Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 26
Bending – 4: Shear Stress in Beams

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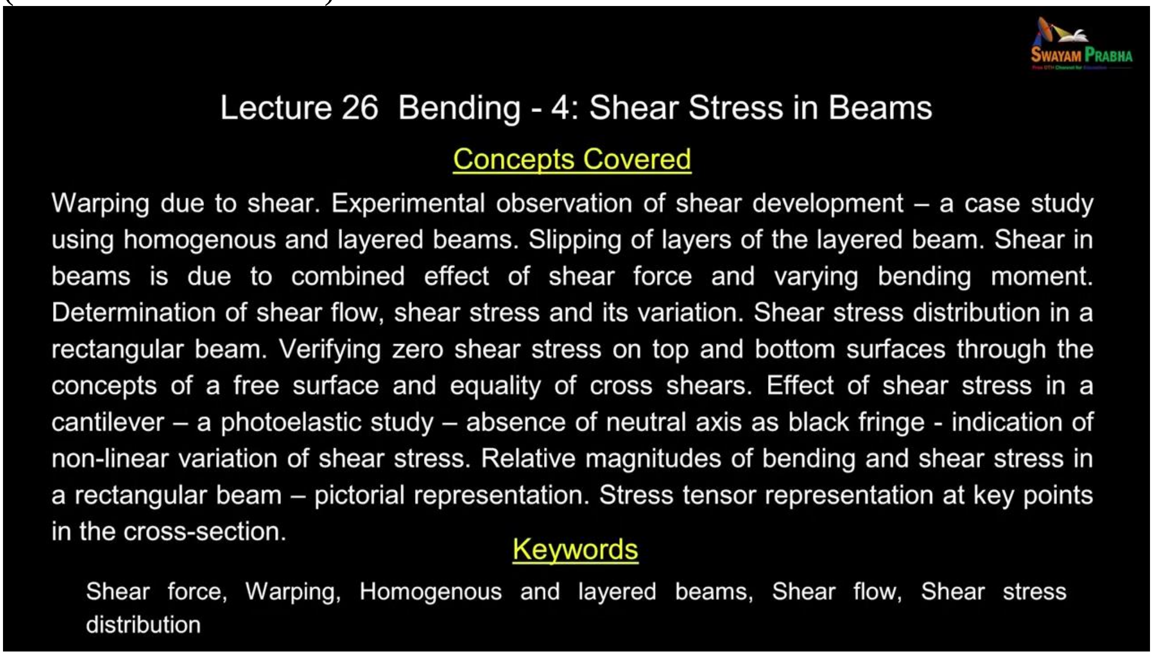

We will move on to the determination of shear stress transmitted by the beam. And you know, you will find, how comfortable it was to take up a beam transmitting only bending moment, discuss the deformation and evaluate the axial strain developed. The moment you come to determination of shear stress; you need to perform experiments very carefully. As I have always emphasized; doing a thought experiment or a direct physical experiment. And what we will do is, we will do some nice experiments and make clear observations on what all things that happen when we apply the load.

