

Lect. 2, Video Lecture on Strength of Materials, Prof. K. Ramesh, IIT Madras

**Strength of Materials**  
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**Lecture - 2**  
**Introduction to Strength of Materials- 2**

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**Lecture 2 Introduction to Strength of Materials**

**Concepts Covered**

Strength of materials in the 17<sup>th</sup>, 18<sup>th</sup> and 19<sup>th</sup> centuries. Madhava's contribution to Calculus. Ecole Polytechnique's role in SOM. Timeline of advances in material science. Demonstration of failure of chalk in tension and torsion; ductile material in tension. Orthotropy, anisotropy, composites for light structures. Nature inspired engineering-strain-induced alignment of collagen, biomimetics in self-healing composites. Functionally graded materials: bones, tooth and even plants! All idealizations are violated in modern engineering - Metallic foam, heterogeneity, manufacturing methodology influencing the strength, Rapid Prototyping – Photoelasticity is indispensable for solving modern problems.

**Keywords**

Strength of materials, Timeline of Western contributions, Indian achievements in Calculus, Failure planes, Biomimetics, Composites, Functionally graded materials, Additive manufacturing, Design for manufacturing, Photoelasticity

Let us continue our discussion on history of strength of materials. In that we have particularly focused on development of the beam theory.

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■ Connection between load and curvature

Jacob Bernoulli (1655 - 1705)

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You know you had Bernoulli coming into the picture. He was between 1655 and 1705. So, he brought in the connection between the load and curvature. See, people were trying to extrapolate their understanding of tension to bending until somebody looked at what happens when you flex it.

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**The Strength of Materials in 17th Century**

- Jacob Bernoulli made calculations of beam deflections.
- “The curvature of an elastic beam at any point is proportional to the bending moment at that point”

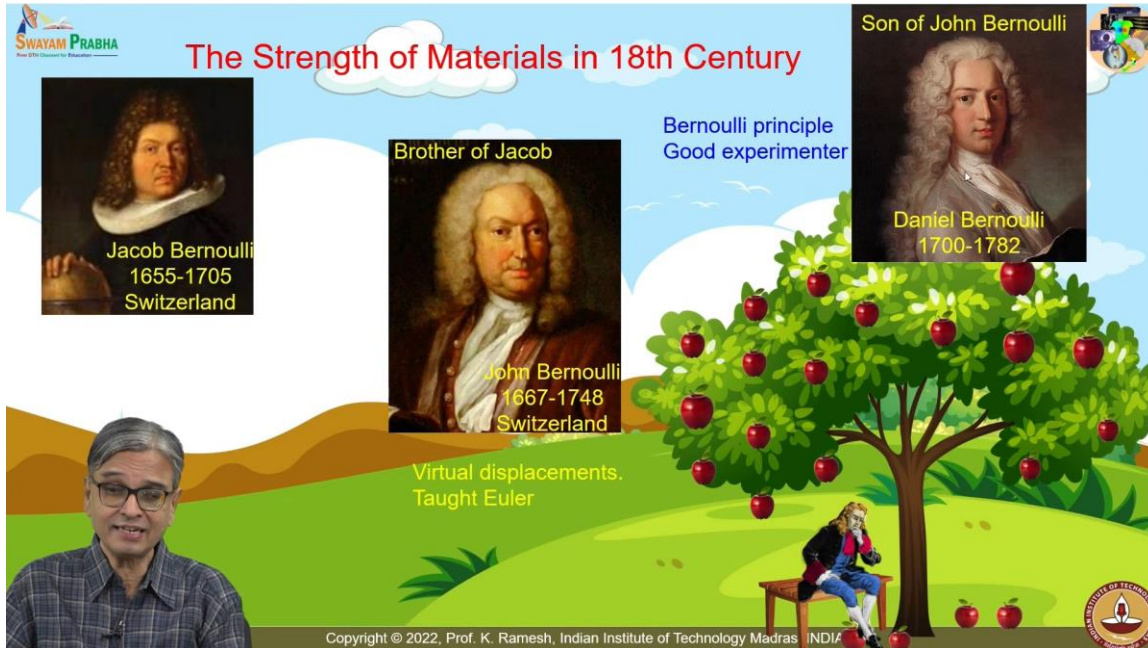
$$\frac{1}{\rho} \propto M_b$$

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See, if I flux the beam; now you recognize that this is a deformable solid. You can easily visualize the top fiber, the way I bend it, stretched and the bottom fibers are compressed. So, they were not doing it earlier. So, only Bernoulli looked at the elastic equation. So, the beam deflection changed the scenario. They were able to appreciate what is the way to handle it. And he was also quite right in coming out with a postulate that the curvature is proportional to the bending moment, but he could not proceed further.

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And when you say Bernoulli, it is not one, it is a family which dominated scientific development in the 18th century. So, you had Jacob Bernoulli, then you have brother of Jacob is John Bernoulli. He is credited for virtual displacements and what is more importantly is he taught Euler. And his son was Daniel Bernoulli. He is known for Bernoulli principle in fluid mechanics. In those days, there is no distinction between solid mechanics, fluid mechanics, light, sound, electricity; everything is like you know natural philosophy. That is the way people have approached it and a bulk of them were mathematicians.

And I also said you know the scientific development has not been poetic. It is recorded history that father and son had plagiarism problems. I do not know, once people become knowledgeable, the human aspect is missing. So, consciously you have to hold on to good human values, fine.

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Found the correct relation and the triangular variation

Antoine Parent (1666 - 1716)

1713

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Then you have Parent, his picture is not available. He found the correct relation and the triangular variation. So, people were taking baby steps towards the correct solution. See, there was nothing like an exam for them to deliver beam theory. So, it was happening at its natural phase. That is why it took 400 years, almost close to 400 years, fine.

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Studied practically about sag and its viability

Jacob Leupold (1674 – 1727)

1726

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Then you had Jacob Leupold. He was also looking at sag. That means, when the beam bends, what happens to it? Ok. So, looking at the deflection picture was the turning point for people to arrive at correct conjecture about bending.

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■ Studied elastic lines and buckling equation.  
 ■ Followed the work of Jacob Bernoulli

Leonhard Euler (1707 – 1783)

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Then you had the famous Euler and whenever you say Euler, people always remember his contribution to buckling. So, he studied elastic lines and buckling equations. He followed the work of Jacob Bernoulli and all these buckling problem became very critical only when people started pumping satellites. You have aircrafts where there is definitely a need for reduction in weight. So, when you reduce the cross section, instead of failing by fracture, the predominant one is when it is subjected to compression, it was losing its stability, fine.

So, optimization brought this new problem. Initially, you want something to work. Later on, you go for optimization, and we will also see when you say buckling, people always attach to column buckling. I want you to break that monotony.

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Beam theory took 400 years!

1726

Stress  $\sigma$ , MPa

Strain in %

UPPER YIELD STRENGTH  
LOWER YIELD STRENGTH

Elastic Region  
Yielding Region  
Strain Hardening Region  
Necking Region

Buckling by direct compressive load

Buckling due to bending. The beam twists as well as bends

Buckling of a stiffened panel by pure shear

Small Deformation

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If you look at my first slide, you would see wherever you have compression, when the thickness is not sufficient enough, you can have buckling. It can happen in bending; it can happen in other situations in shear. We will see that also. So, Euler is credited to the famous column buckling equations.

And if you look at, he has investigated the shapes of the curves with a slender elastic bar will take up under various loading conditions. So, looking at the deflection opened up the possibility and he was also clever looking at a slender bar because I said in a common atta chakki which is seen everywhere, not in an exotic aircraft structure, a simple shaft has many steps. Not only many steps, simultaneously it can have torsion, it can have bending, but each one when you look at, it is difficult to comprehend, fine.

