Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 18
Inter-relations between Elastic Constants

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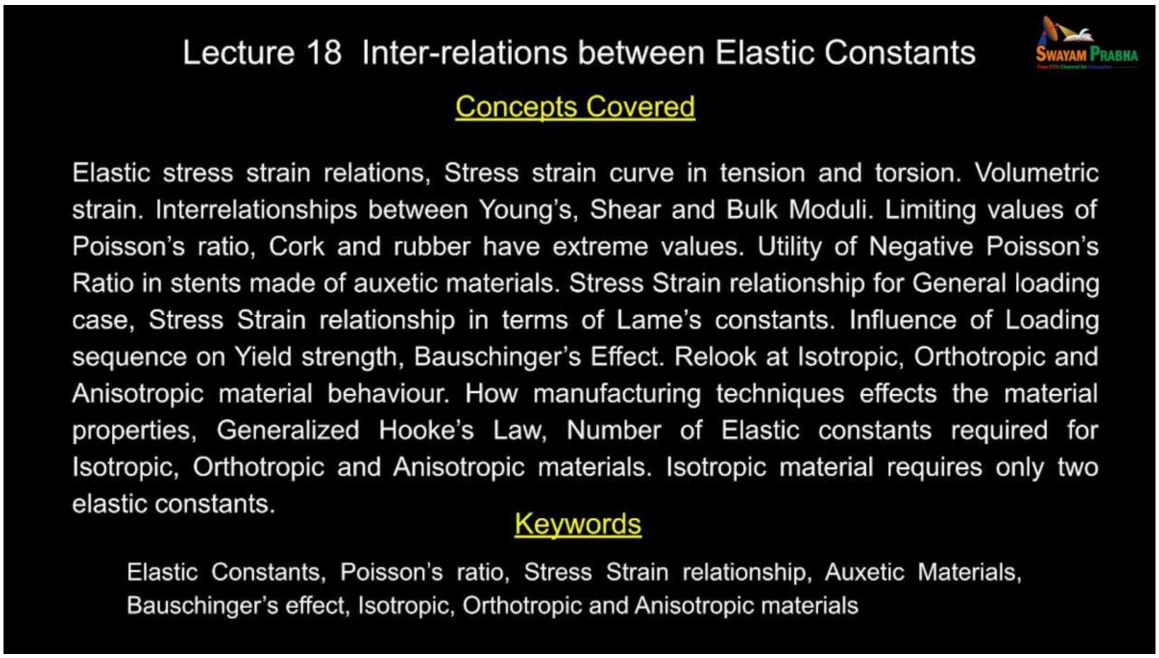

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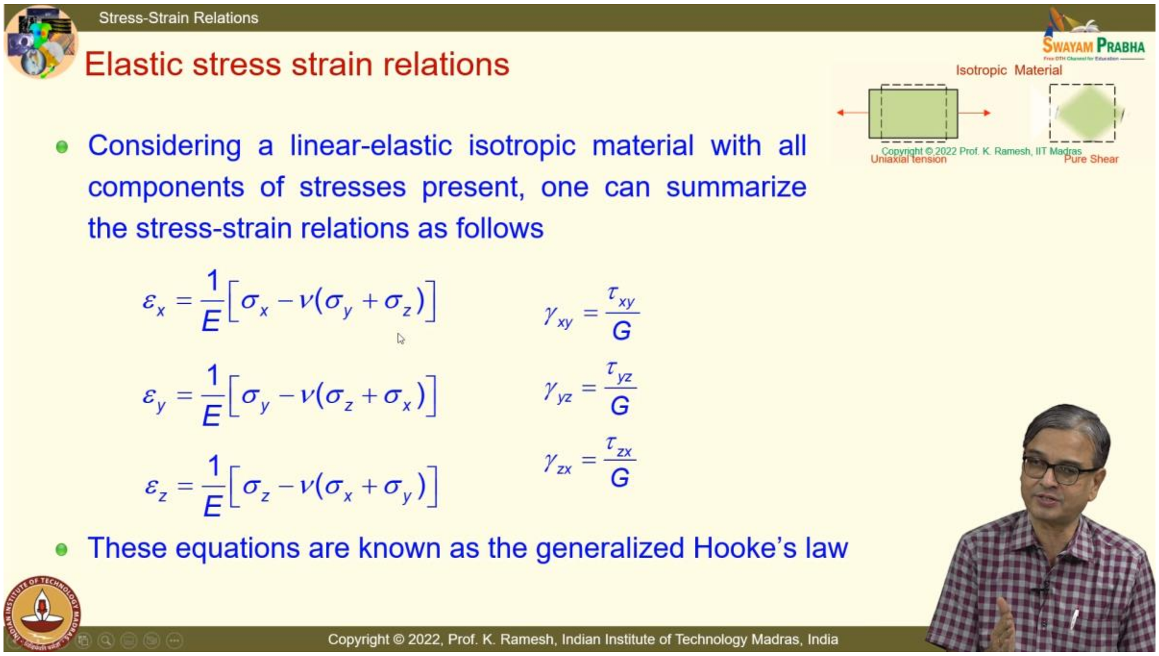

See, we have looked at tension test to characterize the material. Then we also developed in an elementary manner by using the method of superposition, how to get the relationship between strain and stress. Suppose, I want to use a material, I should know have I recorded all the parameters needed experimentally. The question is how many constants do I require to characterize an isotropic material? That is also very important question that needs to be answered. And we got the stress strain relations and an isotropic material when you apply a axial load, it will have an axial extension. When you apply shear, it will have a shear strain.

And we got the strain component in terms of all the normal stresses. And I cautioned you, when you look at the tension test, when stress strain relation is asked, people simply say ε_y equal to σ_y / E. That is valid only in the case of a simple tension test, where apply a uniaxial loading. I apply the uniaxial loading in the vertical direction.

In general, it will have an expression like this,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

And when you look at the relationship between the shear strain and the shear stress, traditionally it is reported with respect to γ_{xy}, $\gamma_{xy} = \dfrac{\tau_{xy}}{G}$ and cyclically you get the all the other strain components. So, when I look at this expression, what are the elastic constants that I see? I see *E*, I see ν, I see *G*. So, how many constant that you require to characterize the isotropic material based on this expression? But they are not really three, we will have to see, that is one issue. Another issue is, we have done it for an isotropic material and how do you perceive the elastic constant *E*? Is it a scalar, vector or a tensor? As it stands now, appears to be a scalar, because every direction is identical in the case of an isotropic material, that is one of the idealizations you make, that is it is elastically isotropic.

We would soon see that this is the tensor of rank 4, not even 2, it is a tensor of rank 4. And you can also say in some sense, this as generalized Hooke's law, but true generalized Hooke's law should cover isotropic material, orthotropic material as well as anisotropic material. We will that will also, we will see towards the end of this lecture, ok. In comparison to what you learn in a tension test, you have little more generalization of the material response.

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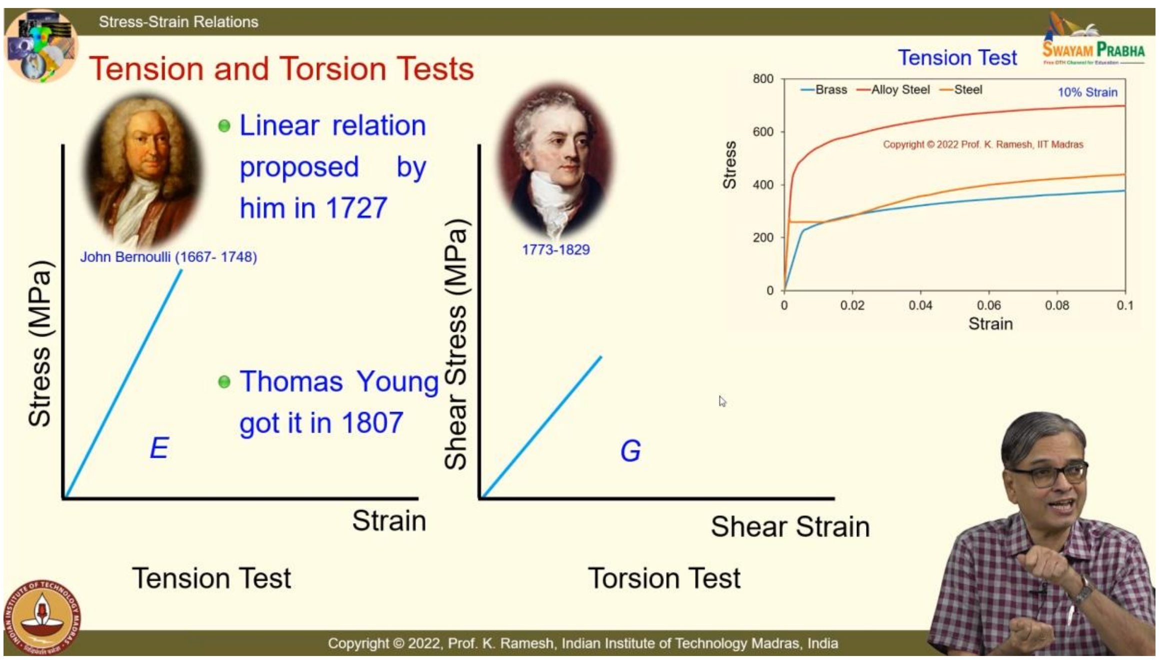

And you know the question is, should I do tension test or should I go for a torsion test? And in both cases, you have a graph drawn and this is a linear portion that we are confining, because we are looking at very small deformation.

And we have seen in the case of a tension test, it is easy to pull, I have to just do the pulling, that is all I have to do. And if I paste a strain gauge appropriately in one test, I can measure Young's modulus as well as Poisson's ratio, is that clear? Suppose I want to do the torsion test, this is just a history, just to give a recognition to Thomas Young, that Young's modulus is in his honor called as Young's modulus. And when you have a torsion test, I have a shear strain plotted on the *x*-axis and shear stress plotted on the *y*-axis. First thing is, I should have a loading jig, where it applies torsion not tension, but you have loading jigs that is capable of doing it. When I come to measure the strain, we have seen, we have also had a lecture on strain gauges.

