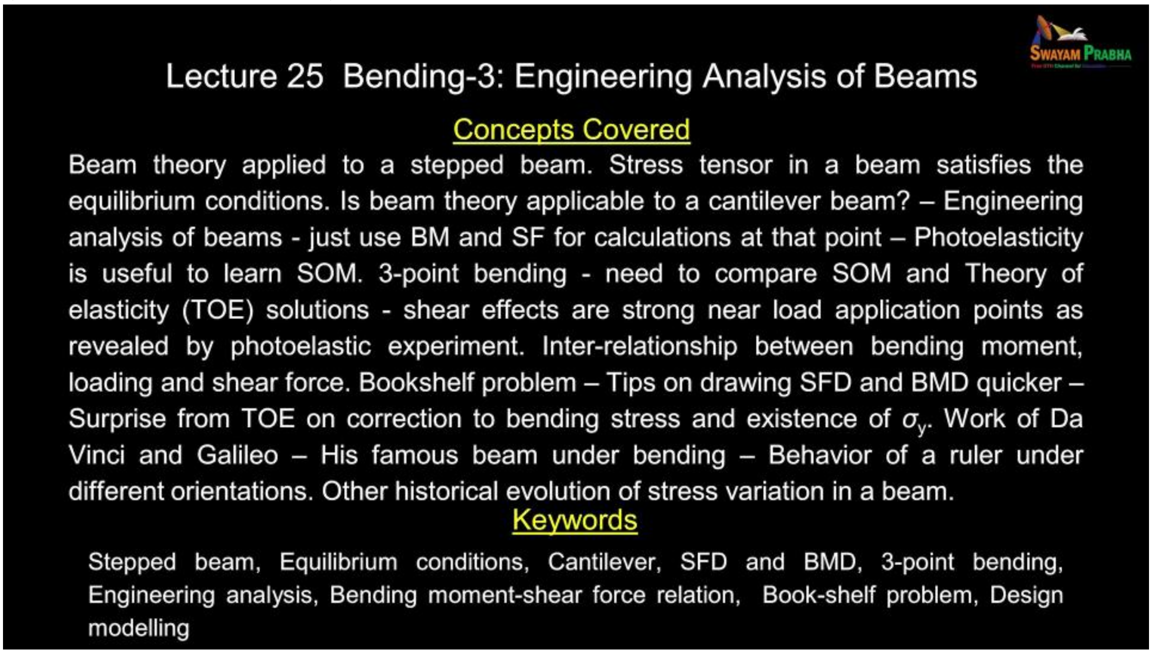
Strength of Materials

Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

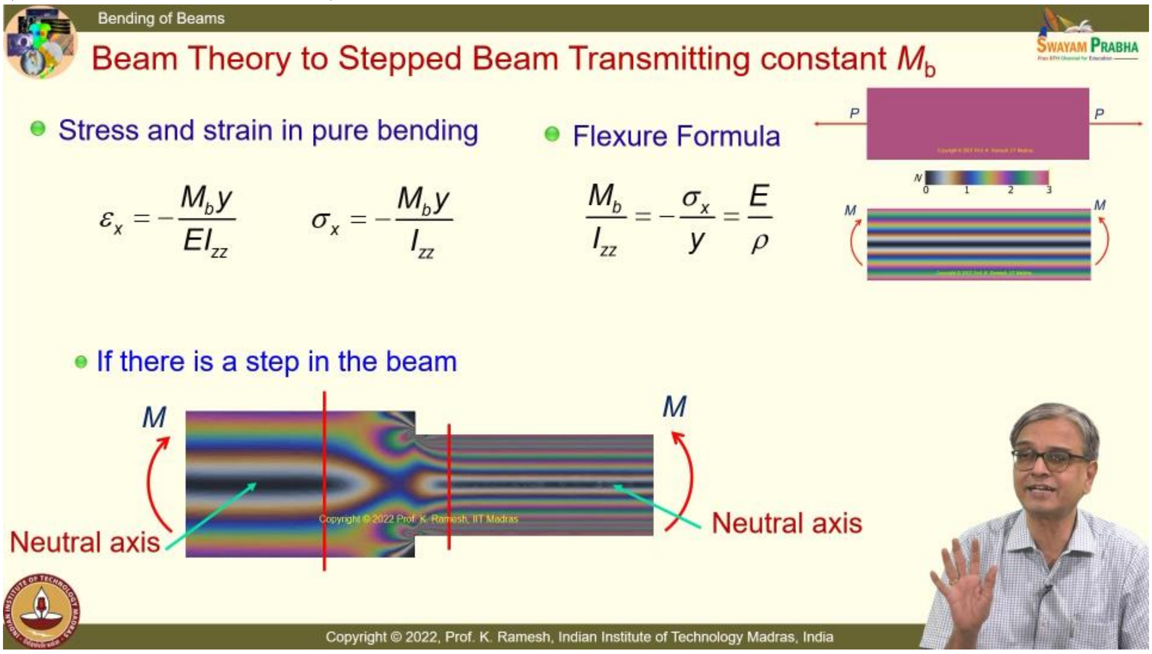
Lecture - 25
Bending 3 - Engineering Analysis of Beams

(Refer Slide Time: 00:20)



Let us continue our discussion on bending beams. In fact, we have developed the basic equations of strain and stress by considering the plane of symmetry.

(Refer Slide Time: 00:35)



We have got the expression of strain as,

$$\varepsilon_x = -\frac{M_b y}{E I_{zz}}$$

and

$$\sigma_x = -\frac{M_b y}{I_{zz}}$$

If you look at the theoretical development, we have recognized it has a plane of symmetry. On the plane of symmetry, we have applied the loading also and we have got these expressions. We did not stop there.

You know, we took a counter example. Suppose, I take a beam like this and then have large deformation. When I consider the thickness, what happens to the Poisson's ratio effect? So, we asked counter questions. When we developed the theory, in order to understand its limitation, we said because of Poisson's ratio; you have anticlastic curvature.

Because we are working in small deformation, these equations are applicable. So, we satisfied. The moment I go to large deformation, I should work with deformed configuration. We have not attempted to do that mathematics. But we satisfied, when I have small deformation, my equations are reasonably good and we have also got the flexure formula.

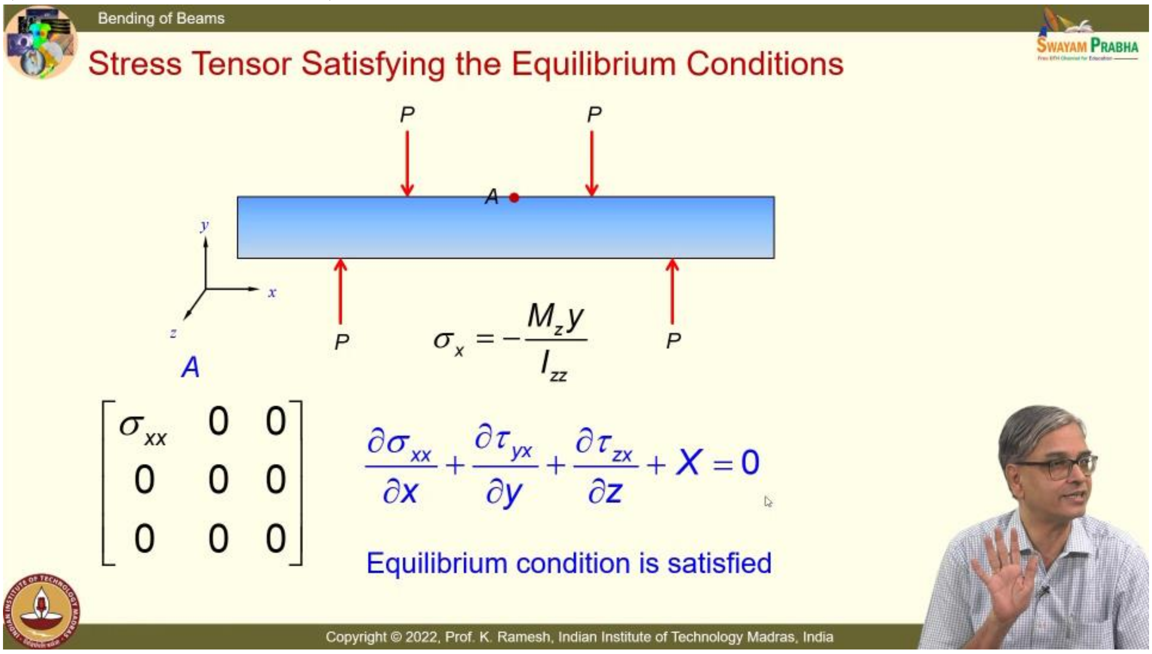
I am not sure how many of you recognize when I apply constant bending moment, the beam bends. These are like arc of a circle. Why? Because the bending moment is constant along the length. The bending moment is related to the curvature. So, you have arc of a circle, ok.

And when we looked at slightly different problem, where I have a step because I said in engineering practice, you need to have a step so that you can locate the bearings. That is a simple modification I have made, but I have not done anything to the loading. The loading is still only pure bending moment. When I have only pure bending moment, I have a very interesting result away from this small zone, which is having this step. I find the fringes are horizontal and I also have the black line forming at the center.

And if you have actually noticed the animation, you would have seen that black line was initially very broad and it was becoming sharper and sharper as we increase the bending moment. Like what we have seen in a beam of constant cross-section subjected to pure bending, you see the same features even in a stepped beam and you also have the definition of neutral axis. In the neutral axis, when I have a triangular variation, the stress value is zero, fine. You know, photoelasticity plots $\sigma_1 - \sigma_2$. Both σ_1 and σ_2 are zero and you also have this neutral axis.

It was a very comforting scenario that whatever we have developed based on simplified appreciation of what is the deformation, we have got the solution and the solution is valid. See, whenever we discuss, we will also have to look at exact solution and compare ourselves are we getting the right solution, which is done.

(Refer Slide Time: 04.35)



And what you do that, you do that in theory of elasticity, fine. Even though I am not going to take theory of elasticity in all its completeness, you just look at what happens to this point *A* and you know from your bending equation that,

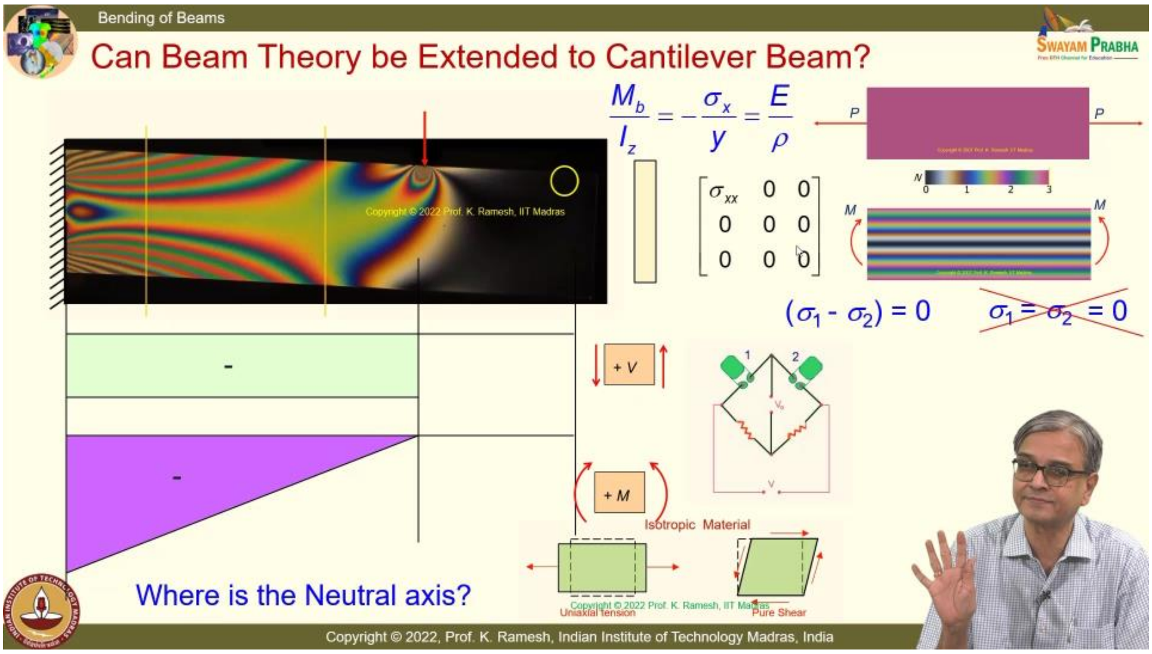
$$\sigma_x = -\frac{M_z y}{I_{zz}}$$

And I said that you should cultivate the habit of looking at the stress component and putting that as a stress tensor.