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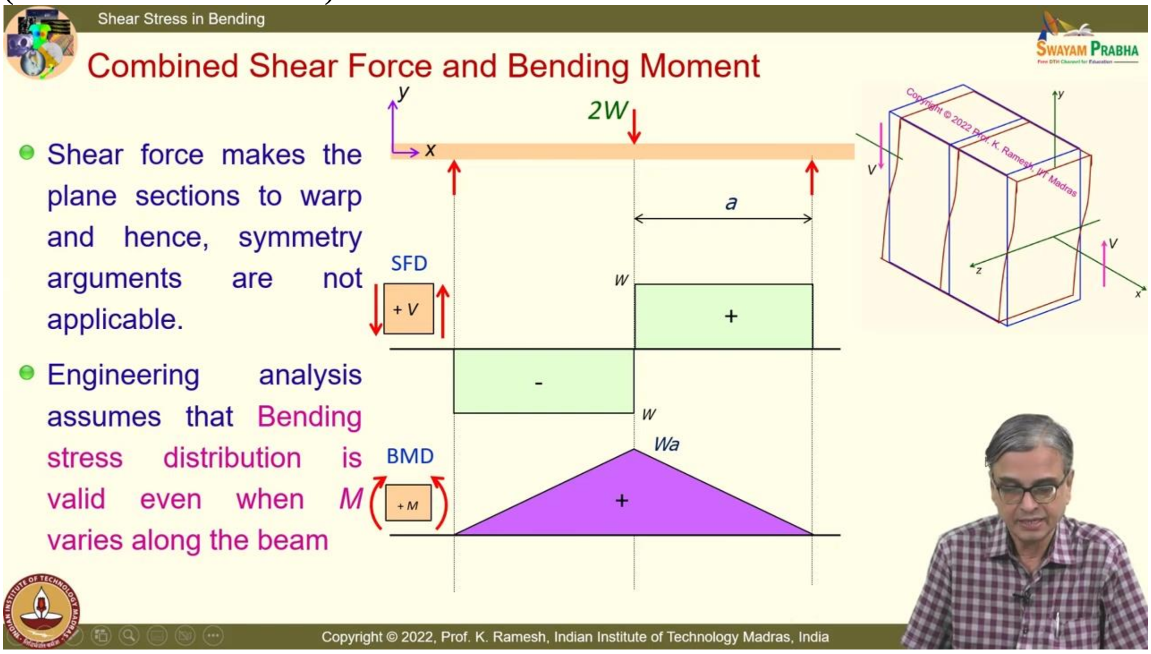

For our discussion, we will take up a problem of a three-point bending. And you know how to draw the shear force and bending moment diagram. And what we have done here is, we have introduced a constant shear force. Earlier we had only constant bending moment transmitted by the beam. Now, I have a constant shear force.

Even if I had a constant shear force, my bending moment varies linearly. And you know, without proof, I have also shown you, how the section will change in the presence of a shear force. We have made certain observations like; the plane section gets distorted like this. And what you have is, here the original 90^o is preserved. So, this indicates shear strain is zero which we have also seen when I have this as a free surface, even though I may have a shear force transmitted by the beam, the distribution which we evaluate from strength of materials should be such, it has to be zero at the top point as well as at the bottom point.

That we have understood from the discussion on free surfaces. And we also note that you have a maximum change in the angle at the middle plane, which is coinciding with the neutral surface. So, your shear strain or shear stress reaches a maximum here. And even though my bending moment varies along the length of the beam, in engineering analysis, you use the flexure formula; and in that, you use the bending moment as what is seen at that particular length of the beam. That is the way that we would use the flexure formula developed primarily for a beam which transmits only bending moment; constant bending moment. So, even when the bending moment varies, we invoke the same formula and substitute this bending moment.

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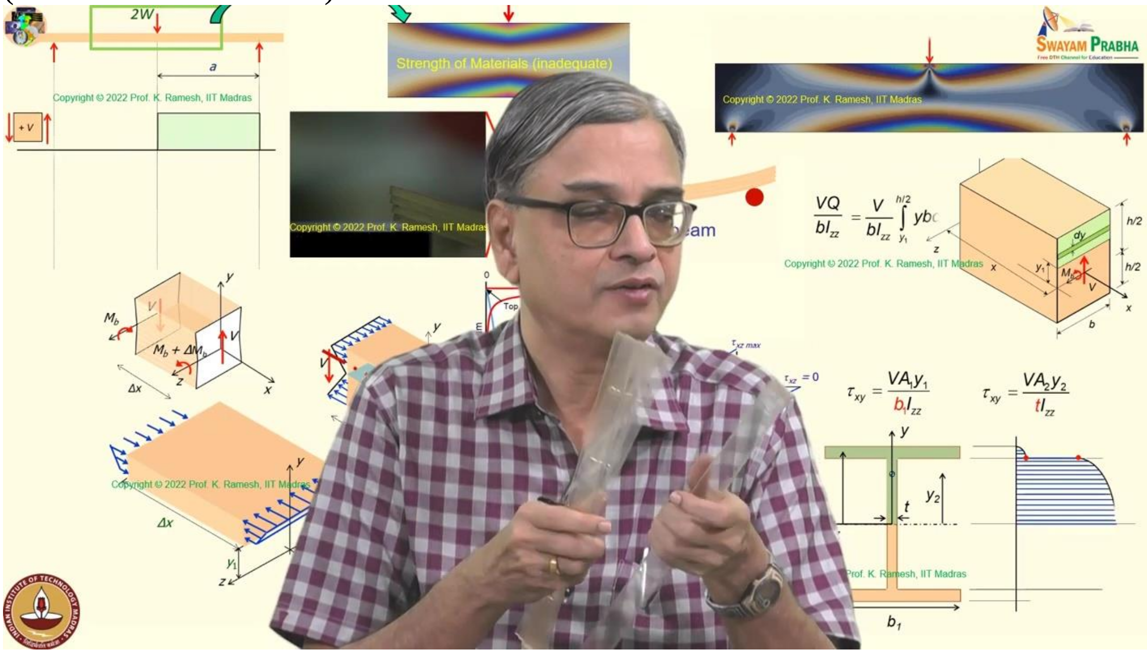

