Strength of Materials
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Lecture - 20
Torsion 1- Thought and Physical Experiments

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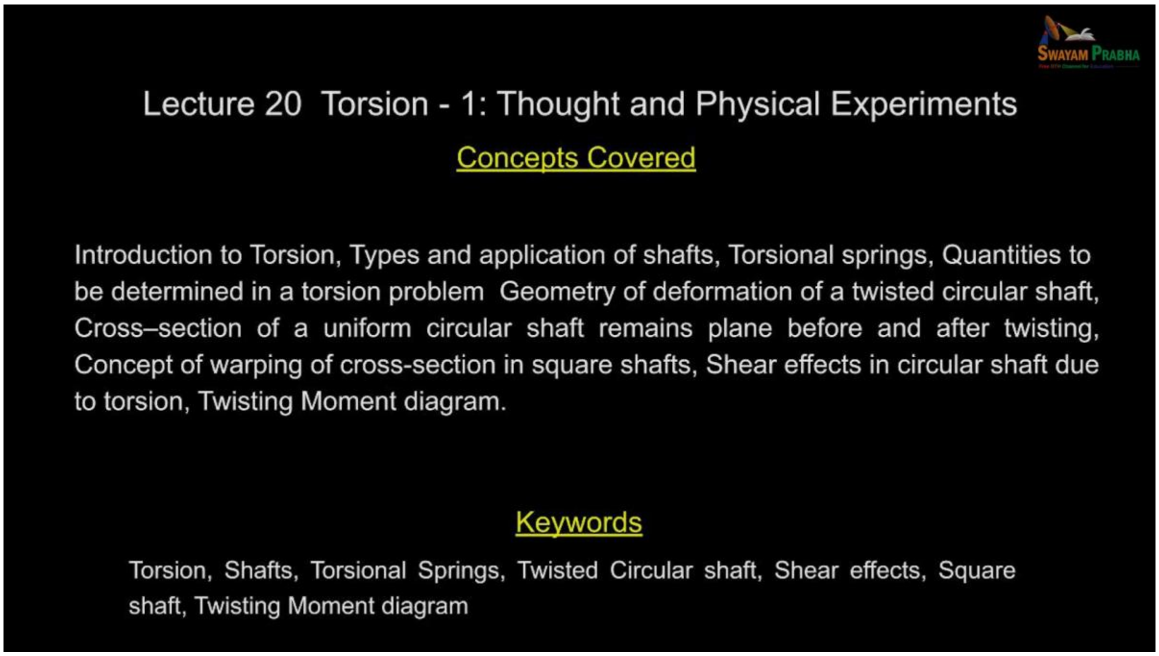

See, one of the important learnings in the course on strength of materials is to see how a shaft resists the torsional load and how a bend, how a beam resists the bending moment, fine. So, we come to the main discussion on how a shaft will resist a torsional load. And before we get into this, you know, you will also have to bring back your old memories on how do you understand what is known as a couple force system.

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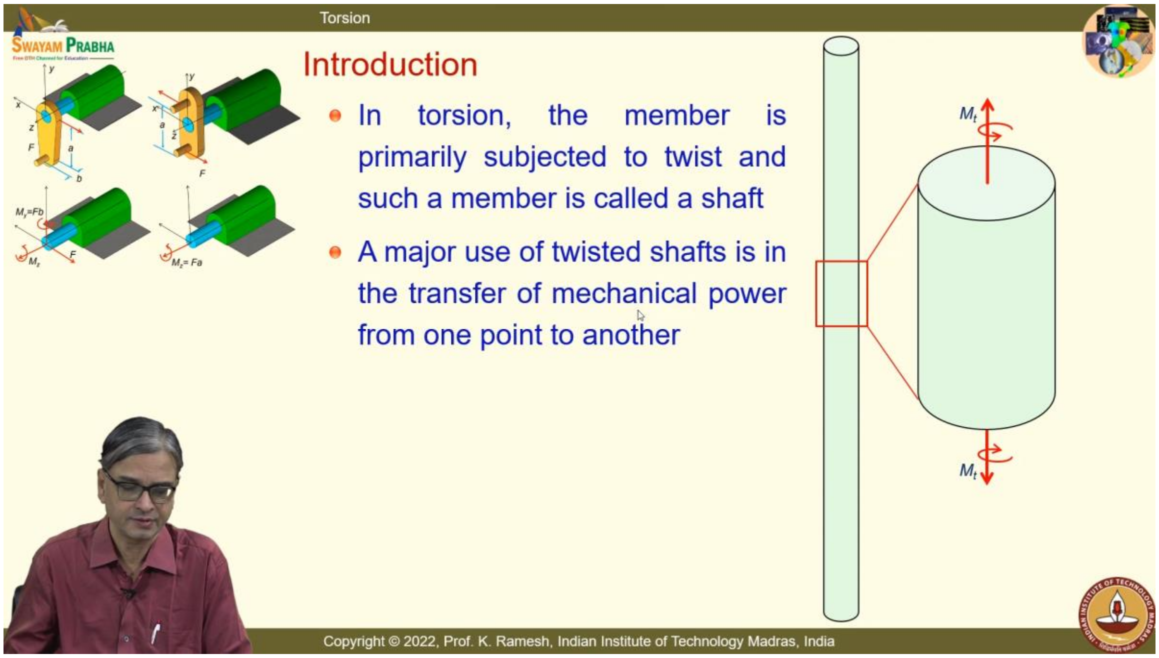

See, if I have a couple force system, when I look at what is the kind of effect on the shaft, you get only a twisting moment. You get the idea? On the other hand, if I rotate the same shaft with a single arm, I get the rotation. In addition, I also have another moment and also a force acting on the shaft. So, there is a subtle difference. And if you look at, you know, in this course, we will confine ourselves to analysis of slender members. It is very, very important. You should not lose track of this idealization that we handle only slender members.

You know, for the purpose of discussion, I might show a portion of the shaft shown big. Not only this, I may take a particular cross section or depending on the need, when I have to discuss certain aspects related to the shaft, those diagrams will be very, very high for you to visualize. So, do not think that we are handling a slab or something like that. It is always a slender member. The cross-sectional dimensions are much smaller than the length of the member.

In torsion, what happens? The member is primarily subjected to a twist and such a member is also called a shaft. The moment you say shaft, you understand that it transmits torque. A major use of twisted shafts is in the transfer of mechanical power. See, it is used in a wide variety of applications and it is a very important mechanical component. So, if you learn how to design it, you can handle a host of problems.

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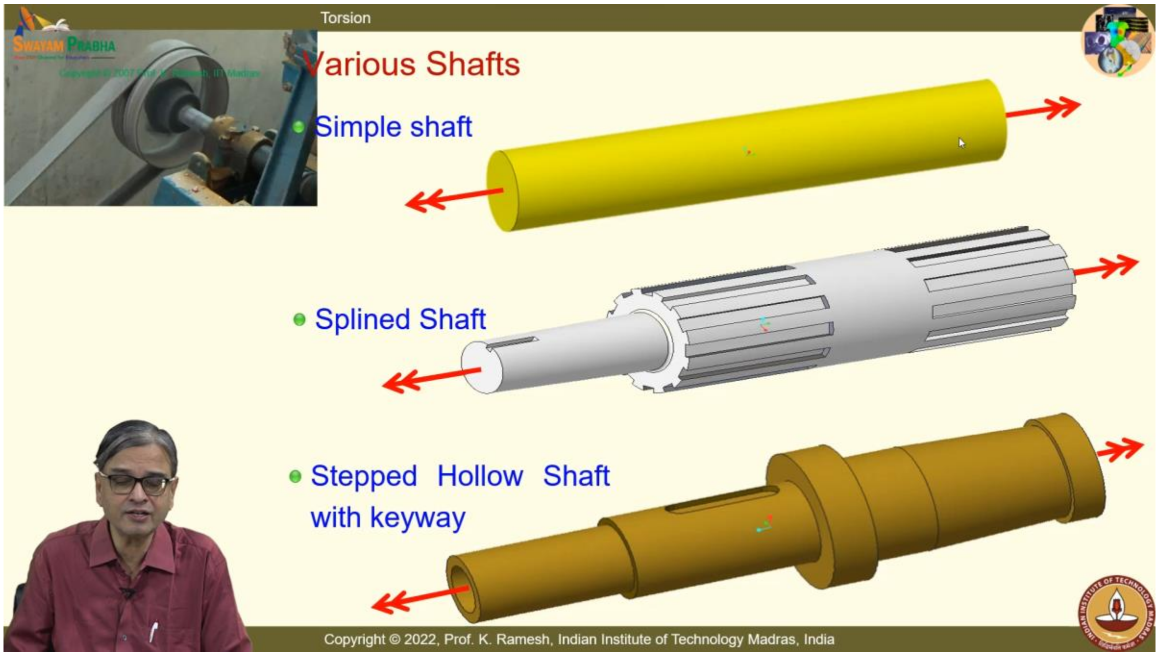

And you would have seen in big trucks, this is a universal joint and this is the axis of rotation from this shaft. And whatever the power from this is transferred to another shaft at an angle. It is not at the same angle as this. And this is very nice to see that like you are applying a couple, you are actually twisting this. You have a better feeling of it, that is why I have shown this.

And we have also seen in the case of simple atta chakki, you have a shaft, and when you take any shaft, it is not simple like a cylinder. But from our analysis point of view, for us to develop the appropriate mathematics, we will consider a very simple shaft. If you go and open up a gearbox, you will have a splined shaft. And some of you have learnt how to drive the conventional car, where you have a gearbox, where you have to open it and then see when you shift the gears, you have splines that help you to shift the gears.

So, this is very complicated. And you know how engineers handle it because you know they will find out a basic solution. Depending on the situation, they can have a factor of safety or some empirical analysis. Only if it is absolutely necessary, they go for a numerical simulation and all the other things. Because when you do all those numerical simulations, the analysis becomes very costly. And we have to mount certain things on the shaft. So, you have a keyway, you have a step. And when you have a keyway, it is going to be a stress riser. So, you have to accommodate that while you plan your design.

The shafts need not be solid, it can also be a hollow shaft. And this has several other features. You can also have a tapered shaft it need not be a cylindrical. For the purpose of developing the basic interrelationship between shear and angle of twist and so on, we would confine ourselves to a cylindrical shaft like this, which is of constant cross-section. And mind you, in strength of materials, we also indirectly do one thing without telling it openly. When we analyze torsion of shafts, we only worry about torsional load on a circular cross-section. But torsional load can happen on any type of structure. I can have a non-circular cross-section. Those analyses become difficult. So, we postpone it to the next level course.