And what we have developed. We have developed differential equations defined in the equilibrium of the solid. We have not used it in this course till now. And whole of theory of elasticity utilizes these differential equations, solve this problem as a boundary value problem and you get the solutions is exact, where you do not make assumptions about the deformation, you evaluate the deformation as part of your solution. And if you look at this in the absence of body force, we have already said in strength of material that shear stresses do not exist because I apply only a pure bending. I have only σ_{xx}, and σ_{xx} is found to be a function of only y. So, when I differentiate with respect to x, it goes to zero. It satisfies the boundary condition. Is the idea clear?

So, any problem that you do, you verify your solution with an experiment, whether it is an experiment based on a bending of a beam like this or when you look at photoelastic fringes. ~~If the experiments are carefully done, what you get in an experiment is truth fringes.~~ If the experiments are carefully done, what you get in an experiment is truth and your theoretical development should confirm to that. If it is not confirming, then you must find out what is its limitation, what way we can improve. So, you should always keep track of that. Experiments are truth if they are performed carefully. If you do not do the experiment carefully, then there is no fun, ok.

(Refer Slide Time: 06.42)



Now, what I do is ask a question, can beam theory be extended to cantilever beam? I make only one small modification. In the earlier problem, we did not have shear force and in this problem, I take a shear force which is constant on the length of the beam. Because it is constant, I find the bending moment varies along the length of the beam. It does not remain constant; it has a linear variation. Whether the solution we have obtained is applicable to ~~the beam, it is not necessarily~~ here is the question.

And you know, I have taken a line, which is away from the point of loading. I have also taken a line away from the fixed-end condition. I have a region here where I can reasonably anticipate my theory should explain the features of the fringe. What is the first observation you look at? What is happening at the center of the beam, where you normally think that there will be a neutral axis? It was black in the case of pure bending as well as a step being subjected to pure bending. What do you find here? What is your first observation? Keep these observations. First of all, this is not black. That means you have stresses there. Is the idea clear? Are you able to see?

Now, what I have here is, I have the bending moment varies linearly along the length of the beam. That means, when I go from this point to this point, the bending moment is increasing. Suppose I say, I can use the flexure formula, which I have it here. And then, I put the bending moment corresponding to what I have in the bending moment diagram and evaluate the stresses. The bending moment is varying linearly and when I move along this direction, you would anticipate stresses also increase.

If I have stress occurring here, the same value of stress occurs at a lower place because the bending moment is higher. Is the idea clear? That is happening. The bending moment is varying linearly, but I find that this is not varying linearly, but this has a curvature. The observation is correct, but you have one more difference.

I have a bending moment varying along the length of the beam. I also have shear stress. I have a new addition because the way the beam is loaded. So, I have to account for what way this shear stress is going to influence. The shear force is going to cause you shear stress.

And long back, we have discussed. You know, you have this as a free surface; this as a free surface. You can find out the distribution of shear stress variation from our strength of materials. Whatever the result I get, because it is a free surface here, free surface here; this should go to zero at those points. That we have made it clear.

Now, what I find is I have raised a question where is the neutral axis? That is question number 1. Our observation is that I have a color other than black, and we know when I have only the bending stress. I will have the stress state as σ_{xx}, rest all are zero. This is what we have evaluated till now. We have not evaluated what is the shear stress variation. That we will hopefully do it tomorrow.

But you can make an observation that the fringe variation what you observe is influenced by the shear stress also, ok. And one can say when I have this as zero here, zero here and the center, the color is not black. So, I have some value of shear stress. So, one conjecture is shear varies from zero and then goes to zero; probably, it attains a maximum at the center of the beam. One conjecture I am throwing, ok.

Now, what you will have to look at is $\sigma_1 - \sigma_2 = 0$, is what my photoelastic fringes gives. In the beam under pure bending, I have a neutral axis whereby theory itself your σ_1 is zero, σ_2 is also zero. And I suppose you appreciate when we label σ_1, σ_2, I always label the highest magnitude as σ_1 algebraically and the lowest one as σ_2. And in the case of a beam when I have bent like this, the top fibers are subjected to tension and bottom fibers are subjected to compression. So, in the top half of the beam, my σ_1 will be same as σ_{xx} and the bottom of the beam, my σ_1 will be zero, fine. σ_2 will be your compressive stress. So, I will only have positive fringes. And what my observation at the center of the beam shows, I do not have the luxury of saying that both are equal to zero. I do not have the luxury at the center of the beam. Now, the question is can I use this expression for me to find out the bending stress? In fact, that is what we do. We simply take this expression and then look at the bending moment diagram, pick out the bending moment at that point in the; in along the length of the beam, use that and calculate the bending stress as if we do not look at what is the influence of shear stress. I get only the bending component by using this expression and that is an accepted and valid practice, fine.

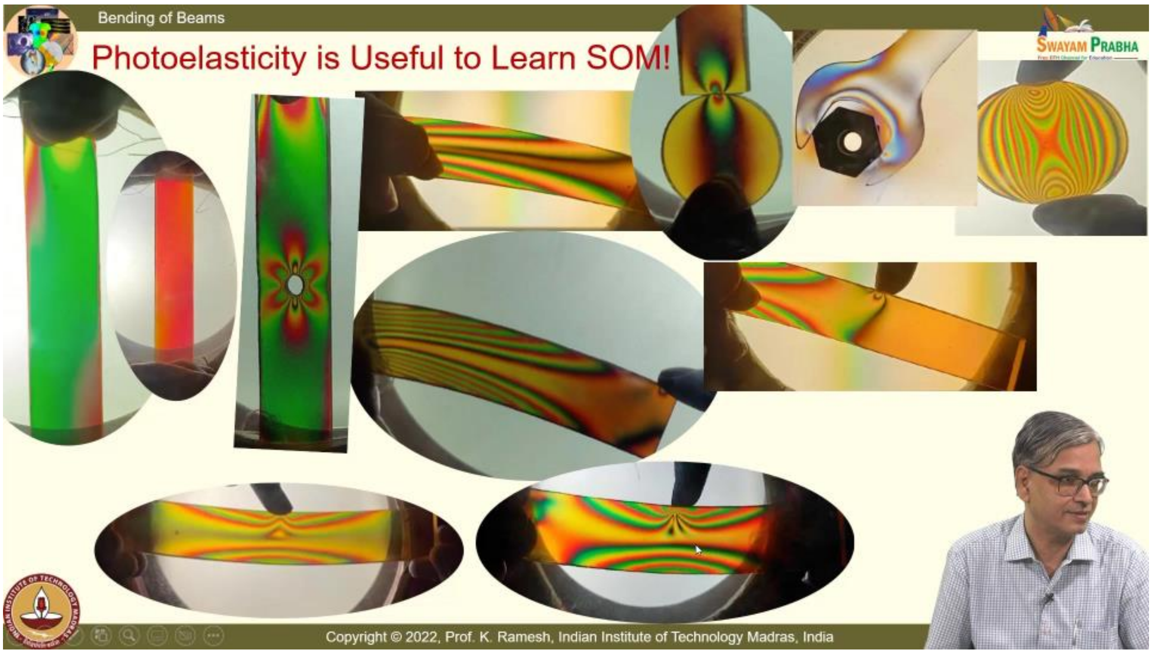
And you know very well along, the if I say that this is applicable in the centroidal axis, my bending stress is zero. And we have seen that I have shear stresses transmitted. So, I have shear at the center of the beam, ok. And in the case of an isotropic material, what happens? Normal stress introduces normal strain; shear stress introduces shear strain. And what we defined as a neutral axis? Neutral axis only elongates. So, it does have no elongation and no contraction. Above the neutral axis, depending on the sign of the bending moment, it can contract or elongate.

So, we are only talking about axial deformation. Even if I have shear present along the horizontal axis, it is not going to influence the strain in isotropic material. I think I also have the isotropic material put, yeah, isotropic material. Just to remind you that normal stress produces normal strain and shear stress produces shear strain.

See, we have been very, very careful in developing the problem. We want to do it only for isotropic materials. Why we do all that? We can understand all this in a simplistic manner and get the core of the problem. The core of the problem is your stress σ_x varies linearly across the beam, ok, and it reaches zero. And you also know from strength of materials now, when this is bent like this, I have maximum tensile stress here and maximum compressive stress here. So, when I put one strain gauge on the top, I put the strain gauge exactly below. I connect them in the Wheatstone bridge in adjacent arms, they subtract.

So, minus of minus becomes plus. So, I have addition of signal. So, all that comes from this understanding. So, at this stage, what we say is you can extend it to analysis of cantilever beam. And the moment you say that you transmit shear, I am going to get shear deformation which will violate plane sections, remain plane before and after loading. So, I must coin whatever the analysis I do as engineering analysis of beams, where exact analysis stops at pure bending. Anything other than that, even though we exploit those expressions, you need to carefully look at that as engineering analysis.

(Refer Slide Time: 16.05)

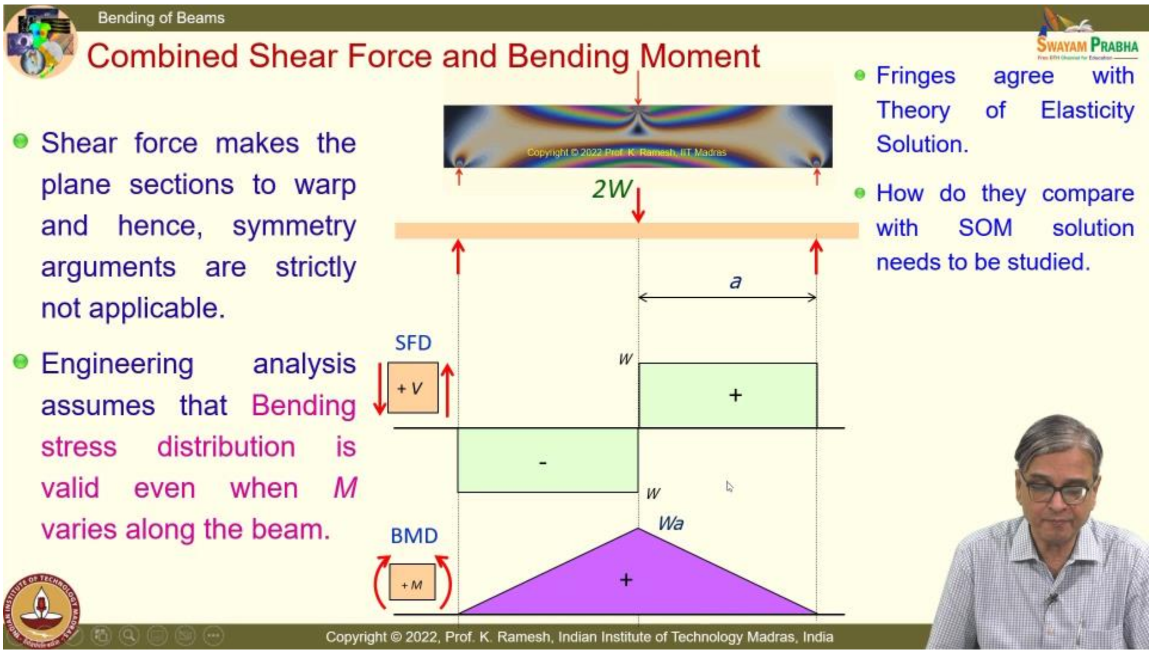


You know, I was quite happy that you have done a beautiful set of experiments. This is what I said, when you do the experiment carefully, experiment is true. If you do not do the experiment carefully, experiment is not true, fine. And in fact, whoever has obtained this fringe pattern here, I should compliment because it is very difficult to handle this model in the polariscope and then apply load with your hand.