And let us do some interesting experiments. You know, I am going to have a beam which is made of Perspex and it has some thickness. And I am also going to do the experiment on four simple strips, which when you put together, it forms the thickness of the beam. In fact, this is also added as one of the experiments in your strength of materials lab long time back when I introduced this; I am not sure how carefully you have done the experiment.

I said, if the experiment is done carefully, it is truth. So, whatever the theoretical development that we make, it should explain what is observed in the carefully done experiment. Let me add that adjective. You have to do the experiment very very carefully.

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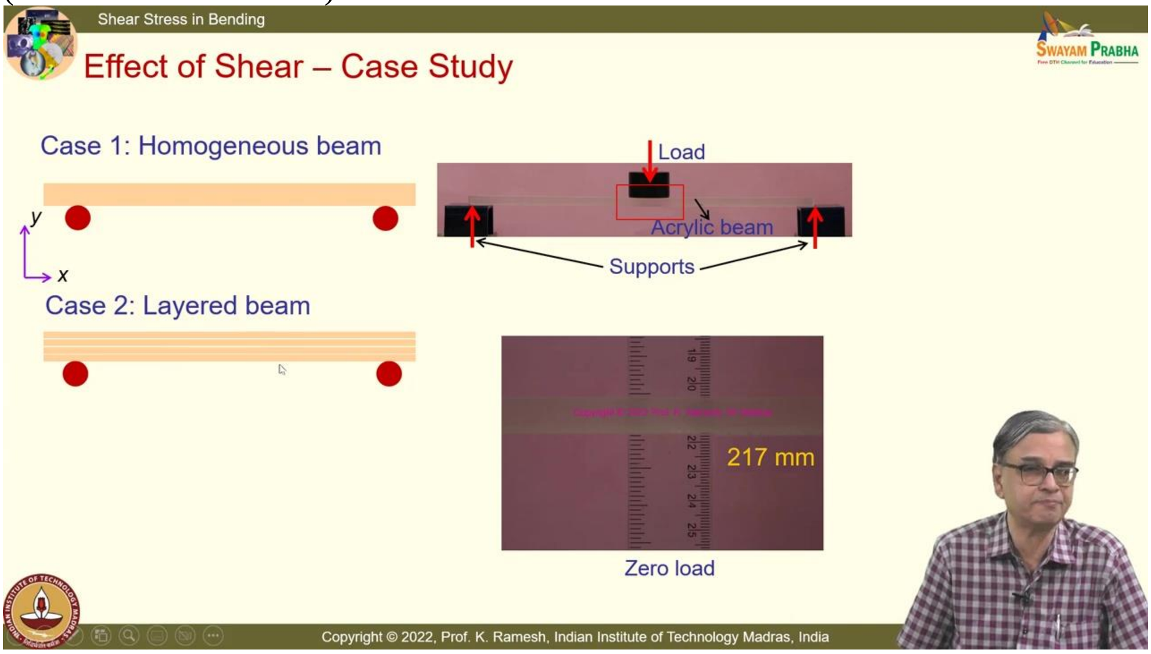

I have the beam subjected to three-point loading. And the second case is; you know, when I put this, I also find out the zero load, I have also put the scale and you can carefully see that this is about 217 mm. The idea is, when I am going to change my beam, I am going to look at what is the deflection that is created by the same load. Supports are same and your load applied is also same. We are going to investigate; case two is a layered beam. When I add the thicknesses, it adds up to the thickness of the original beam.