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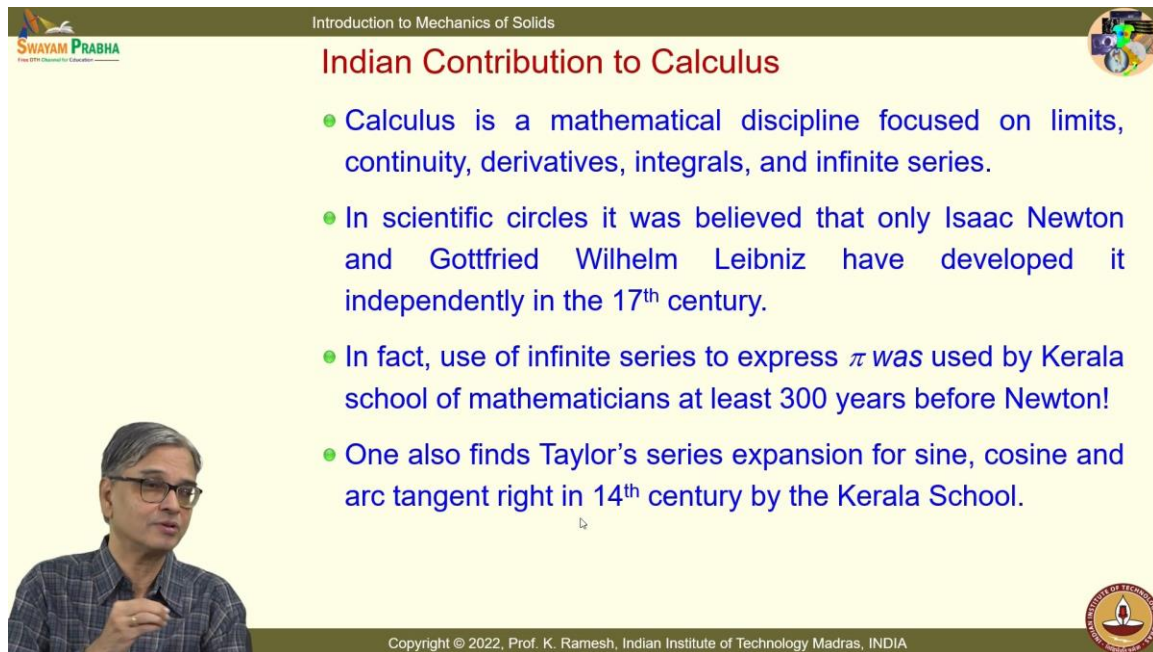
**The Strength of Materials in 18th Century**

- Investigated the shapes of the curves which a slender elastic bar will take up under various loading conditions.
- Introduction to Calculus – 1748
- Differential Calculus (2 Vols., 1755)
- Integral Calculus (3 Vols., 1768-1770)

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So, you will have to appreciate that complexity of the situation, how we break it into simple problems. And this was also the time where people are writing their books. So, you have an introduction to calculus, appeared in 1748. Then differential calculus appeared in 1755 and also integral calculus, three volumes during this period.

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Introduction to Mechanics of Solids

## Indian Contribution to Calculus

- Calculus is a mathematical discipline focused on limits, continuity, derivatives, integrals, and infinite series.
- In scientific circles it was believed that only Isaac Newton and Gottfried Wilhelm Leibniz have developed it independently in the 17<sup>th</sup> century.
- In fact, use of infinite series to express  $\pi$  was used by Kerala school of mathematicians at least 300 years before Newton!
- One also finds Taylor's series expansion for sine, cosine and arc tangent right in 14<sup>th</sup> century by the Kerala School.

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And what is the Indian contribution to calculus? See many times historians, they go back and then find out even if there is a likelihood of similar thinking, they want to go back and give credit to those people, fine.

You find most of this calculus developed somewhere in the 18th century, fine. But what was the contribution in India? You all know calculus means you are talking about differential calculus, integral calculus, infinite series, limits, all that is taught right in your schools these days, I believe. So, it was only Isaac Newton and Leibniz have developed it independently in the 17th century. In fact, use of infinite series to express pi was used by Kerala school of mathematicians at least 300 years before Newton. You have to find out.

See why pi was so important? If you look at history of scientific development, pi has always occupied. And Indian tradition, the Vedic tradition, they have a problem to solve. You have a square altar, you have a semicircular altar and you have a circular altar. All these altars should have same area. So the problem of pi was time immemorial Vedic people have solved it in some way.

If you go and find out, they know how to find, they are all called shulba sutras and so on. And you find in Kerala school of mathematics, that is what we will see, that they have understood about pi and expressed it as an infinite series. One also finds Taylor series expansion for sine, cosine and arc tangent right in 14th century. Whether it is further developed or not, that is a different question, but thought process was there and people said it was taken to Greece, from Greece it spread to other countries. There are, you have to

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have archaeological evidence to do that, but now more and more mathematicians are acknowledging this.

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Introduction to Mechanics of Solids

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Tantrasangraha 1501 CE  
Yuktibhasha 1530 CE  
Karanapadhati 1733 CE

Madhava 1340 -1425 CE  
Parameswara 1360 - 1460 CE  
Nilakantha 1443 -1560 CE  
Jyeshtadeva 1500 - 1610 CE

व्यासे वारिधिनिहते रूपहते व्याससागराभिहते ।  
त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात् ॥

$C = 4 \times D \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots \right)$

~~Gregory-Leibniz series~~

Madhava-Nilakantha series  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots$

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So Madhava was in 1340 to 1425. He had his disciples Parameshwara, Neelakanta, Jaishtadeva and you also have these summarized in the recent book, that is about 2016 it got published. See it also takes time for the scientific society to recognize the contribution. Now more and more people acknowledge this and he has not given the scientific facts in the modern symbolism, fine. And he has given a shloka like this, it says

"Vyaase vaaridhinihathe rupahrithe vyaasa-saagarabhihathe.

Trisharadi-vishama-sankyabhaktamrinam swam prethak kramat kuriyaat."

You know I can read Sanskrit; I cannot interpret it and you have scholars who do that interpretation. And you have professor K. Ramasubramanian in IIT Bombay has given very illustrious lectures on these topics. The shloka refers to an equation like this, where C is the circumference and D is the diameter and the shloka if it is translated.

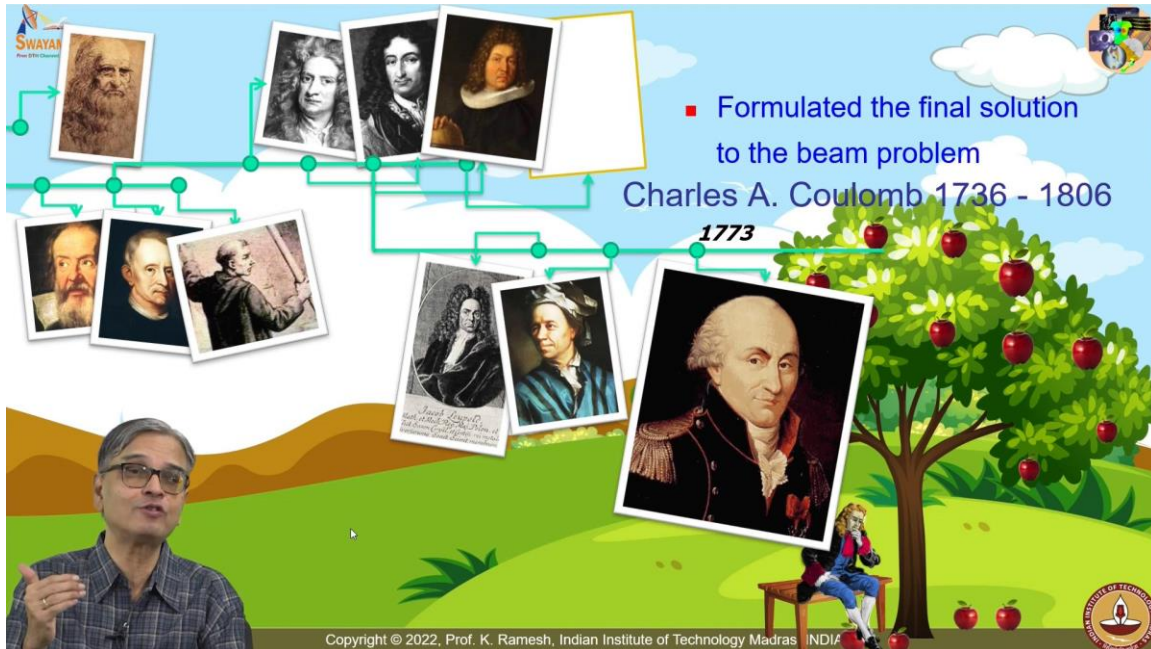
See when you have a knowledge given in a different symbolism you need experts who are well versed in the modern development as well as interpreting it, only then you can uncover the truth. So, a shloka like this results into an equation of this form. And they have also written books. See that is the only way of communicating the accumulated knowledge. So, you have Tantrasangraha, Yuktibasha, Karanapaddhati.

