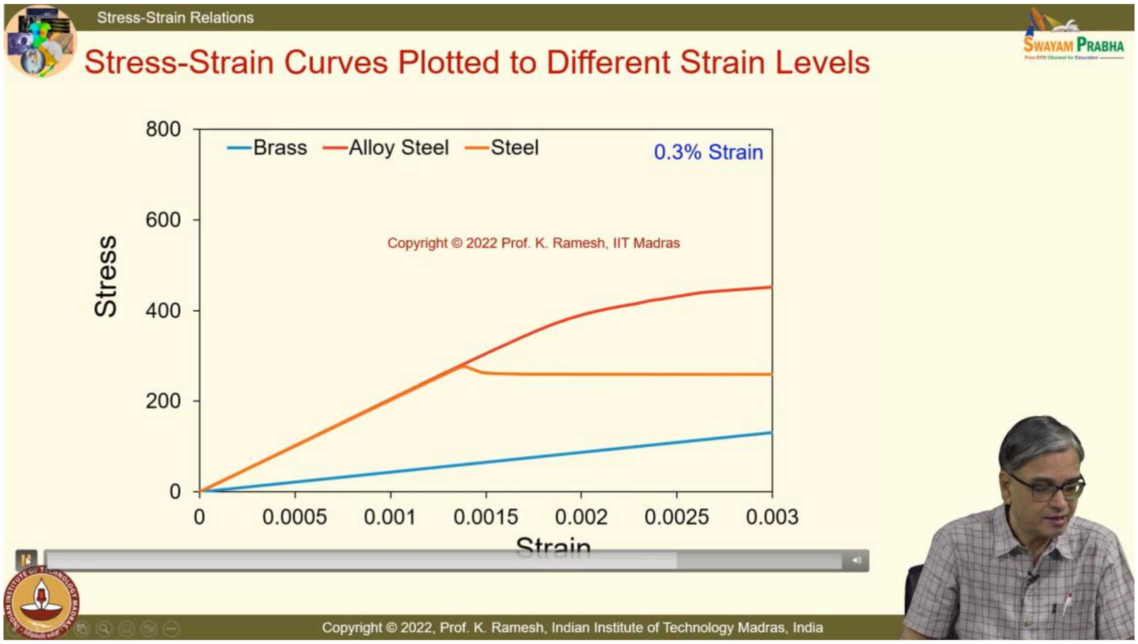

And I think you should also have a graph for 0.3% strain because you have an appreciation what is happening. The same data looks different. See, suppose I also change the *y*-axis parallelly, I would have a different picture. See, one of the lacunae I find in many of the books is, they give stress strain graph without labeling the axis. While discussing the constitutive relations, they give a general picture which is not sufficient from understanding point of view. You also have to associate with these numbers. So, when I look at, in the *x*-axis, a very small strain; all these approximations are valid. So, that gives a better appreciation of what is happening.

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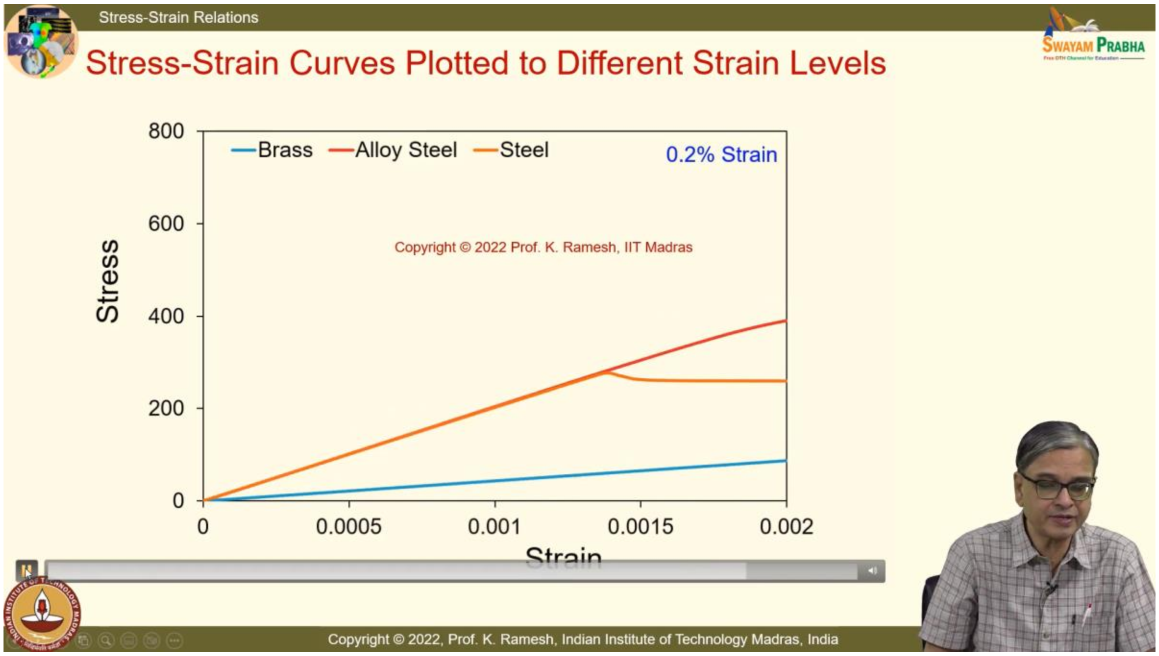

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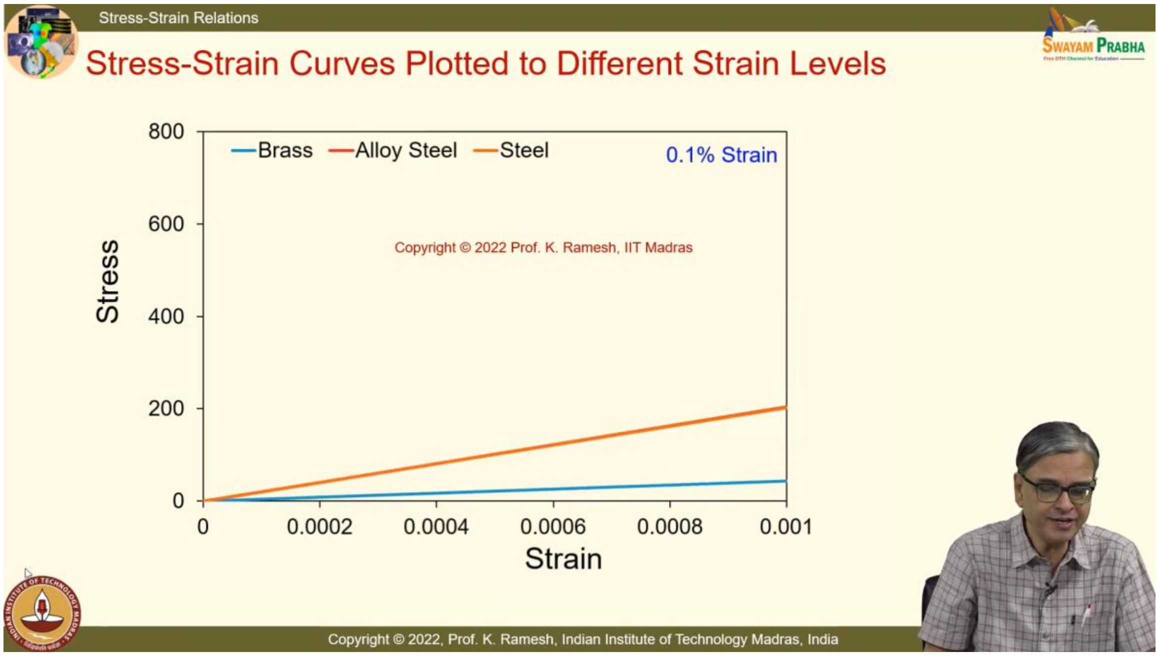