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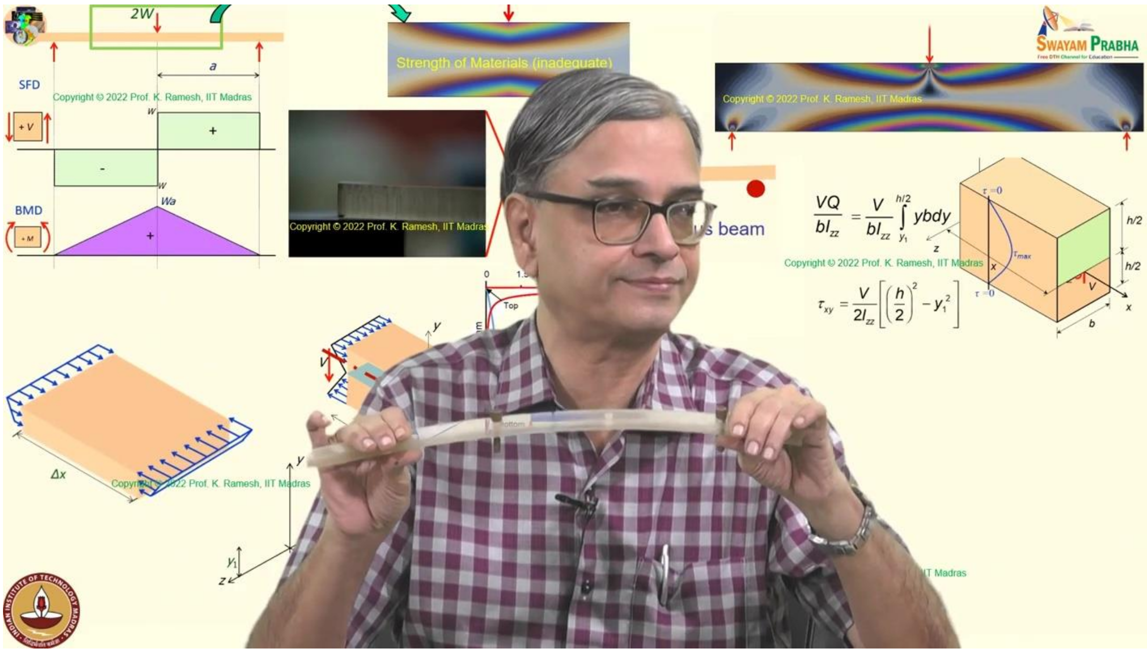

And the third case, what I am going to do is, I am also going to tie these layers. So, that means, they are not allowed to move freely. You know, I have four pins, but I will just insert two of them to illustrate that I am not allowing them to have any relative motion. Just to understand, what way the experiment shows up. So, the third case is, I have a pinned layered beam.