So, these are all written in Sanskrit either in Sanskrit or probably some books are in Malayalam I do not know, but primarily Sanskrit was the language in those times. So, this



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can be rewritten when D is equal to 1 as,  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots$ . The concept that something can be summed up as an infinite series that is a seed, that was seen in the Kerala school of mathematics. It is known as Gregory-Leibniz series, but more and more people accept that this is actually Madhava-Neelakantha series. I think he requires a clap from this class, you should give a clap. So, you have to take pride that there were thought processes in the country about calculus 300 years before the European scientists could come forward with those ideas.



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And as I said, Euler and Bernoulli were able to look at the deflection and come to some understanding about the bending of beams, but the final relationship on finding out the stresses the credit goes to Coulomb. And Coulomb, you also know in electric sciences you have a famous Coulomb's law. So, that is what I said in those days when there were not many branches like this, fine; it is all natural philosophy.

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**The Strength of Materials in 19th Century**

- Coulomb determined the torsional rigidity of a wire by observing the torsional oscillation of a metal cylinder suspended by it.
- In 1826, the first printed edition of Navier's book on Strength of Material was published

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And Coulomb is also credited with one more aspect, he determined the torsional rigidity. See, torsional rigidity means what? When you have applied torsion, whether the machine element or the structure is able to withstand it. See, you are graduating from rigid body mechanics to deformable solids. So, people initially thought whatever the forces that act on it nothing will happen to it. And if they want to feel safe nothing should happen to it. So, that is how these terminologies have come about.

You will have bending rigidity, you will have torsional rigidity, likewise you know this has a origin from that kind of a thought process. So, he determined the torsional rigidity of a wire by observing the torsional oscillation of a metal cylinder suspended by it. And you also have a book by Navier was published in 1826 on strength of materials. So, whatever the understanding is documented and that was available as a text book.

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It actually took around 400 years and 11 gifted men to finally develop the present beam theory.

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So, you have to appreciate it took actually about 400 years and 11 gifted men to finally, develop the present beam theory. It is not trivial! At each stage of development, development of each of these concepts have to be verified by experiments. It is not just you throw a concept and run away, fine. And there has to be a mathematical basis not only that you must also be able to prove it from various stand points. Whatever we observe it should be explainable from the theory that you have built up. From the theory that you have built up you should be able to build certain structures and then you should be able to predict its life.

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● He was a great mathematician under whose leadership an engineering school is established in 1794 which is presently called **Ecole Polytechnique**.

● He was a progressive thinker and held lectures for large groups of students

Gaspard Monge  
 1746-1818  
 France

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And you know you had Gaspard Monge, fine. He was credited as a great mathematician and he started an engineering school in 1794. It is the famous Ecole Polytechnique. See until then engineering was more like apprenticeship people will go to experience people learn certain skills and then come out and practice it as engineers. It was not like they have to be taught basic sciences.

So, he was the leader like you have the current curriculum when you do engineering in IITs or NITs or across the country now you have a basic mathematics, physics and chemistry. So, that kind of a formulation he was responsible for it. Then you know people needed text books on these fundamentals. So, text books were written in those times. He was a progressive thinker and had lectures for large groups of students.

See some account says, they give very large classrooms they lecture it to 2000 people at a time and so on, fine. Because you know knowledge is scarce and communication aspects are less.

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So, many stalwarts in the field of mechanics of solids decorated this Ecole Polytechnique; you have Monge, you have Poisson, you have Navier, you have Poncelet, you have Cauchy and then you have Lamé. We would see their contribution as we go through the course, fine.

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- He has made important contributions to mechanics of materials and wrote a book “**Mecanique industrielle**”.
- The most complete presentation of the knowledge of mechanical properties of structural materials of that time is found in that book.
- He has introduced the effect of shearing force in the formulas for deflection of beams.

J V Poncelet  
1788-1867  
France

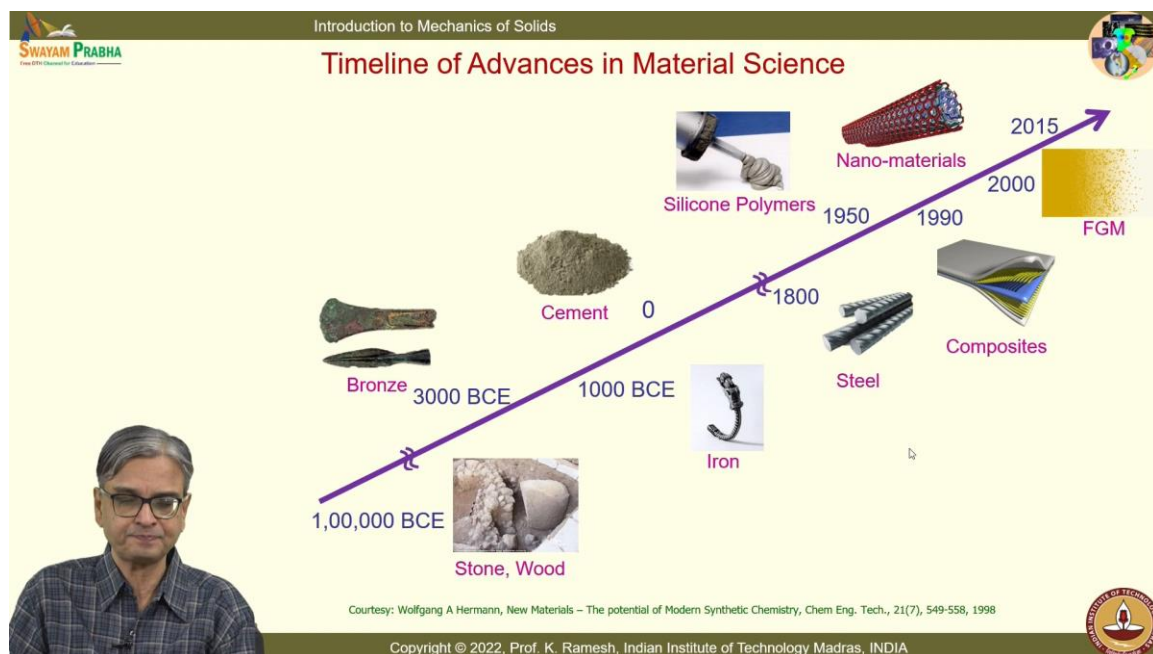
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And Poncelet is immediately important because you know when you say strength of materials, what you find is here documented what is the knowledge of the material that they use at that point in time.

So, the most complete presentation of the knowledge of mechanical properties or structural materials of that time is found in that book. So, that was his contribution, and you will also see in this course I have already said why Galileo could not comprehend bending completely, because he took a problem which is complex from a mathematical perspective. From a practical perspective, cantilevers are seen everywhere. You see more cantilevers than simply supported beams, fine. He had taken a common example, but a cantilever you all know it has a shear variation and you will also see in our subject development also we will develop deflection only due to stresses developed in bending then we will try to find out how to accommodate shear.

So, Poncelet is credited in introducing the effect of shearing force in the formula for deflection of beams. We will take it up towards the end. So, introduction of shear is not trivial. These are all important developments, ok.

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And you should also look at what is the timeline of material development. So, this is like 100,000 years back, I am going to recall this 100,000 years later, fine. So, you should know we are talking about what is happening currently and look at the genesis of various materials. What was the material available in those times? You had only stone; you had wood. That is how you had many gadgets and structures have been made. Then people went into bronze stage, you have tools out of bronze; then they had iron.