And I also have for 0.2% strain, 0.1% strain.

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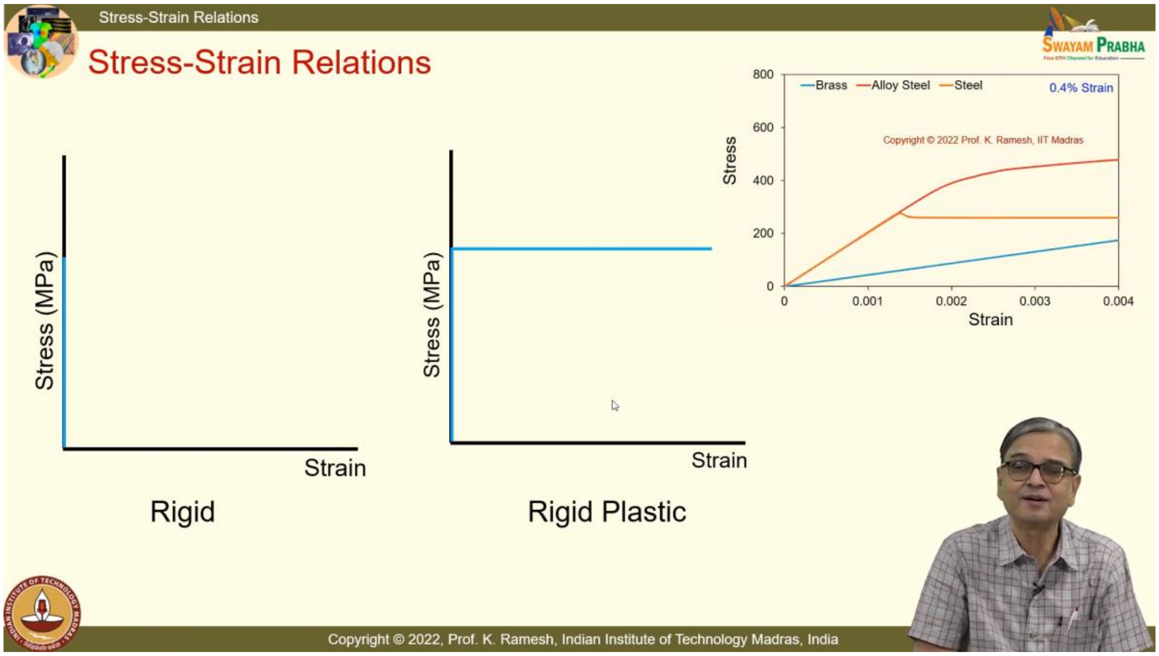

And what are the ways that you have a simplification to the stress strain behavior? You know you have graduated from rigid body mechanics, yes or no? See, you can argue as a mathematician, you can argue.

If I have a 1000 mm length, if it extends by 1 mm, I will simply close my eyes and say it is rigid. You can also take that stand. You can take that stand. See, if there is an elephant standing on granite, you may not be able to see the deformation even though granite you considered as more rigid compared to a plastic. When elephant stands on it, when you make sensitive measurement, it will also have some deformation, fine? So, one of the simplest one and you have also had one full course on rigid body mechanics.

That means whatever the stress you apply, only the stress keeps increasing, there is no corresponding deformation at all. That is the definition, ok? It was a very convenient approximation. And the second one is you have a system where it is rigid and perfectly plastic. See, many of the initial plasticity theories started based on this kind of a modeling. It was easier to focus only on plasticity behavior, not on combining it with elasticity, fine? You need to have graphs drawn.

What is a rigid idealization? What is a rigid plastic? Please make a graph. The idea why I give a discussion is to give you sufficient time to have these graphs plotted in your notebooks. So, please make a simple sketch on this.

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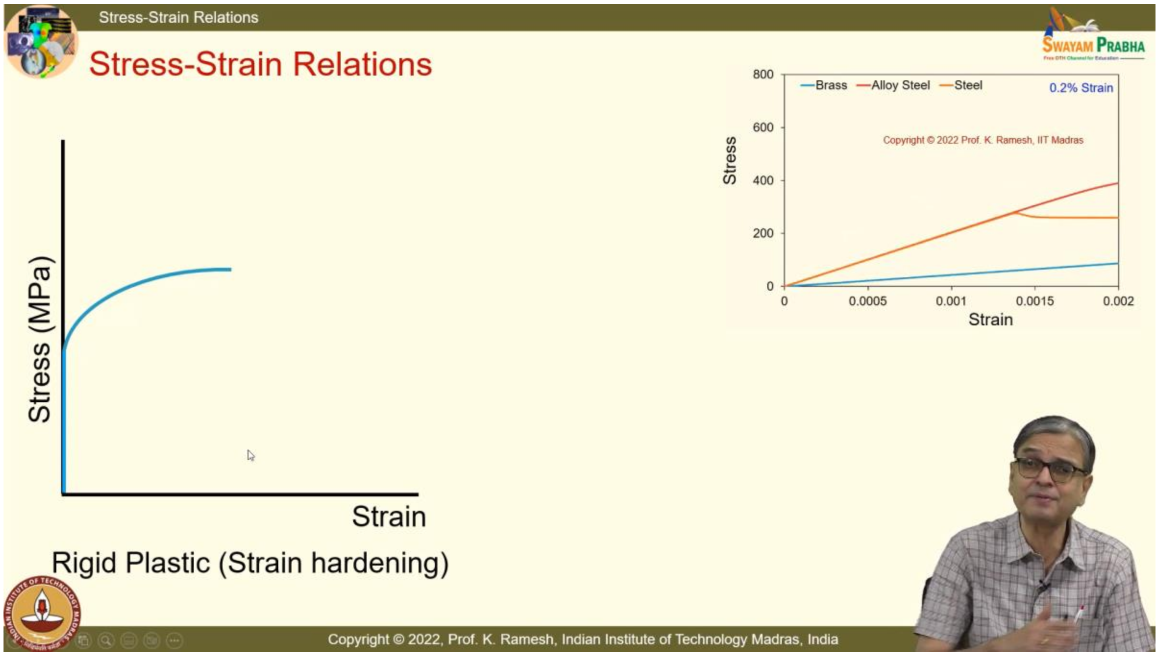

And you also have a system with strain hardening. I have a rigid plastic and then strain hardening. See, this is how mathematical theory is developed. They took baby steps before they want to analyze what happens to the material.