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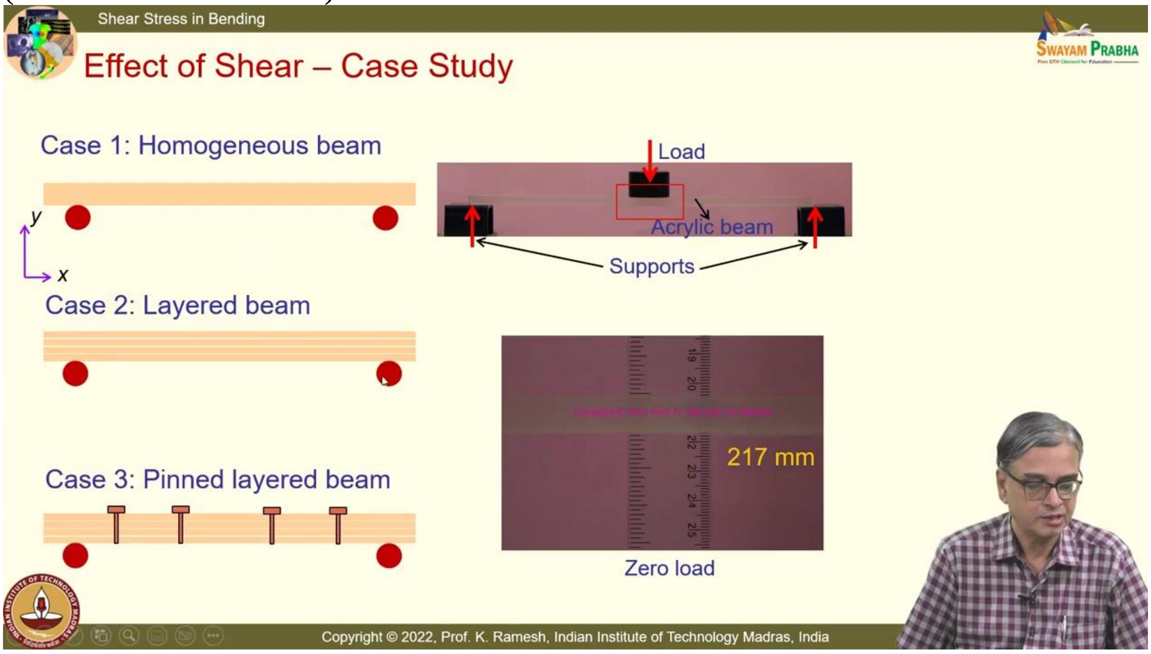

So, let us do the experiment.

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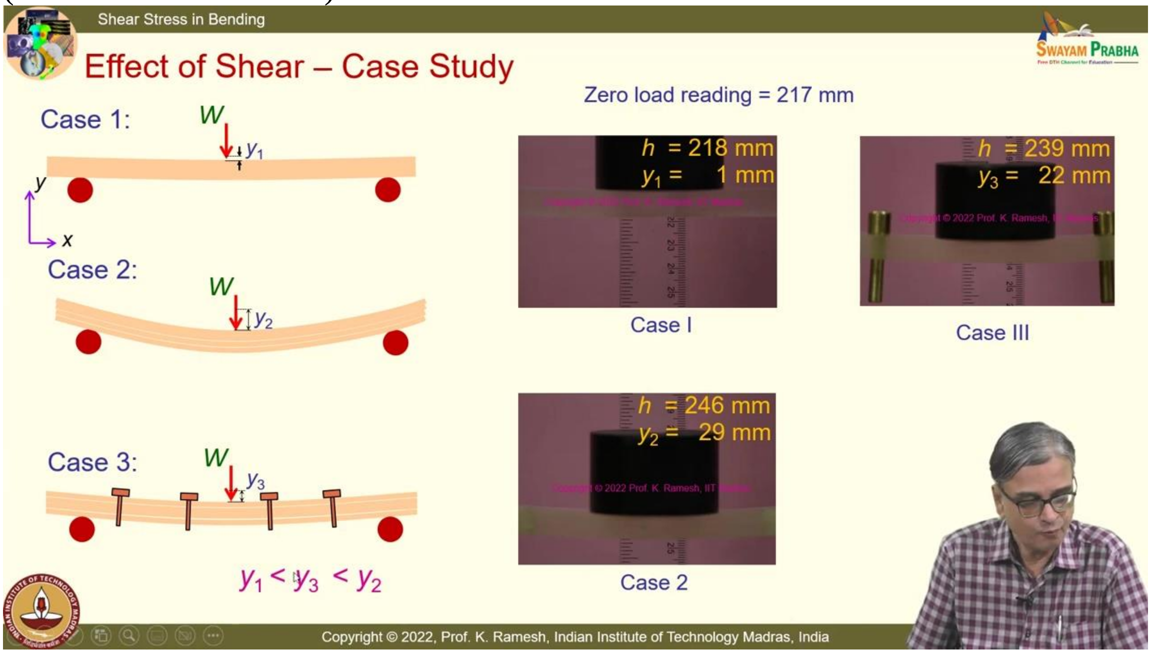