How many strain gauges we use to measure the torque? We use four of them, we wanted to amplify the signal, even in principle, one strain gauge also you can use, but it should be aligned properly. So, even the measurement of shear strain is not going to be simple, when I do the torsion test. So, from the torsion test, I will get a linear graph and the slope gives you shear modulus, it is also known as modulus of rigidity. And from the tension test, you get the Young's modulus and it is also relatively shown, you know the stiffness of G is smaller than E. So, the slope is different here and when you look at what is drawn here is a tension test result, all the material model that we have simplified are all embedded in the data.

If I have a linear elastic and perfectly plastic, you see it here. If I have a linear elastic and a bilinear curve to represent the plastic region, you have it here. And if I have a straight line followed by strain hardening, it is there. So, the only trick is you confine your strain to a very small value, where you see all these simplified material model, they are very useful. And you should also understand that this is plotted for the bulk material and how the loading is done? The loading is gradually applied and it is done at room temperature.

See, now we work on extreme environments, we work at very high temperatures as well as very low temperature like your cryogenic engine and so on. So, those applications you have to redo the tension test and characterize the material. And these days, there is also another threat, people do bombing and in that case, the strain rates are very very high. How do the materials behave when I have high strain rates? You have specific tests meant for that. So, what you have learnt in the simple tension test is very basic, this is for a bulk material. Done in a manner that the loading is gradually applied at room temperature, fine. That is how you start learning how to characterize a material.

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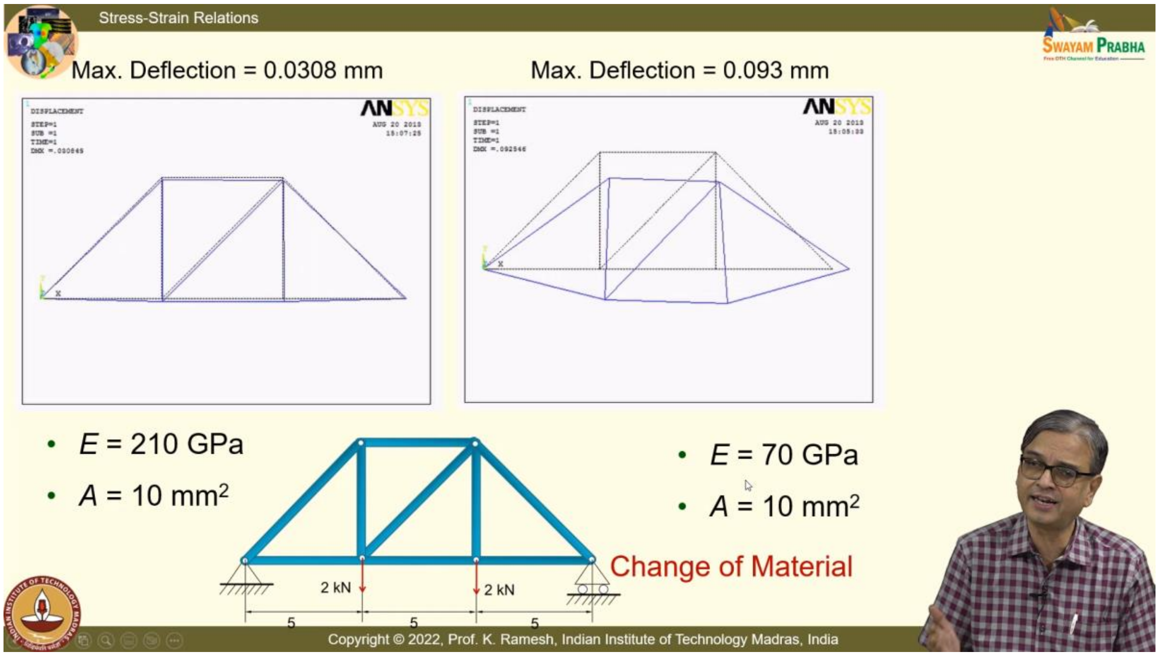

In in strength of materials, what you want to see? Why I use different materials for a given application? It is in a very exaggerated picture, I have the truss and the truss is loaded like this. In your previous course, you have learnt how to find out the member forces by method of joints and method of sections and you are not bothered to find out how the truss will deform. Suppose, I plot an exaggerated picture, see the maximum deflection is only about 0.03 millimeter, it is very very small. I have one material, this is by looking at the Young's modulus, you should be able to say that this is steel. By looking at the Young's modulus here, you should say that this is aluminum. And you find when I change the material, you could see distinctly the deformations are much much higher. This is an exaggerated picture to drive home the point material selection is also a purpose, why do you use a course on strength of materials? So, you should know what is the difference in this, but the maximum deflection are still very small. Now, the question is we have seen in the stress strain relations, we saw Young's modulus, shear modulus as well as Poisson's ratio. Are they distinct? Is there an interrelationship between them? Are all of them are independent or only two of them are independent?

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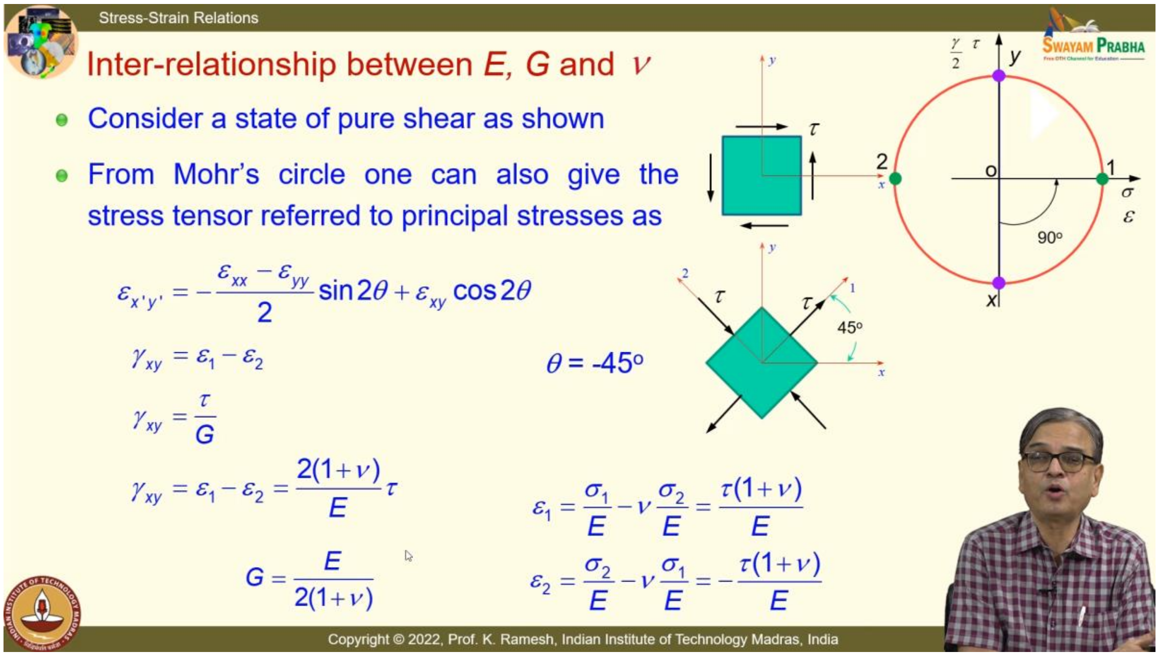

Now, we take up a simple situation, where I take a pure shear state. See, this is also an application of the concepts that you have learnt so far. So, please draw the Mohr's circle and label the points. Another very important aspect, see most of our discussion will be confined to isotropic materials.

In the case of isotropic materials, what you have as principal stress directions, what you have as principal strain directions, they are identical. That gives you lot of simplification in your understanding of the material behavior and we would use that. Have you drawn the Mohr's circle? Check it with mine. So, I can plot on the horizontal axis σ, on the vertical axis shear stress and for this on the x plane, using our convention you plot it downwards and your y plane is plotted upwards and you draw the circle. So, this gives a complete picture of what happens at the point of interest.

We have already seen, I can represent the stress state by taking values from any diameter, fine. We are going to use that to establish. I can also replace this stress state by using the principal stresses, which are marked as 1 and 2 and you know that in the Mohr's circle it is 90°, in the physical plane they are separated at 45°. I can rewrite this stress state with respect to the principal stresses. Can you draw the sketch and also label the values? From the Mohr's circle you can find out what is the value of σ_1 and what also the what is the value of σ_2 for a pure shear stress state.

I want you to draw the Mohr's the pictorial representation with respect to the principal stresses and also the label the magnitudes, then check with my diagram. Because that will help you to appreciate how do we go about in establishing the interrelationship. Can you do that? Have you done that? You try to do it. See there should not be hesitation if I make a mistake what will happen? Make mistakes that is how society has learnt many of the happenings. There have been spectacular failures. If scientists have designed it correctly why there is failure? From failure and the failure analysis they learnt where they have gone wrong, what idealization was wrong and then go and improve the theory. So, you make a mistake that is the best way to learn a subject. Suppose I draw it with respect to the principal stresses, you have at 45° from x you go to σ in anti-clockwise direction, in the real plane you go by 45°, I put the principal stress σ_1, the magnitude from the Mohr's circle you get that as τ, fine. And you have in the two direction a compressive stress whose magnitude is also τ. And I said Mohr's circle of stress and Mohr's circle of strain are identical if you adjust your axis properly.

So, I can also visualize the same circle as Mohr's circle of strain where I have the normal strain here and where I have the shear strain here. The idea is to inter relate what is the relationship between Young's modulus and shear modulus, fine. I can also look at the Mohr's circle of strain from your transformation law,

$$\varepsilon_{x'y'} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\sin 2\theta + \varepsilon_{xy}\cos 2\theta$$

Now, what I am going to look at is, I want to express γ_{xy} in terms of the principal strains. So, I am going to look at write my strain in the principal strain direction that means I will have only ε_1 and ε_2, is the idea clear? From that I want to express what happens in plane x and plane y.