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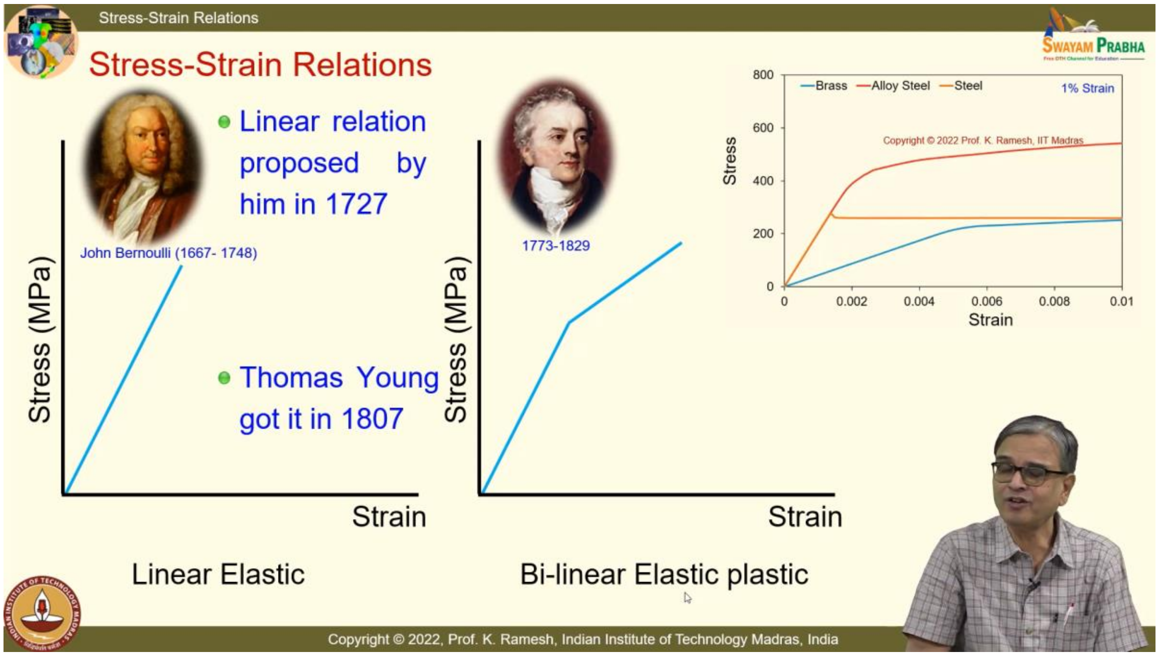

And you also have our famous linear elastic. You know, I have also followed the books. I have not labeled what is the strain here, what is the stress here. But you know from this graph, when I look at in a region of very small strain, the stress strain behavior is linear. What is important here is the behavior is linear. And when it is a linear relationship, we have many many advantages. We will exploit those advantages and use it for our discussion. And if you look at history, this linear relationship was first proposed by John Bernoulli.

He is the brother of your other Bernoulli who was responsible for the beam theory. So, this was proposed by him in 1727. And I am sure that this is also taught in your high schools and you are told that you have what is known as Young's modulus. Thomas Young was only between 1773 and 1829. He got this only in 1807. I do not know how the labeling of some of these and giving the credit to a scientist is operated. Whatever you have, this is known as Young's modulus in his honor, ok?

And you also have another simplistic modification, ok? Can you see here? I have this as a straight line followed by another line with a different slope. You see that, that is embedded in the data.

See, I want you to draw this. Please do not leave it. These are all very very important. These are all the best modeling that you will be doing when you have your constitutive relations. Even if you have a straight line for elastic and another straight line for plastic behavior, the mathematics is back breaking. Please understand; it is not simple.

From a mathematical perspective, it is not a simple task at all. Even if you have modeled it, even when people do finite element analysis where the computer faithfully reproduces whatever the input you give, only if the modeling is right, you will get good results. Otherwise, it is garbage. Even there, if they have to model some material behavior, they would do linear elastic and change the slope to model some aspect of plasticity, not all aspects of it.

And this is called bilinear elastic plastic. It is a very famous approximation. In material constitutive law, when people want to go beyond yield, the first baby step is to have it horizontal perfectly plastic. Second baby step is, have that as a straight line of a different slope.

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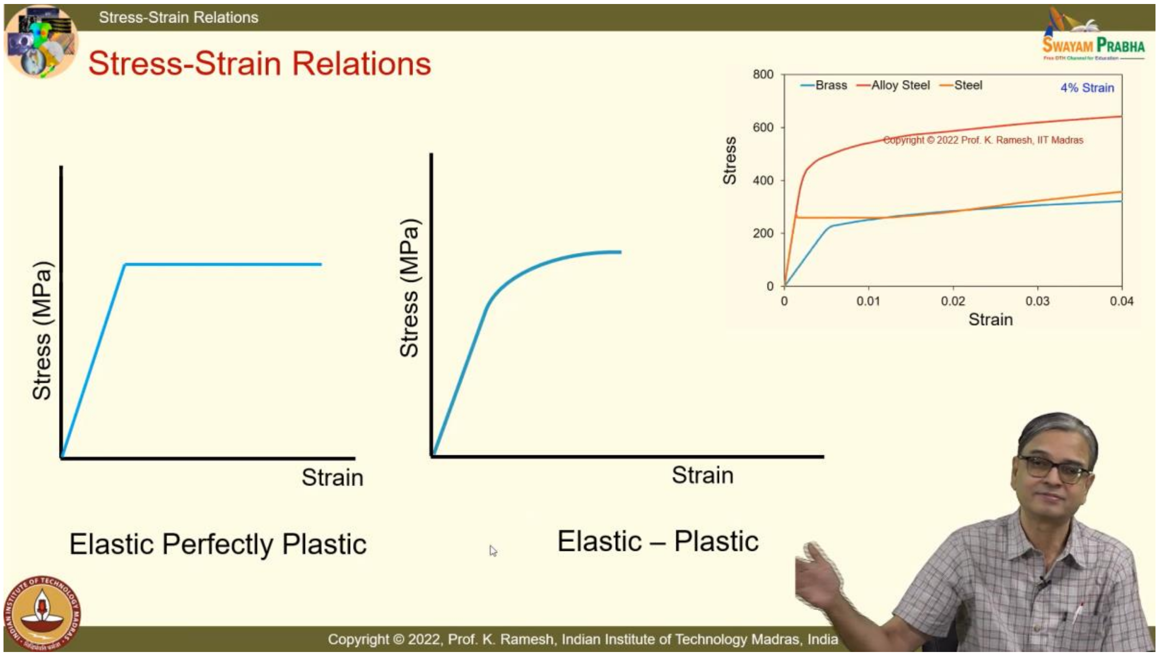