And zero loading is put as 217 mm and then I have, when I apply the load, it gets deflected. And now the deflection is 218 mm. So, it has deflected only by 1 mm. It is very, very small. When I have the Perspex beam, when I put the load; see, it is not as flexible like the plastic beam, it is very strong. And you have a deflection of only 1 millimeter. Suppose, I put the load on the layered beam, you find that it has deflected to 246 mm. That is the scale reading. The initial reading is 217 mm and I find that the deflection, what you find is very large. It is about 29 mm. See, yesterday I said, I have to double the load carrying capacity of the glass plate. And I said, from a design perspective, I can make multiple options. One option is to buy another glass plate, put on top of it. Another option is, increase the thickness of the glass plate. The third option was put one more support. After this experiment, what would you say about putting another glass plate? It is not found to be effective. Let us see, if I pin them, what happens? That means, I am restraining it. And when you find this, the deflection is 239 mm. You know, these are all very carefully done photography because if you look at this, it is not just the figure is shown, you also see the reading from the scale to convince you.

And you find the deflection is reduced compared to the earlier case by about 7 mm. This is 22 mm and this is 29 mm. Whereas, if I have the solid beam, that had hardly any deflection. See, for the purpose of illustration, what happens when I have a layered beam? I have taken four layers. And if you really look at the beam, I can imagine infinite layers of very thin cross-section, very thin depth.

And even when I do the pinned case, I have just put four pins. Suppose, I had put instead of four, if I had put ten pins, then what would you anticipate? This deflection would be still smaller. So, the beam becomes and behaves more and more stiffer. So, something is happening in the beam. So, we will have to find out; our interest is to find out what happens in this solid beam.

To appreciate what happens in the solid beam, we have taken layers to understand what happens in between the layers because this is very subtle. The issue what we discussed today is a very subtle concept. And you can also make a record that y_1 is the least, y_3 comes in between and y_2 is the largest deflection.

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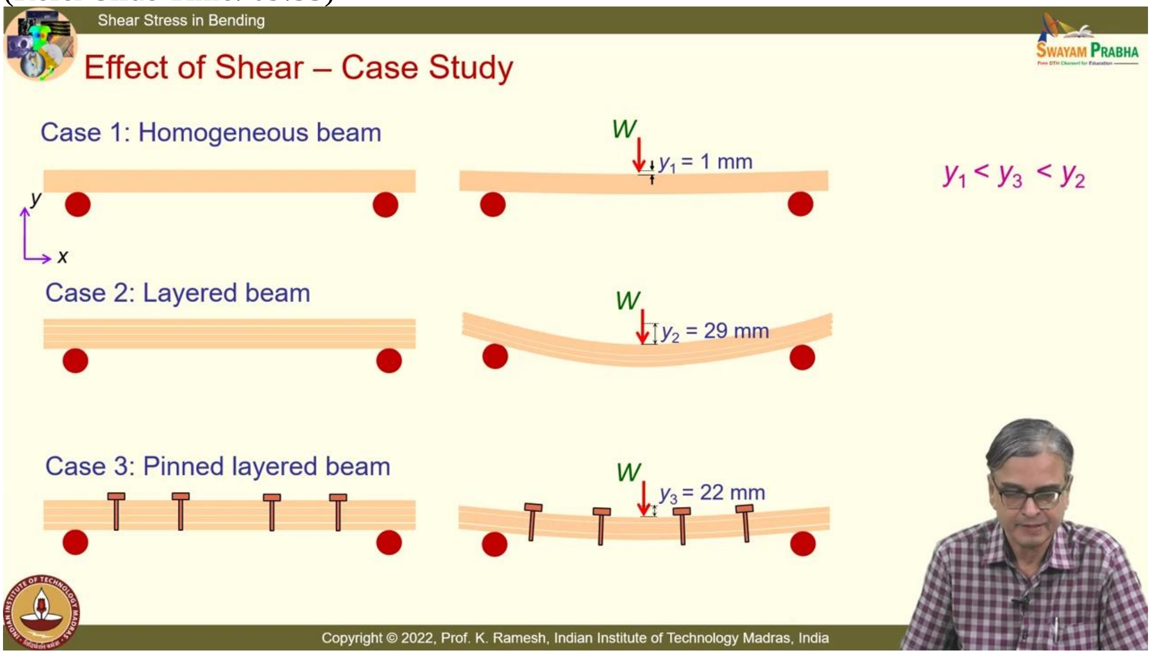