So, that means my xx will go to 11 and yy will go to 22 and then $x'y'$ will be xy. And tensorial transformation you have to write $\varepsilon_{x'y'}$ in a engineering sense I can write this as $\frac{\gamma_{x'y'}}{2}$. So, you should understand that. From the Mohr's circle can you tell me what is the value of γ? You look at the same circle as a Mohr's circle of strain can you tell me what is the value of gamma? Very conveniently you can say. What is the value of γ in terms of ε_1 and ε_2, is the question clear? See you have the radius, radius is $\gamma/2$ the diameter is γ I have given you enough clues.

How do I get γ value? $\varepsilon_1 - \varepsilon_2$ ok, same thing you can get from multiple ways I can substitute theta equal to -45 because I am going from the principal direction to x direction. So, I rotate in the clockwise direction, clockwise is given as minus sign I can write from there or I can directly write from Mohr's circle $\gamma_{xy} = \varepsilon_1 - \varepsilon_2$ is the idea clear? Can you write what is ε_1 and ε_2 in terms of the normal stresses? Can you do that because we have just now seen the strain stress relation. Now, I am looking at the stress quantities which are τ and $-\tau$ you should keep track of what is the stress state that we are having. So, I can also write ε_1 and ε_2 in terms of normal stresses let us see those expressions. Before that you know we have the identity that $\gamma_{xy} = \frac{\tau_{xy}}{G}$ you should understand this is τ_{xy} Since we are dealing with interrelationship and even if I do not write the symbol you should implicitly understand in the context it goes to $\frac{\tau_{xy}}{G}$. I can write ε_1 and ε_2 in terms of the normal stresses. Can you write that and check it with me? So, I get this as ε_1 equal to

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E}$$

When I substitute what is σ_1 and σ_2? σ_1 is τ and σ_2 is $-\tau$. So, when I substitute I get

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \frac{\tau(1+\nu)}{E}$$

Is the idea clear? See, when we have learnt it in terms of *x, y, z* you should know how to convert it from *x, y, z* to 1, 2, 3. So, I can also compute what is $\varepsilon_1 - \varepsilon_2$ from this that gives me

$$\gamma_{xy} = \varepsilon_1 - \varepsilon_2 = \frac{2(1+\nu)}{E}\tau$$

And you already have another definition of γ_{xy}. So, when I equate the two I get an identity that shear modulus G as

$$G = \frac{E}{2(1+\nu)}$$

It is a very famous relationship. So, from this what do I understand? Even though for me to write a generalized strain stress relationship I needed E, ν and G of the three only two are independent. The other one is interrelated. So, for isotropic materials the greatest convenience is it is enough you get any two elastic constants. See there is also another strain that is discussed in strength of materials known as volumetric strain. The moment I go to volumetric strain I cannot remain in plane *xy* I have to go to z direction also, fine.

So, there again we will develop one more elastic quantity and we will find an interrelationship. And we will also find out what should be the limiting values of the Poisson's ratio ν, fine.

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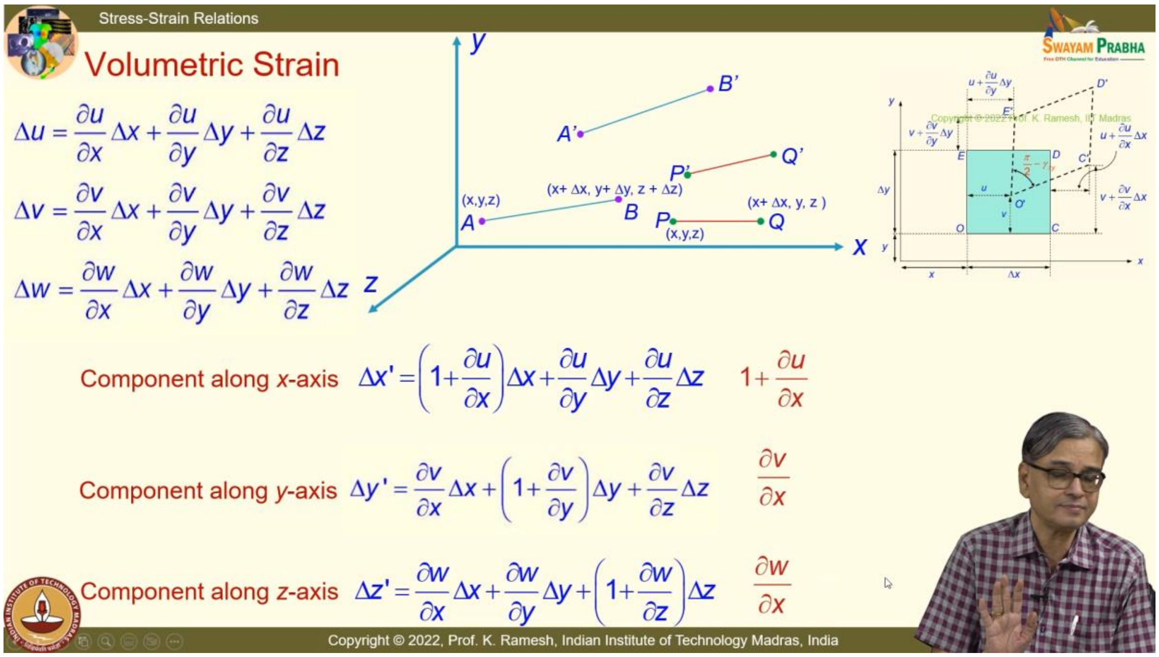

See, the moment I go to volumetric strain I have to work in three dimensions because I am talking about volume. And we have already seen in your strain displacement we have labeled it and we have taken deliberately a planar situation. We have now looked at the *z* direction we have confined ourselves to *xy* only.

But when we developed the finite strain we also discussed what happen in three dimensions. And you know in my drawing it is difficult for me to show the points *AB* somewhere in the three-dimensional space. But I have labeled the coordinates appropriately a is *x*, *y*, *z*, *B* is $x+\Delta x$, $y+\Delta y$, $z+\Delta z$. What is the difference between this and this? There will not be any *z* quantities in this. In this I have all the three coordinates are available.

So, the points *A* and *B* are situated in space, fine. And they get deformed and then moved to new positions $A'B'$. We have also seen that when we discuss the finite strain. And I have this will go to a displacement u, this will go to displacement $u+\Delta u$. Since we have the displacements u, v and w are also functions of x, y and z.

We have said that Δu should be from your Taylor's approximation is

$$\Delta u = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

And cyclically you write it for Δv and Δw, fine. It is fair enough, I mean that is how we have looked at the quantity we are now looking at in a three dimensional situation. Now, what I am going to do is I am going to take an element which is along the x direction. Before that let us also understand when I go from AB to $A'B'$ the B has got shifted, B' has got shifted.

And if you look at what are the components along the x-axis, y-axis and z-axis they will become

$$\Delta x' = \left(1+\frac{\partial u}{\partial x}\right)\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

Primarily because I have the coordinates of B' as shifted by $\Delta x+\Delta u$, fine. You have this as $x+\Delta x$, $y+\Delta y$ and $z+\Delta z$ and when I go to this point B' you will have all these quantities plus Δu , Δv and Δw. So, when I look at what happens to point B' this is what would happen. When I look at what are the component along y-axis I will have

$$\Delta y' = \frac{\partial v}{\partial x}\Delta x + \left(1+\frac{\partial v}{\partial y}\right)\Delta y + \frac{\partial v}{\partial z}\Delta z$$

And cyclically I can also write what happens along z-axis I have

$$\Delta z' = \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \left(1+\frac{\partial w}{\partial z}\right)\Delta z$$

When you look at this it looks as if it is too complicated it is not too complicated. Suppose I take an element like I have taken OC, I will also take points P and Q where I say the point Q is shifted only by Δx that y and z remains same. I am taking a line which is parallel to x-axis. Suppose I look at Δy Δz goes to 0 and look at the component along x-axis, y-axis and z-axis I will get what is that deformed position $P'Q'$.

See because it is a two-dimensional space where I am plotting I have not shown the distinction between AB and PQ. The distinction is brought out in terms of labeling the coordinates. So, you will have to have your visualization helping you to see that. And I get this as $1+\frac{\partial u}{\partial x}$ *x* component, *y* component is $\frac{\partial v}{\partial x}$ and *z* component is $\frac{\partial w}{\partial x}$.

This is what you have seen here. See here we have shift taken the two-dimensional situation, ok it has got distorted like this. And when you look at what happens to this it is $u+\frac{\partial u}{\partial x}\Delta x$ that is what you are putting it here, fine. And then here you have shifted by $\frac{\partial v}{\partial x}$. We have missed out the what happens in the *z* direction. So, the extra quantity $\frac{\partial w}{\partial x}$ comes, leaving that it is very similar to what we have done it for a planar situation. Now, we have graduated to a three-dimensional situation. Suppose I know what happens for a line parallel to *x*-axis. Now, I take a line parallel to *y*-axis and I take a line parallel to *z*-axis, I can construct an elemental cube, is idea clear? And I want to see what is the volume before deformation what is the volume after the deformation that is my interest.