And the third one is you have elastic and elastic and you have strain hardening. You have a law called Ramberg-Osgood law which explains this. You do not have a law which explains from the elastic region to fracture, ok? You have portions of this stress-strain graph; you have mathematical models.

So, plasticity is also very important. So, when people wanted to develop theories related to plasticity, they started with elastic and perfectly plastic, then elastic and strain hardening, ok?

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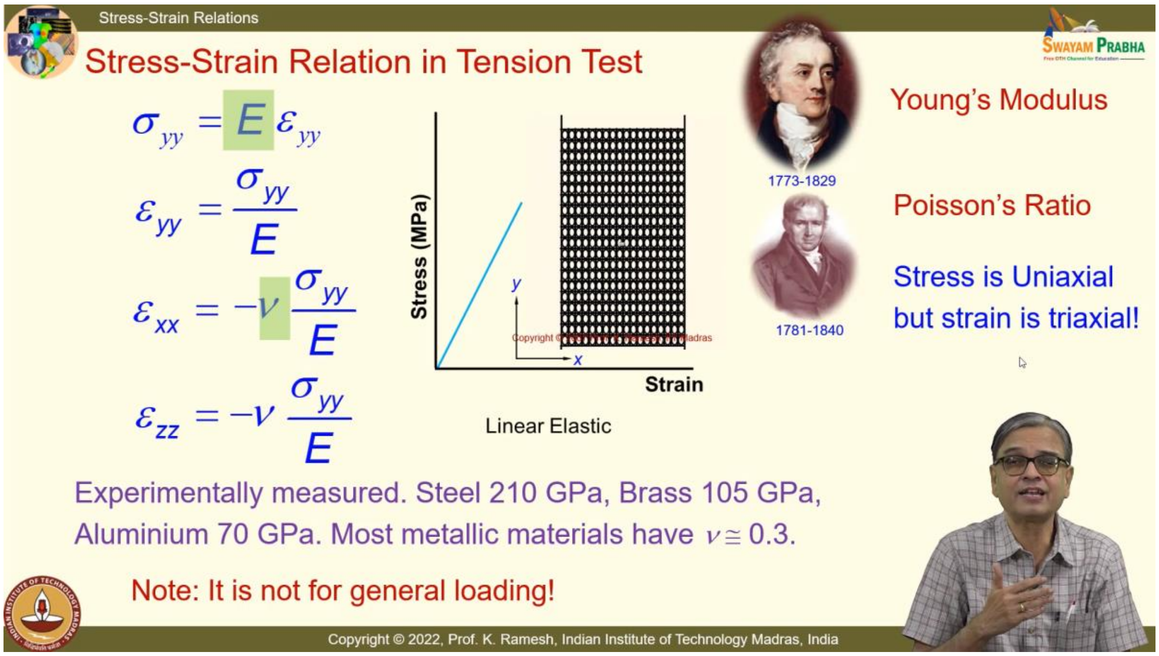

Now, let us go and look at, from a mathematical perspective. You know, I have shown it many times that when you have a specimen which is made of soft material and I stretch it, you are able to visually see the lateral contraction. So, that cannot be ignored. That is what is seen. Apart from the lateral contraction, the thickness also changes, ok? Thickness also changes.

And when I have plotted strain and stress, they can be related. I have pulling in the *y*-direction. So, I essentially apply σ_{yy} that is related to the strain ε_{yy} by a material parameter called Young's modulus. It is labeled as *E* in honor of Young, Thomas Young, ok? But we have seen that this linear relationship was first hypothesized by Bernoulli, the brother of the first Bernoulli, ok? And you not only have ε_{yy}, you also have ε_{xx}, you also have ε_{zz}, lateral strain. You have ε_{yy}, the same equation is put in a different form where I have $\varepsilon_{yy} = \sigma_{yy} / E$. Initial material characterization, I have the transverse strain to longitudinal strain is a ratio. They took a constant value of 0.25. People did not know that it changes from material to material. And they were thinking that you can characterize the isotropic material with one elastic constant. Because if it is one elastic constant, then my life also becomes very simple. So, people have made crude assumptions like that and only later they found out. So, people have found that ε_{xx} is related to the strain in the *y*-direction by a coefficient what you have this as a Poisson's ratio which is given a symbol *ν*.

This is credited to Poisson. He was between 1781 to 1840. He also decorated that Ecoli polytechnic. And you have a minus sign mainly because you know when you stretch it, the other strain contract that is what the natural behavior. If I compress, then it will bulge out, ok? And you have ε_{xx}, you also have ε_{zz}; that is also $-\nu \frac{\sigma_{yy}}{E}$. So, when you conduct a simple uniaxial tension test, you find the strain is triaxial.

See, while we discussed strain, I also said, imposing uniaxial stress is very simple. Imposing uniaxial strain, I have to take extra effort to constrain. And for that what I did was I simply compressed; bulging out, I prevented it by a rigid block and you had uniaxial strain.

But you will have biaxial stress. Stress will be developed. That is a different issue. And when you have this Young's modulus, you know you should also know the relative numbers. Please write down this. Young's modulus is always quoted in GPa, Giga Pascal. And your stress is usually quoted in Mega Pascals (MPa) and strain as micro strain ($\mu\varepsilon$).

You should know the relative values for different material. For steel, it is 210 GPa. Brass is 105 GPa. Aluminum is 70 GPa. Do you see something in the way that I have written the numbers? See, what way we remember is, brass is one half of steel and aluminum is one third of steel. Some of these numbers you should remember because the stiffness is also very important. If I have a design scenario where I have to limit my displacement, then I will have to use a stiffer material or I should increase the stiffness of my structure by some means. Either of the two I have to do. And you should know how the Young's modulus varies for some of the standard materials. And most metallic materials have Poisson's ratio as 0.3. If nothing is given, you can have that as a starting point and then do it.