And you know, when you do experiment, you also anticipate results. No one does the experiment blindfolded. Before you do the experiment, you have an hypothesis. You want to see whether my experiment gives that hypothesis, ok. Because I have been repeatedly showing that you get constant color in the case of axial force, that has prompted you to adjust your loading in such a manner that you got constant color. The same experiment done by some other group, you did not get constant color for the entire length.

And you have also done three-point bending. See, this needs little more attention. We have to see whether my theory, my strength of material solution is giving me the result in all its completeness. We will initiate the discussion. You have to wait for shear force development for you to complete the discussion.

(Refer Slide Time: 17.24)

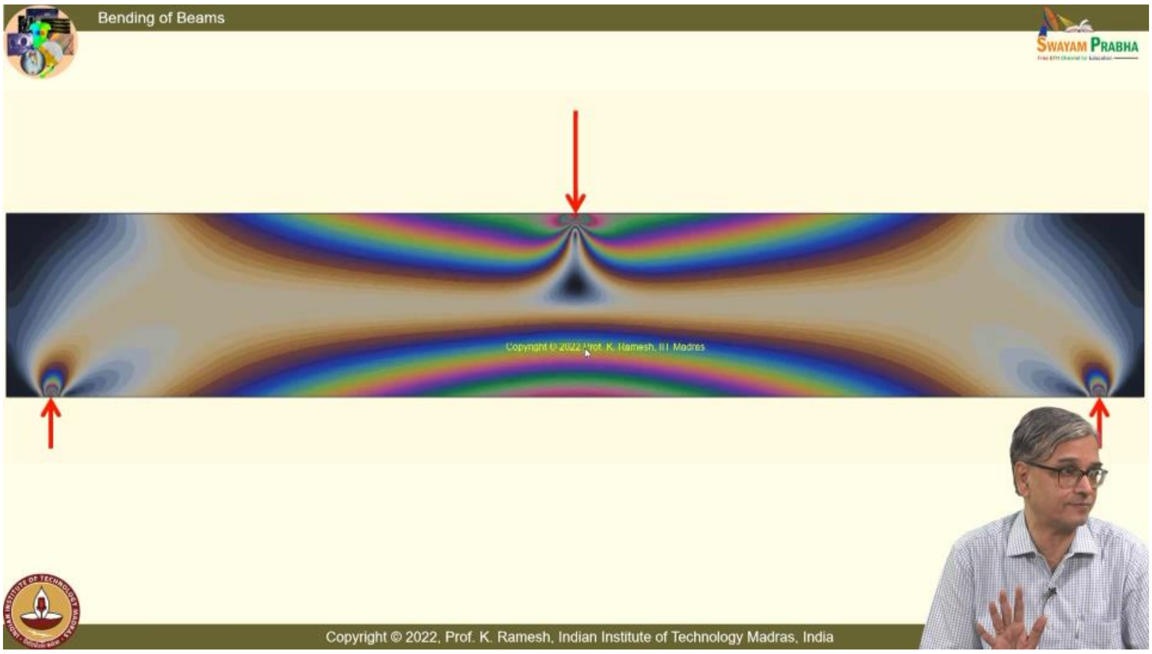


And you know, I have this as a three-point bend, very similar to your cantilever because when you look at the shear force, shear force is constant and your bending moment is varying linearly. So, if I can apply my strength of material solution to cantilever beam, I can also apply to three-point bending because this shares the same feature here, ok, on both the sections. And I would like you to take a photograph of this because these fringe patterns are very, very interesting that needs to be looked at and discussed, ok. The moment I have shear force which I have used, it is going to warp the section. So, symmetry arguments are strictly not applicable.

And I want to emphasize, we do not give up as engineers. We do the analysis. We recognize that we have made an approximation. We label this as engineering analysis. So, the bending stress distribution is valid even when M varies along the beam.

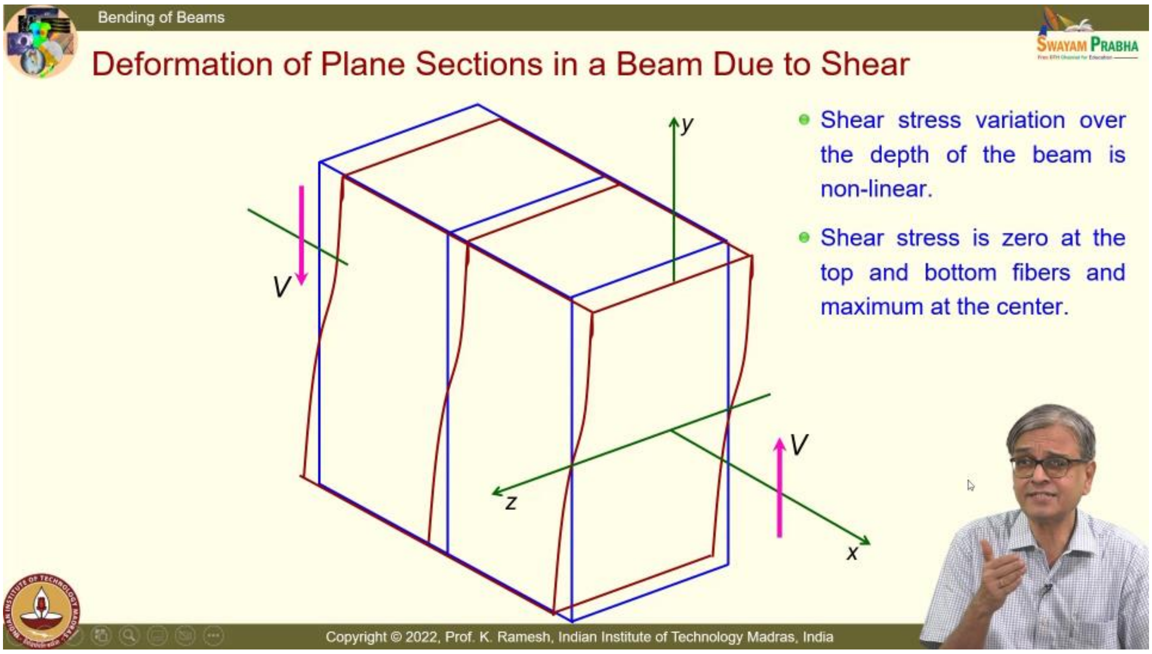
Now, we will have to investigate that this is influenced by the shear stress introduced. And what you will have to look at is whatever the fringe pattern I have shown here, they agree with theory of elasticity solution. And we have to investigate how do they compare with strength of material solution. If I have to compare it with strength of material solution, I should also know how to calculate shear stress variation across the depth of the beam which we have not done till now, fine. We know only the bending stress distribution. And at this stage, I want you to observe how the fringe patterns are.

(Refer Slide Time: 19.26)



I am also having a bigger blow-up of the fringe patterns developed. And what I want you to observe is, you know, the bending moment is varying linearly across the length, it is increasing. And we have seen that fringe is having a curvature. And when you come closer to the load application point, I have this curvature and it has a sharp turn. Is the idea clear? Do you make that observation? And we have to understand whether my strength of material solution what we are going to develop in terms of shear stress is good enough to capture these or not. You will have to wait, but have this observation in mind. When we develop the discussion, we will find out an answer, ok.

(Refer Slide Time: 20.18)

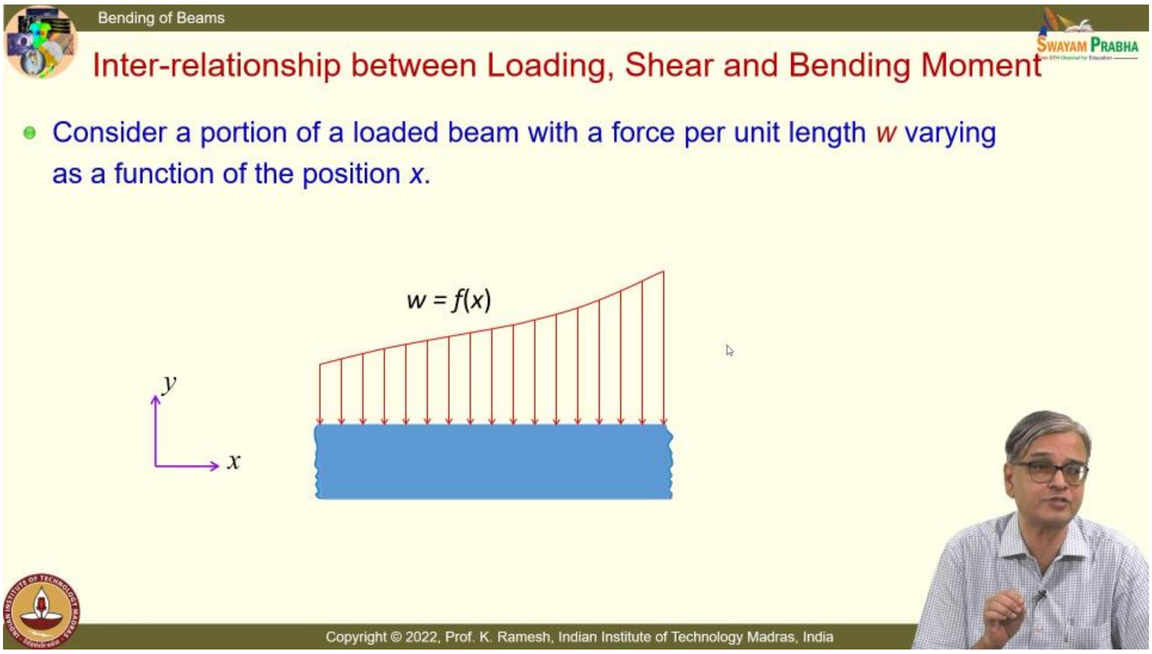


And the moment I introduce a shear stress, I am not going to have these sections remain plane, they get deformed. And this is based on the final solution, ok. You can make certain observation of this. See, this line is perpendicular to this free surface. So, when the deformed section is perpendicular, that means the original 90 degree is preserved; what would you say about shear? Shear is zero. Shear is change of the original rectangle. Is the idea clear? And I have also made a conjecture that it is maximum at the center of the beam and I have the maximum deviation from 90 degrees at the center zone as a deviation.

And what I have is the plane sections have warped. Whole of our beam analysis was based on the premise plane sections remain plane before and after loading. On that basis, we calculated the bending stress distribution, bending strain distribution and that is clearly violated even in the simplest case of cantilever beam. Where does this influence? That is question number 1. And question number 2 is what is the variation of shear stress? Question number 3 is what is its magnitude in comparison to bending stresses? All these aspects are important. First answer is as long as I am having a slender beam, that means, the depth is very small compared to its length, shear effects can be ignored. When the depth increases, you have a coupling between shear and axial. There unless you bring in the shear effects, your beam theory solution is not complete. You also have idealizations for that. Bringing in shear effects, you have what is known as Timoshenko beam, ok. These are all postponed for your higher level of education.

For this course, we will confine to strength of materials and shear stress variation over the depth of the beam is non-linear. You know, we have always been looking in the case of a twisting of shaft, shear stress was varying linearly. And in the case of a beam, axial stress was varying linearly. And we find that shear stress variation is non-linear. As we have conjectured based on free surface argument, shear stress is zero on the top surface as well as the bottom surface and reaches the maximum at the center. That reaches the maximum at the center; I gave evidence from photoelasticity fringes. It was no longer black; I have a color.