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So, cement was available at the start of the common era and then you had steel somewhere around 1800; mind you that this is a non-linear scale, fine. Then in 1950s you had polymers. You have polycarbonate and so on and so forth they are all used in many structural applications. And then composites developed because there is more and more dearth of fuel, people thought that I should spend less fuel one way of doing it is reduce the weight, yet retain the strength, fine. Reducing the weight means it is not that compromise on safety, you have to go for advanced materials which provide you strength where you are needed.

So, people found that if I have fibers inserted which are stronger than the matrix I can make a composite and do it. Then you had nano-materials; I have not put here. You also have smart materials, fine. You have smart materials. And there are also material like this which are functionally graded material.

We started saying in our idealizations we will keep seeing that because you have to know we make very simplifying assumptions to understand the basics. So, this has been the timeline for development of materials. So, we will come back to this.

Now, what we will do is we will learn some practical experience of what is the strength of a material, fine. So, I would like you to do the experiment on chalk I would first ask this section of students to do the experiment you just watch.

What we will do is you would have broken the chalk many times randomly, but I want you to do it in a particular fashion. Now, you take the chalk hold it in front of you all of you take it! take it out! Pull it until it fractures. Pull it! Pull it until, hold it like this! You are being recorded. Pull it! Pull it! Pull it, yeah! Ok. So, I will also now pull it, fine. Everybody has pulled it, and what do you find it is more or less perpendicular to it. We have not done any prior adjustments with people I should break it only like this it has broken like this.

Now, what we would see is we would break it in a different manner now. I want this section of students to take the chalk and I want half of them. See, what I would suggest is I want this row to break it in one fashion, that row to break in another fashion.

So, you take the chalk, all of you take the chalk and do it anti-clockwise, ok. Hold it and do it, rotate it anti-clockwise. Can you break the chalk? Rotate until the chalk breaks. Just see what is the shape, is it perpendicular? Can you guess what could be the angle? roughly. Roughly, roughly it is 45.

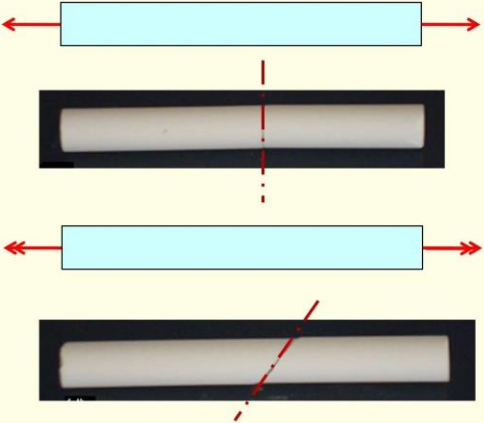
Now, what I will do is I will ask the other section now you take the chalk rotate it clockwise. Then also what happens there is something very interesting, the chalk is very smart. It understands somebody is pulling me; when somebody pulls me, it breaks like this, fine! When somebody twists it, whether it is clockwise or anti-clockwise, the 45 degree will be in one of the two complementary directions, ok.

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Introduction to Mechanics of Solids

SWAYAM PRABHA

### Failure of Brittle Materials Subjected to Tension or Torsion



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So, how do you comprehend this? Chalk is definitely not smart, fine. Something happens to the material on this plane or on this plane which reaches the critical value and the material separates.

See, we have graduating from rigid body mechanics to deformable solids and we said the body resists and we also call the resistance as stress. We have still not developed what is the concept of stress, but we have to have in our mind if I have to develop the concept of stress, I must bring in the planes passing through the point. Is the idea clear? I should know what happens in any of the possible planes. If I do not bring in that aspect of it then I would not be in the position to appreciate or reason out mathematically how do I explain the failure of the chalk. Is idea clear? So, now we know when I have to develop the concept of stress because Galileo did the experiment and then found out and gave a conclusion it depends only on the cross section not on the length. A very useful. It may appear very childish for you now.


So, obvious because it is taught to you may be in tenth class, you have a tension test that is what I understand; tenth or twelfth you do a tension test study elasticity all that is introduced. I do not know how much you have comprehended that, but it is very subtle. So, now, if I have to develop the concept of stress, the concept will be complete only I bring in the planes until then the concept will not be complete.



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Introduction to Mechanics of Solids  
 SWAYAM PRABHA  
 EXPERIMENTAL STRESS ANALYSIS  
 Overview of Mechanics of Solids

### Tensile test for a Mild Steel specimen



play stop

Elastic Region Plastic Region

Stress  $\sigma$ , MPa

Strain,  $\epsilon$  (in %)

UPPER YIELD STRENGTH  
 LOWER YIELD STRENGTH  
 ULTIMATE TENSILE STRENGTH  
 FRACTURE STRENGTH

Yielding was considered to be a failure to be avoided at all costs in practical structures.

Back to main

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Now, let us see if I have a steel rod and I pull it cannot be done with hand you have to go to a machine and you will be doing it as part of your laboratory course. When I take it and faithfully record my data, I am going to get only a non-linear relationship like this, I am not going to get a linear relationship.

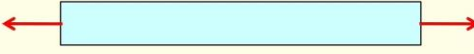

And we have already looked at what are the goals for an idealization. See, idealization is very, very important to simplify your problem. And you find when I have a non-linear curve like this, the red portion is the portion where I apply the load when I release the load it comes back to its original position and fortunately this is linear, but it is very, very small. But that is a golden goose for us because once I have a linear relationship there are many, many things I can do mathematically it makes my life lot more simpler.

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Introduction to Mechanics of Solids

SWAYAM PRABHA

## Failure of Ductile Material Subjected to Tension

Cup and Cone Fracture



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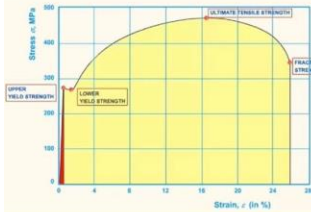
So, when I have a failure of a ductile material, it is not failing in the same way like chalk. The ductile material fails by a famous cup and cone fracture. In a brittle material was flat whereas in ductile material, this is at 45 degrees. You will get the answer yourself once we develop the relevant concepts in this course, fine. You have a cup and cone fracture.

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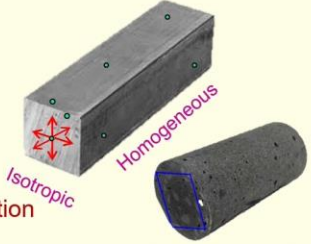
Introduction to Mechanics of Solids

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## Idealisations



- **Small deformation**
  - Apply the equilibrium requirements to the *undeformed* configuration.
- **Material is Homogeneous**
  - Elastic property is same at every point
- **Material is Isotropic**
  - Elastic property is same along any direction
- **Material is an elastic continuum**
  - No defects!



Concrete cylinder – heterogeneous with pores etc.



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I said that we have to go back to these idealizations again and again. My interest is to emphasize two points. First, when I say small deformation, I can write the governing equations based on undeformed configuration, that is one advantage. The second crucial advantage is I can hang on to the linear relationship. I do not have to worry about combined loading of axial pull, torsion and bending simultaneously. I can understand them. I can understand them one by one and then simply use the principle of superposition and then look at the combined effect. So, it makes our life very, very simple.

See another one what we have looked at is isotropy. I said you have a simple spanner that is done by forging. All manufacturing operations modify the isotropic property. You would no longer have the material as isotropic and you have to learn how to live with it and that is what we are going to see.

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The slide, titled "How Manufacturing Method Modifies the Material...", illustrates the grain structure of a material under different manufacturing processes. It features a central image of a rectangular bar labeled "Homogeneous" with red arrows indicating isotropic properties. To the right, four diagrams show grain orientations:

- Casting:** Shows a V-shaped cross-section with a random, irregular grain structure, labeled "No Grain flow".
- Forging:** Shows a V-shaped cross-section with a uniform, parallel grain structure, labeled "True Grain flow".
- Machining:** Shows a V-shaped cross-section with a grain structure that is broken and irregular, labeled "Machining breaks Grain flow".
- Cold and Hot Forging:** Shows two T-shaped cross-sections. The "Cold" forging shows a very fine, parallel grain structure, and the "Hot" forging shows a slightly larger, parallel grain structure. Both are labeled "Higher Strength".