Plastics have very high value 0.36 or 0.37. And the caution is, whenever we ask students after learning this strength of material, what is the stress-strain relation? They would give only this as a stress strain relation. This is for a very specific loading of uniaxial case, ok? Only when I apply a uniaxial loading, your stress is related to strain in a linear fashion like this. When I have other components existing, the stress strain relations you have to look at, you should know how to write particularly the strain component. How the strain component is, you have to write? You will have contribution from other stress components as well.

So, I want to emphasize, you can apply uniaxial stress comfortably. A uniaxial stress gives, in general, strain as triaxial. It may appear trivial; it is not trivial. People do not get this in the first level course. You have to appreciate when I apply stress, it can get deformation in all the three directions.

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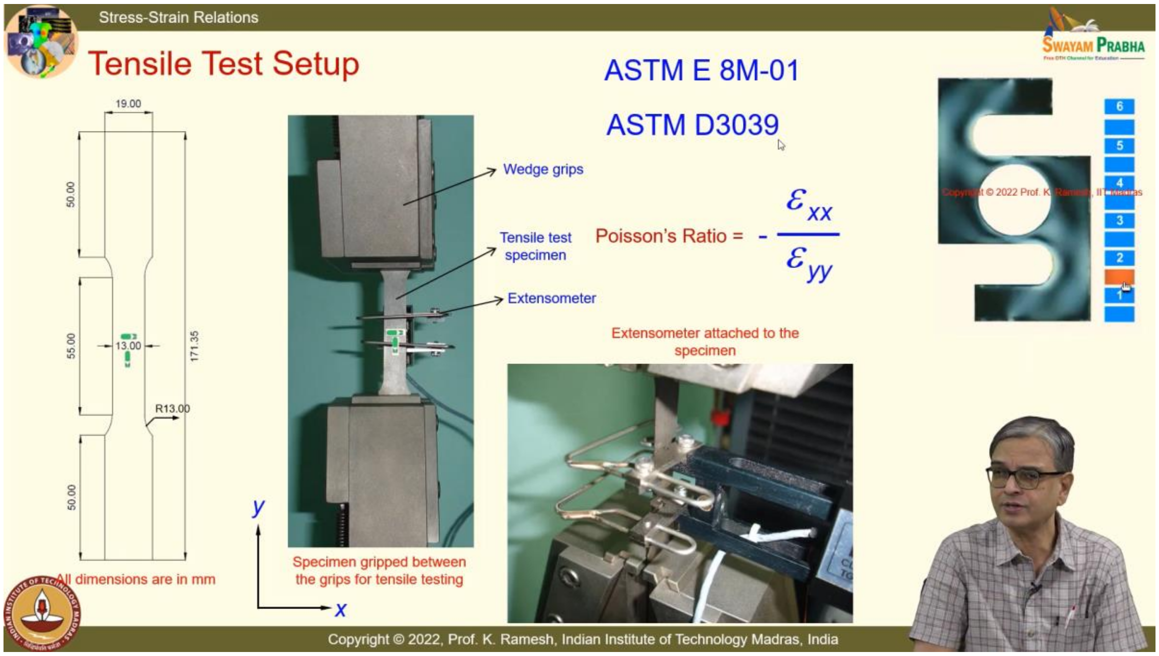

The next question is, how do I measure this Poisson's ratio? See, in the tensile setup which we have seen earlier, I have put an extensometer to find out only the strain in the axis of the member.

I was measuring only ε_{yy} and you had a load cell which was giving you what is the load. So, divided by area you are able to get the stress. So, you could plot stress versus strain. And we also saw there are standards available for this. And similarly, you also have a standard available for finding out Poisson's ratio particularly using strain gauges.

Suppose I give you a strain gauge and my extensometer gives me the strain along the *y*-direction. Can you guess how will you go and paste a strain gauge on the specimen so that I can measure one more strain so that I could get the ratio of transverse strain to the longitudinal strain? Please make a guess. You know strain gauge measures. Can you draw a strain gauge? Where will I paste the strain gauge? How will I paste the strain gauge? Please make a guess.

Now, you must make an attempt. Only then assimilation of knowledge will be there. We have looked at strain gauge just in the last class and I said a strain gauge will give you the component of strain along the gauge length. So, you draw the strain. What strain I have to measure? I have reference axis *y* and *x*. What strain I need to measure? I need to measure ε_{xx} because that is easier to do because thickness is very small.

There is no sufficient space for me to paste a strain gauge on the thickness direction.

Now, once you know this, which way will I paste a strain gauge? Because alignment is also very important. Along the *x*-axis. So, I will put the strain gauge horizontal. I will have the strain gauge horizontal.

I have also shown it on this specimen. I can also put it on the horizontal. So, I can use the extensometer and then find out ε_{yy} and use a strain gauge and find out ε_{xx}. So, if I take $-\varepsilon_{xx}$ I have these expressions, ok? I have, you know, if I want to do this by strain gauge without an extensometer, I can also paste a strain gauge in the vertical direction and then connect two strain gauges and do my job.

So, I can find out Poisson's ratio as $-\frac{\varepsilon_{xx}}{\varepsilon_{yy}}$. That is the definition. You have a transverse strain; you have a longitudinal strain and you have a minus sign. This is how Poisson's ratio is defined and you have a standard ASTM D3039 explains what are the precautions that you have to take when you use two strain gauges to find out the Poisson's ratio. Here, I have simply put it on this, but when you follow the standards, you will not put the extensometer there, fine? So, you should know how to get Young's modulus and also how to get Poisson's ratio from a tensile test. Suppose I use digital image correlation, what was the advantage? In one go, I can measure ε_{xx} as well as ε_{yy}. I can get Poisson's ratio also without putting any additional instrumentation.