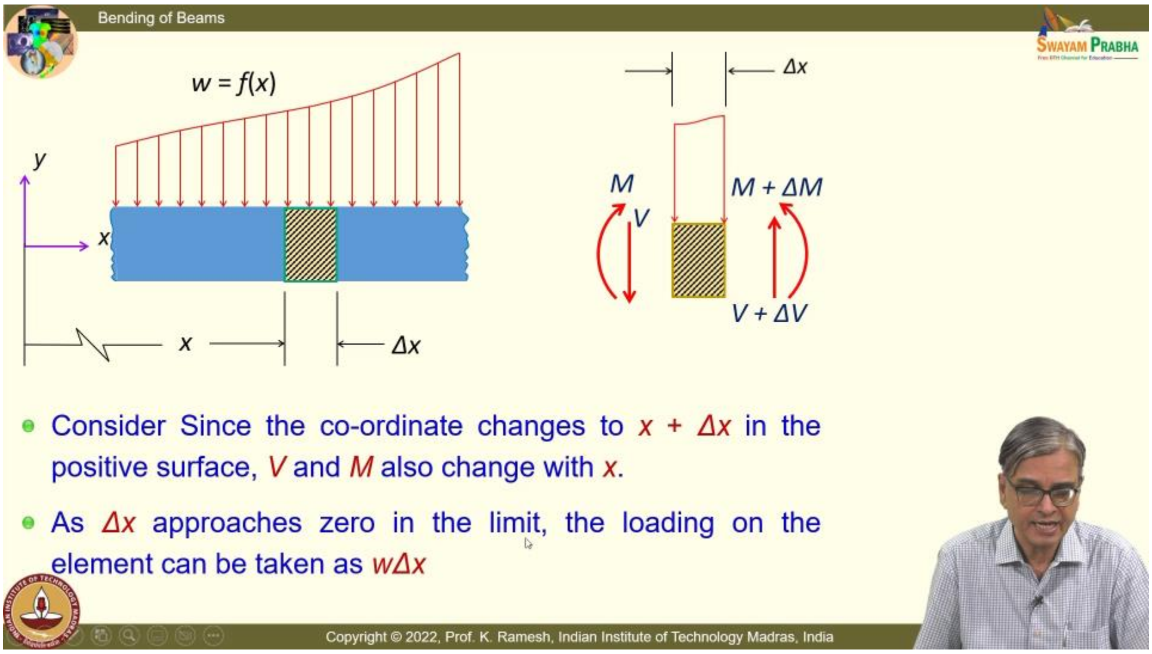
(Refer Slide Time: 23.28)



And you know, we will also have to graduate when I have a loading that is varying on the beam. And one of the things that you normally take is particularly when you deal with civil engineering construction, self-weight itself is a load that is acting on the beam. And this you have already developed in your earlier rigid body mechanics.

And this is the way you have used how to find out whether you have drawn the bending moment diagram, shear force diagram correctly in comparison to the loading diagram. It is only a recapitulation, ok. It is not a new derivation.

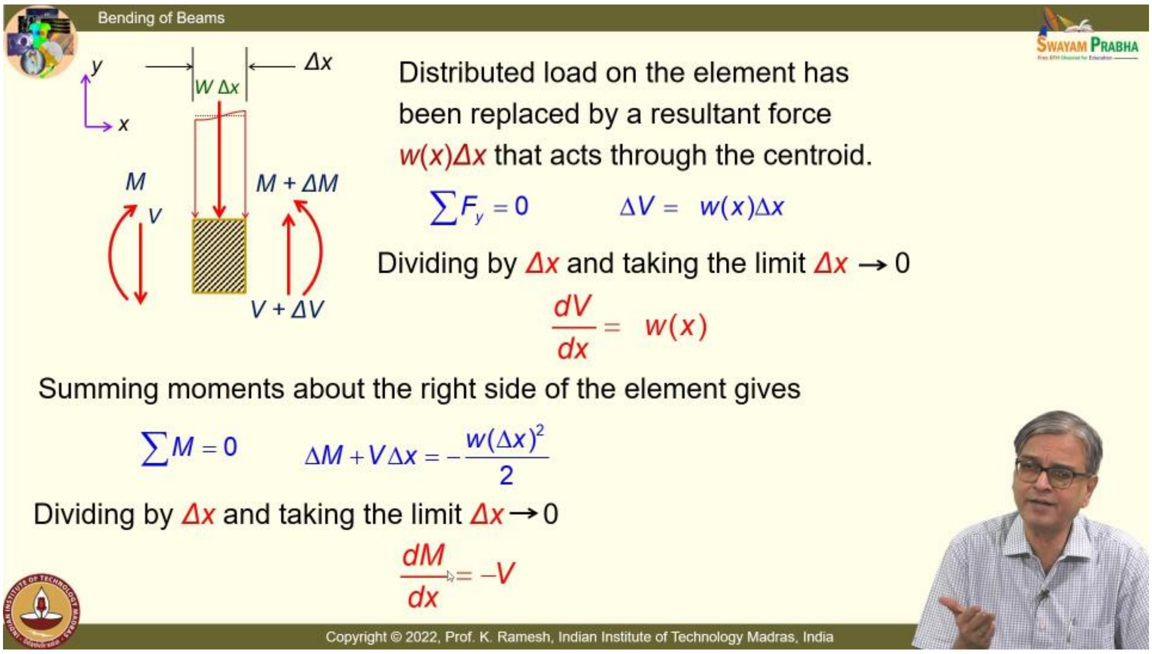
(Refer Slide Time: 24.11)



Recapitulation with the symbol on how do we label positive shear stress and positive bending moment. That is the only difference. Otherwise, this you have already done in your earlier course. So, I take a small section and then I write all the quantities I know. I recognize that this is having a finite length of Δx . So, I anticipate there is a variation of shear along the axis. There is also a variation of bending moment along the axis.

And we have taken on a positive surface, positive direction of shear is positive. The only difference is we have learnt it in a different symbolism of sign convention. That is the only difference. Otherwise, you have already been taught how to get this. And you know I can also say that in a short distance Δx , I can simply put the value as $w\Delta x$, ok. And I can replace this as a force.

(Refer Slide Time: 25.26)



I can write the force balance and then get me the interrelationship. So, if I do $\sum F_y = 0$, I get a very important relation in the limit Δx tends to 0,

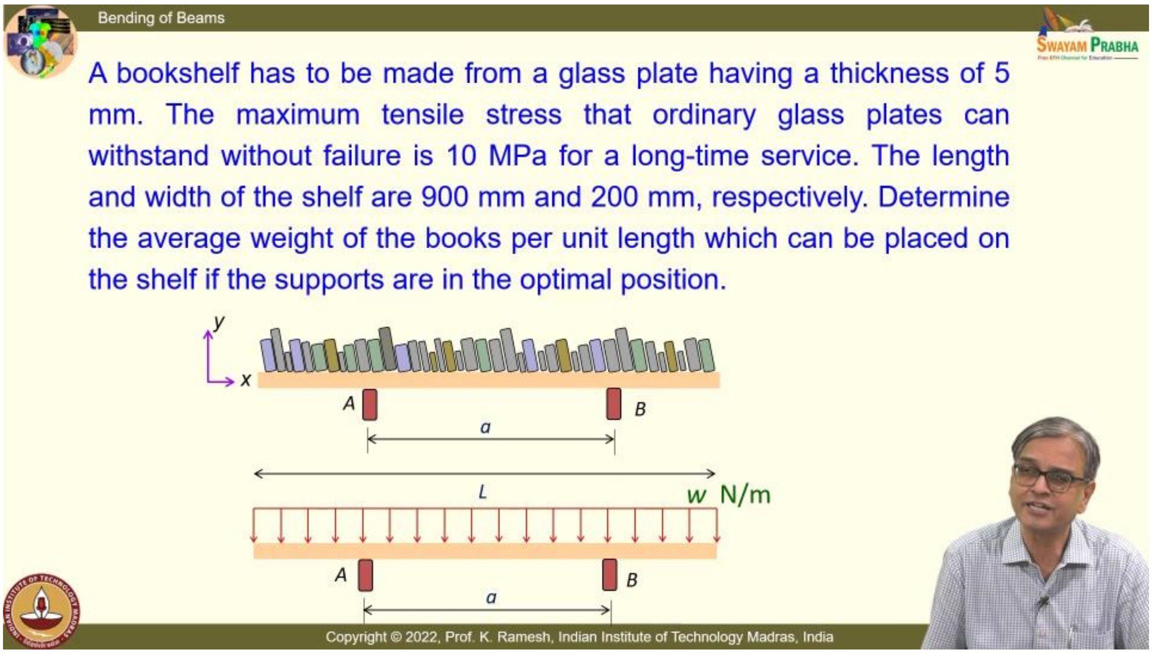
$$\frac{dV}{dx} = w(x)$$

intensity of loading, ok. And when I do the same thing for moment equilibrium, I get another very interesting result,

$$\frac{dM}{dx} = -V$$

If you have used opposite end convention, you would have got this as $dM/dx = V$. That is where the difference comes. And what it says is, if I know the shear force diagram, I can comment about the slope of the bending moment diagram. So, that is one of the ways that you verify your plotting. Hand plotting you have to do all that. See, if you have an equation, put it in your software and then want to plot, the software will faithfully fit in the curve. But when you do the hand plotting, you have to know how to do it, ok.

(Refer Slide Time: 26.35)



And let us solve one very down to earth simple problem. At the same time, it is very interesting. It also has components of modeling. See, consciously I want to bring in, you become better engineers only when you know how to do modeling. First modeling is, you know, when you have a stack of books, none of the books, if you take 5 books, each of them will have different weight, different size. You do not have one uniform book unless you go and buy an encyclopedic volume which are published from one same publisher. If you take 10 of them, 10 of them will have the same size, same weight and all that. But if you buy books in fluid mechanics, solid mechanics, material science, all of them will be different.

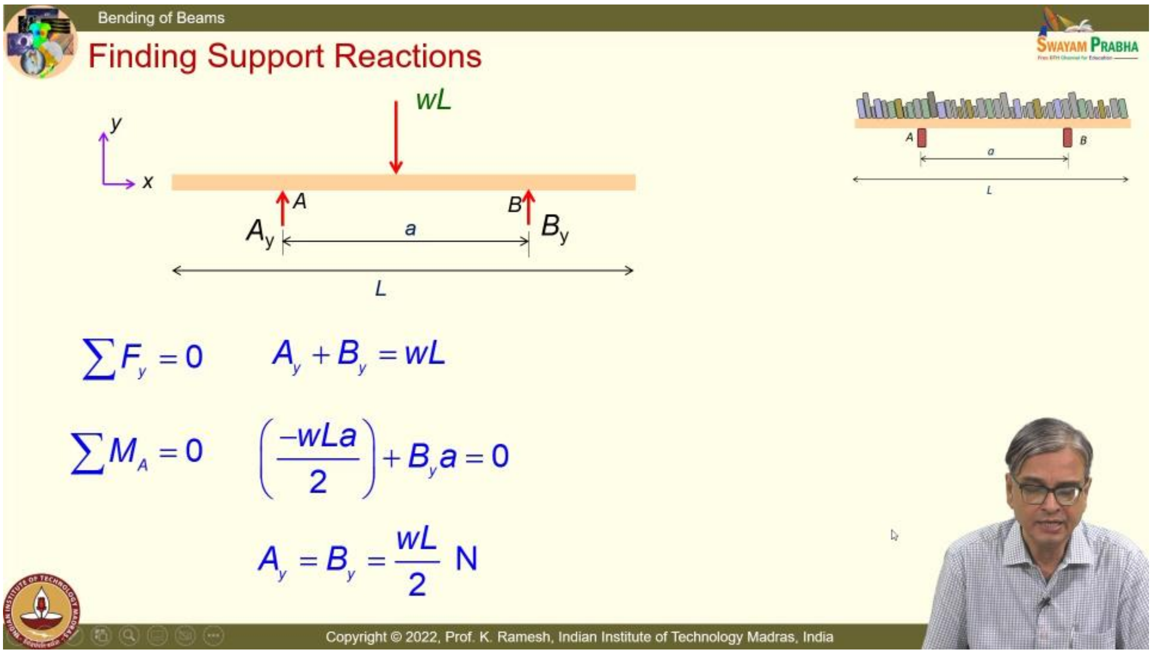
So, the first idealization you have to make is translate this into a uniform loading. You should not exceed that, that is what I want you to recognize, ok. And what is given is, you can make a nice elegant support by having a simple glass plate. This is of length 900 mm and depth 200 mm. It can accommodate many of your books. And it also says, determine the average weight of the books per unit length which can be placed on the shelf, if the supports are in the optimal position. You have to understand what is an optimal position.

In fact, there is embedded clue in your question itself. The supports are not put at the end. The supports are put in a manner that this whatever the length $a > L/2$. You are given a clue and as engineers, you have to find out where do you put the supports.

See, if you are not done engineering, you would normally put the support at the ends. You think that is stronger. That is what you would do. What we call as common sense, we jump to such conclusions from based on prior training.