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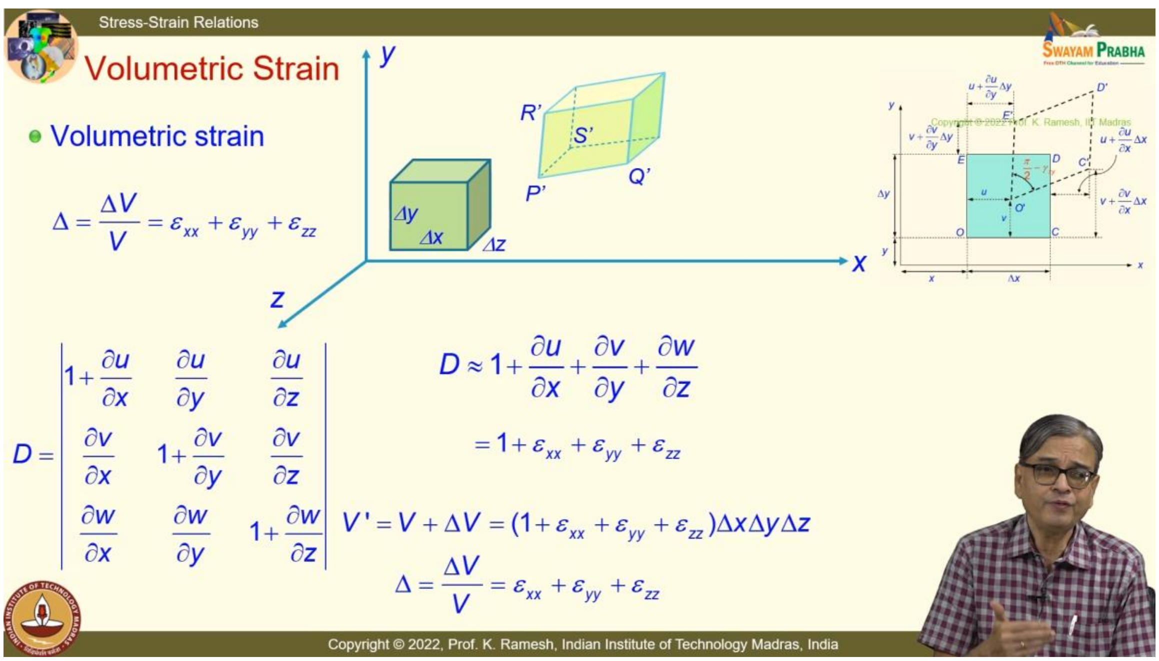

So, what I am going to see here is instead of just a line I will take a cube. I will take a cube that has dimensions Δx, Δy, Δz. So, that means I have a line which is parallel to *x*-axis to start with parallel to *y*-axis to start with parallel to *z*-axis to start with. And whatever we have discussed earlier we will borrow that result and then find out what is the deformed quantities, ok. Now, let me also show the deformed nature of the cube, I label this as $P'Q'$ and we have already discussed what happens to PQ which was originally parallel to *x*-axis. Now, following that discussion we will have to write for $P'R'$ as well as $P'S'$, is the idea clear? So, I am going to write $P'Q'$ $P'R'$ and $P'S'$ and we have seen already for $P'Q'$ in the previous slide. They are nothing but $1+\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$, $\frac{\partial w}{\partial x}$.

In the generic expression we have written only Δx we have made Δy and Δz zero, for a line which was originally parallel to *y*-axis that is this line, I have no Δx I have Δy and Δz is also zero. So, when I do that I get this as $\frac{\partial u}{\partial y}$ and component along *y*-axis as $1+\frac{\partial v}{\partial y}$, component along *z*-axis as $\frac{\partial w}{\partial y}$. And similarly, I can also write it for $P'S'$. I get this as $\frac{\partial u}{\partial z}$ I get this by $\frac{\partial v}{\partial z}$ and I get this as $1+\frac{\partial w}{\partial z}$.

Now, go back to your analytic geometry. See this is a deformed parallelepiped, ok. How do I find out the volume of this? I know the vectors $P'Q'$ $P'R'$ and $P'S'$. If I know the three vectors, how do you find out the volume? you remember? triple products scalar triple product. So, that is what I am also going to do, fine. So, you know when you learn engineering you cannot afford to forget the mathematics many times what happens is you study mathematics as a separate course to get grades you have to remember some of those concepts when you want to learn engineering subjects also.

So, that is nothing but your determinant, ok, into Δx, Δy, Δz and I have to get the determinant of this whatever the quantities that we saw earlier and when we find out the determinant we will use the identity we are working on very small deformation. So, product of small quantities will be much smaller. So, neglect all of them. So, then I will intelligently write this product as the determinant value as

$$D \approx 1+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

neglecting all the higher order terms. If I have product of $\frac{\partial u}{\partial x}\times\frac{\partial v}{\partial y}$, I neglect because product of small quantity is much smaller because I am dealing with very small quantities.

So, this when you go back to your original definition of what is strain I can $\frac{\partial u}{\partial x}$ as ε_{xx}, $\frac{\partial v}{\partial y}$ as ε_{yy}, $\frac{\partial w}{\partial z}$ as ε_{zz} and the volume is demultiplied by Δx, Δy, Δz this is the deformed volume the original volume is $\Delta x\ \Delta y\ \Delta z$. So,

$$V' = V + \Delta V = (1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})\Delta x \Delta y \Delta z$$

and when you want to write Δ which is labeled as $\frac{\Delta v}{v}$ like you have the extensional strain change in length divided by original length here change in volume divided by original volume. I get this is a very famous expression it is nothing but addition of the axial strains

$$\Delta = \frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

So, this is known as a volumetric strain and this is a very famous expression that we have determined it in a very systematic fashion.

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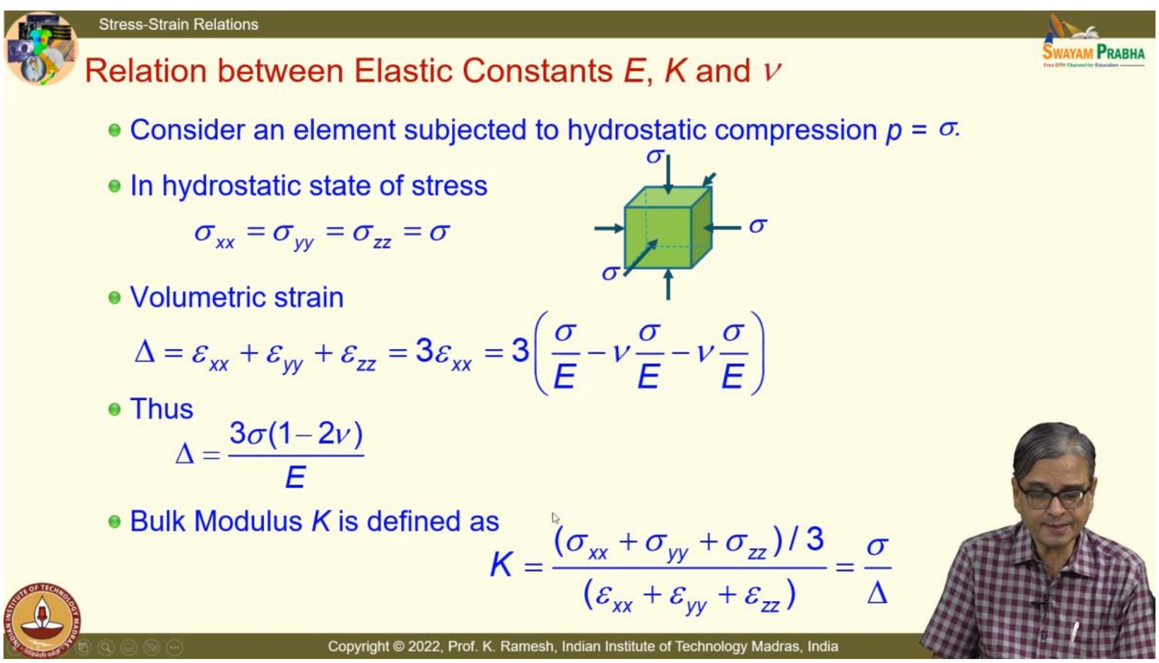

Now, we will go and see one more elastic constant what we call it as bulk modulus and here you take an element which is subjected to a compression the pressure is p since the direction is already shown only the value is shown as σ and I have stresses acting on all the three faces they are of equal magnitude that is why it is called hydrostatic the idea is if you immerse anything inside a liquid, fine, you will have equal pressure acting on all sides. So, it is a hydrostatic state of stress and what is the strain quantities? because our objective is to find out $\varepsilon_x + \varepsilon_y + \varepsilon_z$. Since I have all these quantities identical on all the three phases if I say ε_y will be identical to ε_x. So, I can say $\varepsilon_x + \varepsilon_y + \varepsilon_z$ as three times ε_x and write down write out the values. So, volumetric strain is $\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ since the stresses are equal, I can write this as three times ε_{xx} and I can write what is ε_{xx} based on my original proper strain stress relation where I have to count for σ_x as σ_y as well as σ_z in this case all of them equal to σ.

So, this reduces to

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 3\varepsilon_{xx} = 3\left(\frac{\sigma}{E} - \nu\frac{\sigma}{E} - \nu\frac{\sigma}{E}\right)$$

You should not forget these quantities, that is a common mistakes people do when they learn strength of materials for the first time the tension test is so dominant you do not see beyond that. So, please do not make this mistake you should account for the other stress components their contribution to the strain. So, now I know the volumetric strain and you have the definition I have the volumetric strain when it is simplified I get this

$$\Delta = \frac{3\sigma(1-2\nu)}{E}$$

You have a definition, there is a definition of bulk modulus it is defined as

$$\frac{(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3}{(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})} = \frac{\sigma}{\Delta}$$

So, this is a definition of what is the bulk modulus now we have all the quantities we have Δ in terms of σ.