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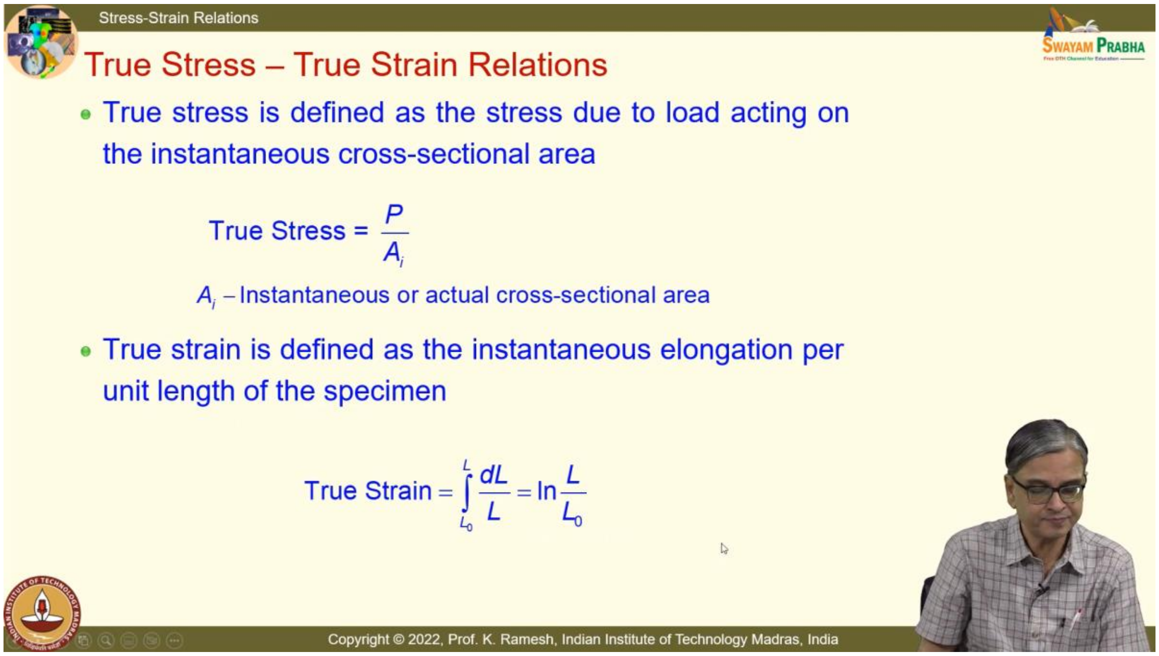

And I have also mentioned that there is something called true stress and true strain. The definition is, you know, we have said *P*/*A*. You improve the definition of cross-section *A* as instantaneous or actual cross-sectional area. See, this becomes important when the material starts necking.

When the material starts necking, you visibly see a change in the cross-section. But if you hang on to your original cross-sectional area, you are not going to make any headway with that. That is why the graph was going down, ok? So, this is another measure and we have also seen when we looked at finite strain, we have labeled this as Hencky strain, that is, $\int_{L_0}^{L} \frac{dL}{L}$. I have got this as $\ln \frac{L}{L_0}$. We defined that as true strain.

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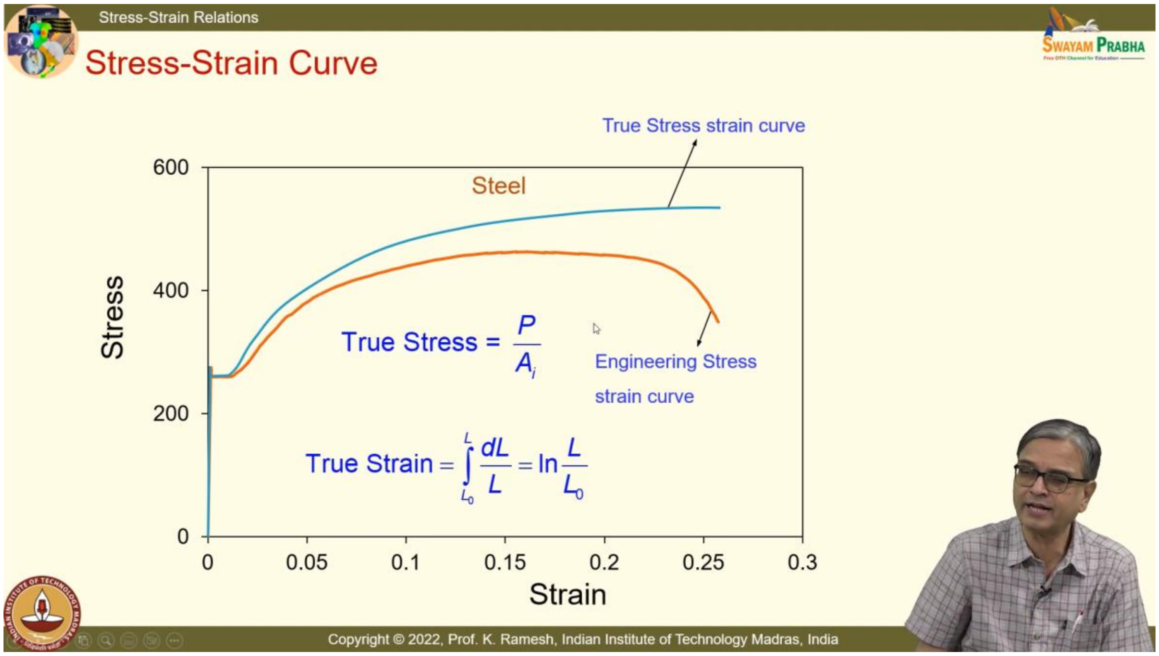

And I have used this and plotted for mild steel. Please make a sketch of this. You should know how does the graph change. After necking, this is drooping down because there is a visible change in the cross-sectional area. And when I plot it with the new definition of strain, the graph will keep moving up because you are also correspondingly changing the area of cross-section. And this is called a true stress strain curve and you have a definition of true stress as P/A_i and true strain is $\int_{L_0}^{L} \frac{dL}{L}$. Just to distinguish between your conventional stress-strain curve, they have labeled this as true stress-strain curve.

Other than this academic interest, I do not find any other utility of this straight away for your course. But you should know there are other measures by which you can measure strain and the graph appears differently. It is very simple; you have to just draw this graph above that. Even if you have the old graph of mild steel, you can just draw the true stress strain. You do not have to draw afresh, ok?