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So, when I have the casting, you have grains there is no grain flow, there is no great advantage. And many times you extrude from the material from the bar stock you machine it like this. So, you have grain, but there is breakage in this gap whereas, if I do the forging the grains are aligned like this. So, whenever that is that is like you know you have a fiber aligned like this. So, it is much stronger, and I can do forging either by a cold forging operation or by an hot forging operation. And if you do cold forging, that has the highest strength.

So, now, people do not talk about design; they say design to manufacture. So, the manufacturing process has to be integrated with the design thinking and final performance of the component depends on both the material as well as the manufacturing process.

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Introduction to Mechanics of Solids

SWAYAM PRABHA  
Free ETE Online for Education

## Orthotropy and Anisotropy

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And you know you go to your carpenter; he has not come to IIT to study engineering mechanics or strength of materials. He will know if I have to make a shelf, he will cut the wood like this, where the grains are aligned horizontally.

So, you will put the shelf like this by experiential knowledge. He knows! And we have also said unfortunately, Galileo worked on wood which is a very complex material; it is an orthotropic material. And if I have to visualize an an orthotropic material, how do I do it? What I can do is I can have imagined as these lines are indicating fibers. I align these fibers in different direction and sandwich them. The behavior of a material like this would be different in every direction.

You call that kind of material as anisotropic. You call this kind of material as orthotropic, fine. So, isotropy is a convenience for us to live in a happy domain. That is the way that we will do and develop this course.

(Refer Slide Time: 30.17)

Introduction to Mechanics of Solids

SWAYAM PRABHA

## Composites to Reduce Weight in Aerospace

GLARE® in Upper Fuselage

CFRP J-Nose

Vertical Tail Plane

Horizontal Tail Plane

Section 19.1

Section 19

CFRP center wing box

wing ribs

Rear Pressure Bulkhead

Courtesy www.airbus.com

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And essentially these composites are developed to reduce weight and what you have is; can you recognize this plane? This is our LCA aircraft, ok. Now it is also being exported and you also have a light helicopter. See many of these first developments happen in military hardware. Once it is tested in military hardware then it comes to passenger plane because a person sitting in a LCA can eject out. You and I cannot eject out from a 300 seater plane, fine. You can't have trajectory for all those people.

So, It percolated down to your passenger aircraft, you have airbus A380. There are many components you have a center wing box, wing ribs, rear pressure bulkhead, horizontal tail plane, vertical tail plane, CFRP J-Nose. So, bulk of the components are now made of composites. So, you have to graduate from handling isotropic materials; at least to orthotropic materials and then graduate to anisotropic material.


You cannot live in the comfort domain. We have made simplification one course and nothing else to be learnt. But this concepts what we develop, they form a foundation for you to hopefully helps you to understand the concept associated with it. So, these are all what you find in engineering development.

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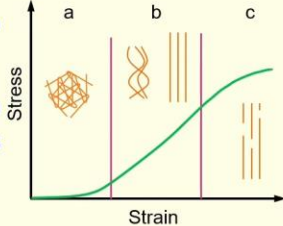
Introduction to Mechanics of Solids

SWAYAM PRABHA

## Strain-induced Alignment of Collagen



- Collagen is a protein responsible for skin elasticity and healthy joints.
- It is comprising of three quarters of the skin and is also in bones, muscles, tendons and cartilage.
- Collagen deficiency leads to wrinkles and brittle bones!
- Collagen is oriented in a random pattern initially.
- As the applied load is increased, they realign parallel to each other, so that higher load is required for further elongation.



Reference:

1. S Nesbitt, W Scott, J Macrone and S Kotha, Collagen Fibrils in Skin Orient in the Direction of Applied Uniaxial Load in Proportion to Stress while Exhibiting Differential Strains around Hair Follicles, *Materials*, 8, 2015, 1841-1857; doi: 10.3390/ma8041841
2. D Vader, A Kabla, D Weitz and L Mahadevan, Strain-induced Alignment in Collagen Gels, *PLoS ONE* 4(6): e5902, 2009. doi:10.1371/journal.pone.0005902

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Now, let us look at what happens in our human body. What we have said, if I want to have strength in a particular direction, put a fiber which is strong enough and then have your weight reduced.

But look at collagen is a protein responsible for skin elasticity in healthy joints. It is present everywhere; it is present in three-quarters of the skin, it is also available in bones, muscles, tendons and cartilage. And collagen deficiency leads to wrinkles and this you find in many of the people who live in the hills. See, what people say is as you go up different altitudes, they seem to have different deficiencies, ok that shows up.

But you know wrinkling is also very good. It is a very graceful ageing process, fine. But what is important from a scientific approach, is collagen is oriented in a random pattern, initially. See, in the case of composite you want to align it, you pre-align it depending on the load anticipated. But what happens in a human body? As the applied load is increased they realign parallel to each other, so that higher load is required for further elongation. Do not you feel we have still not come any way closer to this kind of fiber composite, fine. Need-based! So, that is what you find, and it is a very, very interesting stuff, fine. But people have come closer to this; what is it that they have done?

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Introduction to Mechanics of Solids

SWAYAM PRABHA

### New Material Developments Inspired by Nature

DiCoFRTS with CoFRTS

Flow of sheet moulding compound (SMC)

Release

CoFRTS bonded with DiCoFRTS

Demonstrator Product

CT-scan fiber arrangement

Interface

Polyurethane tendon

Functionally Graded Materials

Ker D F E, Wang D, Behn A W, Wang E T H, Zhang X, Zhou B Y, Mercado-Pagán A E, Kim S, Kleinmeyer J, Gharabab B, Shanjani Y, Nelson D, Safir M, Cheung E, Campbell P and Yang Y P 2018 Functionally Graded, Bone- and Tendon-Like Polyurethane for Rotator Cuff Repair *Adv. Funct. Mater.* 28 1707107-1-16

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They will have a block of material with short fiber randomly oriented, fine. But that is not sufficient. The edges have to have continuous fiber for strength. And this is a development somewhere around 3 or 4 years back; I am talking about very recent developments.

So, you put those here and then make a hard press. So, I have this black ones or continuous fiber where you need to have improved strength. So, the final composite is like this, these are all short fiber composite and where you want the line, it is possible to mimic it in a particular way, but it is not dictated by the stress locally, fine. That is the best we are able to go closer to it.

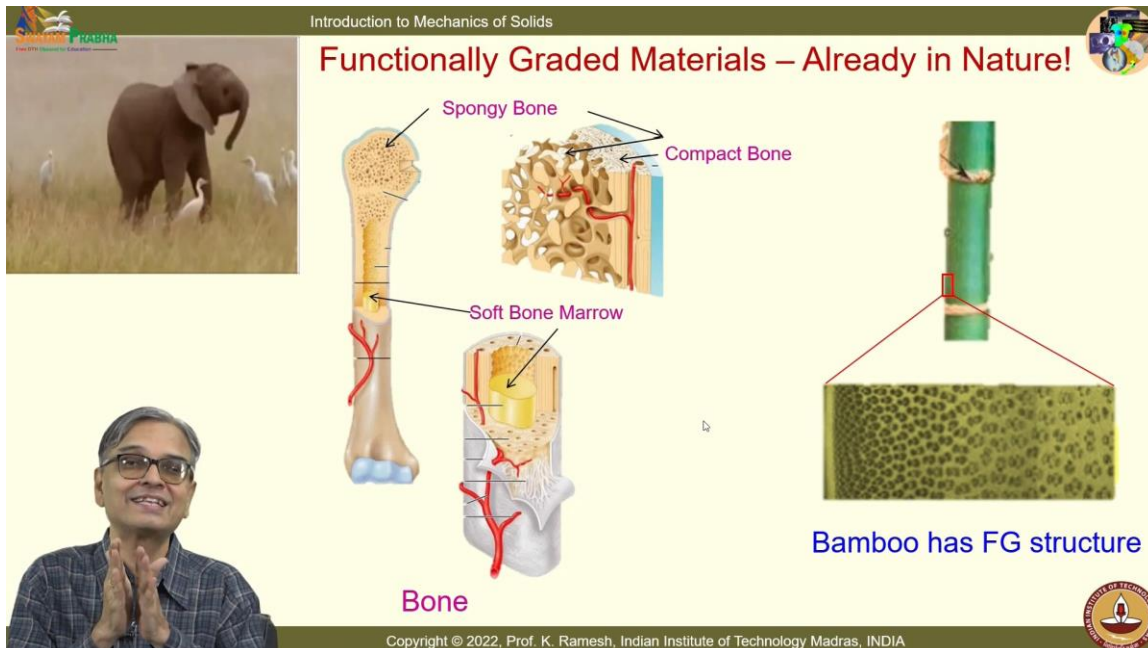
So, the next material development is people also wanted to have instead of putting fibers if they have particulates, particles. And if they are dispersed like this, the elastic properties would change as a function along the length. You will not have one Young's modulus, even though I have not introduced in the course, you have already heard the term in your schools. So, the elastic properties varies as a; what would you think scientists are more smarter they have learnt it from composites, and then they want to have functionally graded material? Do not you feel that you get excited? Whether the excitement is short-lived or not, we will see, ok.