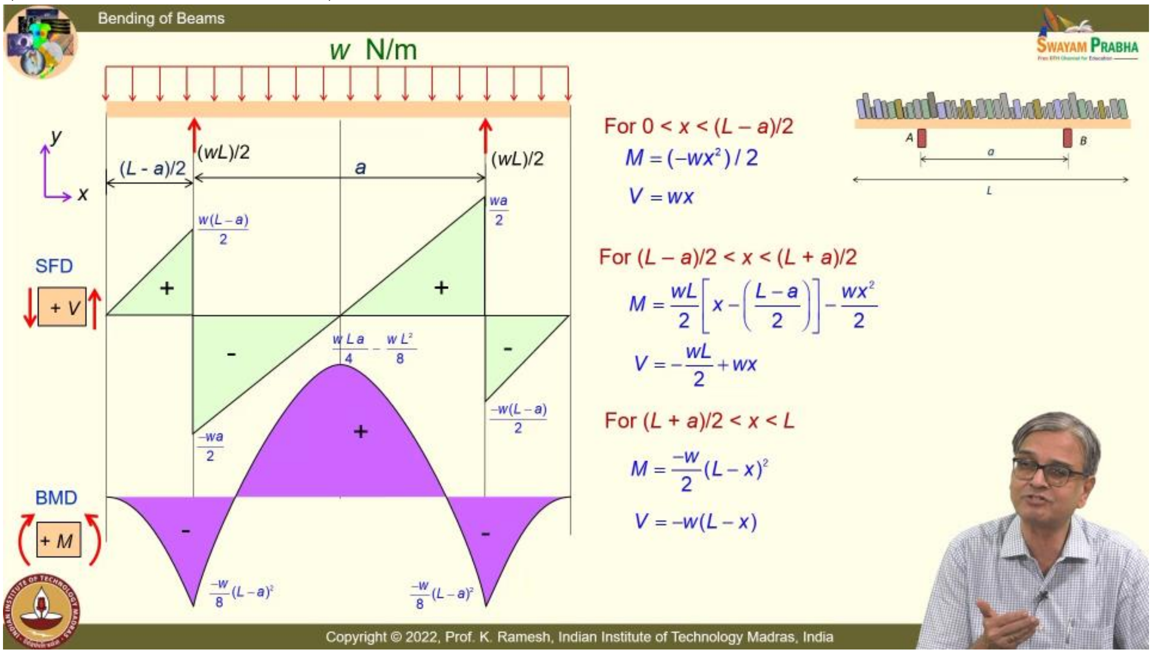
And I have also asked you to do this as your tutorial to get the shear force and bending moment diagram, ok. So, that I can go and handle those portions little faster, ok.

(Refer Slide Time: 29.02)



First thing is you have to get the reactions and that is you do by $\sum F_y = 0$ and $\sum M_A = 0$. It is very straightforward. So, I get the reactions as $wL/2$ at both the supports.

(Refer Slide Time: 29.23)



You know, I have also been saying that when you have a problem like this, you should be able to draw the bending moment and shear force diagram quickly, fine. So, if you give me this, I will find out for drawing my shear force. I will find out what is the shear force at this point; I will find out the shear force at this point. That is good enough for me to draw the complete shear force diagram, ok. And if I have to do the bending moment, I will also find out what is happening at the center. So, two points are sufficient for me to draw, three points are sufficient for me to draw the bending moment. Because I have the loading is w N/m, I anticipate my shear force for the entire length of the beam to be the first-degree curve because I know $dV/dx = -w(x)$. I use that. And when I go to bending moment, I have $dM/dx = -V$.

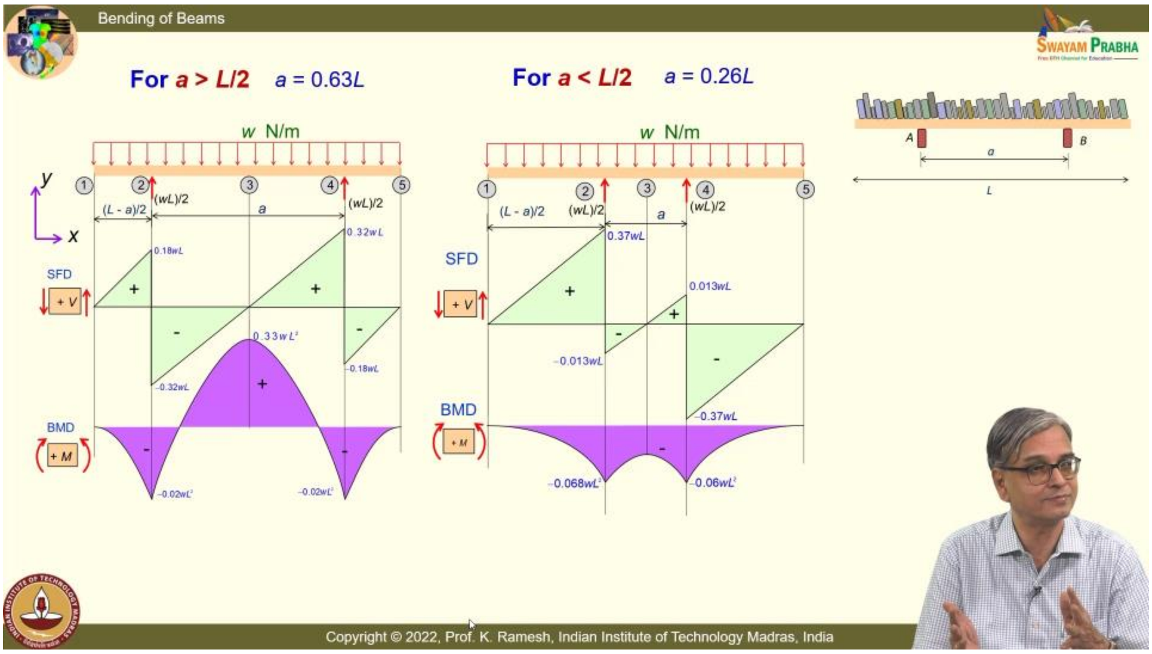
So, if the shear force is a linear curve, bending moment will be a parabolic curve. So, all that information you use it. And you know, if you do from the conventional point of view, I can find out what is the expression for M. If I take a section and then do it, I can do this.

So, I will do that for one section, another section and the third section. That is for you know for completeness. Completeness you have these expressions. I can get it for this. But when I want to plot, you know when I start here, here the shear force is zero as well as bending moment is zero. So, I know two points already to start with. So, if I know this, at this support I have $wL/2$. Actually, from this point, I will go up by $wL/2$. That is what your mathematics also will give. I will go up $wL/2$. And if I know this point, I know this is a straight line, I will join these two. Again go up by $wL/2$ and then come back to zero. So, you can draw this very quickly. You have to build up that acumen. You know as you learn the course, you cannot be saying that you will take half an hour to solve this problem, ok. And similarly, when I go for the bending moment, I know that this is a second-degree curve. The question is whether I should draw it like this or draw it like this. That you look at based on your slope. See, slope is zero here, slope is positive here. You have $dM/dx = -V$. When $V = 0$, slope is also zero. When V is more and more negative, slope is also more and more positive. So, that is how you decide why I have to plot the curve like this. And now, you can quickly say as an engineer, suppose I put the support at the end, what would be my bending moment diagram? Bending moment diagram will be a parabola from this end to this and your bending moment peak value will be much higher than this. Do you see that difference?

So, one clue you get is by adjusting the support, you can achieve a situation where you can minimize the level of bending moment maximum occurring in the plate. So, that is a design tool. See, you do not worry about whether it is a positive bending moment or a negative bending moment. Because, whether it is a positive bending moment or negative bending moment, you are definitely going to have same maximum stress and maximum compressive stress. Instead of occurring at the top fiber, it may occur at the bottom fiber. That is the only difference. So, even the sign of the bending moment does not matter as long as my cross-section remains same and symmetrical. Suppose I have a cross-section where the distance also matters, ok. There you may have to find out whether the compressive stress is more or tensile stress is more if I have a brittle material. If I do not have a brittle material, I do not have a problem. For ductile material, your tensile strength and compressive strength are identical. For a brittle material opposite, you have tensile strength is hardly anything and compressive strength is 10 to 12 times. So, there it matters.

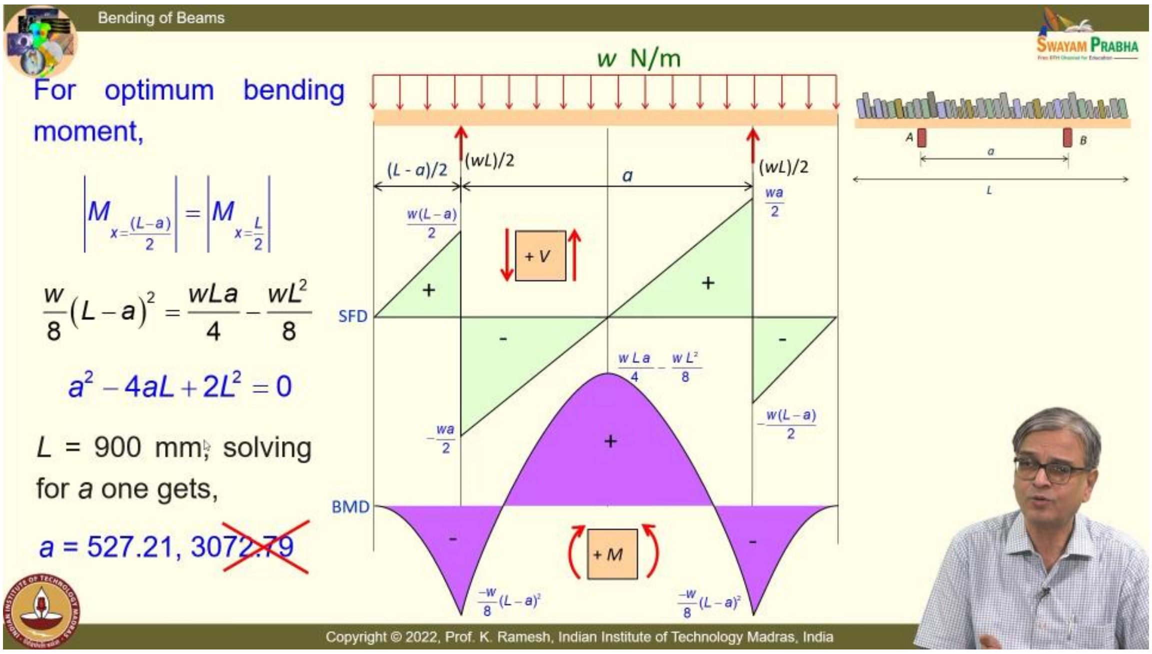
But you know glass plate you have taken, it is of a constant cross-section. And with this itself, you can find out by equating these two, you can find out the optimal position.

(Refer Slide Time: 34.10)



In order to aid your thinking better, I am also plotting this when I have $a > L/2$ and when I have $a < L/2$. Greater than $L/2$ is what we have got earlier also. When you substitute the numbers, the numbers are easy to recognize and then see how they vary. And when I have $a < L/2$, I have everything on the one side. So, if I start from this, I will not be able to find out the optimal position at all. So, only if I start from this kind of a bending moment diagram, I have an opportunity to find out the positive bending moment and negative bending moment and equate the two. So, it is a design scenario.