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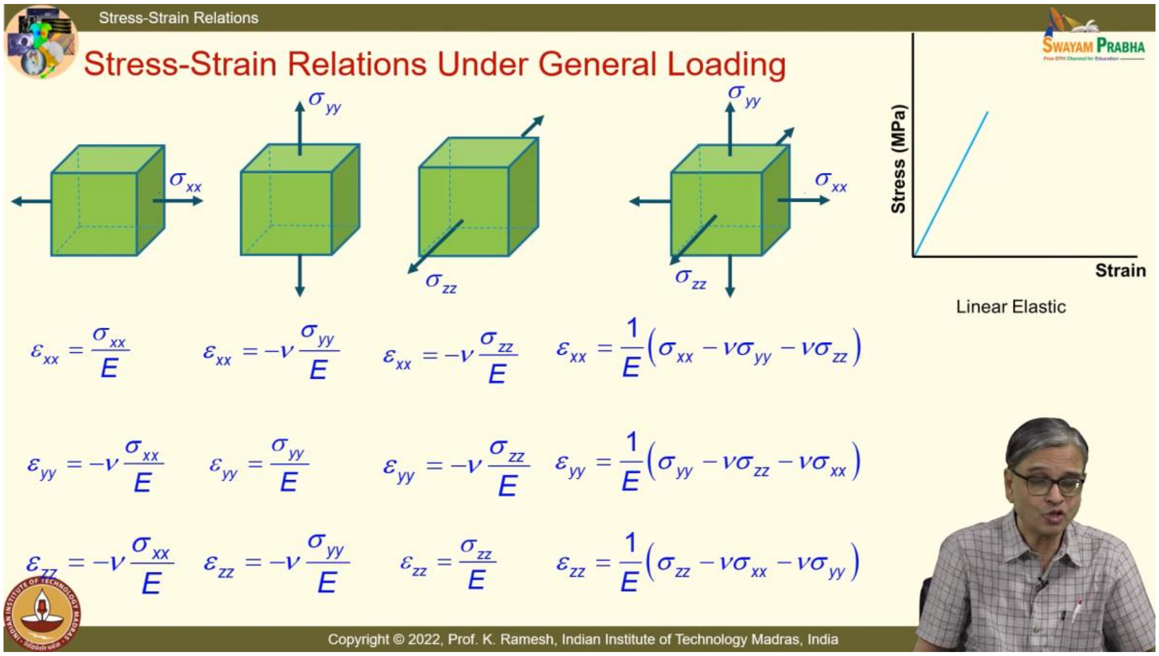

Now, let us summarize what we have learnt in uniaxial loading. In uniaxial loading, I have got $\varepsilon_{yy} = \frac{\sigma_{yy}}{E}$, $\varepsilon_{xx} = -\nu\frac{\sigma_{yy}}{E}$, and $\varepsilon_{zz} = -\nu\frac{\sigma_{yy}}{E}$. Suppose I apply the load in the x-direction, can you write down the strain in all the three directions? Make an attempt and then check it with my slide. Because once you do this for one direction, you can similarly write, ok? Cyclically you can write. Apply the load only in the x-direction. You have σ_{xx} applied and tell me what is the ε_{xx}, what is ε_{yy} and what is ε_{zz}. What you will have to recognize is, I apply a uniaxial stress, but strain is triaxial.

It is not difficult, it is simple and straightforward. So, I will show it for you. I have load applied in x-direction. So, I have $\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$, $\varepsilon_{yy} = -\nu\frac{\sigma_{xx}}{E}$, and $\varepsilon_{zz} = -\nu\frac{\sigma_{xx}}{E}$. Can you repeat the same for σ_{zz}? You should now realize where I am going towards. Because our ultimate idea is, we would like to know under a generic loading, how do I write the stress strain relationship. Is the idea clear? And now, we will take advantage that we are looking at a linear relationship.

The linear relationship gives me a mathematical advantage. And σ_{zz} you can write in similar manner. I will have $\varepsilon_{xx} = -\nu \frac{\sigma_{zz}}{E}$, $\varepsilon_{yy} = -\nu \frac{\sigma_{zz}}{E}$ and $\varepsilon_{zz} = \frac{\sigma_{zz}}{E}$. Now, let me ask a question. Suppose, I have simultaneously σ_{xx}, σ_{yy} and σ_{zz} acting on the material, can you write the stress strain relationship? How will you go about? Superposition; that is very good. That comes from the strength that the behavior is linear. That is the reason why we hang on to small deformation. The greatest advantage I have in small deformation is the relationship is linear, it gives me a via-media for the mathematical simplification.

I want you to simplify and then verify it from my result. I want you to get the expressions yourself. And we have also seen with sufficient discussion, even though it is a convenient way to say small deformation, in reality, in many practical applications, we work under very very small level of strain in actual loading. Because everything is dynamic, you are having only fatigue loading everywhere. If you come in your car, there are many moving parts. Even if you are sitting in your chair, you get up and then come back. That means, chair is also experiencing a fatigue loading. As a function of time the load changes. So, whenever there is a function of time the load changes, the maximum load that I can apply is smaller and smaller, which you have seen it by experience.

You could not have pulled that jump clip and then failed it, but if you bend it several times, you are able to fail it and you know what is your capacity. So, you would have applied only smaller load, fine? So, there is every justification that we say that we are looking at small deformation and the greatest advantage is this linear relationship. How many of you have got ε_{xx}, ε_{yy} and ε_{zz} expressions? Have you got? Can I show? I have a system where I have σ_{xx}, σ_{yy} and σ_{zz} acting.

And I write by superposition:

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}\right)$$

$$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\sigma_{zz} - \nu\sigma_{xx}\right)$$

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}\right)$$

See, strength of materials is a course you will always have to fall back on experiments. We are handling an isotropic material.

You know an isotropic material, we have seen, when you pull it, you have axial deformation. There is no shear deformation. So, we have accounted for all the normal stresses. Am I right?

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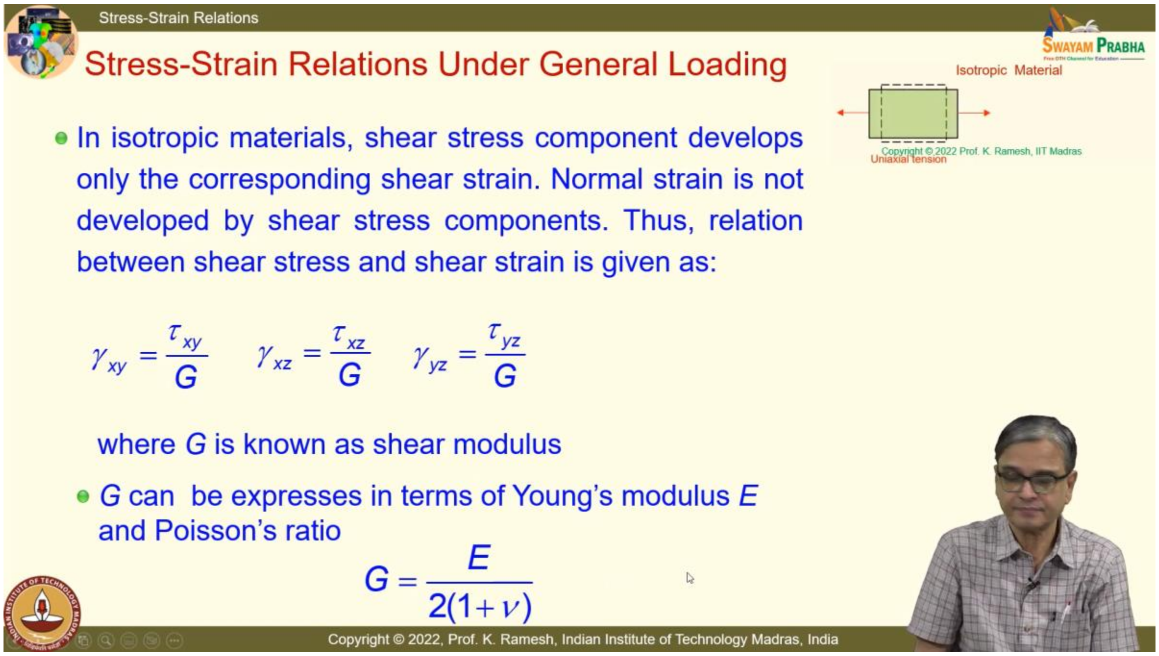

When I apply shear, I get shear deformation. So, that is the advantage of an isotropic material. So, you understand why we idealize the material to be isotropic. It makes our life simple. I have said, suppose I go and extrude the material, the fibers are aligned. Even a crystalline material, you will have alignment of grains. The material becomes anisotropic, but we still analyze it as isotropic. We will also know how to handle it.