So, in the first level course, we would like to develop how the shaft resists torsion. That is a circular cross-section. It has a wide variety of application. And it is difficult if I have anything other than circular when you want to do a simplistic analysis.

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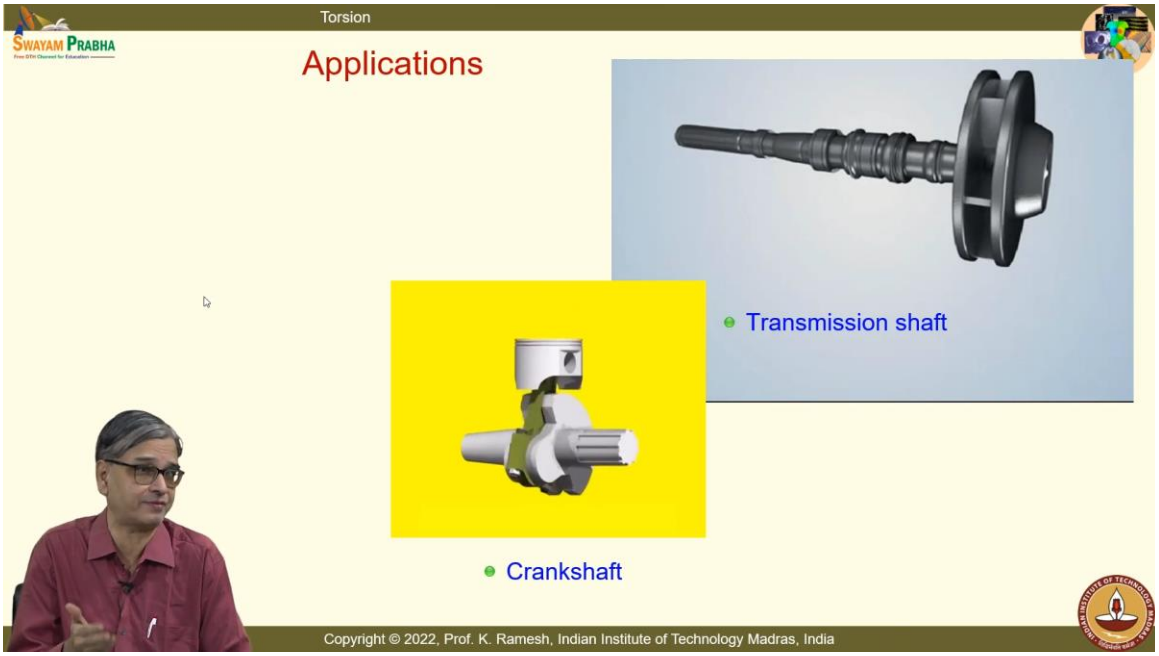

And what are the applications? You know, we have already seen the IC engine where you have looked at the Gudgeon pin, how it is made. You need at very high plasticity to do that. And you have a shaft and this also shows the, you know, balancing wheel. So, I have a tapered shaft here, I have a splined end here and you have a gas turbine here which also rotates and this has a splines. So, any application will have steps, splines, key base. Without that you will not be able to use a shaft.

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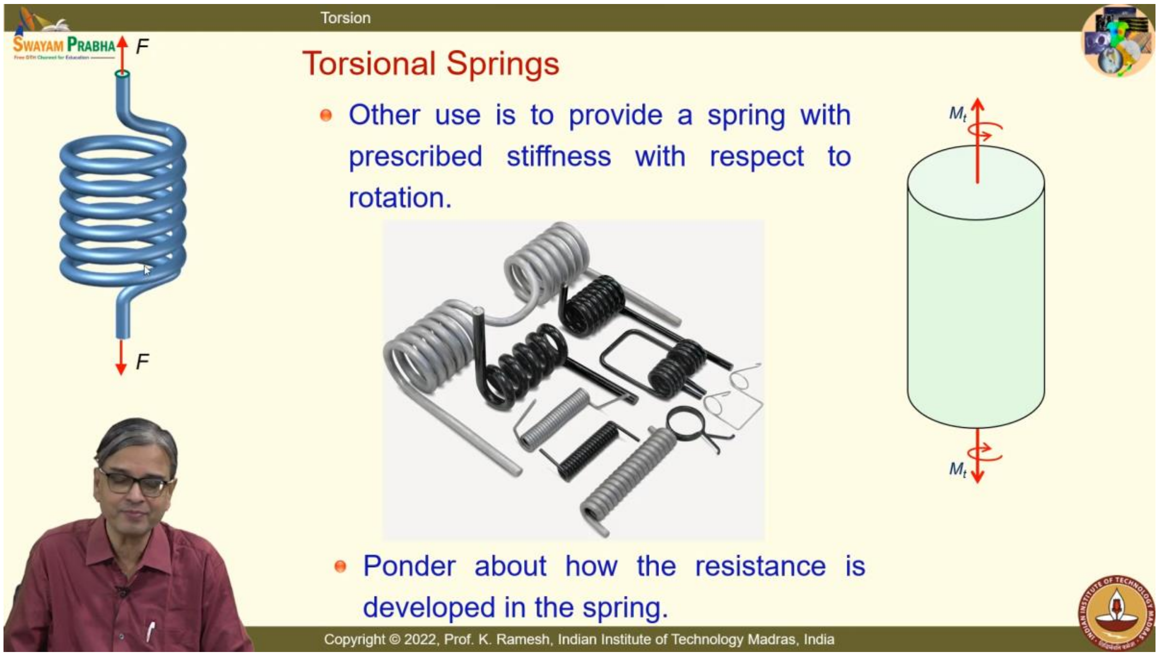

On the other hand, when I go and look at springs, you know, it is primarily a constant cross-section. And you know, once you come to deformable solids, the understanding is it can store energy because it is deformable, it is not rigid, it can store energy. Have you seen some of these springs in your day-to-day life? Have you used it? Can you tell me which application you have used these kinds of springs? Writing pad, I do not know. How many of you are having your cycle? You ride your cycle, in cycle many places you have such springs. If you have a carrier, you have a beautiful spring like this. And you know, what is it that it does? It provides a stiffness and it resists rotation because you want to lift the carrier and when you leave it, it comes and grips whatever that you have kept and it should not fall back when you ride the cycle. So, it should have sufficient force. This is named as a torsional spring. But I would like you to go back and think, what is the resisting force developed in this member? Long back I asked you, we have a compression spring or a tension spring like this. I do not know how many of you even attempted.

When I pull this or compress this, take a generic cross section and find out what is the way resistance is developed? What is the actual force system acting on a cross section of the spring? It is not trivial. I have a reason for it. You know, I have a reason for showing a tension or compression spring as well as this spring. I want you to ponder about how the resistance is developed in these two categories of spring. They are different. They are not one and the same. From a functional approach, you label this as a torsional spring. It is not resisting torsion. I have given you the clue. So, look at what happens in this spring and also look at what happens in this spring. These two are different. Which one is relevant to this course? You have a look at it.

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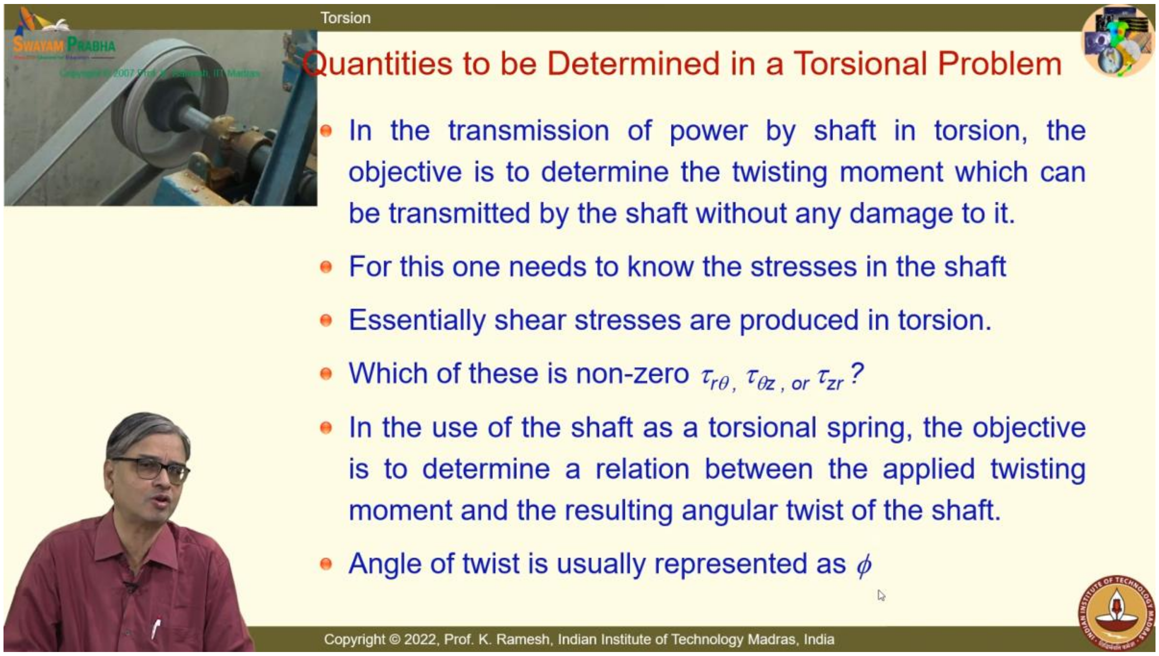

And what are the quantities that you need to find in a torsional problem? First is transmission of power and you are a mechanical engineer. I have a power and then power is equal to torque and ω speed. And why do you have a gear box? When I want to start the car, I need more torque, fine, So, I operate at a lower speed and when I go in the road highways, I would like to have only speed. Minimal torque is sufficient. So, you manipulate your speed by using the gear box. The power still remains the same, fine.

So, one of the main objectives of a shaft is to transmission of power. So, that is where you have to find out whether the cross section is sufficient and it does not fail. And it is also subjected to a fatigue loading. In this course, we may not focus on fatigue. So, the shaft design has to take care of torsional loads, bending loads and if you have helical gear mounted on it, you may also have an axial load. So, you will have a combination of all the three loads are possible.