(Refer Slide Time: 34.59)



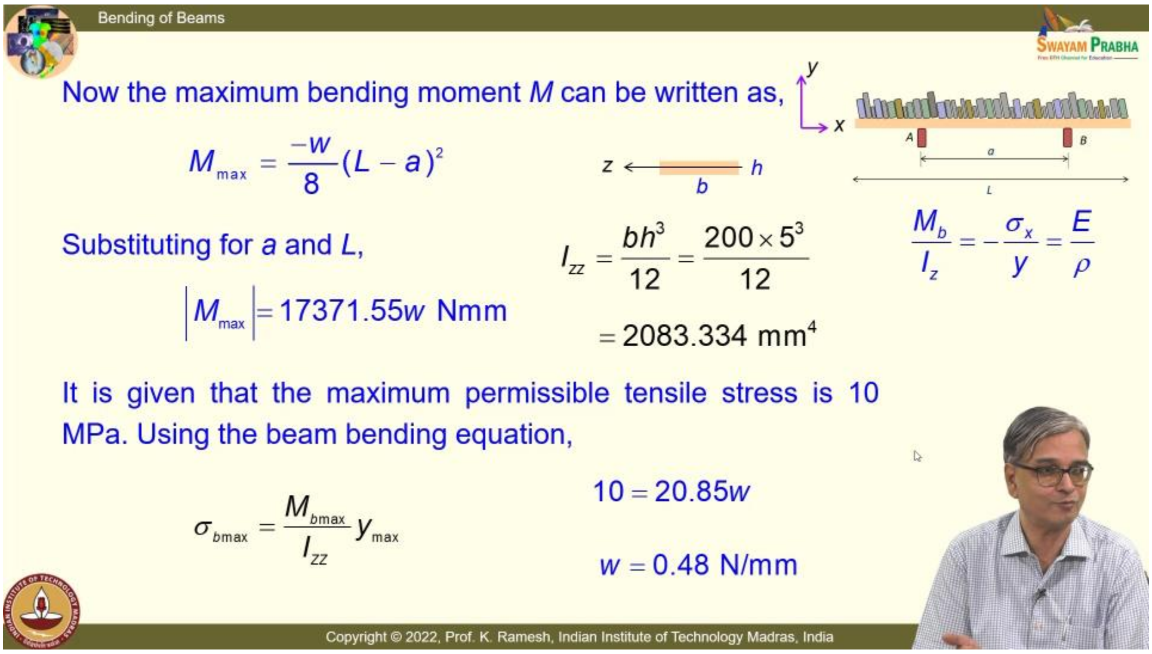
So, what we do is, we simply equate the two, equate the magnitudes, ok. Equate the magnitudes. The same shear force and bending bond diagram is re-plotted. I equate these two values, I get a quadratic expression of *a*,

$$a^2 - 4aL + 2L^2 = 0$$

From this, I can solve for what is the value of *a*. So, that means, I have adjusted my supports in such a manner, whatever the bending moment I get here, I ensure that this peak and this peak are of the same magnitude. So, that is the best optimization that you can do. For the given problem, that is the best optimization I can do.

So, I get, when I substitute this being a quadratic expression, I will get two roots. You have to select the root which is appropriate. Only this is, will lie within the beam, ok. So, I have this as 527.21 mm . And once you substitute here, once you apply the beam theory; hopefully, the beam theory is applicable. We have to verify; we have to verify with theory of elasticity. We have not done that verification. We will also do the verification and if the deviation is small, keep our eyes half closed, pat our back, I have got the value which is good enough for my design. This is how engineers operate, ok. You have to get the core of the problem, that is more important. The core of the problem here is bending stress varies linearly over the depth of the beam. When I apply an axial load, it is constant. To recognize that varies linearly was a biggest challenge the scientific community faced. It took 400 years. You should recognize that. We will also revisit the history today because you are now more equipped with what is beam theory. You can go back and appreciate what aspects they could reveal, what aspects they could not find out in the development, ok.

(Refer Slide Time: 37.13)



So, I have this bending moment is equated and you have got this expression. So, when I substitute and then substitute the value of a which I have calculated, I get the maximum bending moment magnitude is like this and I have the bending equation is available. So, whenever you solve a problem in bending, please write this in one corner and start because you are going to exploit this equation. So, I can write,

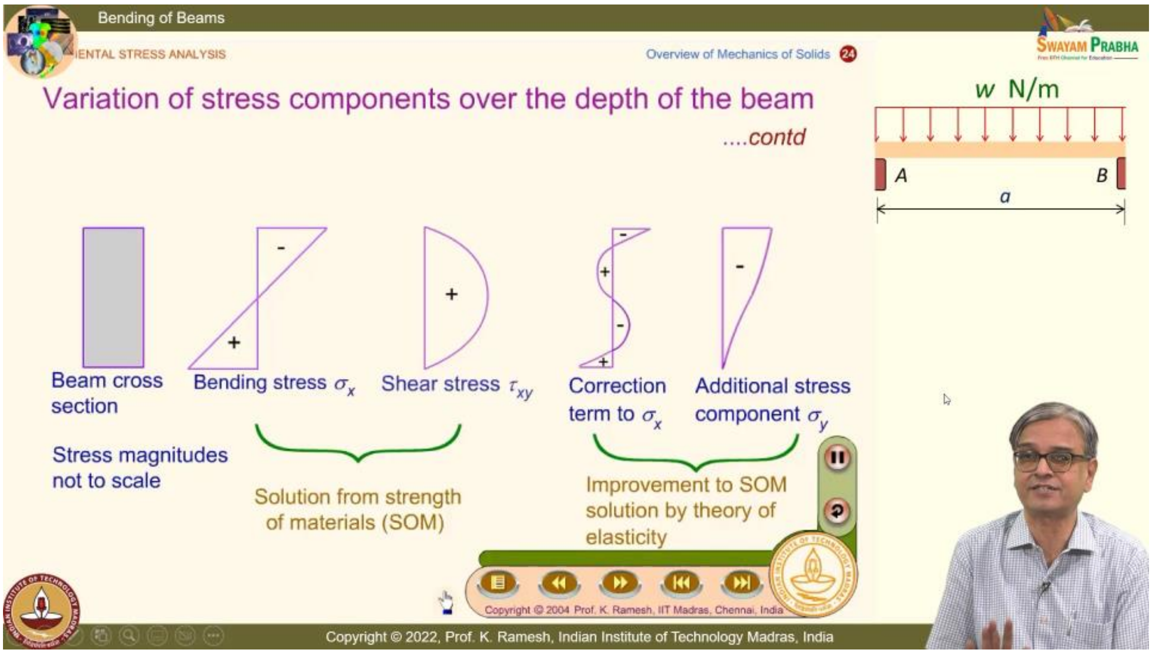
$$\sigma_{bmax} = \frac{M_{bmax}}{I_{zz}} y_{max}$$

And then you have a cross-section like this. I have this as the breadth and height. So, I have for this rectangular cross-section how to calculate the moment of inertia. And when I use the information given in the problem that it cannot exit 10 MPa, I find out what is the value of *w*. I have this as 0.48 N/mm.

Let me ask you few more questions. I am not going to give you answer, fine. I want you to think. Suppose, I want my w to be; not 0.48, right. I want this to be doubled. What are the options that I have for me to do? It is obviously going to exceed the 10 MPa limit. That means, I have to strengthen the beam in some form, isn't it? One possibility is you take another glass plate, put it. That is one possibility. The other possibility is instead of a 5 mm glass plate, find out what is the thickness of the glass plate you have to go. That is option number two. What is option number three? Simply increase one more support. You get the point?

See, I should also provoke you to find out when you go to design scenario, there is no one solution for a design. You will have multiple solutions depending on the context, one of them may be really applicable. So, I want you to exercise this. So, you find out what is when I want to double it, what should be the thickness I should have with the same support because that is the optimal support. Suppose I have three supports, you know you have to wait until you develop a deflection because it is a continuous beam. You need to find out the forces. Only then you can solve the problem. So, keep that at the back of your mind. So, it is a very interesting problem, ok.

(Refer Slide Time: 39.55)



Now, let us look at whether this is supported completely by theory of elasticity. There are surprises, ok. You have not calculated the shear stress, ok. And you can take it that this is varying parabolically. I said non-linear. Anything other than linear, we will say non-linear. It may be even a simple second-degree curve, ok. And we have obtained from strength of materials that bending stress varies linearly. Please make a sketch. This you will not find it in any books that you have access to. And also appreciate stress magnitudes or not to scale. That is also very important. We are only discussing the variation over the depth of the beam. And you will find that what we get from strength of material up to this point, it is perfectly correct.

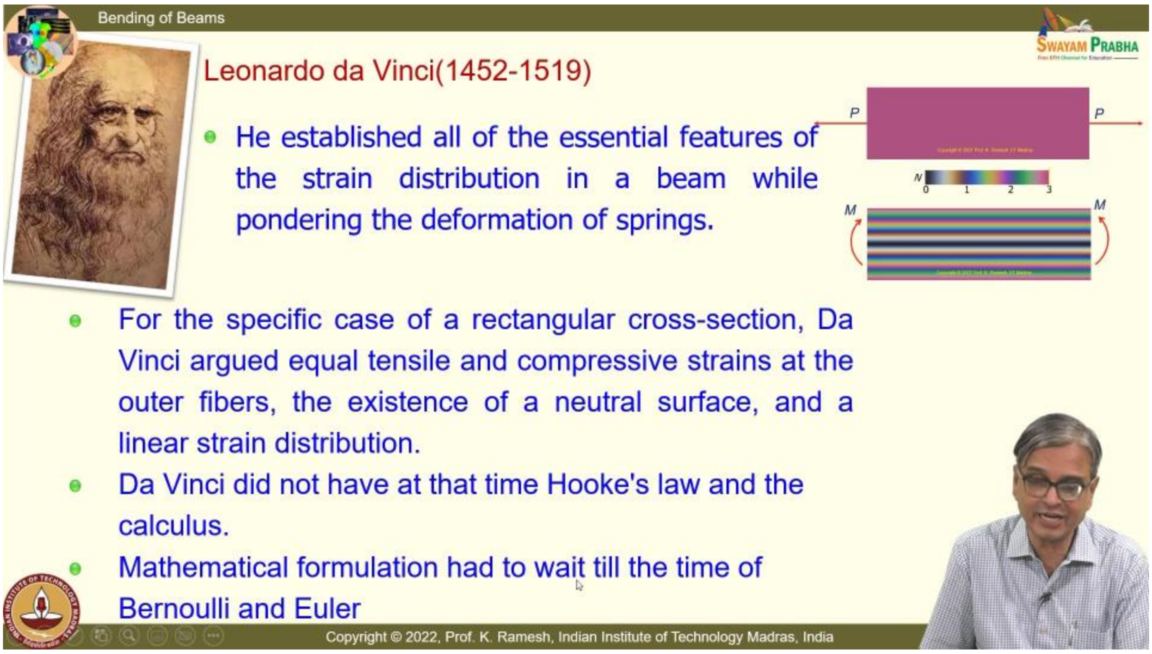
We have also done discussion on free surface. Suppose I want to do the discussion on what happens to this surface. Is this surface free? I am deliberately applying a vertical load. How can that be free? In the development of bending of a beam with constant bending moment, we have idealized σ_y as zero. Isn't it? σ_y as zero. But this is one of the most common loading that you have and you are definitely applying σ_y. You are applying a force to generate σ_y.

So, when I go and solve the problem as boundary value problem, take in the differential equations, I am definitely going to have σ_y and it will die down to zero at the other edge. And there is also another surprise. Even the linear variation of bending stress is no longer supported by theory of elasticity. You have a correction term to σ_x.

So, you have two additional quantities σ_y exists. And I have not drawn these to scale. You will find that these are second-order effects. This small variation can be ignored when you use the expression for bending stress which we have developed in sigma materials. Is the idea clear?