We will take the specimen along the direction where you have fiber and conduct a tension test and find out the strength. So, what you have the advantage is, in isotropic materials, shear stress component develops only the corresponding shear strain. Similarly, a normal stress develops only the corresponding normal strains, ok? It is not one axis, but multiple axes. So, if I do this, I can also relate $\gamma_{xy} = \frac{\tau_{xy}}{G}$. See, even though tensorially we say $\gamma_{xy}/2$, ε_{xy} is what we are using it now. Traditionally, when the strength of material developed, people have visualized some of this from the experiments only based on γ_{xy}.

So, you cannot throw it out. So, you have a relationship between shear stress and shear strain, which is labeled like this. So, that completes the material characterization. And now, what we have introduced is, instead of Young's modulus, we have introduced another symbol G, which is known as shear modulus. See, like I have done a tension test, you also have a provision that you can take a shaft and then twist it. I can find out the shear stress and also the shear strain and draw a graph and you will get a straight line.

And your G is much smaller than Young's modulus. See, G is related to Young's modulus as

$$G = \frac{E}{2(1+\nu)}$$

It is a very famous relationship. We will also see how to prove this relationship. It is not that you have to look at this relationship and then remember it. You can also mathematically prove from your background of Mohr's circle, look at strain and stress and find out how this can be related.

And from this, what you find? See, ν is around 0.3. So, your rigidity modulus, this is also known as modulus of rigidity and you call this as shear modulus. There are multiple names and this is smaller than the Young's modulus.

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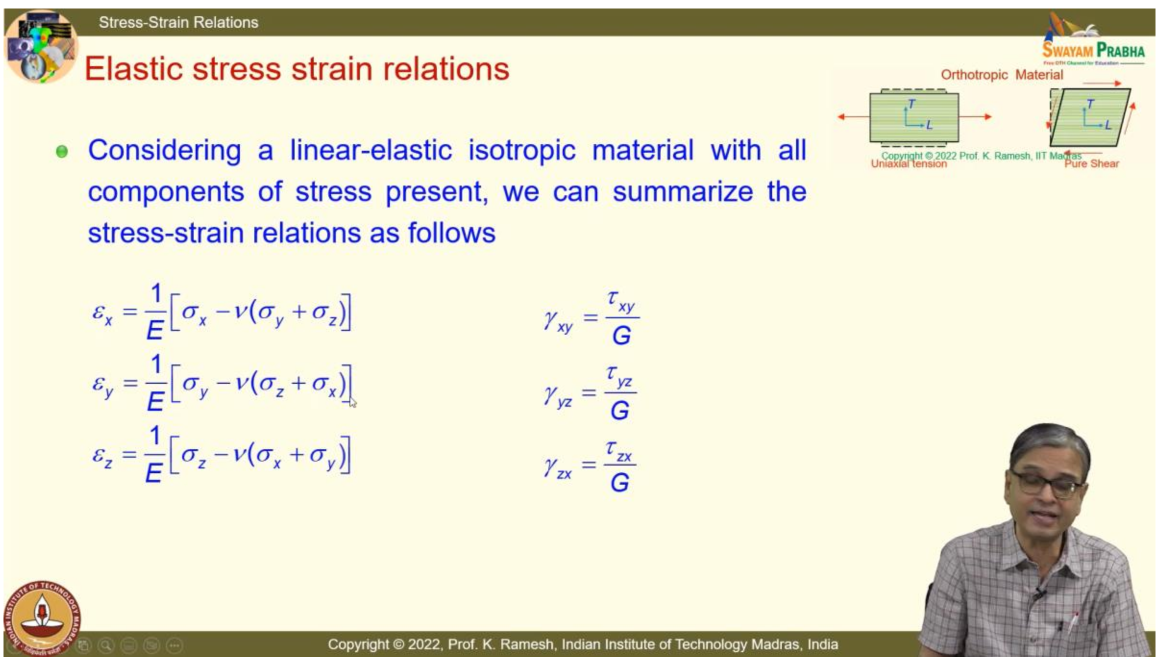

And let us summarize these relations. So, I have the elastic stress-strain relations. So, when you say stress-strain relation, you will have to write the complete expression for ε_x, ε_y and ε_z.

Do not stop at σ_x or σ_y. That can happen if I apply stress in the x-direction or y-direction alone. When you say stress strain relation, this is the stress strain relation, ok? And you have shear strain related to shear stress and what happens in an isotropic material? When I apply tension, it elongates. When I apply shear, it gets deformed. Suppose, I go to an orthotropic material and I call this as *L*-direction and *T*-direction, it gets elongated.

It also behaves like an isotropic material in these two directions. But when I go to an anisotropic material, if I pull this, it will elongate, also deform. Similarly, when you apply shear, it can have elongation and deformation. It is very very complex. That is why we do not go near an anisotropic material in the first development of the course.

We live in the comfort of isotropic materials because the material behavior is simple. You are in a position to model it comfortably. And whenever taking the stress strain relations, you should write these stress strain relations completely and knock off the terms that are zero. Do not say from your tension test, $\varepsilon_{yy} = \frac{\sigma_{yy}}{E}$, do not write! That is applicable only for a tension test. In a generic situation, $\varepsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\sigma_{zz} - \nu\sigma_{xx}\right)$.

So, with this, we have looked at and graduated to what is the simplest constitutive relation that you can establish between stress and strain in the case of a isotropic material. And this is also known as Hooke's law. We have given credit to Robert Hooke who was doing all the experiments when these aspects were developed in those ancient times. And we will look at this further and we will also establish interrelationship between elastic constants. In the process, we will also know what are the ranges of these that are possible, what is the range of Poisson's ratio, we will develop in the subsequent lectures. Thank you.