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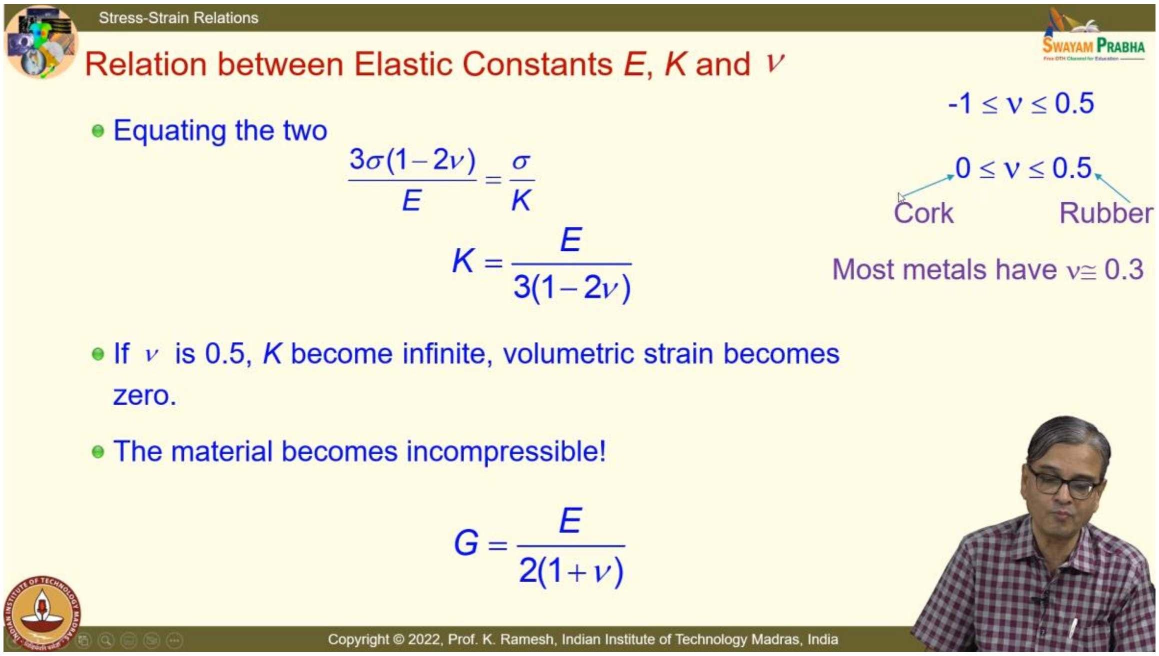

So, when I substitute this I get an expression for bulk modulus K in terms of Young's modulus E as $G = \frac{E}{2(1+\nu)}$ .So, what is your first take what will be the value of K in comparison to Young's modulus? We have already seen shear modulus is smaller than Young's modulus. So, similarly your bulk modulus is also smaller than Young's modulus and what you have here this gives me a limiting value of what is the value of ν, what is the highest value ν can take because I cannot have this go to zero if it goes to zero, I get this as infinity.

So, when ν is 0.5, K becomes infinite volumetric strain becomes zero. So, one limiting value of ν is 0.5 and you have this idealization see these are all justification that you say the material is incompressible and then do the analysis particularly in the development of plasticity you know it is very mathematically highly demanding. One of the aspects we have also looked at when we discussed strain gauges when the material becomes plastic the Poisson's ratio approaches 0.5. So, one of the idealization they do in simple plasticity theory is the volume remains constant that is one of the conveniences in simplifying the material model that comes from a discussion like this.

We have looked at what is the highest value of Poisson's ratio, is there a way that you can find out what is the lowest value possible from whatever we have learnt till now is there anything that you can take up and then say based on this the lower value is also fixed for Poisson's ratio, any guesses? See we have just now seen a relationship between Young's modulus and shear modulus what was it like it was it like $G = \frac{E}{2(1+\nu)}$. So, this gives me the other limit that it cannot go beyond -1. So, I have the range of Poisson ratio -1 to 0.5, but most materials have between 0 and 0.5 and you have a very interesting material cork. Cork has Poisson's ratio close to 0 it may have some small value, but we declare that it is 0. Similarly, rubber is also very close to 0.5, ok and most metals have ν equal to 0.3. So, what we find here is when we discuss shear modulus when we look at bulk modulus we have been able to establish a relationship between Young's modulus, bulk modulus and Poisson's ratio.

Similarly, an interrelationship between Young's modulus, shear modulus and Poisson's ratio and this interrelationships declare any two elastic constants are sufficient, you may give Young's modulus and Poisson's ratio or bulk modulus and shear modulus you can find out all other quantities. An isotropic material requires only any two of the four elastic constants that we have brought into focus and we have also seen what is the limiting value of the Poisson's ratio. And if you look at cork it is naturally occurring material it is actually bark of a tree that they use it and some of the applications of cork which has never changed even with other material development. One of the things that they use is or alcoholic seasoning they put the sealant by using a cork you are going to apply compressive stress and then insert it in the bottle. Normally, when you look at Poisson ratio what we imagine that when I stretch the lateral part gets opposite strain when I stretch it, it gets shrunk when I compress it, it can bulge out.

So, if you are sealing a bottle when you are applying a compressive force if it bulges out it will not it will be difficult for you to insert it that is one of the advantages. Another advantage what they say is that it also allows oxygen into the bottle that is one of the reasons and also in some of the shoes that they have the heel stillness cork and it is a green material in the sense you know you can replenish it. It appears that every 12 years or 13 years you can remove the bark carefully and the tree will again grow the bark as cork. So, it is a green that is also very important we now become environmentally very very sensitive, we want everything to be recyclable.

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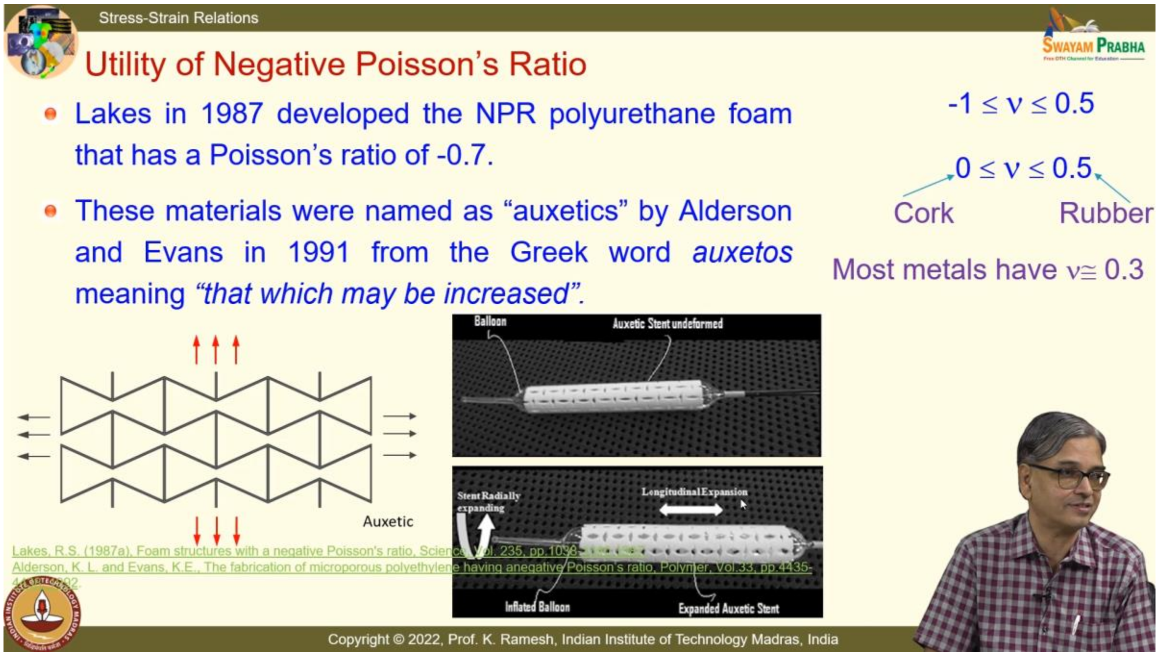

And what is the utility of negative Poisson's ratio? Say I said you also have the range as -1, people have developed exotic materials that is he has developed polyurethane which has a negative Poisson ratio it is put as NPR, he was able to develop it with Poisson's ratio of -0.7 and such materials were named as auxetics that comes from the Greek word auxitas means that which may be increased. So that means, when I stretch it instead of laterally contracting it should bulge out you get the idea. So, that is achieved by a different structure of the material. This finds application in these stents. See you hear these days instead of going for a bypass surgery many of the elders in the family will go to the hospital and get a stent implanted in their arteries.

And one of the ways that they implant is they have a balloon and they will blow the balloon. So, what will do is it will bulge out at the same time it will also elongate in the longitudinal direction that is a desirable characteristic for a stent to stay in place. So, people have developed such type of material particularly the medical profession uses the negative Poisson's ratio so well.

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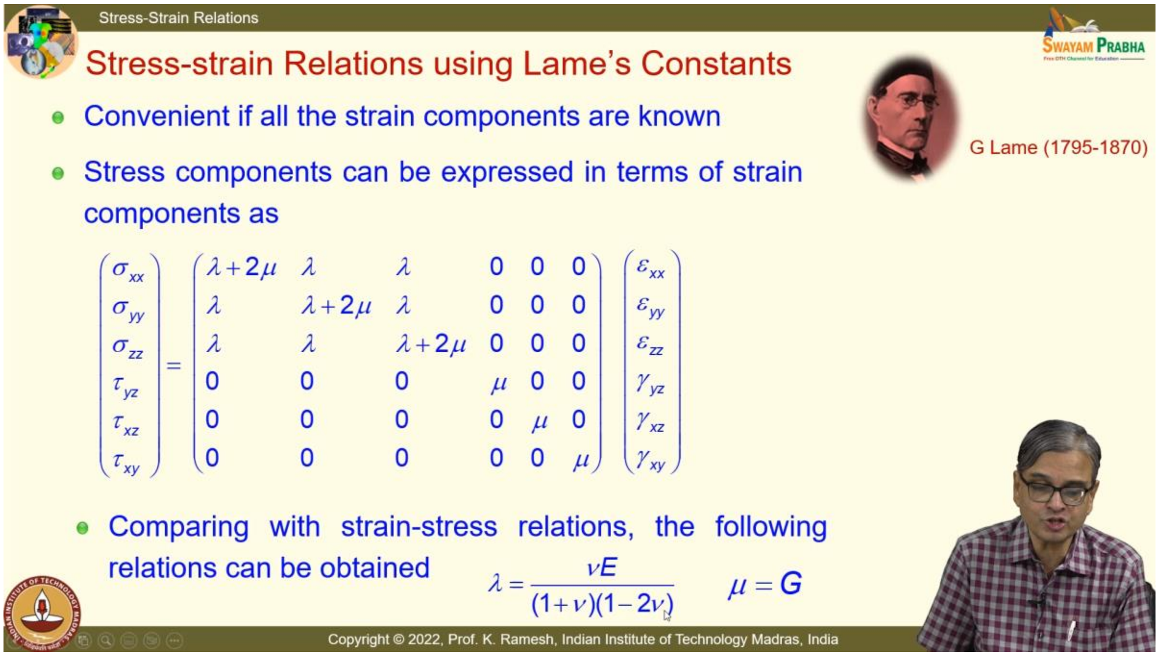

See, I can also write the same thing what we have looked at in a matrix relationship because my interest is to show later, you know I can have what I have this as elasticity matrix is actually a tensor of rank 4. We saw that as a scalar in isotropic material whether you take it as a number or tensor it does not matter because you are never going to worry about the direction. And I can also express strains related to stress instead of writing it as long hand it is written in a matrix form there is nothing new here, ok. See, I have a relationship between strain and stress I can also ask a question can I write the relationship between stress and strain? This was also attempted. This was attempted by Lame and you know here again I emphasize that E, G and ν you have and $G=\frac{E}{2(1+\nu)}$ again to remind you that when we deal with isotropic materials you have a greatest advantage you have only two elastic constants that are required even though I have three in the matrix here only two are sufficient because the third one is dependent on the other two. I can also write it in terms of Lame's constants it is all together by a different approach and the idea is if I know the strain quantities how do I relate the stresses.