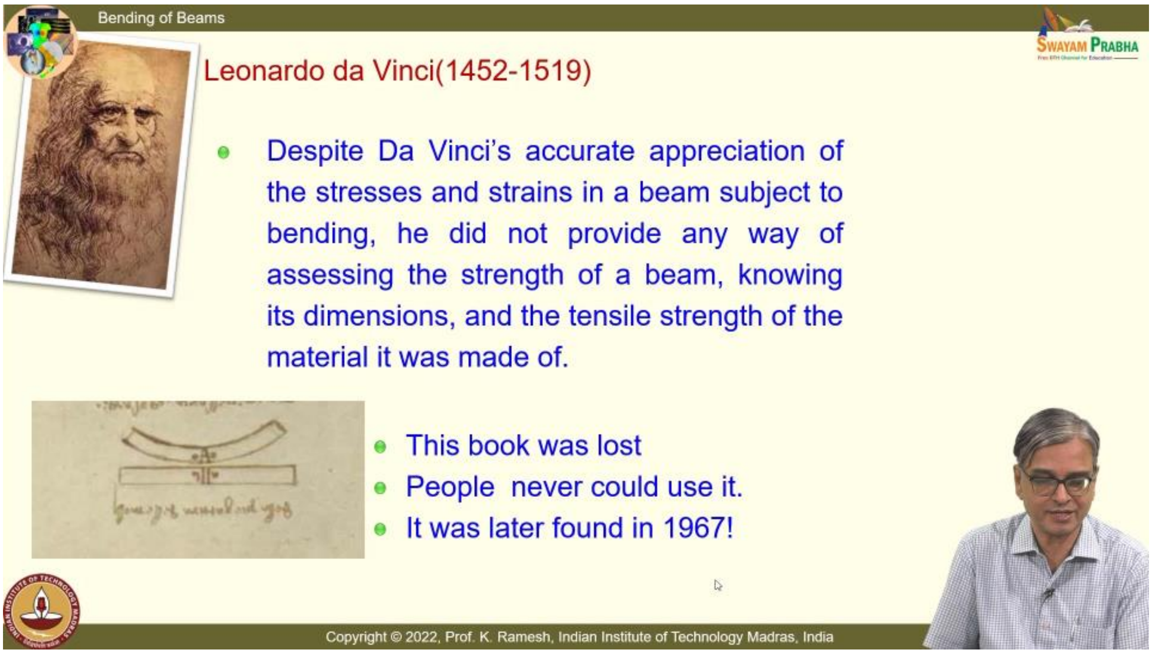
So, you have to understand that we are doing only engineering analysis. So, you have to appreciate the limitation and I have raised a point in a three-point bending. There is something we have to discuss about the shear stress, whether strength of material predictions correctly or not. You have to wait until we develop what we do that in strength of materials, ok. Then compare it with theory of elasticity. So, you should appreciate that whatever the expression that we have got in strength of material is very useful, but it has its own limitations; appreciate the limitations.

(Refer Slide Time: 43.09)



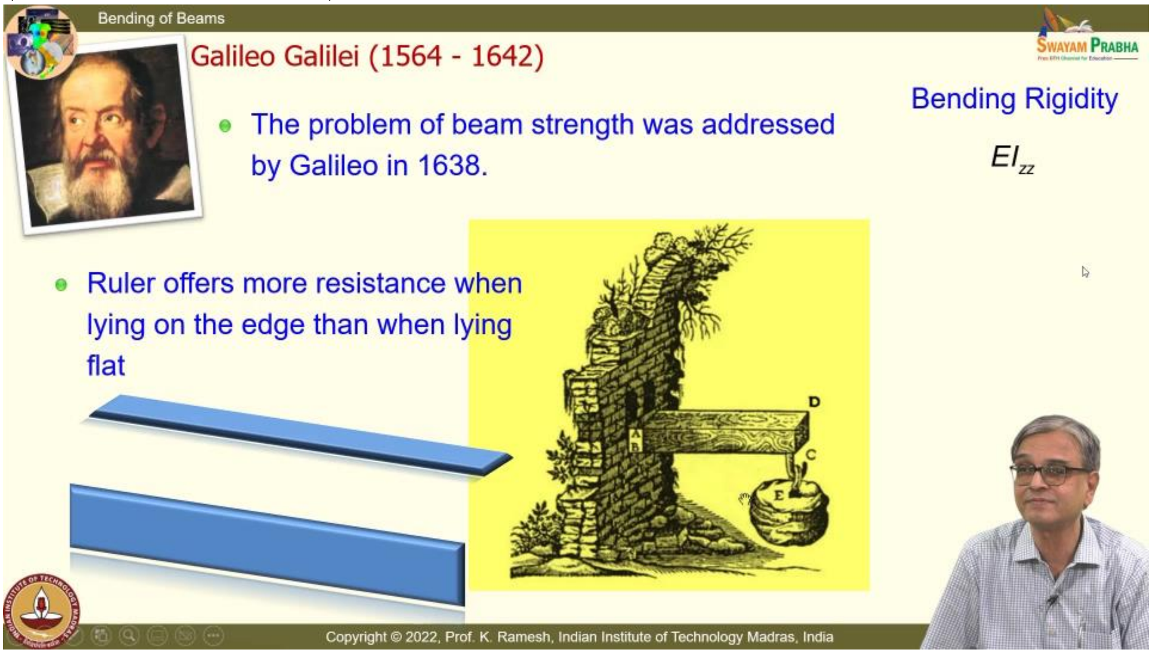
And let us go back and quickly go through the history because we have already seen this. You can appreciate certain aspects now better. See, Leonardo da Vinci was the first one. He established all the essential features of the strain distribution while pondering the deformation of springs; that is his wisdom, we have to say. And for the specific case of rectangular cross-section, Da Vinci argued equal tensile and compressive strains at the outer fibers. That is what we also say now in strength of materials. He could also say that the existence of a neutral surface and a linear strain distribution. What he could not achieve? He did not have Hooke's law. So, he could not translate this understanding into stresses, ok. And second one is, you know, he has to wait for the mathematical formulation of Bernoulli and Euler.

(Refer Slide Time: 44.08)



And he was also not in a position to give recommendation how to design a beam. How to, if I know the strength of the material, how to get the dimensions of the beam? And when you dig the history, you find that he has written a book that he has all this information, but this was found out only in 1967. So, all the early researchers without his book, they had to struggle and find out what is the beam theory.

(Refer Slide Time: 44.38)

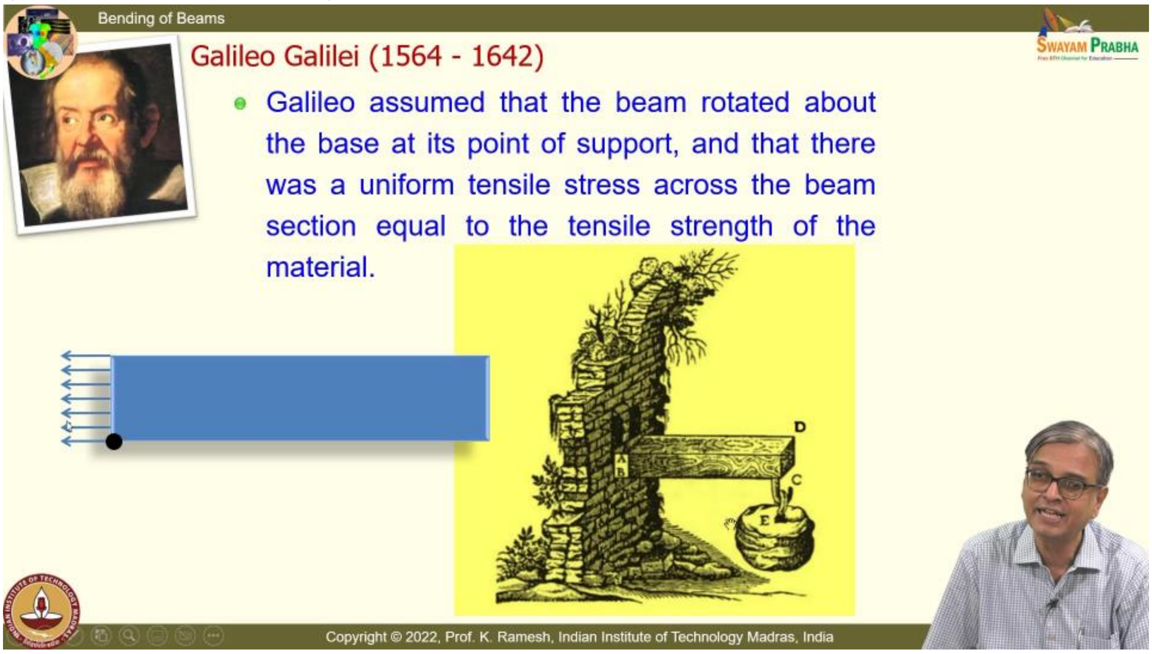


So, Galileo was the first person. He said the beam strength was addressed and he was able to give some numbers based on his experiment. Even at that time, I said this was a very difficult experiment, ok. Can you recognize he made a statement, Rulers offers more resistance when lying on the edge than when lying flat. Do you agree with this observation or not? Do you agree with this observation or differ? I do not get the answer. You nod your head, and what do I make out of it? Do you agree or not? You agree? Can you tell me how you say that this is better than this? What is the aspect that you can say mathematically? Very good, very good; I am happy! So, bending rigidity and I have EI_{zz} and you know very well I_{zz} for this cross-section is very high compared to this.

So, this observation was correct. They were learning it experiential. Experiential knowledge was very good, ok. I have been using this beam. This is also made of plastic. This has some cross-section and this has cross-section which is much, much, much larger. And you know very well that this has hardly any bending rigidity.

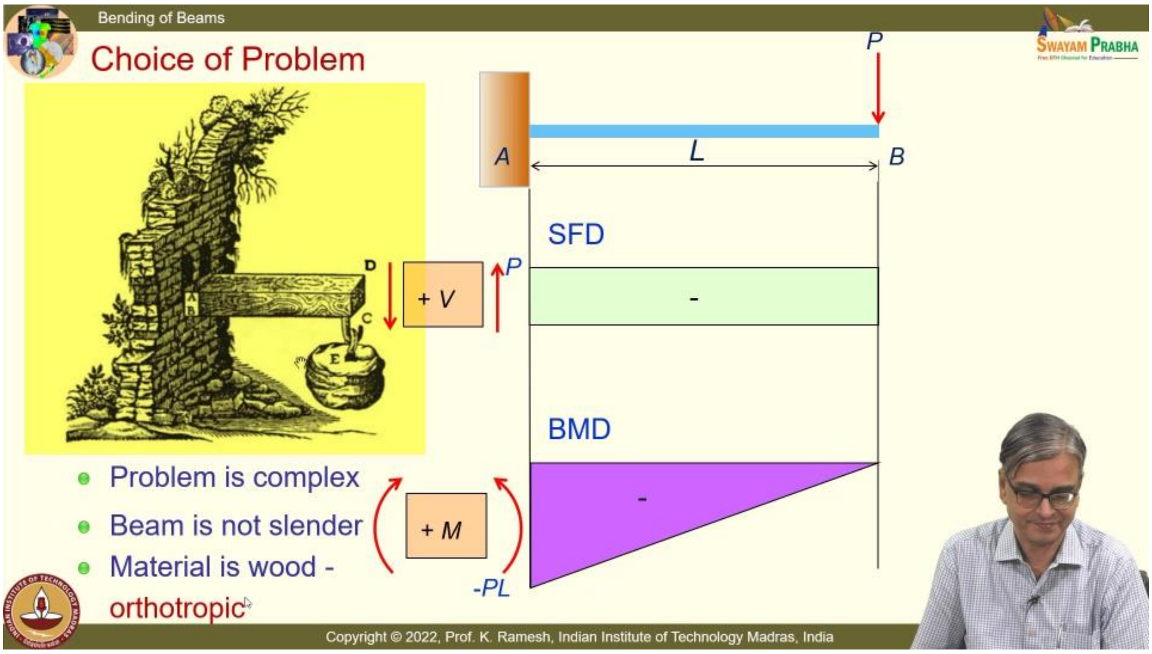
And I needed this for me to show your warping in case of twisting. And also for anti-clastic curvature, ok. So, what made the difference between the two? This has more rigidity than this because of its Young's modulus, ok. So, $EI_{zz}$is a very, very important aspect when you are looking at anything to do with bending. Similarly, GI_p for torsion.

(Refer Slide Time: 46.36)



And he had a good idea of how the beam failed and things like that, ok. And he could not find out the variation of stress. The variation of stress was very difficult. He had some idea that it has failed here; it said it is because they were accustomed only axial force transmitted. So, the same idea of equal force was obscuring their thinking.