So, one needs to know how the stresses are developed due to torsion. And even before we go and try to find out formally, we have been saying this that it essentially transmits shear stress. Even in my first class, I have shown you what is the nature of shear stress distribution. Since I am having a circular cross section, it is convenient to analyze it in a polar coordinate system. So, once you go to polar coordinate system, the question arises among the shear stresses $\tau_{r\theta}$, $\tau_{\theta z}$ or τ_{zr} , which one will be non-zero or whether all these stresses are existing. And in the case of a torsional spring, the objective is to determine a relation between the applied twisting moment and the resulting angular twist of the spring.

And angle of twist is usually represented as an angle ϕ that is a symbol you normally come across. So, these are the questions for which we need to find an answer.

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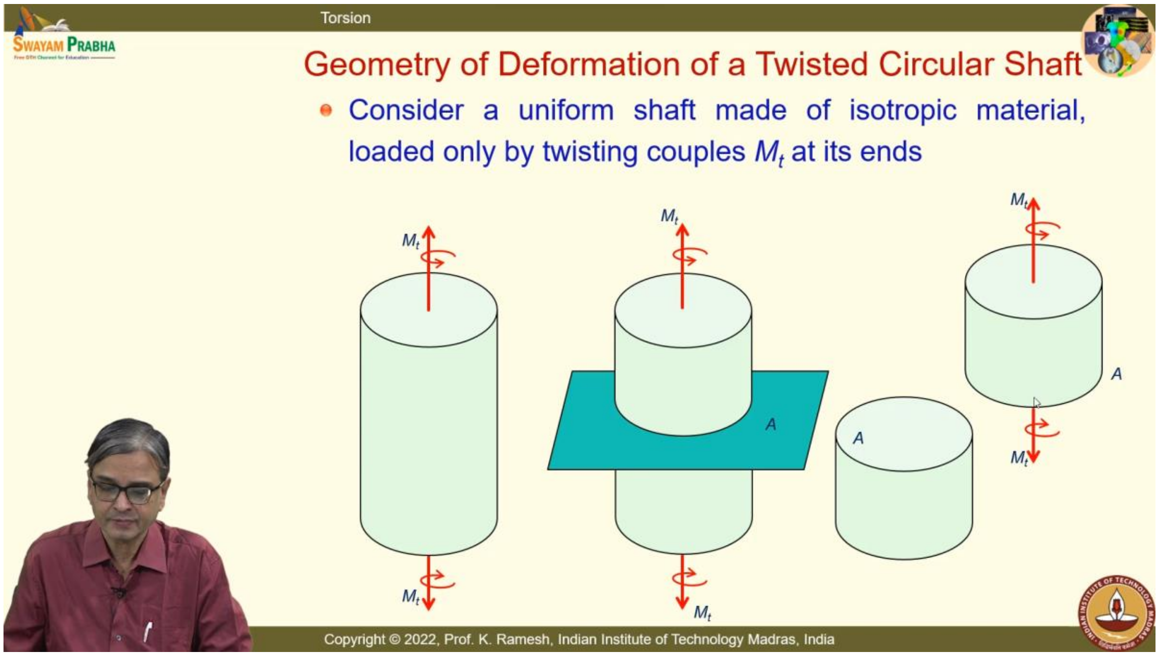

And you know what we are going to do is, see in those days when people graduated from rigid body mechanics to deformable solids, they were essentially handling stone and wood. They were not having soft material. See, one of the simplest ways to do is, if I take a soft material, put lines and then twist it and then find out what happens, then I can write my mathematics immediately. And you know, we have also made a square member. This is soft, but it is not soft enough for me to show certain properties. And for that to be done, actually we got this model specially made for me to show this. We will see all this towards the end of this class.

Now, we will have to go back and see how our forefathers when they were developing torsion, they were also doing a thought experiment. See, in the learning process, experiential knowledge is very important. It is also very simple provided you have access to soft material. Suppose, you do not have access to soft material where the deformations can be visualized in perceptible levels, then you have to do a thought experiment. And what we are going to hang on is, this is an isotropic material.

It is an isotropic material; it is very important. It is loaded only by twisting moment M_t at its ends. First thing is, what is it transmitted? See, if it is an axial force, you all agree that when you go from one section to another section, it is transmitting the same force. A similar thing also happens in the case of a torque applied to a shaft. First, we will understand what is transmitted across the cross-section.

So, that means, I pass a generic section and then split it and then see what happens in the top portion and what happens in the bottom portion. And I would like you to draw many sketches today. You know you have to draw many sketches. So, the idea is to develop a thought experiment and then look at what displacements are possible, what deformations are possible and whether it violates our isotropic material behavior. If it violates, then go back and then see and change your hypothesis.

So, when I have a small portion cut, the top portion also will transmit the same torque. See, there are two ways of indicating the torque. I can have the arrow like this and then show a twisting direction or I can also have double-headed arrows. So, the top portion also will transmit the torque and bottom portion also will transmit the same torque. There is no change in the value of torque when it is a circular cross-section.

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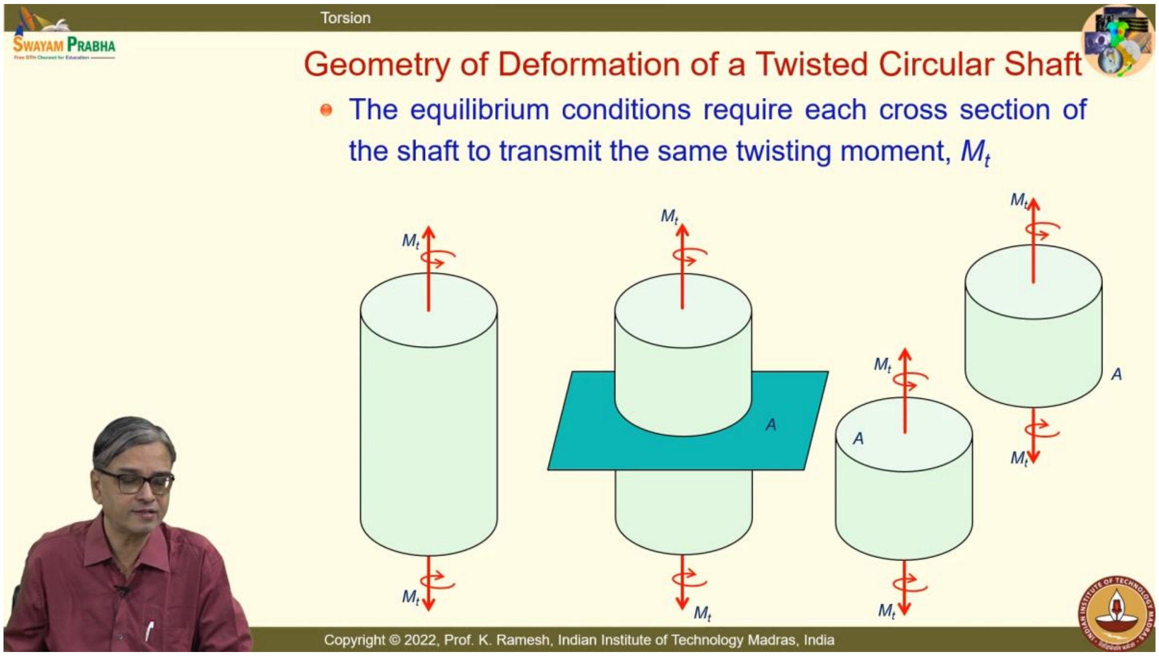

So, that is what is summarized here. The equilibrium conditions require each cross-section of the shaft to transmit the same twisting moment M_t. So, that is the first understanding.

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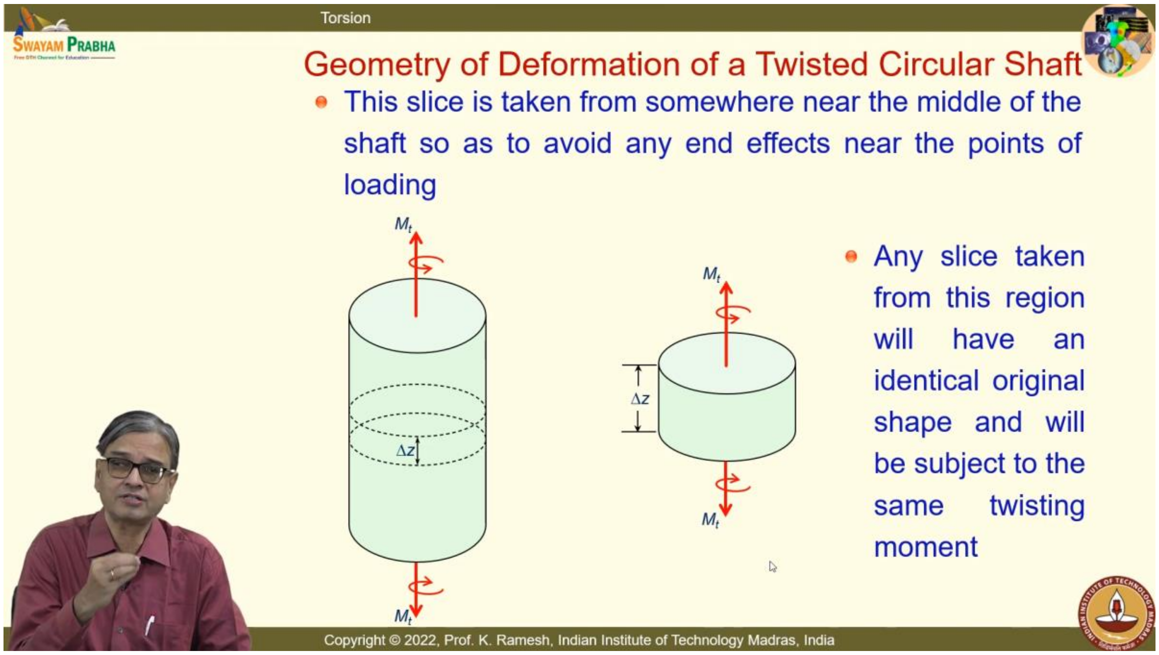

Then we would take a small slice. I am going to take a small slice of height delta z. See, I have already told you if I have to show something in this slice, I will show this length larger. But you should not lose track that we are still analyzing a slender member. For the purpose of discussion, certain portions I enlarge for us to visualize what kind of deformations take place. So, I take this slice out of it and I have the torque transmitted. Any slice taken from this region will have an identical original shape and will be subjected to the same twisting moment. That you should recognize.

Now, we will have to hypothesize. Suppose I take a radial line, how this line can deform? So, you start from a hypothesis and find out whether it satisfies the isotropic and material behavior.