He came up with some other elastic constants. So, this you write the diagonals as $\lambda+2\mu$ this is the symbol that he has used and we have to find out what is the interrelationship between the symbols λ and μ to what we have learnt as E G and ν, ok. This is credited to Lame and easily you can relate what you have as μ is nothing but your shear modulus that you can easily say the other one is difficult to look at from the expression you have to derive and this lambda is related to Young's modulus as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

So, this is also another set of expressions. See, when people approach the same problem from different perspectives, if you agree on the essential analysis that you require only two elastic constants that gets reinforced because I told you at the start of strength of materials people started that you need only one elastic constant because they assume for all material Poisson's ratio is a fixed value of 0.25, by looking at the Poisson's ratio ratio now if you look at many of the materials hover around that you may make a small error that is one aspect but from a fundamental aspect you need a minimum of two elastic constants but you can appreciate that you require only two elastic constants in an isotropic material only when you look at an anisotropic material how many elastic constants that you require I will also give you a brief outline of that for you to appreciate we are living in a very comfortable domain isotropic materials where two elastic constants are sufficient, ok.

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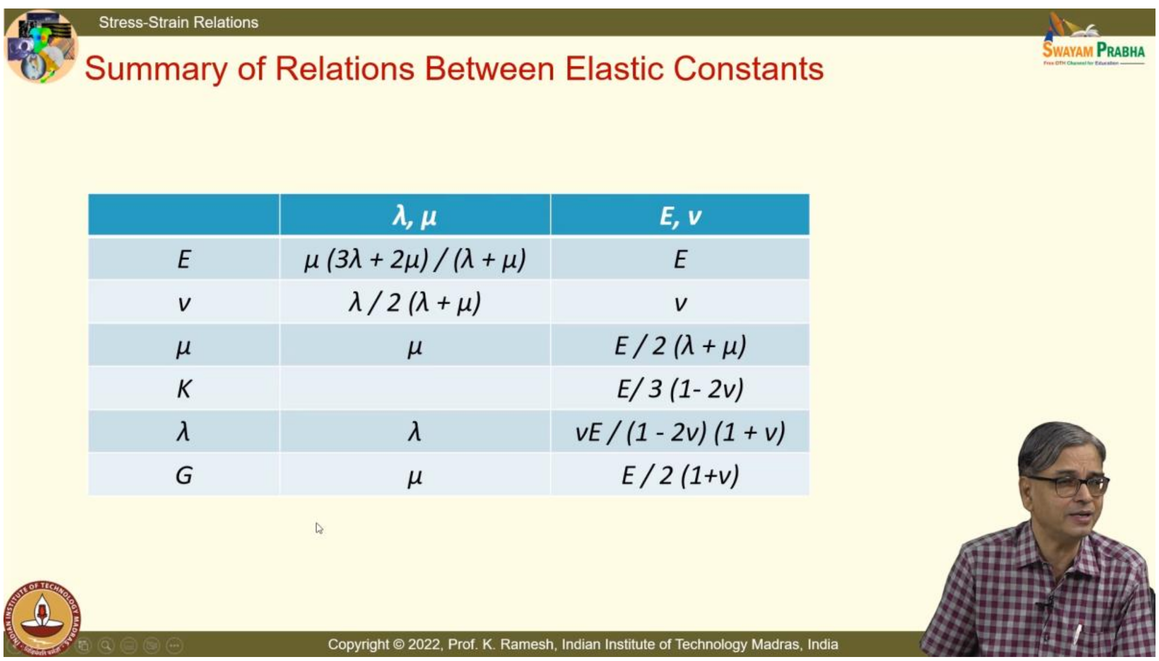

You can just take a snap of it I am not going to read this these are all the interrelationship that is available from an academic interest you need to have this table in your notes, fine but we all know that $G = \frac{E}{2(1+\nu)}$ that is a very famous expression that we will repeatedly use for many of our problem solving and simplifications other quantities are from an academic interest point of view.

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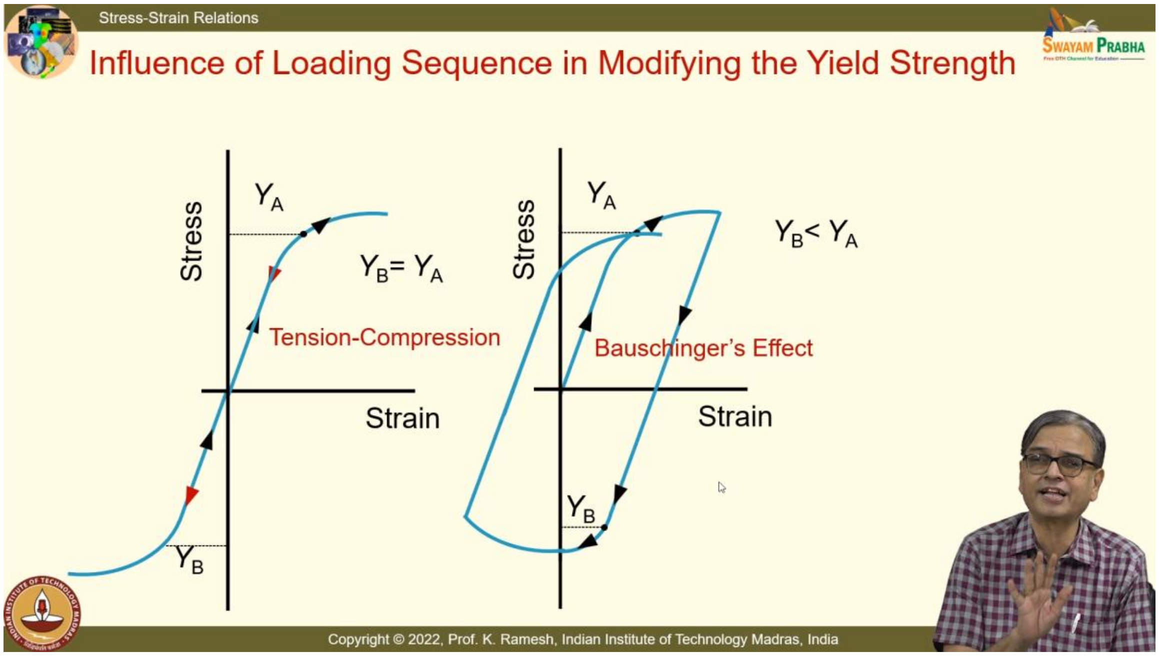

And you know before we move on to the generic material, see people also have looked at suppose I take a ductile material, I pull it I go to yielding what happens then I also compress it and then go to yielding in that compressive zone see in the case of brittle materials we saw young's modulus in tension and compression are widely different they are very strong in compression, they are very weak in tension, unlike that a ductile material is having similar yield strength both in tension and compression when you load it like this. Suppose I load it to plastic region then I unload it and go to compression people have found that young's modulus has decreased then I load it unload it and then load it to elastic region, I find this Y_A also decreases Y_B also decreases. So, this is credited to a scientist called Böschinger it is known as a Böschinger's effect and people have looked at all of this. See, the moment you want to go to plasticity history of loading becomes very important you may think why history is important? They are transporting material in a truck and then truck has an accident and all the material fell on the road that means they have an impact load some of them would have deformed plastically and if I want to make a component out of it then I want to see what would be its failure strength its behavior may be totally different.

So, if you want to model plasticity history of loading is also very important and people have done many tests they will do tension compression, tension shear all these combinations you have. So, they are all special situations special situations need additional experiment and additional understanding that is the take from this discussion.