We also have tendons, fine. And this is the recent research it was done in, I think it was published in 2018. This is photo-tunable polyurethane. There is an interface, and since you have seen photoelastic fringes, these are all photoelastic fringes. If there is a gradual change of elasticity, the color is also gradually changing from a photoelastic perspective. From a photoelastic perspective, this is a step change where the elasticity is not changing smoothly.

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See, the fundamental idea here is why you have functionally graded is, stress concentration will be smaller if I have a functionally gradient application, fine.

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Now, I said that whether this is for the engineers to have a look at it or what or what is there in nature. Right your bone is functionally graded. That means what? Hundred thousand years back, when people had stone, your bodies were having functionally graded material. We are now thumping our chest and then say we are developing functionally graded material and how this is done is by microstructural change and bone is also hollow. It is hollow and many of you might know that hemoglobin is generated inside the bone fine, and when you climb up the stairs, your bone thigh bone is subjected to bending as well as torsion depending on how you climb up.

So, we have already looked at by inference I have shown you that the core of either shaft or core of the beam is not transmitting any load. We have taken 400 years when we analyzed the bending. God has understood 100,000 years back because bones were there when somebody whether animal is there or anyone is there, they were all having it and you have a softer core, where hemoglobin can beautifully develop, ok. Not only in this! Plant life also you have this bamboo, and this bamboo if you look at and then see there is a microstructural change, and it is a porous structure.

See, we have said elastic continuum and then made everything as very simple for us to analyze because when I say area goes to zero, I have still material point, and I can do the differentiation and integration. That is how we will develop the mathematics. But we cannot live with that comfort. We see that it is available in nature so readily. Bamboos



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were there from time immemorial. We have understood and if you look at some of the publication. I think I have a publication detail the next one.

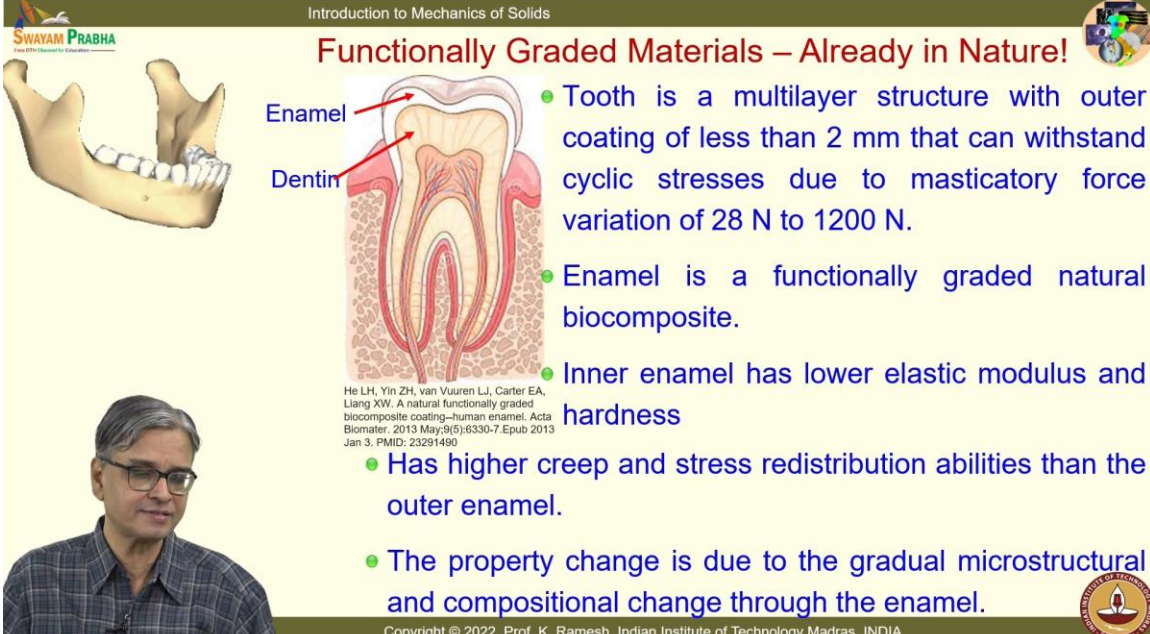
I was so enamoured by this nice child elephant, you know. It is flaunting its trunk, ok. You know what is the beauty of trunk, do you know? It can handle a needle which is very, very small and also carry several tons of log. See, human-made any equipment has a range. The range is limited and if you go to robotics, it is extremely difficult to make a elephant trunk. It is not so simple, that is why I thought that, you know we have to really look at nature and salute, we are still far behind nature.

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Introduction to Mechanics of Solids

SWAYAM PRABHA

### Functionally Graded Materials – Already in Nature!



- Tooth is a multilayer structure with outer coating of less than 2 mm that can withstand cyclic stresses due to masticatory force variation of 28 N to 1200 N.
- Enamel is a functionally graded natural biocomposite.
- Inner enamel has lower elastic modulus and hardness
- Has higher creep and stress redistribution abilities than the outer enamel.
- The property change is due to the gradual microstructural and compositional change through the enamel.

He LH, Yin ZH, van Vuuren LJ, Carter EA, Liang XW. A natural functionally graded biocomposite coating—human enamel. Acta Biomater. 2013 May;9(5):6330-7. Epub 2013 Jan 3. PMID: 23291490

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Let us look at your human tooth. What is happening? Tooth is a multilayer structure with outer coating of less than 2 millimeter. See I have an enamel, I have a dentin and you do the mastication depending on what you eat, fine. If you go and try to chew your sugar cane, your masticatory force is going to be very, very high. So, the load is varying; load is not constant. So, when you have a tooth, it should withstand all of that, and people have estimated the masticatory force variation of 28 Newtons to 1200 Newtons.

And what you find is enamel is a functionally graded natural biocomposite. It is very, very surprising and when scientists have recognized this fact. Papers appeared somewhere around 2010, it is in 2013. There were also some papers earlier than that. Until they looked at from this perspective, they were not able to appreciate. So, within a span of 2 millimeters it has to become softer and merge with this, ok. So, inner enamel has lower elastic modulus and hardness. Has higher creep and stress redistribution abilities than the outer enamel, that needs to be there. Because it has to be soft, fine. It has to give a cushioning. And many of, if you go to any of the IITs, some of the leading laboratories would be working on

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functionally graded material. They will be doing a mathematical analysis of it. They would be physically preparing it. Now, you can go and flaunt, I have right in my mouth, I have 32 teeth normally. But, I think these days children have more than 32, fine. You can flaunt each of that teeth the top layer of enamel is functionally graded.

So, which is great, scientists are trying to come closer to nature, nature is far more superior. We are not still understood you know when you have an injury, automatically you know the skin becomes rough and then after few days it gets healed. So, you people, they have identified this and developed composites that can self-heal themselves. Where they are needed, you are having a space station in the space. If there is a problem recently, that telescope is they lunched, they had you know the meteoroid hit that and then some portions are got damaged. So, there you cannot go and repair it. So, it has to self-heal it.