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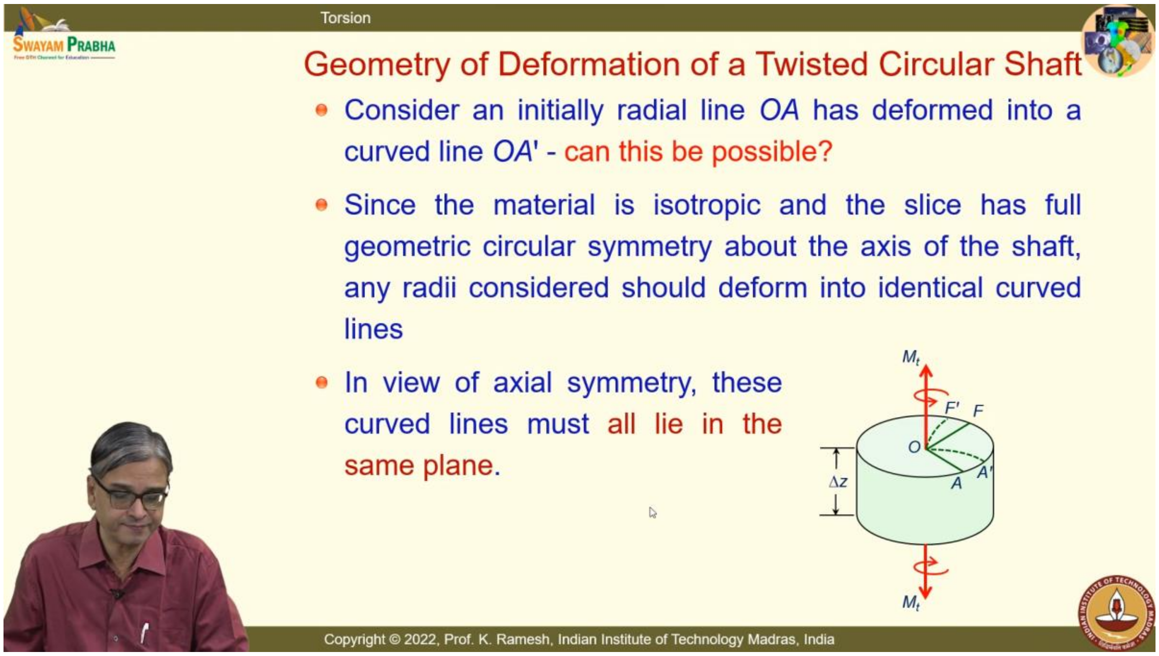

And you know, when you say anything related to symmetry, you should recognize. Suppose I take this as an axis, the geometry is symmetric to that. Boundary conditions are symmetric to that and material behavior is also symmetric to that. That is the reason why we bring in the symmetry argument. So, consider a radial line like this OA and let us say that it has deformed into a curved line. This is a hypothesis. See, hypothesis need not be absolutely correct to start with. We would investigate the hypothesis based on the other aspects of the problem. Here we are going to invoke the symmetry conditions and we have taken as a hypothesis that I have a deformation into a curved line What we are going to investigate is, can this be possible? So, you have to invoke that the material is isotropic. The slice has full geometric circular symmetry about the axis of the shaft. And any radii considered also should have same similar deformation.

Suppose I take another radial line here. If I hypothesize that this can be a curve, the line *OF* also should deform into a curve. And we also make one another very silent hypothesis which is very, very important in the first level course of strength of materials. In view of axial symmetry, these curved lines must all lie in the same plane. That means, if I have this as one plane, all these lines when they deform, in general we start you know we want to pat our back that we have not started with a constraint, we will have that as a curve. Which curve? We do not know. Whether it is a curve or a straight plane is what is to be investigated. Fine. Hypothesis you have to start like that. And what we are going to look at in the complete discussion is we will assume that this is satisfied. This is not a simple statement; it is a very important statement. You can appreciate it only when I twist a square shaft. We will see that also today. Now, we are going to investigate whether this line can remain curved under deformation or not.

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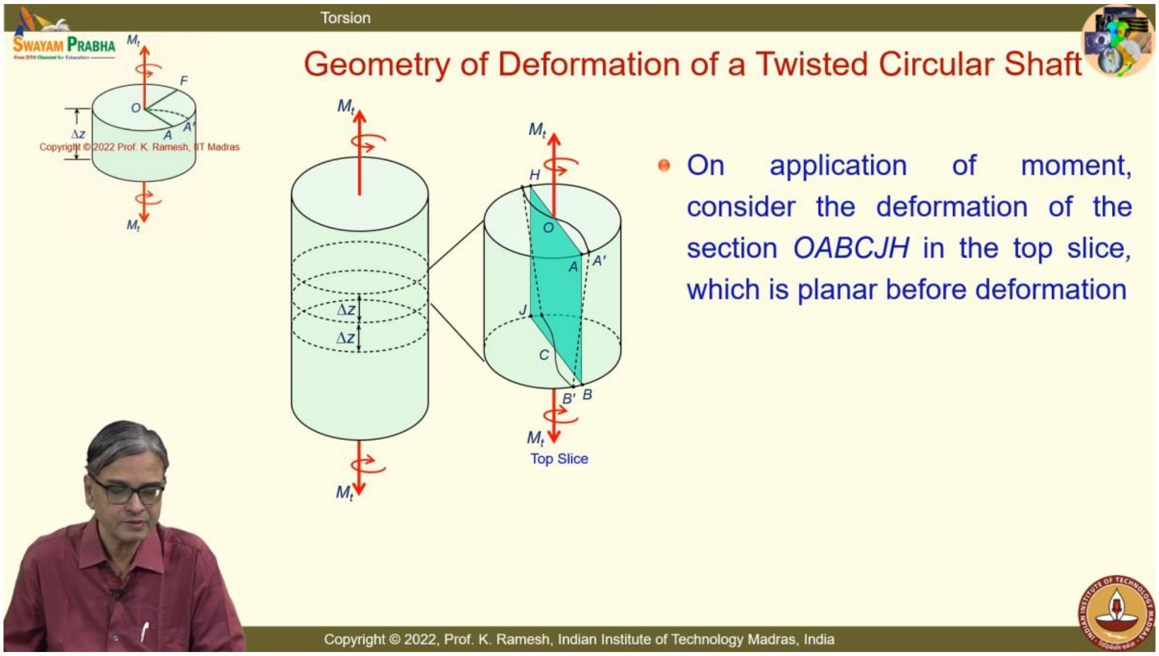

So, we have to look at and when I want to investigate this, I will look at two slices adjacent slices. I have slice 1, I have another slice just below this. And now I want to analyze this slice so I will take this Δz very large for me to illustrate the idea. Otherwise, it is difficult for you to visualize. And when I say this as an original line, I also see a plane. I have this as $OABCJH$. I read this so that you can draw it. I would like you to make a sketch of it.

Now, the idea is when this line under the action of the twisting moment, let us state that this is a curve. See, I have shown some other curve here. Do not worry about it. The idea is we are saying that it is not a straight line, but as a curve, some curve I have taken. Now, the question is if I have a deformation like this, Question number 1: What way can I visualize the deformation at the bottom surface? Question number 2 is if this is deformed like this, what way the next slice will be deformed? Your anticipation is when I take out a slice, whatever the deformation picture I visualize, it should be identical for every slice. That should not be violated. So, that is the way that we will investigate to find out whether the deformation can be a curve. You should understand the question clearly.

See, while learning a subject, it is also better you exercise your brain to do some thinking. So, that rational thinking habit has to come. So, for that reason this is a good discussion. And you know it is getting twisted. I am twisting it like this, I am twisting it like this. So, it is logical to visualize the bottom portion can have a curve like this and these are joined by straight lines. That is the hypothesis. So, we have this.

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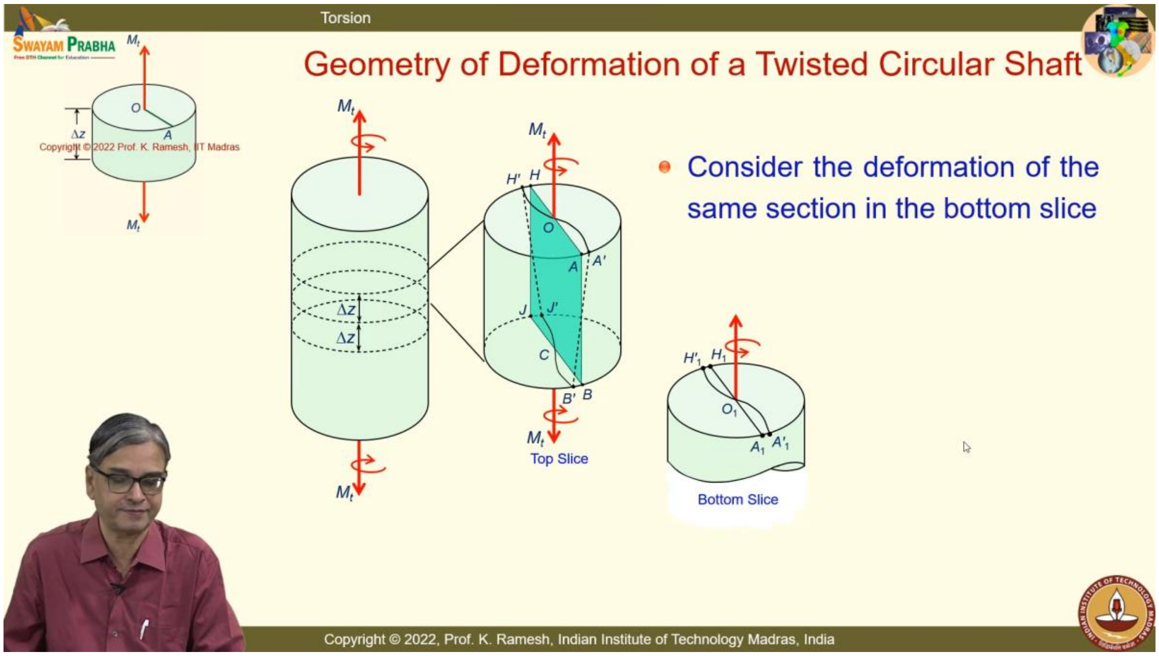

And let us see, this is also labeled as OA', B', J' and H'. Consider the deformation of the same section at the bottom slice. And we have to see is there a contradiction between the two. If I take this slice, the top portion should be identical to this. So, I have labeled this to distinguish between the top slice to bottom slice as A_1, O_1 and H_1. And I have also shown very similar to this. Now, the question is we have to compare what happens here and what happens here because these two are same surfaces. I get one result from this; I get another requirement from this. Let us investigate.