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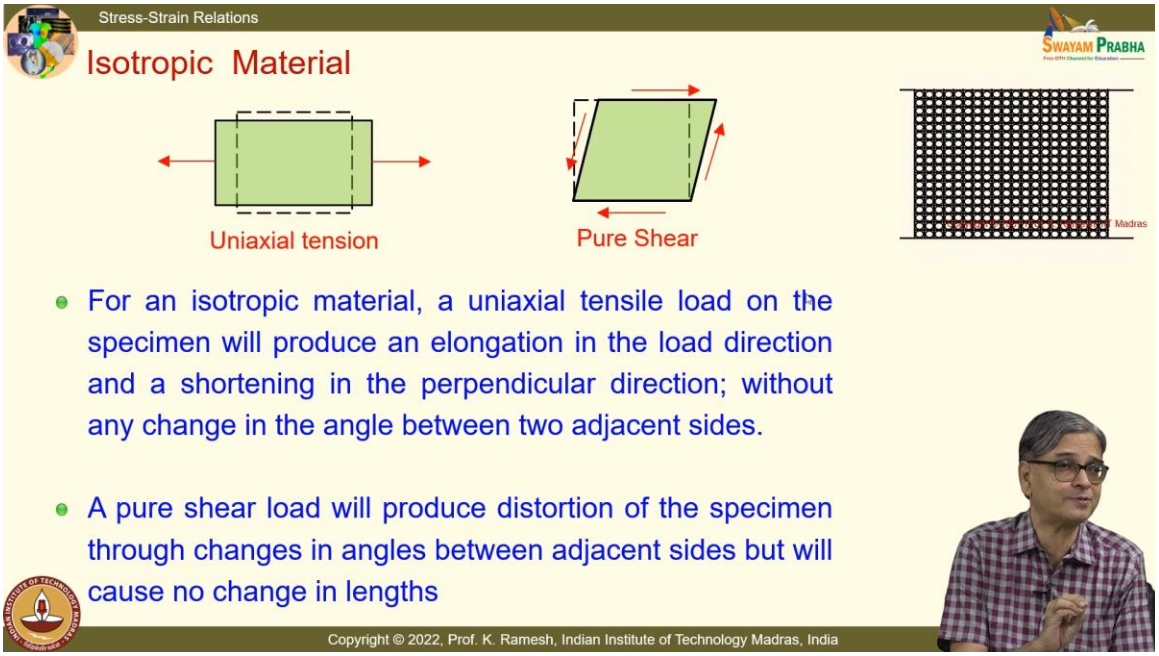

So, we have a comfort of an isotropic material, when I apply tension, it has only axial strain and when I apply pure shear it has only shear deformation the tension and shear are distinguishable tension is provided the axial strain is precipitated by an axial stress and a shear strain is precipitated by a shear stress. Same thing happens even in the case of a orthotropic material, but with a restriction when you have an orthotropic material see what I show here is lines is it is easy to visualize if I have a fiber composite and the lines indicates the direction of the fiber. And you have labeled this as a longitudinal direction whatever you have this perpendicular to that you call this as a transverse direction.

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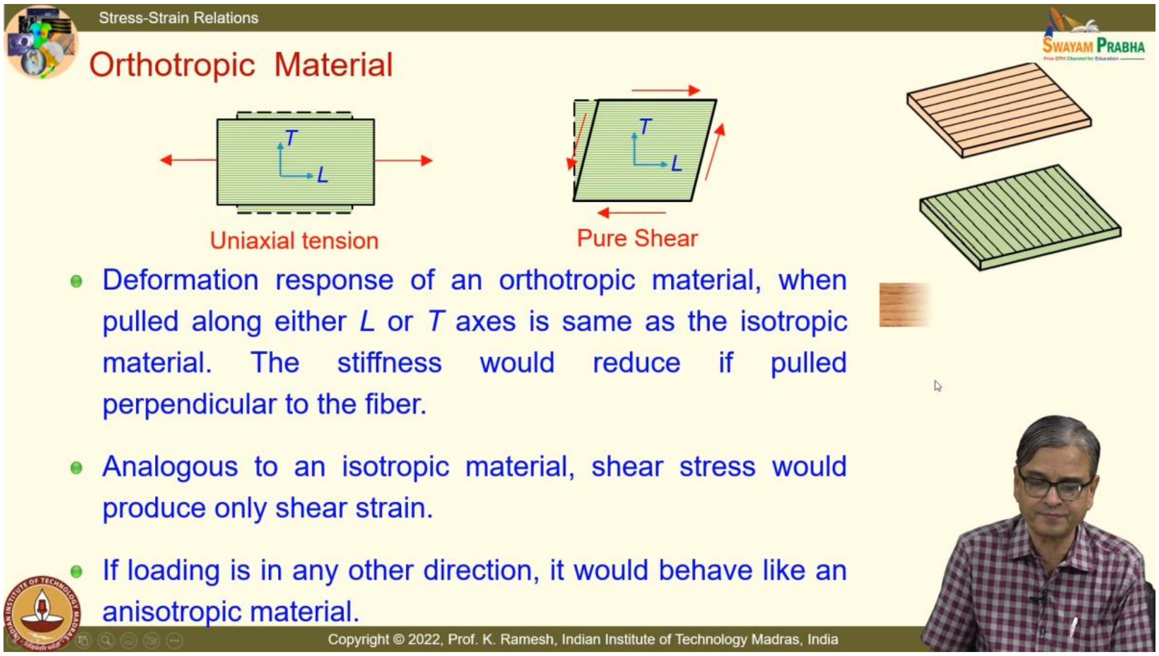

So, you label this as *L* and the symbol *T*, if I load it along *L*, I will have only axial strain produces axial stress produces only axial strain. Similarly, your shear will produce the shear, suppose I rotate it that means I pull it along the transverse direction, the transverse direction will have a lesser Young's modulus than the longitudinal direction that is what is seen in an orthotropic material. So, orthotropic material is also similar to isotropic material under certain directions in a generic direction it will behave differently. So, that is one step from isotropic go to orthotropic and you also have the natural material as wood, wood is an orthotropic material. And you know your carpenter will know if he has to develop a shelf, he will know how to put the shelf for it to withstand, he has not studied solid mechanics, but he will put it all the fiber is along the longitudinal direction he knows that from experience.

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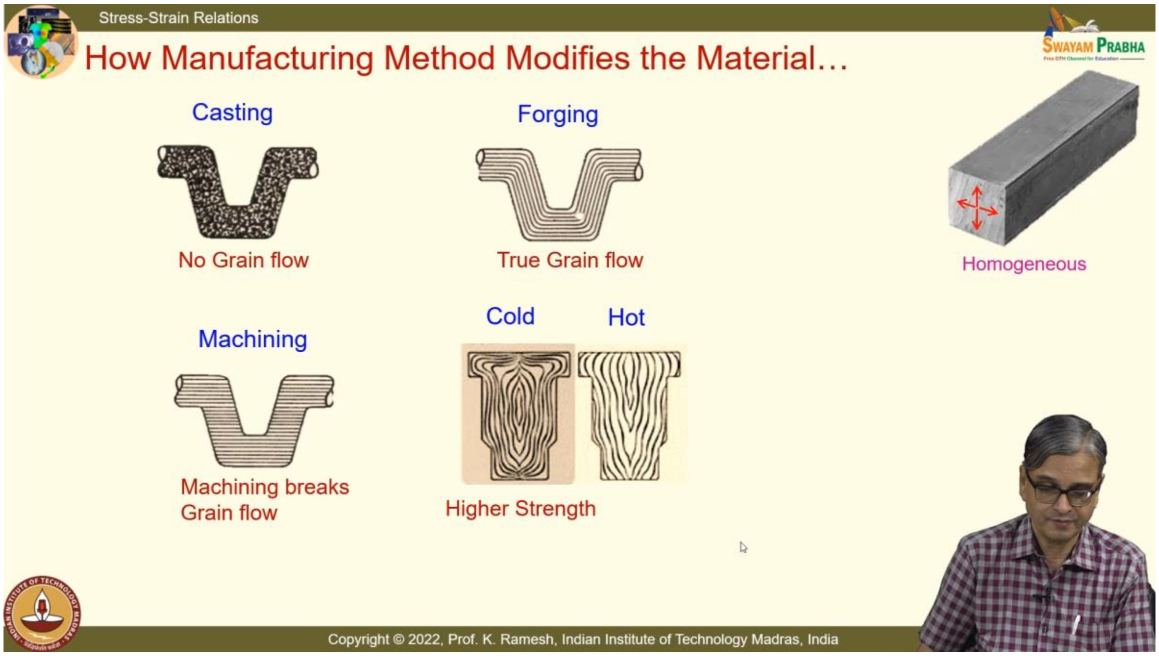

And if you look at any of the engineering material processed, I have said that you have a grain flow in the measuring and the grain flow is cut forging is the best process and I can have a hot forging or a cold forging and I when I have a cold forging, I have a higher strength. And you know you have a recent announcement by the government we are having the Vande Bharat trains, those wheels they need about lack of them per year. So, the 80,000 they were importing it from Ukraine now because of the war they want to have this manufactured in India you know what is the manufacturing process that is going to be done by forging. Forging is a very common I told you your spanner is made by forging. So, the wheels are also going to be made by forging and forging alters the grain flow that means, the bulk material property what we have done by simple test will not be sufficient to predict the behavior of a forged component.

So, you have to take a specimen along the fiber direction and find out what is the maximum stress it can stand. So, tension test is from a bulk material point of view that is good for simple day to day design you can have a bulk mark figure, but for actual structures you will have to do additional test and get the properties. Not only that people also have qualification test for all of this they do not just go by your design alone, very exhaustive testing are made.

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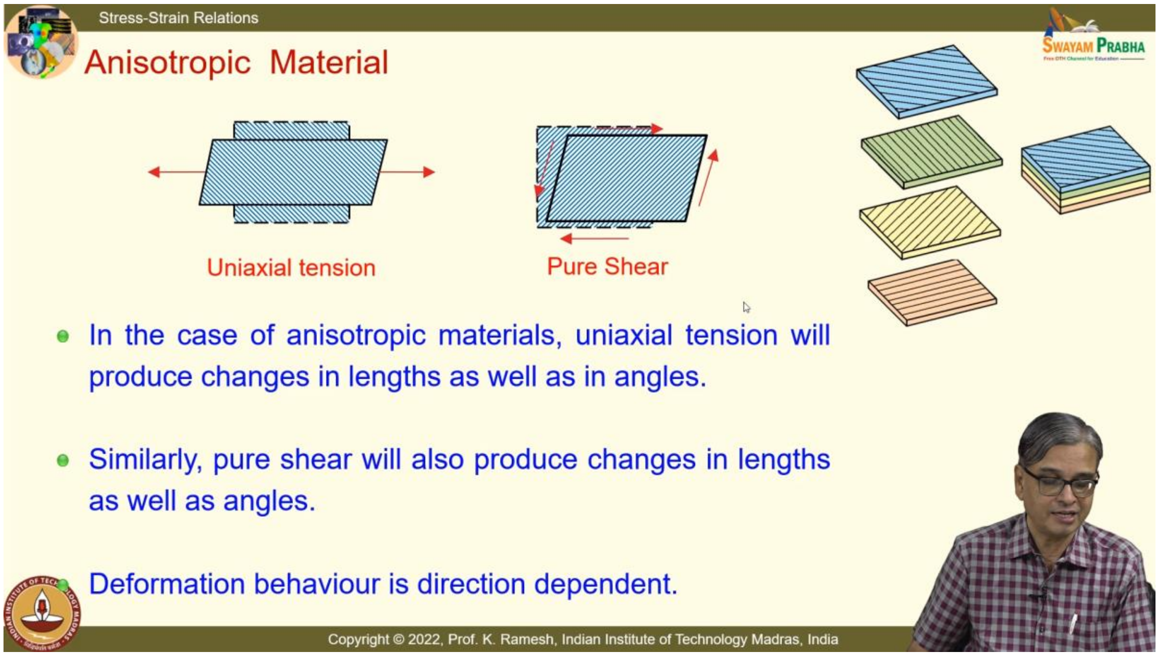

Another way to look at the material is an anisotropic material, it is very simple to visualize if I have fiber composites at different orientations are assembled and you make a lamina like this. This will behave in an anisotropic manner that means, every direction is different. Suppose, I pull this material please make a sketch out of it, it will elongate it will also shear it is very difficult to handle. In anisotropic material and orthotropic material in special directions you have a very comfortable material behavior, but when I go to anisotropic material all crystals are anisotropic. Here to illustrate a simple anisotropic material I have taken plies of different angle orientation I have made a ply like this. When I apply axial tension it has elongation as well as shear, when I apply shear it will have shear strain as well as axial strain. So, that makes the behavior very very complex and if I have to model this material behavior then what I have is the generalized Hooke's law and I am going to write it in indicial notation.