So, what you have to understand is the current materials are very complex. And you will have to find out many things and learn from nature.

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Introduction to Mechanics of Solids

## Rapid Prototyping – Additive Manufacturing

EXPERIMENTAL STRESS ANALYSIS Digital Photoelasticity 03

### Model Fabrication in Stereolithography

- A beam of ultraviolet (UV) light is focused on the top layer of liquid photo-curable resin contained in a vat.
- Earlier systems used He-Cd lasers (40 mW, 325 nm) which are now replaced by high-powered Argon-ion lasers (600 mw, 351 nm).
- The beam is positioned and moved in horizontal x and y-directions using a set of mirrors to crosshatch the area contained within the boundary of a particular layer of the model.

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And people also have looked at additive manufacturing. I said that manufacturing is closely linked to design now. People do not want to have large inventory they would like to have just a CAD diagram and then generate the component when there is a need.

Here, what you do is you can make very complex models by adding layer by layer. These are all photo-curable resins. See, the idea is photoelasticity has played a very significant role in understanding simple solid mechanics, also in the development of fracture mechanics it also has a role to play in the development of rapid prototyping and also handling problems of the future. That is what I am going to get that to you.

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So, here you know people also had the plastic ribbons for them to make it, initially it was all used for designing your chapels and it has become it started somewhere around 1990s. Now it has rapidly picked up, and you have a layer by layer, and this is called stereolithography and a very recently, what I am talking about in the last two years, people have also used additive manufacturing for the composites. That means, I can lay the fibers at different orientation even though I cannot make the fiber respond to stress like collagen does, ok.

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Introduction to Mechanics of Solids

SWAYAM PRABHA

## Rapid Prototyping – Additive Manufacturing

EXPERIMENTAL STRESS ANALYSIS Digital Photoelasticity 05

**Curing over the depth of the layer**

- Over the depth of the layer, the curing of the resin is not rectangular.
- For the case of a Gaussian laser and a resin obeying the Beer-Lambert law, over the depth, the shape of the cured resin will be parabolic.
- This leads to pockets of uncured resin within the model.
- This trapped resin has to be cured during post-processing and hence the RP model becomes elastically inhomogeneous.

Staggered hatch

Cross-sectional view of the cured layers obtained by WEAVE™

Act like rivets to secure the layers

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Something closer to that in a different perspective. So, more complex manufacturing processes have are getting developed, and in additive manufacturing one of the issue is what is the depth of the layer. If you reduce the depth of the layer, the precision goes up, fine. And you are going to have a laser to cure it. So, it is not going to cure it in a rectangular fashion, it is going to be a parabolic fashion.

How to put the layers and what happens why you want to have one layer over the other, these green act like rivets. So, that hold the layer and you also see this is an highly exaggerated picture, you have resin and hardener queued here and you have gaps. So, what you find is in the early stages of rapid prototyping, the you were able to get the model that was used for visualization. You want to explain the customer what is it that you want to design, it was used for that purpose to start where there you do not need anything about the strength or anything like that you want only the shape. So, they had pores they had internal stresses and so on and so forth and people also solve this issue. See, if you have a problem it is a nice problem for the research. So, people do not give up and you have a different way of scanning have come about and different ways of putting the layers and now you have much better machines.

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Introduction to Mechanics of Solids

SWAYAM PRABHA

EXPERIMENTAL STRESS ANALYSIS

Digital Photoelasticity

### Fusion of Rapid Prototyping, Photoelasticity & Finite Elements

3D Photoelasticity

CAD

3D FE model

FE Analysis

Isochromatics of slices

RP & RT

Model making

Stress Freezing

Slices

Analysis of splined shafts

A 3 mm slice cut from a stereolithography built stress frozen model.

Courtesy: K. Ramesh, M.A. Kumar and S.G. Dhande: Fusion of digital photoelasticity, rapid prototyping and rapid tooling technologies, Exp. Tech. 23(2) (1999) 36-38

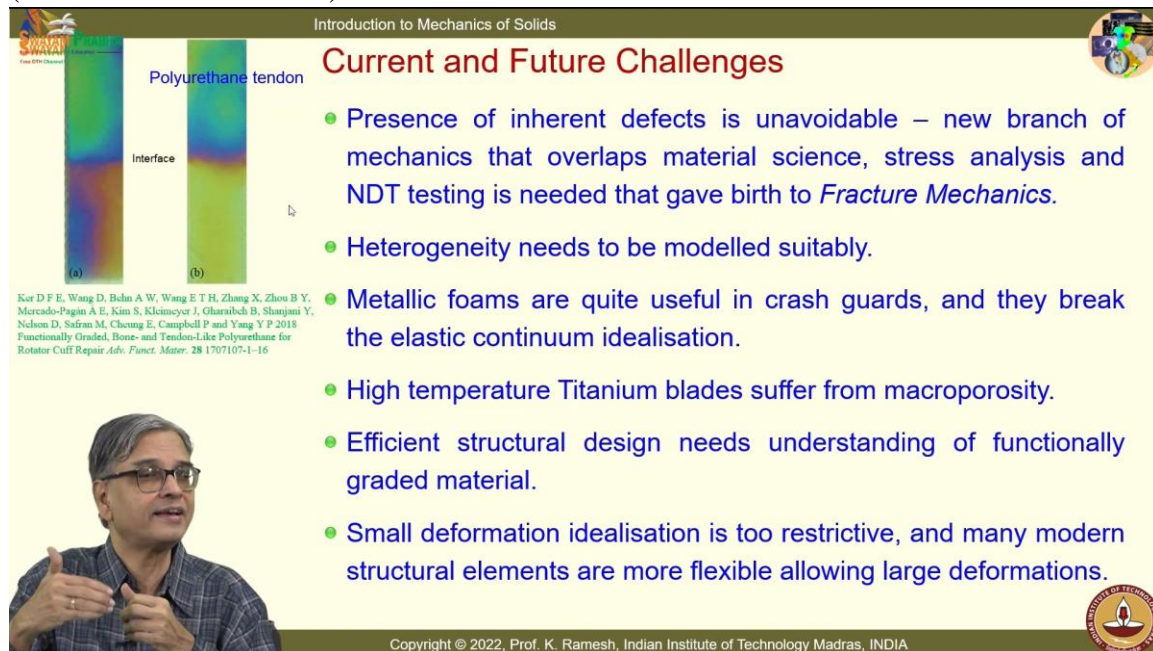
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So, somewhere in the 90s, we proposed- myself and my group at IIT Kanpur when I was working there we proposed fusion of rapid prototyping, photoelasticity and finite elements. So, I can do a three-dimensional photoelasticity based on a CAD model. Either, I could do it by rapid prototyping like additive manufacturing or rapid tooling, where I make a mold and then make a cast. The same CAD model can also go for a numerical simulation. With a numerical simulation, you do a finite element analysis, and we have developed sophisticated software to post process a finite element to plot fringe patterns. So, I have an experimental route I have the numerical route, and I can verify it. This was proposed by us way back in 1999. And this was done somewhere around 2005 or so, 2005 to 10. This was done in our lab, and this is for the LCA aircraft.

So, you have a rapid prototyping of the shaft that is used in the Cauvery engine. You have the three-dimensional photoelastic model and it was done several years back. So, you had this photoelastic fringes for a gear slice. You see black dots randomly in a regular fashion; I would not say random. It is mainly because the rapid prototyping process was developing. So, that had its initial hiccups that is seen here.

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Introduction to Mechanics of Solids

Polyurethane tendon

Interface

(a) (b)

**Current and Future Challenges**

- Presence of inherent defects is unavoidable – new branch of mechanics that overlaps material science, stress analysis and NDT testing is needed that gave birth to *Fracture Mechanics*.
- Heterogeneity needs to be modelled suitably.
- Metallic foams are quite useful in crash guards, and they break the elastic continuum idealisation.
- High temperature Titanium blades suffer from macroporosity.
- Efficient structural design needs understanding of functionally graded material.
- Small deformation idealisation is too restrictive, and many modern structural elements are more flexible allowing large deformations.