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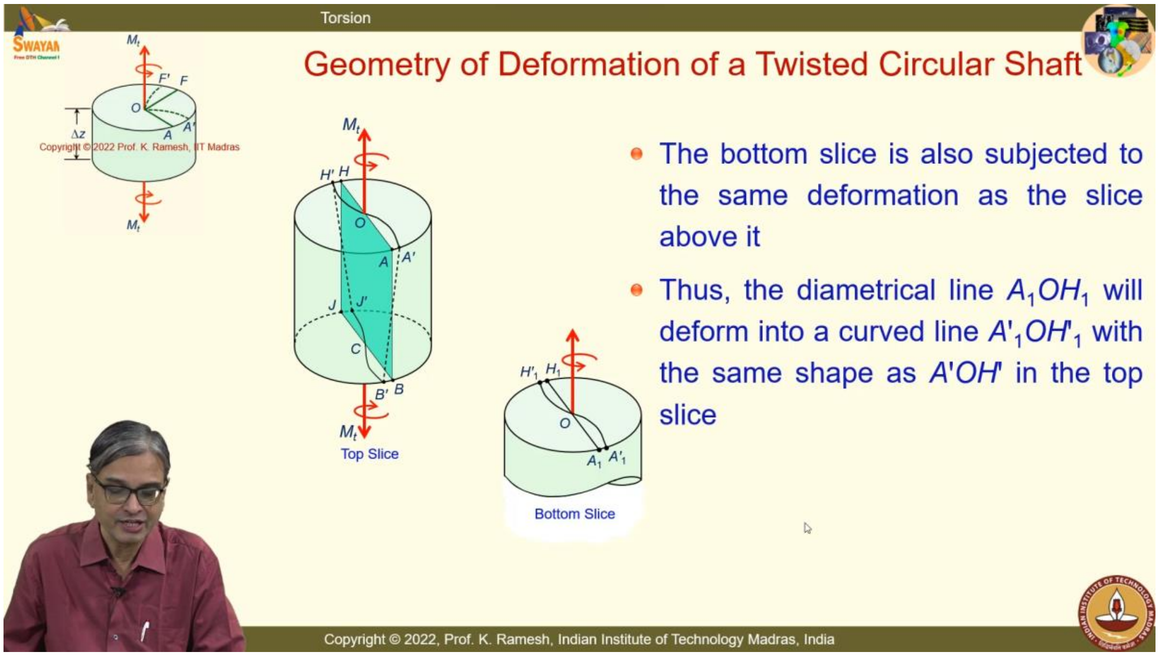

See the bottom slice is also subject to the same deformation as the slice above it which we have already noted and it should also be like this. So, the diametrical line $A_1 O_1 H_1$ will deform into this. But these two are different.

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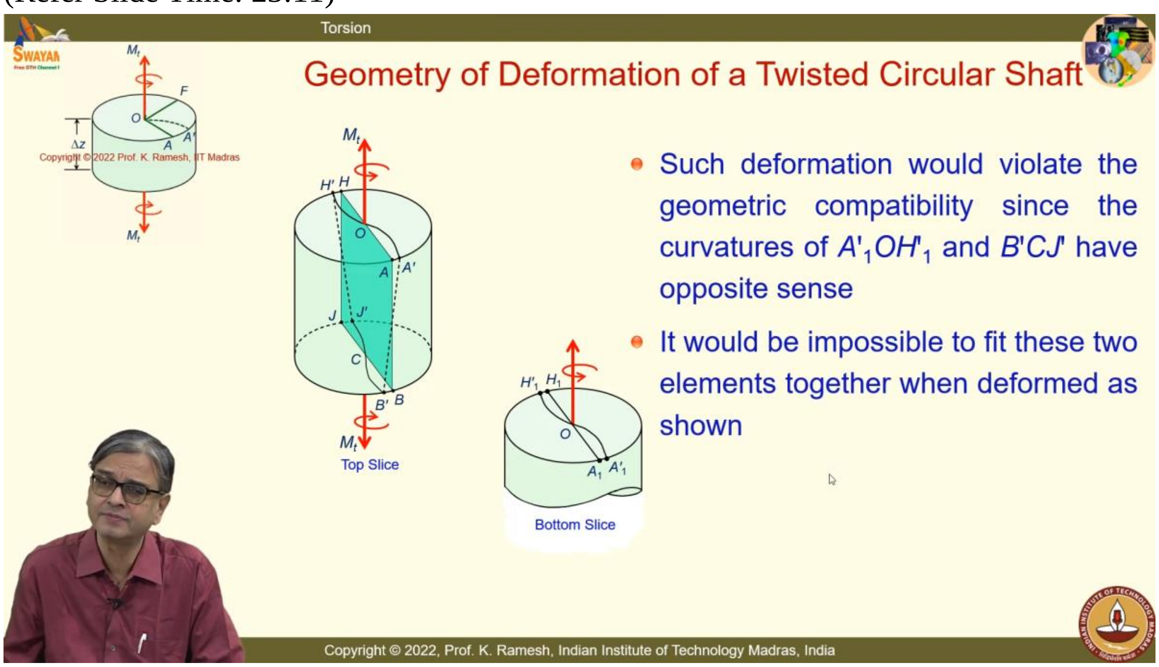

So, in order to satisfy this, let us hypothesis another step. What we will do is we will change the curvature. They are in opposite sense. So, we will change the curvature of this. It would be impossible to fit these two elements together when deformed as shown.

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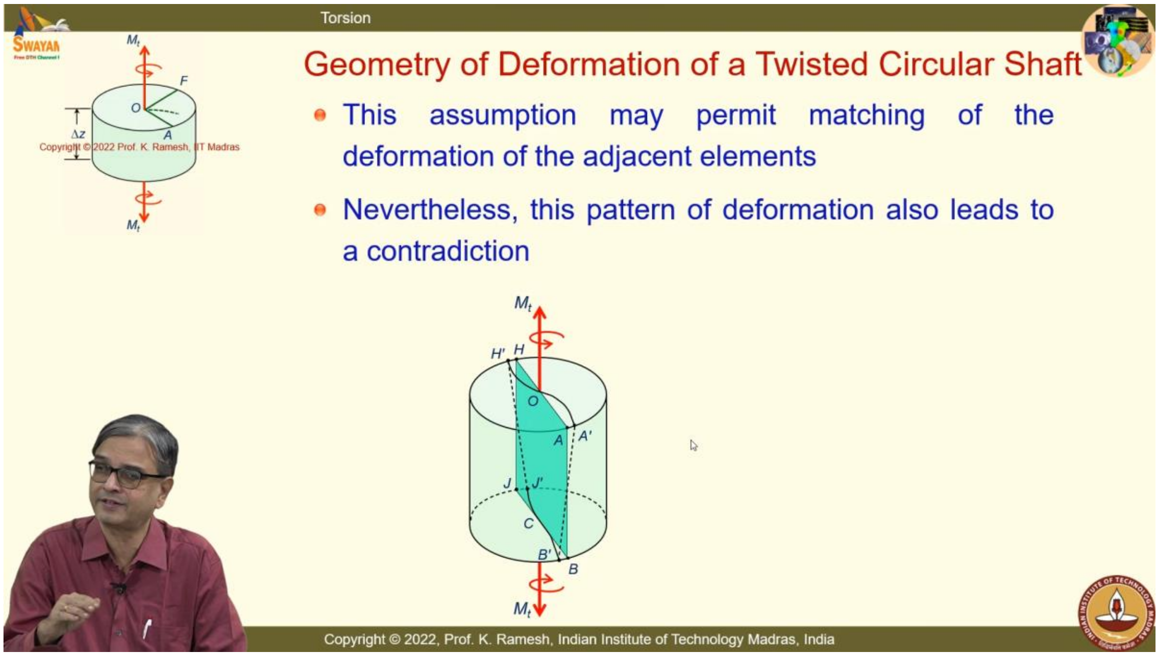

And we would change the curvature. Instead of a curvature like this, in the thought experiment for the bottom, let me change the curvature like this. They are different. So, we hypothesize that the curvature becomes different when you go from top surface to bottom surface. This may permit matching. It does not say that when you hypothesize a priori you know whether it is going to match or not. It may permit. But when you investigate it very closely, what you will find is even this pattern of deformation also leads to a contradiction. How do you verify this? See the top portion should remain same whether it is the top slice or the bottom slice. There should not be any difference.

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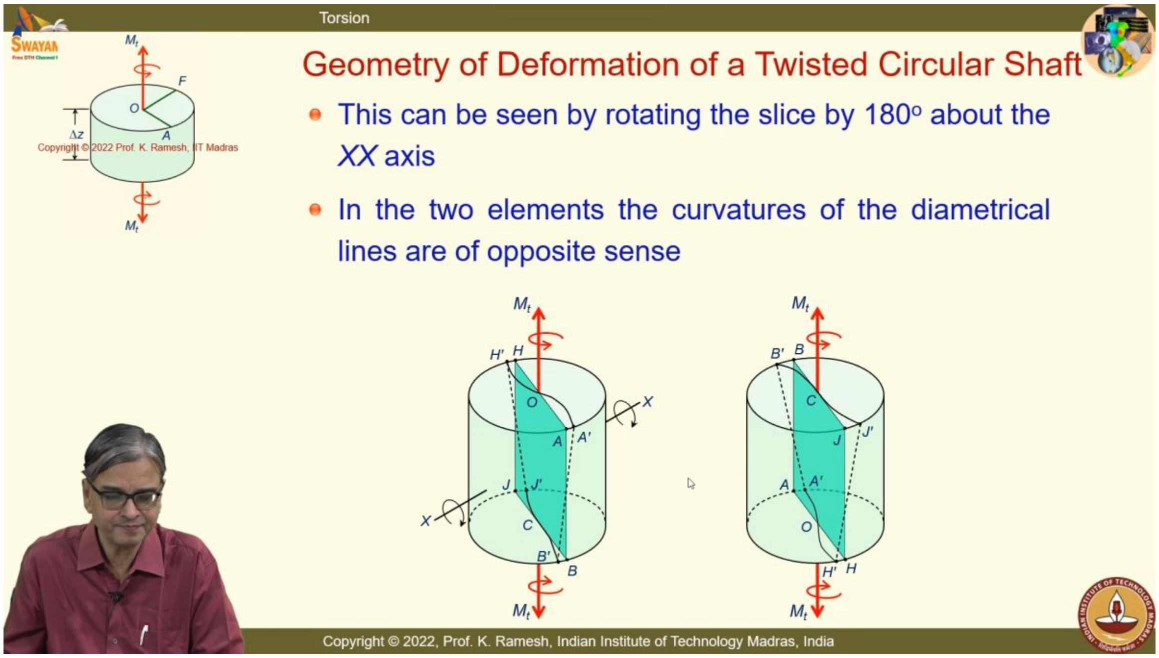

So, one way of doing this is in the thought experiment is rotate this by 180°. Rotate this by 180°. Is the idea clear? Please make your sketch. See we are developing a logical framework to give some foot for thought to your brain. Fine. At the end of it, we will understand what is the deformation that is possible. You started with a generic curve and then as we proceed looking at that there could be a difference between the top and bottom, we hypothesis again to change the curvature to see whether it satisfies. If it does not satisfy, we will have to go back and see what way it can deform. That is the logic behind this.