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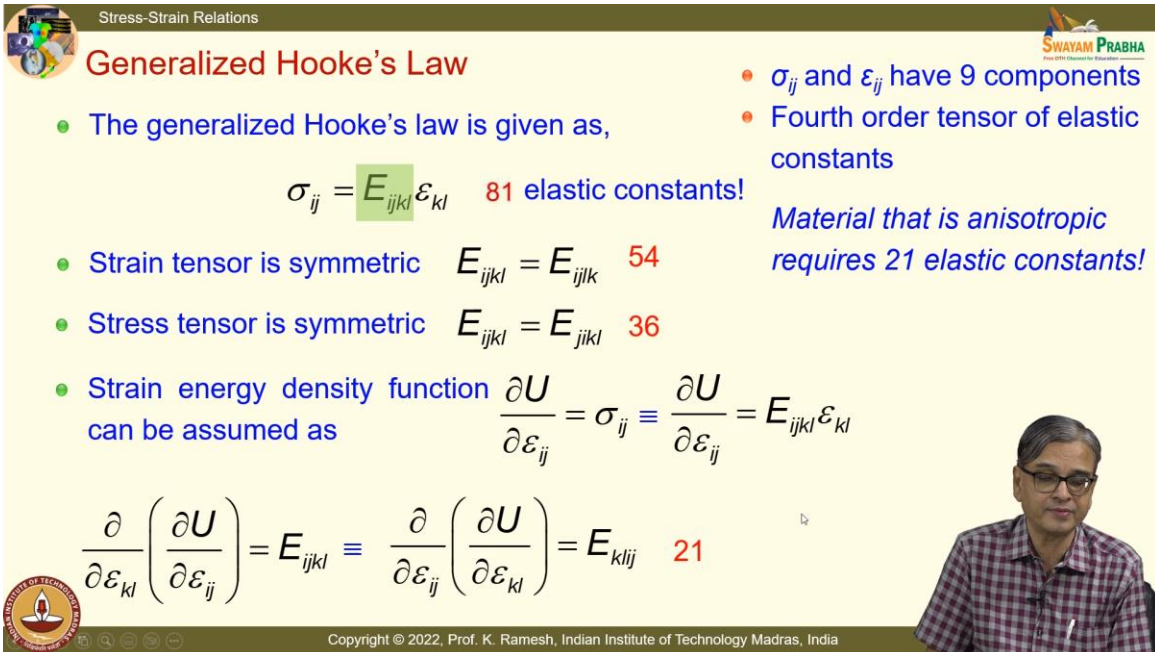

I have σ_{ij}, i goes to 1-3, j goes to 1-3 and I have in general nine stress components. This is related to the strain component ε_{kl} and what you have here is E_{ijkl} with four subscripts. Here, you will recognize the Young's modulus is a function of the direction, it is a tensor of rank 4. And if you look at this how many coefficients I need to have when I have this in a matrix form? I will have nine independent stress components and nine independent strain components. So, I will have a whooping 81 elastic constants, it is too big to handle is the idea clear? I have this as tensor of rank 4, ok, 81 elastic constants and you know stress tensor is symmetric, strain tensor is symmetric, all these symmetries reduce the number of elastic constants.

So, instead of nine suppose I say strain is symmetric, I have only six independent components. So, I have this as 9 into 6 as 54 elastic constants, *ijkl* here *ijlk* when it is reverse it is symmetric. So, I have 54 elastic constants, when I bring in stress tensor is symmetric, I get *ij* becomes *ji*, I will have 36 elastic constants. But the next step you have to go to thermodynamics find out the energy and all that you do not have a background right now, but just listen because you have to appreciate an isotropic material requires only two, do not think two is very high. For you to illustrate that I say if you have to have an anisotropic material, I can still simplify go to thermodynamical conditions. So, a strain energy density function can be assumed as the energy is U when I differentiate I get

$$\frac{\partial U}{\partial \varepsilon_{ij}} = \sigma_{ij} \equiv \frac{\partial U}{\partial \varepsilon_{ij}} = E_{ijkl}\varepsilon_{kl}$$

If I differentiate it with respect to the other subscript the order of subscript does not matter. So, that is what I am going to do I am going to differentiate with respect to ε_{kl} and then I am going to differentiate with respect to ε_{ij} the order does not matter. So, this gives me a condition *ijkl* equal to *ijlk* that reduces my elastic constants to 21. So, an anisotropic material requires a minimum of 21 elastic constants to characterize where is 21 and where is 2, do not you feel that you are handling a isotropic material why we want to idealize, even though your extrusion process aligns the fibers in one manner we still live in the comfort of isotropic material and handle it from an engineering point of view. You do a different test, use a different young's modulus for you to handle the situation. So, I have 81 elastic constants, 54 elastic constants, 36 and finally, with all this discussion an anisotropic anisotropic material the number of elastic constants reduces to 21.

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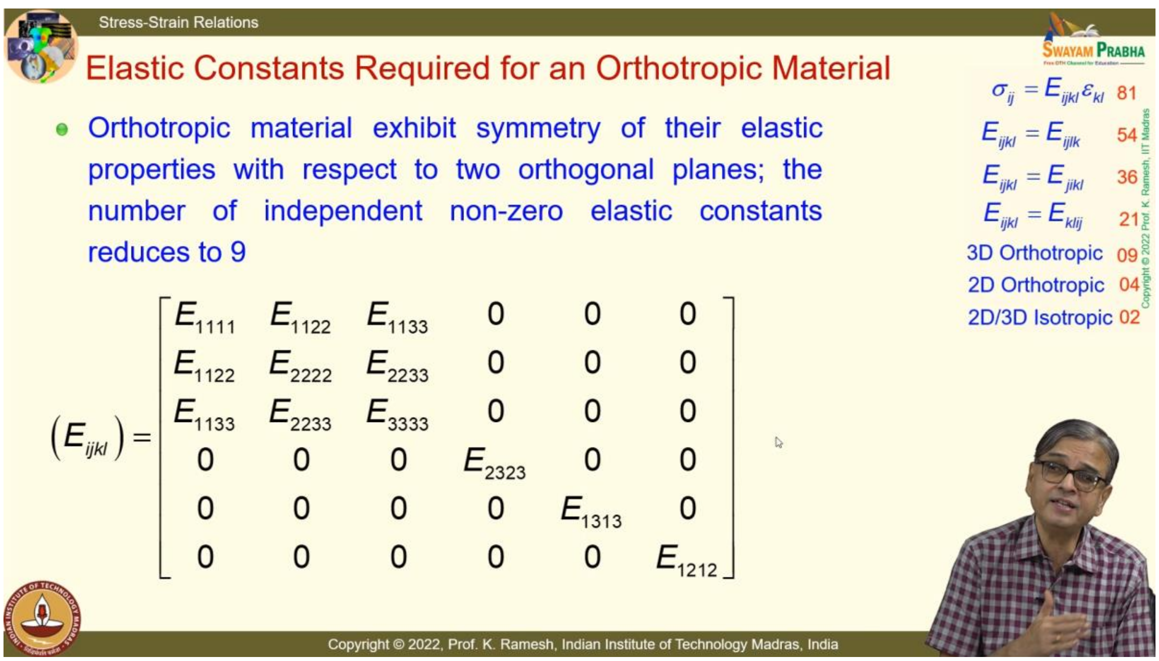

Suppose, I go to an orthotropic material I will stop with that, ok. So, this emphasizes anisotropic material requires 21 elastic constants. When I go to an orthotropic material just to give you a idea that what I have as the matrix here I will have for a three-dimensional situation only 9 elastic constants because it has 2 orthogonal planes in which the elastic properties are symmetric. So, you bring in the symmetric condition you go further because you are going to look at elasticity is a tensor of rank 4. So, when I look at symmetry on the symmetric axis the elastic constants cannot change you will have some contradictions that contradictions will say that these constants should go to 0 that is how you get the 0's here.

So, you extend this argument, you can also show at isotropic material will have only 2 elastic constants. So, what you have is an anisotropic material has 81 to start with the discussion, but we all know that stress tensor is symmetric strain tensor is symmetric when I apply all this and also bring in the thermodynamical condition it reduces to 21. A 3D orthotropic has 9 elastic constants that you see here a 2D orthotropic will have 4 elastic constants, whether it is a 2D or 3D, I have 2 elastic constants only which makes our life extremely simpler. So, in this class we have looked at interrelationship between Young's modulus, shear modulus as well as bulk modulus. We also looked at what are the limiting values of the Poisson's ratio and we have looked at where the negative Poisson's ratio is used medical industry takes benefit out of it. Then we have also emphasized that in the case of an anisotropic material you need 21 elastic constants whereas, isotropic material requires only 2 elastic constants. Thank you.