Kor D F E, Wang D, Behn A W, Wang E T H, Zhang X, Zhou B Y, Mercado-Pagan A E, Kim S, Kleimeyer J, Gharabchi B, Shanjani Y, Nelson D, Safran M, Chung E, Campbell P and Yang Y P 2018 Functionally Graded, Bone- and Tendon-Like Polyurethane for Rotator Cuff Repair *Adv. Funct. Mater.* 28 1707107-1-16

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So, with modern developments you can develop much smoother models. So, you have to look at what are the current and future challenges. The presence of inherent defects is unavoidable. And I already said from the failure of liberty ships new branch of mechanics called fracture mechanics got developed. And you need to model heterogeneity you cannot always say homogeneous, homogeneous and live your entire life, fine. And in India also the speeds are going up in the roads. So, you have crashes and it is very important that you have crash protection in all these automobiles, and what people are doing is they are developing metallic foams that means, there are lot of pores. And magnesium is used for that and magnesium alloys are more and more finding application in automobiles and you are not going to have a material which is continuous. So, all the idealization that we have made in this course are breaking and we have already seen any mathematical model which I want to develop, it can be accepted only if it is verified by an experiment.

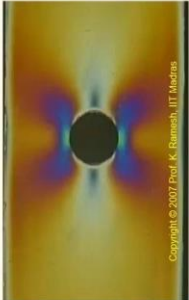
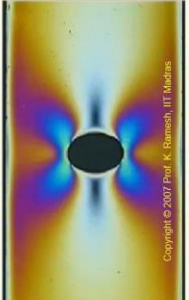

What experiment can I use it to do this complex situations where there is heterogeneity, where there is pores; we have to look at. And we also find, when I want to have high efficiency for engines, we go for high temperatures and then titanium blades are made they suffer from macro porosity not micro pores, but macro porosity and how the crack interacts with holes you need to know. An efficient structural design needs understanding of functionally graded material, I have already shown if you make it out of a polyurethane you are able to visualize the need for the functionally graded material. We have already seen when there is a step in the beam, there was lot of concentration of fringes something like a stress concentration. So, functionally graded material when you look at a bone, bone is connected to your muscle, muscle is soft, bone is hard. That is done by a tendon. So, tendon has to be functionally graded, ok.

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Introduction to Mechanics of Solids  
 SWAYAM PRABHA  
 ENGINEERING FRACTURE MECHANICS  
 Overview of Fracture

### Photoelastic Appreciation of Severity of a Crack

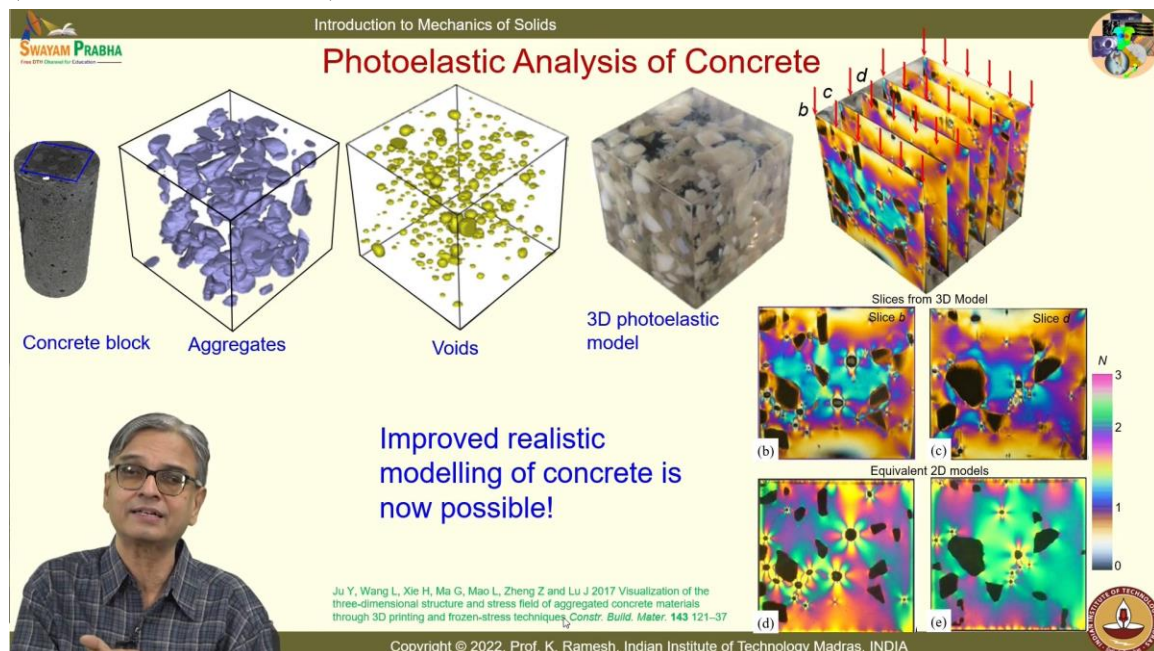
- Fringe density is indicative of the severity of the stress field.

Circular hole	Elliptical hole	Edge crack
		
Circular hole dia = 12 mm	Length of major axis = 12 mm	Crack length = 12 mm

Basics of Photoelasticity  
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So, for all this, you know there is also another thing when people, were looking at mathematicians developed all of this if I have a plate with a infinite plate with a small hole which is bad. People said the small hole is very small; you can completely ignore it. People had fist fights in conferences, mind you. They were not registering that hole can be dangerous, ok. And what you see here is, comparison of a hole, an elliptical flaw and a crack, ok. So, what you find here is, you see the play of fringes until photoelasticity were very useful to convince the mathematicians; you cannot ignore the hole, there are lot of fringes very close to the hole, it indicates stress concentration it has to be handled properly. And if you compare a hole to a crack the amount of material removed in a crack is what? very, very less! Your intuition will say only small material is removed; I do not have to worry about it. What does this picture say? This picture says among the circular hole, elliptical and a crack, you see the largest fringe for the same load. So, it is an indication that the stress levels are very, very high, fine.

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And you also have I said heterogeneity you cannot hide away, you have to recognize heterogeneity, and we have very nicely developed CAD scanners these days. So, people have scanned this concrete block, identified that it this has aggregates. It also has pores and if I want to do an experiment, I must mimic this verbatim like this. Only then whatever the experiment I do the results can be extrapolated for analysis.

People have done it using CAD scanners and by rapid prototyping with several materials and this is a 3D photoelastic model, and this is analyzed with a compressive load, and what you see as black are the aggregates. So, you compare what is happening in a 2D model and a 3D model, you understand what happens in the three dimension.

So, you are in a better position to analyze heterogeneous materials; functionally graded materials. If I want to analyze the metallic form with CAD scanning, I can mimic that precisely using photoelasticity, and this is the work done in 2017 that is about 5 years back. I talked about some of the rapid prototyping then 20 years back. Now the technology has very well developed and photoelasticity holds the key, please understand that! And you are going to have an experiment on photoelasticity in your lab.

(Refer Slide Time: 53.15)

Introduction to Mechanics of Solids

### Usefulness of Plotting Photoelastic Contours from Numerical Techniques

Experimental

Tresca

von Mises

Numerical Postprocessed

MPa

N

Postprocessing software developed at Digital Photomechanics Laboratory, IIT Madras

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And I also said there is a crack interacting with macro porosity this is an experimental result. And many of the finite element packages will give you only a plot like this either this or this.

They are not good to convince you that your numerical modeling is precise. On the other hand if you post process in plotting fringe patterns, you can easily explain to your mother, mother I have done this work, fine. You do you find that they are identical, I am also going to get my MS thesis, fine. You can do that.

So, with this opening remarks let me complete the introduction to strength of materials. The idea is to see what is the scope of this course. The scope of this course is very limited. We want to have small deformation, we want to have homogeneity, we want to have isotropy, but you cannot stop there. The current needs are widely different; fortunately the concepts that we are going to develop as stress or strain will have the elements which would form the foundation for you to extrapolate this knowledge and understand the current developments better. And we have also seen convincingly, we are inching towards nature is still far ahead of us. Thank you very much.