So, when I rotate it by 180°, I will have a situation like this. So, this is not same as this. Is the idea clear? Is the idea clear? When you are investigating symmetry, we find that this violates the symmetry. This aspect violates the symmetry. So, what is the way it can deform? It can deform only as a straight line. That is the conclusion that we are going to arrive at. The curvature, even when I change the curvature, they are not identical.

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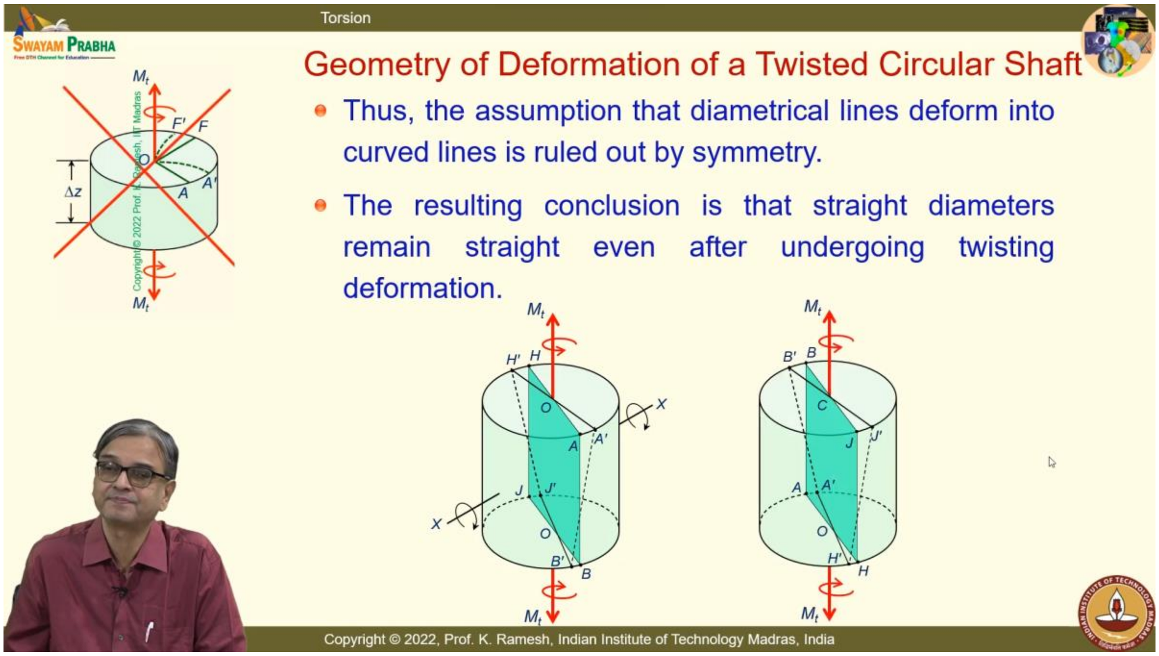

The only way that it can deform is a line OA will deform to another straight-line OA'. Is the idea clear? This is by thought experiment.

Thus, the assumption that diametrical lines deform into curved lines is ruled out by symmetry. The resulting conclusion is that straight diameters remain straight even after undergoing twisting deformation. So, this is when I rotate it by 180°, I would get identical shape. So, this is the only possibility of deformation when you apply the twisting moment.

You see, you should also imagine those days when they were having very stiff material. And I told you that for me to get this soft, we have to do it very, take special efforts to get it done in IIT. Commercially, what is available is not possible for me to show what I want to show, fine, So, in those days they were not having the material. So, they were only conjecturing based on this. See, when deformations are exaggerated, your thinking process is also better for you to visualize what are the interrelationships and so on. For you to develop the theory, you need to have a picture of the deformation very clear.

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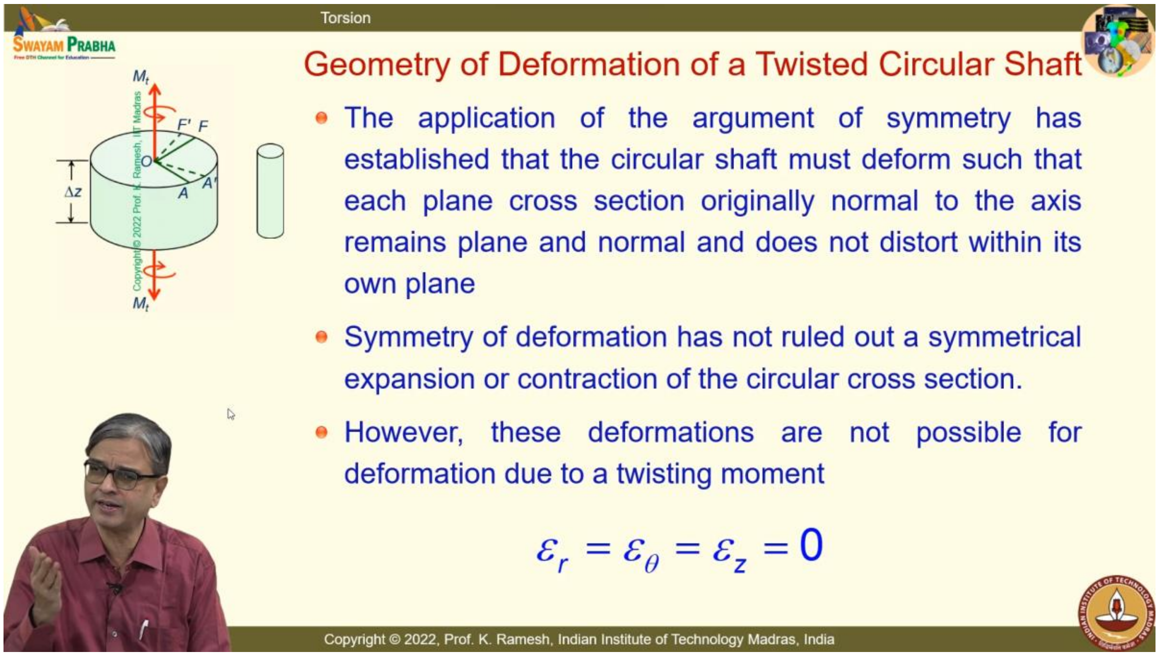

So, the application of the argument of symmetry has established that the circular shaft must deform such that each plane cross section originally normal to the axis remains plane and normal and does not distort within its own plane. You know, I have said two things. We have looked at the radial line and we found that this radial line cannot have a curved deformation. It can have only a straight line. We also made another statement that all these lines remain in the plane that we have not even investigated. Whether it can be in a different plane or not, we have not even investigated. We have taken it for granted as a result that all these lines remain in a plane. I will show you an experiment and then convince you.

Before that, when I look at only symmetry, see the symmetry of deformation can also say it can have an expansion and contraction. Yeah, it can have an expansion or contraction. Symmetry is still maintained. But you know, when you are handling an isotropic material, we have been discussing when I pull it axially, it will have only axial deformation. When I shear it, it will have only shear deformation. That is the beauty of isotropic material. So, even though from a symmetry argument, it can have expansion or contraction.

Since you have applied only torsion to the member, it can have only shear stresses developed. You cannot have normal strains. So, I will not have ε_r , ε_θ and ε_z , they go to 0. Is the idea clear? See, since our hypothesis was based on symmetry, how do we investigate the deformation? So, when you investigate that, symmetry or deformation is still there when you have expansion or contraction. But because the nature of load that I applied is such, you cannot have expansion or contraction. So, that rules out ε_r , ε_θ , ε_z equal to 0. I suppose you appreciate that this is the axis, I call this as z axis. And then when you have a circle geometry, I have this as r and θ coordinate. They are not shown here, but you can visualize that they can be taken like this.

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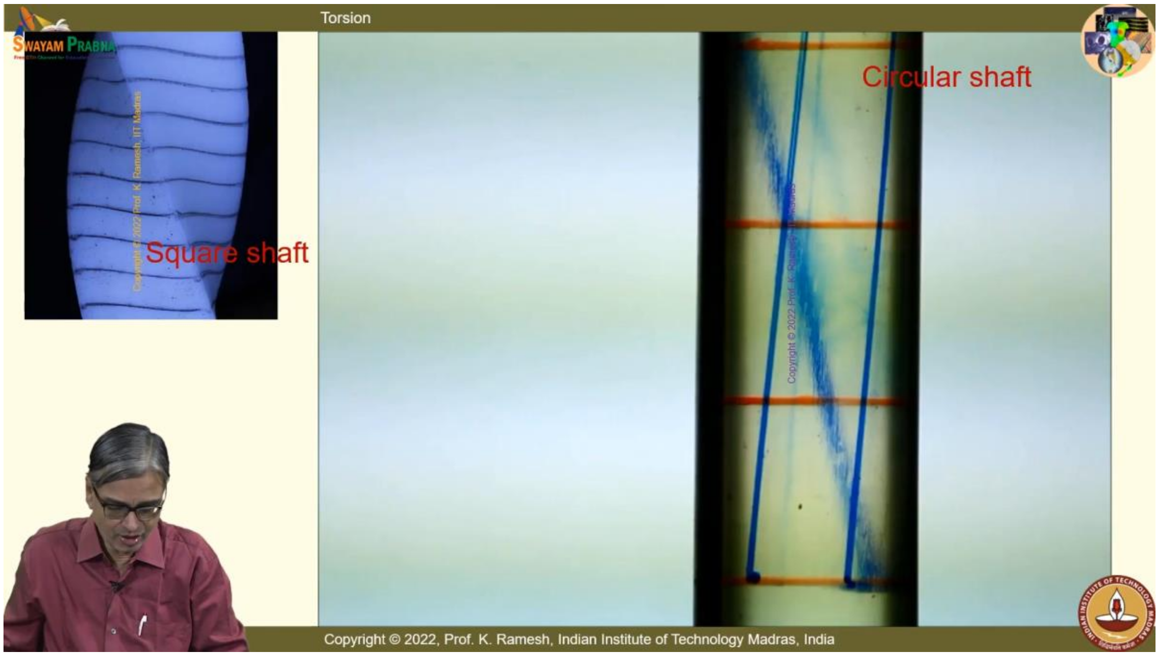

So, now what I am going to do is, I want to show you the shaft and this is a very nice demonstration. You look at the red line as well as the blue line. See, the red line is like the cross section. We said all the deformed line should lie in that plane. We have not even investigated it based on symmetry while developing the argument. But when I take the shaft and twist it, I find this line remains horizontal. This gives the hypothesis that this cross section remains plane before and after loading. See, you see a small shift because of the focusing, because you know when I am taking the shaft and then twisting it, we are doing it with hand. We are not doing it in an equipment and even when you put a camera stable, you know you have this difficulty in focusing.