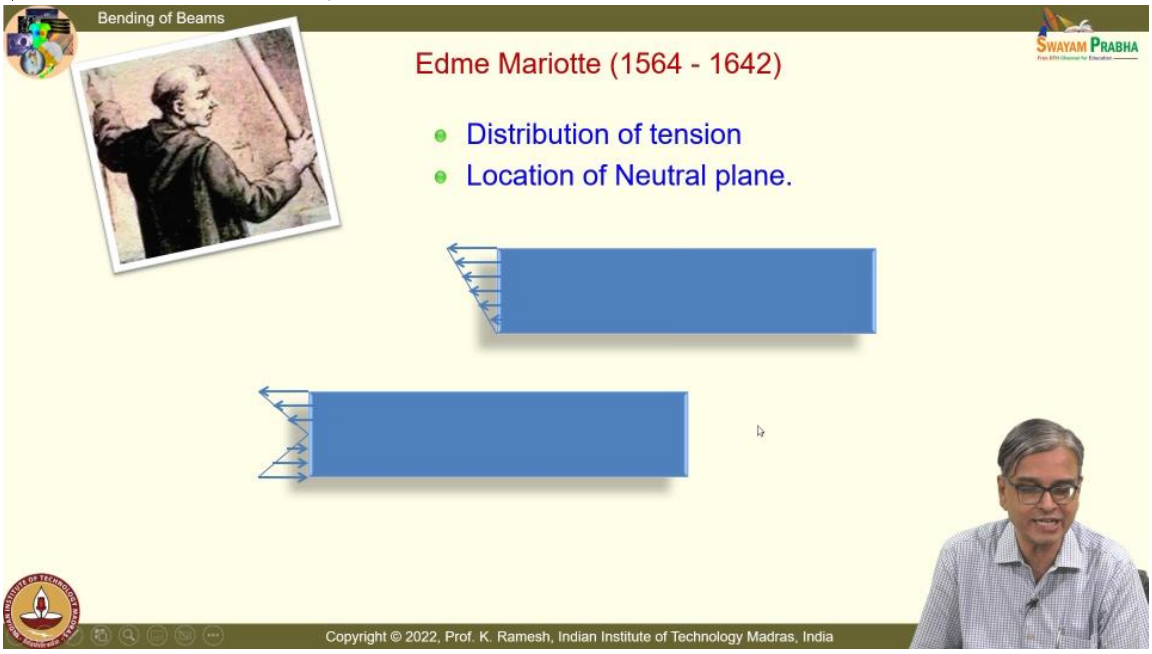
(Refer Slide Time: 47.05)



And now you can say that he had taken a complex problem, ok. Because I explained how your cantilever beam is difficult to handle because it has shear force. And shear force introduces plane sections do not remain plane and you have bending moment varying. We were very clever! We took a problem deliberately that it will transmit only constant bending moment. He did not have that luxury. So, he had he did not have that luxury that is why he faltered, he could not proceed beyond a particular point.

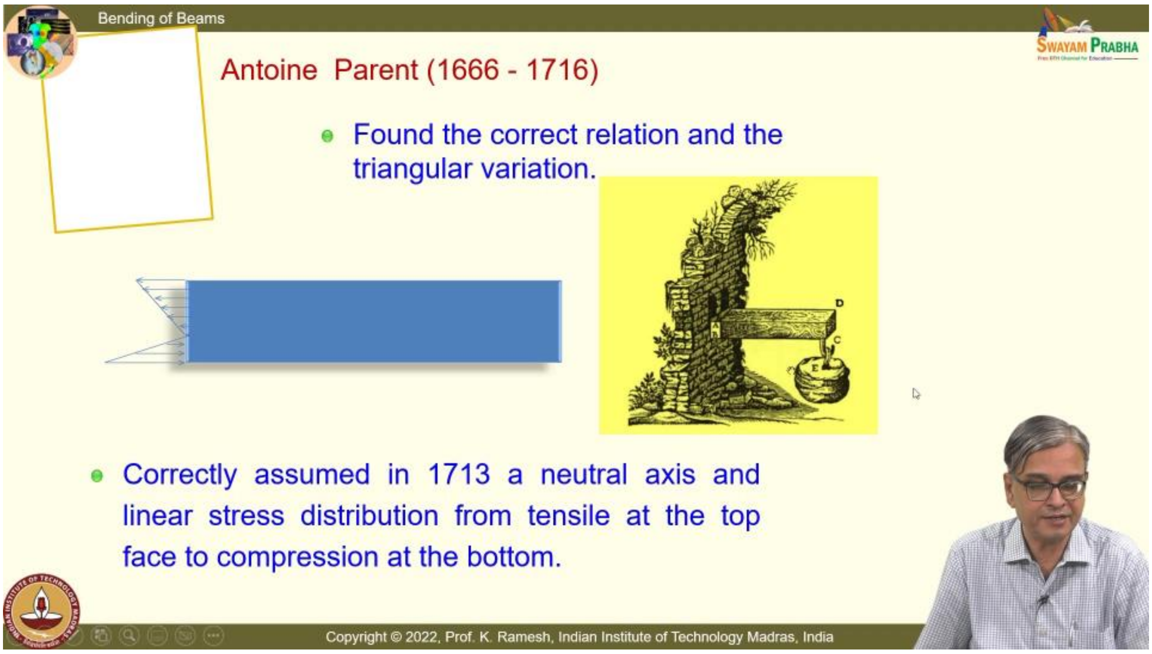
And you also see that this beam is not slender, ok. The problem is complex because it is not a slender beam; the depth is important. I said the shear and the axial force, the coupling takes place when these two are; the depth is comparable to the length. And material, he was not having isotropic material; he was having wood. So, they could not proceed. So, you when you want to solve your problem also you should know how to simplify and get the core of it.

(Refer Slide Time: 48.19)



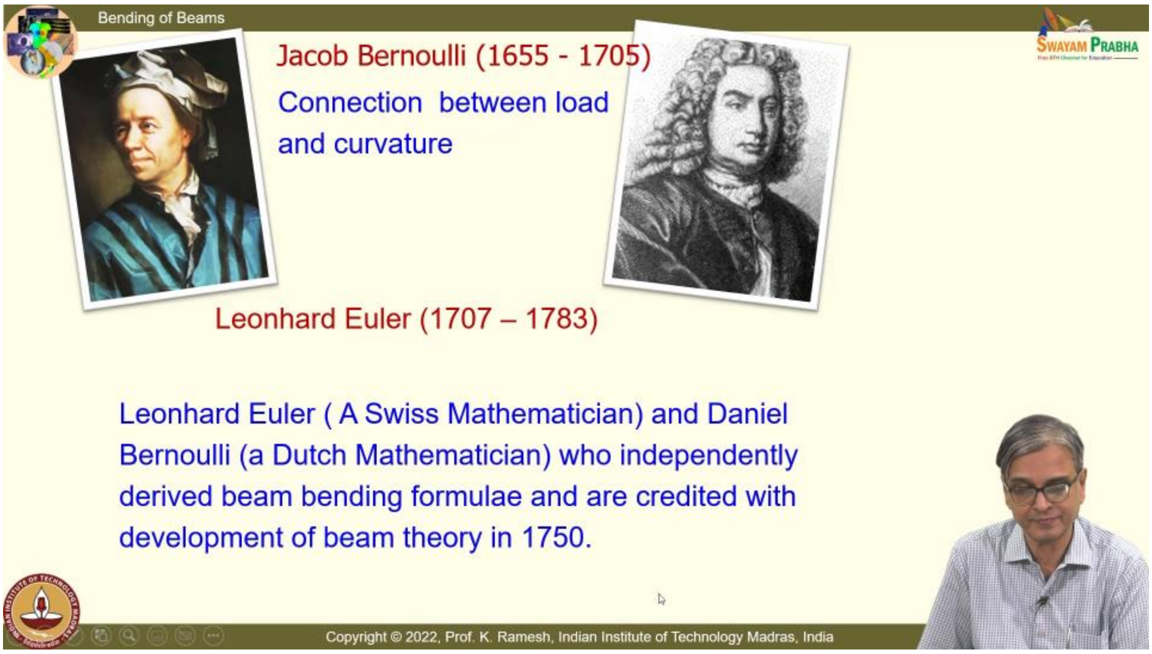
And Mariotte had some idea that there is a variation is linear, he was also not able to succeed. But he showed both variation like this as well as variation like this, but it is not clear why he obtained two of them. So, they were very close to the solution, but not achieved it. He could also say the neutral plane.

(Refer Slide Time: 48.45)



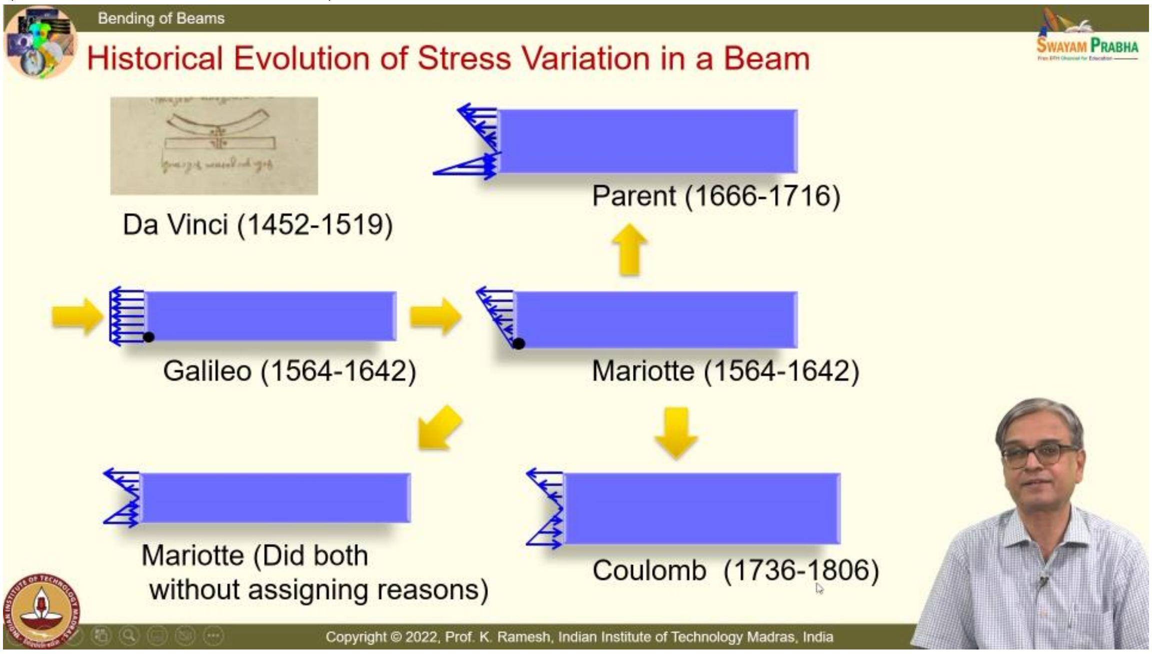
And Parent, he got this as triangular, and he had this neutral axis shifted. And stress distribution from tensile at the top face to compression at the bottom. See they were always looking at bending like this, ok, like what I was showing. So, the top surfaces in tensile and bottom surfaces in compression. So, he was able to do it. And so, the triangular variation is credited to Antoine Parent.

(Refer Slide Time: 49.15)



The breakthrough came only with Euler and Bernoulli; they linked load and curvature. Unless they could bring in the deformation, the beam problem was not solved, ok. So, that took such a long time to develop. So, he was a Swiss mathematician, another is a Dutch mathematician. They independently derived beam bending formulae and are credited with the development of beam theory in 1750.

(Refer Slide Time: 49.48)



But the actual distribution was credited to Coulomb. And if you look at the distribution, the contribution by various scientists; we summarize. Da Vinci had all the idea, but his book got lost. Galileo could design beams, could solve problem half way, but he assumed

that the stress is uniformly distributed and you have first appearance that it is varying in a triangular fashion in one form.
He also had mentioned this, ok; without assigning reasons. And Parent correctly said that you only have this kind of variation, his neutral axis is shifted. And finally, you have Coulomb, who has solved the problem of beam bending. So, what you have learnt in the few lectures has taken 400 years for its evolution. It is not something simple. So, if you have doubts, do not feel annoyed. Great minds have these doubts, ok. And they did not have access to simplified experimentation. And in fact, even in IIT, see this I could get easily in a commercial shop. And in order to show your warping in torque, I had to get a material which is very soft. We had to take some effort to get this material done, ok. So, in this class we have looked at what are the limitations of strength of materials approach to beam theory.

We said it is derived only for a beam of constant bending moment transmitting. Anything other than that, it can be considered only as a good approximation of engineering analysis. Thank you.