On the other hand, if I take a square shaft and then I twist it, which is also shown in the animation in the left. So, I have a horizontal line. This horizontal line, does it remain horizontal in a square shaft? Let us see this.

I take a square shaft; I have a horizontal line and this is twisted. Do you see that this is moved up and then there is a slant and then it is coming down? Do you see that it is like this? It is very, very important, it is a very subtle aspect. And this has a bombastic name in literature. It is called warping of the cross section. Please write down. See what we do in strength of materials is, we take up problems, we take up material, we take up cross sections in such a manner that you can solve it comfortably in this course.

Yes, square shaft also can transmit torsion and if you go to an automobile chase, it has all funny cross-sectional shapes. They all transmit torsion. But at this level of the course, we do not have the mathematical background to analyze such structures. We confine ourselves to circular cross section mainly because you see the animation again. There are many things are illustrated here. See, I am holding it top and bottom. I am holding it top and bottom and I do the twisting. And I have the twisted shaft and in this, I have this red line is the horizontal, remains horizontal. It is not moving up and down. Whereas, the same horizontal line in the case of a square shaft has moved up and down. That is called warping of cross section. Because of warping of cross section, we do not analyze it in this course.

And this solution was first proposed by Saint-Venant and you know Saint Venant from Saint Venant's principle. He had solved it using the differential equations approach. He has, so you have solution for this. Please do not think that you do not have solution for what are the stresses developed in a square shaft. You have solution, but it is beyond the level of this course. So, and you also see that the straight lines which are drawn, they are modified like this. Because of twisting, you can see that again the straight lines how they are twisted.

The straight line, it is sheared. You can very well see that this is sheared. Is the idea clear? You can very well see that the rectangle has become a parallelogram. You can see right from the beginning; I have been saying that a shaft transmits shear stresses and you see the deformation as due to what happens in a shear. And another aspect what you have to look at is, we have also discussed free surface.

The entire shaft is free on the outer boundary. See how do I apply torsion? It depends on various aspects. When I want to do a demonstration, I have to hold it on either end and then apply it. And we are not applying whatever the equations developed in the zone where I apply torque. You all know Saint Venant's principle.

We move away from the load application points. We analyze what happens somewhere in the middle. And we have already seen when I apply a torque, every cross section here transmits the same torque. And you should recognize that this complete surface is free. See, one of the common confusions people have is because you have seen that you have to apply torque by holding this. When you discuss what shear stress can exist, people have confusion. Whether you have shear stress on the surface or on a different plane. We have discussed that threadbare when we talked about free surface. You have to go back to that lecture and find out what we have discussed and understand the nature of free surface in the case of torsion.

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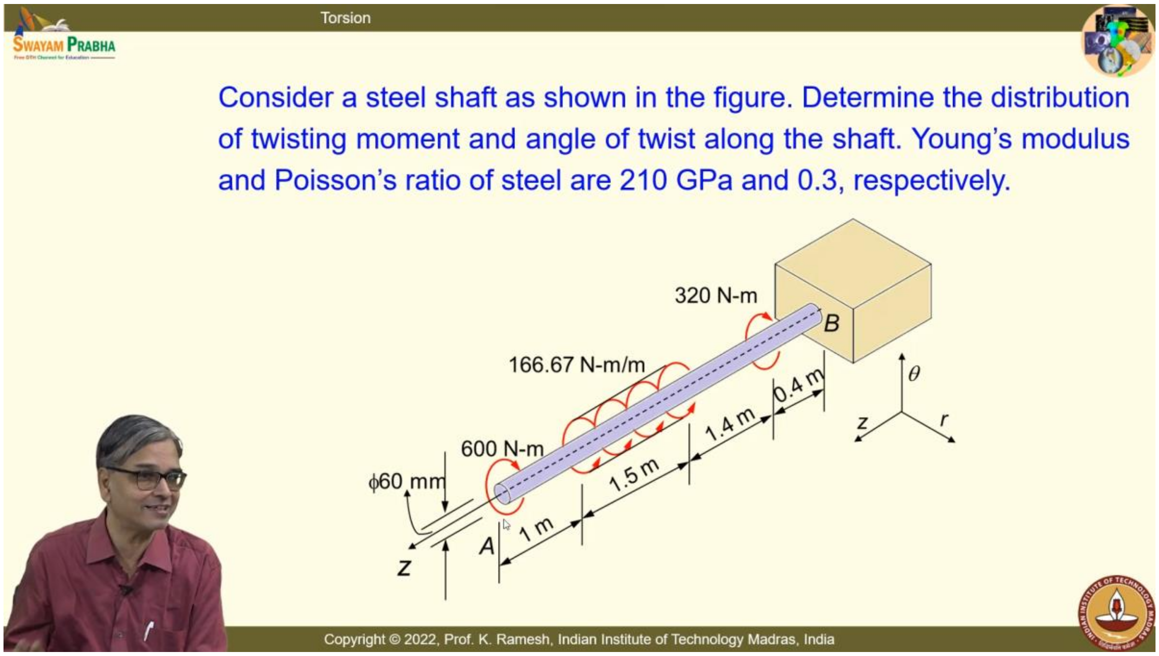

See, in your course, you have done repeatedly shear force and bending moment diagram. You have not looked at twisting moment diagram. Though it is listed, that is also one of the possibilities. So, before we take up torsion in a great way, please make a sketch of this. Let us get the twisting moment. The question is get the twisting moment as well as angle of twist. We will postpone the angle of twist after we develop the relevant equations.

In this class, we would see how do we get the twisting moment diagram for this. Like when you want to draw the shear force diagram, you had a sign convention for the shear force and you also had the sign convention for the bending moment. Similarly, we should also have a sign convention for the twisting moment. And you can see, see when I apply the torque in the clockwise direction, it is negative, anticlockwise is positive and this is a distributed torque over a length and there is a torque applied here.

So, you have the end B is fixed and end A is free. Your angle of twist will be maximum where? Along the length of the shaft. It is a free end, but when we graduate and then come for interview, when you ask question, students always get confused because they see there is a maximum twisting moment here. They also think that that is where the maximum torque is there. This confusion is there. So, I am warning you. So, learn it clearly. So, visualization should be very clear. So, even before we solve, we have an appreciation of what the anticipated result is.

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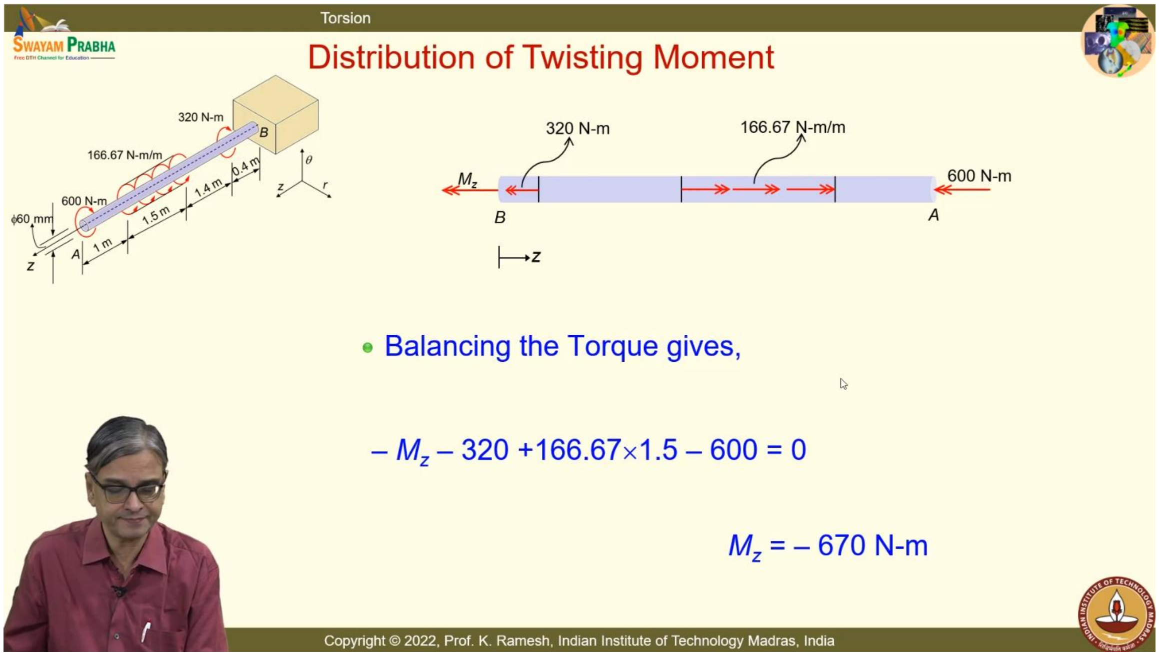

And you know, to make your mathematics simple, I have shown this torque as double headed arrows. It becomes much easier to handle and also helps in visualization very comfortably.

And you have the reference axis. The z-axis is taken along the axis of the shaft. I have the direction r and θ shown and I take a cross section and first is to find out what is the torque at the fixed end.

So, balancing the torque gives, I have - M_z - 320 + 166.67 * 1.5 - 600 goes to 0. That gives me the torque and the fixed end is -670 Newton meter. When I say twisting moment diagram, I want to find out what way the twisting moment changes as a function of the length. In principle, I can take a generic cross section, analyze the left portion or the right portion. But in this problem, your mathematics is much simpler and straightforward if I analyze the left portion because I have already found out what is the twisting moment at the fixed end.

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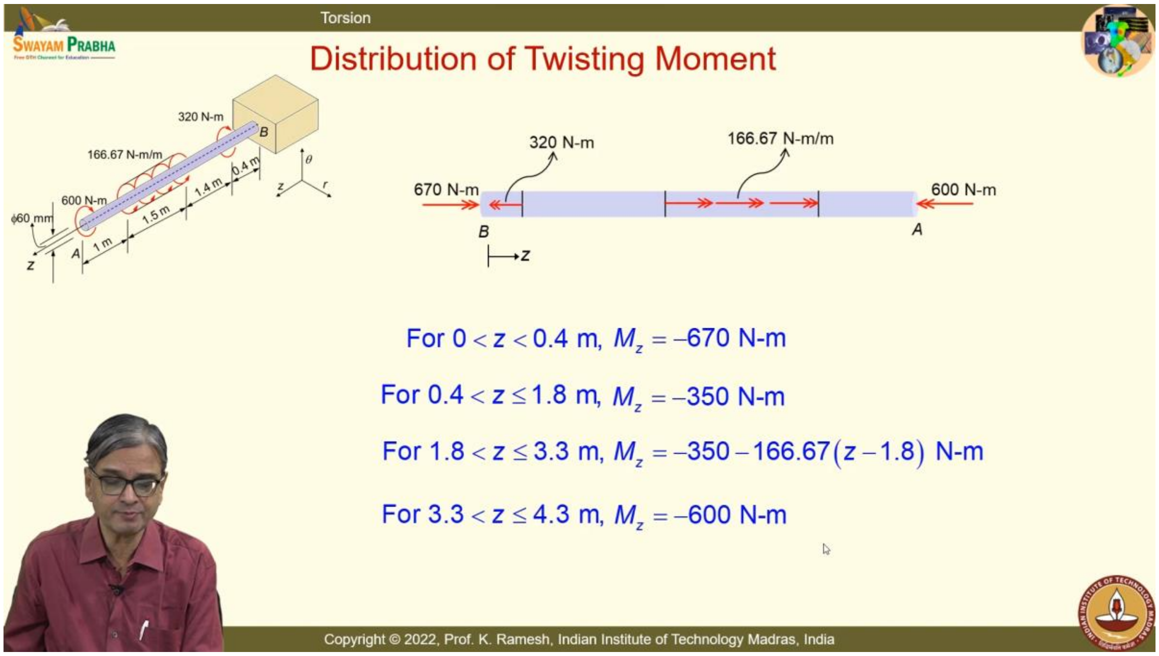

So, that is shown in the right direction in the diagram. So, for the zone 0 to 0.4, see we are analyzing from end B. So, we are analyzing this section. So, you should understand how the drawing is drawn for convenience. So, in this section, you take a cross section and then write the free body diagram.

I know this and I have to find out the twisting moment M_z. It is fairly straightforward. You get that as -670 Newton meters. That is what we are going to do for every cross section. Like what you have done the bending moment diagram, you will also have to do for the twisting moment diagram. As many loads as you have, as many sections you have to pass and then get them and plot that as a diagram.

And then I move to the next section that is 1.4 meters. That is 0.4 to 1.8 meters. I expect you to write. That is the reason why I am reading it out so that you have sufficient time to draw the sketch and also write that. Like what you have learnt in engineering mechanics, the rigid body mechanics course, you take the isolated section, draw the free body diagram, put all the quantities, then your job is done. And you get M_z as -350 Newton meters.

And I go to the next section where I have a distributed torque and you draw the free body diagram. You take a section, draw the free body diagram and put the quantities very systematically. Please take your time to draw the free body diagram. That culture should be there. Whenever you analyze a problem, quickly you should be able to draw the free body diagram. And free body diagram, if you draw it without a mistake, your solutions will be straight forward. And we have also followed a convention.

See, on the positive surface, I write it anticlockwise as positive. Unknown quantity, I write it as anticlockwise and put it on the positive surface as positive. On the negative surface, clockwise moment will be positive. So, you need to follow a sign convention. That is done. And this is again you can easily handle this. So, I have this as z -1.8 is what, because you take z from this and then find out how it varies.

So, when I get this M_z, finally, I have an expression as a function of z. And the last section is simple. So, you take a cross section and again take the free body diagram. And here to simplify the mathematics, I have considered the right section. And I have appropriately put the unknown twisting moment. Unknown twisting moment is labeled correctly. So, this is the way that you have to label it. And while do the mathematics, you should do the mathematics properly. So, M_z comes to be -600 Newton meters. Once you have all these values, it is a child's play to plot the twisting moment variation.

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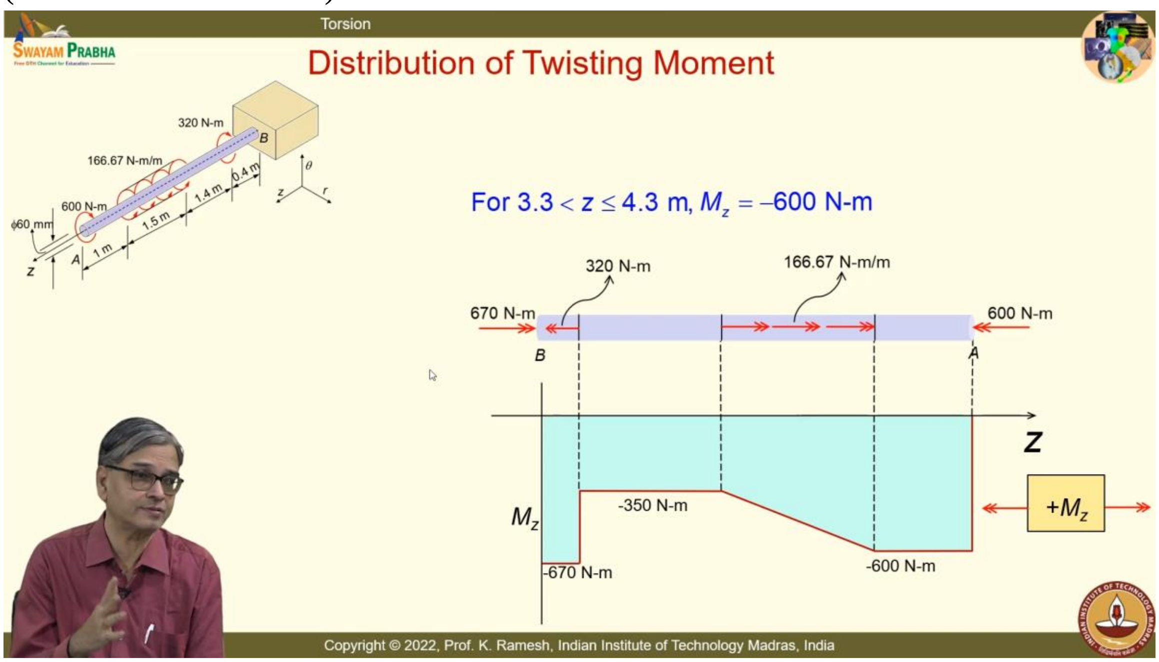

So, I have these expressions. And this is the sign convention. Please write down the sign convention, because I have also emphasized in engineering mechanics. If you look at the video lectures, when you draw the shear force diagram or a bending moment diagram, you should put the sign convention used. If the sign convention used is different, these diagrams will appear different. But what happens on the actual structure? What is the maximum torque or maximum bending moment? That will remain same, whichever sign convention you use. Sign convention is only to interpret when I have the diagram, how to find out what is the twisting moment in this context acting on the shaft.

So, I have a constant value in this section. So, I have writing it from the fixed end. Then it drops down in this section, because I have a concentrated twisting moment applied. How this is applied? We are not worrying now. It is applied by some means; something is mounted there and that is giving a different twisting moment.

So, I have this as a straight line. Then I have this as a linear variation, which is dictated by this equation. So, I have this as a linear line. And this section again it becomes constant. And I have the twisting moment diagram for this shaft. Why do you do this? The bending moment or shear force or twisting moment diagram, when you have to design, you design for the maximum load, so that your cross section selected is appropriate to withstand that load. And let me also give you an idea to think.

See when you physically look at, you are able to see it is a circular cross section. When you physically look at, you are able to see that this is a square cross section. Can you imagine you have learnt many quantities in your engineering mechanics also? Mathematics, when you do, how does the mathematics understand the cross section? You have any idea? You are doing mathematics. When you are developing the mathematical equations, depending on the cross section, the stress levels also will vary. Which quantity that we use mathematically to represent a cross section? Think about it. I have three exercises for you to do the homework. One is, if I have a spring loaded in the axial direction, what is the resisting force that it has to withstand? How it is resisted? Please think about, I need the solution by next class. And we have also looked at a torsional spring.

We say torsional spring because I would like to have stiffness in the, when I rotate, it should give the resistance. From that context, from the functional aspect, it is labeled as torsional spring. It is not transmitting torsion. So, find out what is the resisting force and how the resistance is developed in a torsional spring. And the third one is, in your mathematical development, which you have already done, which you have calculated left and right in your engineering mechanics course, without realizing that this is the quantity that represents the cross section in a mathematical sense, fine.

So, with this, in this class, you know, we have looked at basics of applying torsion to a slender circular shaft. We have actually done a thought experiment. The idea is to find out, when you have an isotropic material, when you have axis of symmetry because the geometry is symmetric by taking a circular cross section. If you take a square cross- section, it is not symmetric about the axis, fine, it has sharp edges and it has four edges, fine.

Whereas, a circular shaft is completely symmetric about the circular cross section, about the axis. And the boundary conditions also symmetric. So, by symmetry argument, we have been able to establish only one aspect. If I have a radial line, the radial line can deform into only a radial line. We have taken it for granted that the plane in which the radial line lies remains plane, which we have not even investigated in the thought experiment. But I have definitely shown that when I have taken the shaft and twisted it, I have definitely shown that, that you have these red lines representing the horizontal plane remains horizontal in the case of a circular shaft. They do not remain horizontal when you have a square shaft. And what you have to understand is, people have solved stresses developed in a square shaft under torsion, but this is beyond the purview of this course.

Thank you.