And you know, this is a summary; I have 1 mm and then in the second case, I have this as 29 mm and in the third case, you have this as 22 mm. See, even before we take up deflection, these slides illustrate, when I have a three-point bent specimen, how the beam is deflecting.

You know, you have to do that because we are going to study principle of superposition even in deflection, where you will have to visualize, how a beam can possibly deflect. Only then, you understand. See, as engineers, you have to have lot of visualization skills.

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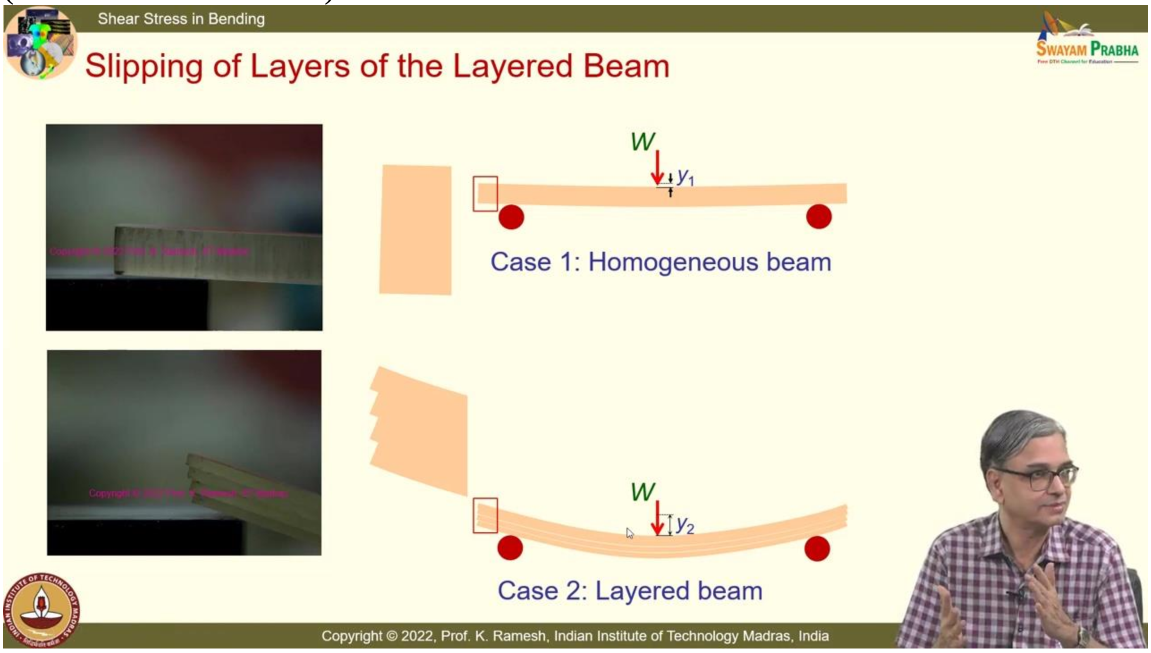

And now, we would see very closely, what happens at the edge, fine. You observe this video and what you find when I apply the load, I am showing only this portion. You could see that this is getting lifted, fine. This is for a homogeneous beam. Now, I have a layered beam. So, you have this only like this. There is no significant change on this.

This remains straight on the edge. Now, I have a layered beam. Even here, I only focus my attention on this and find out what happens, ok. Can you see? There is something interesting happening. Was this edge remaining straight? It does not remain straight, ok.

And what you have is, you have these layers slipped to different extents. That is why you see this as an edge like this. So, conversely, what we can understand is, something was there to keep them together in the homogeneous beam. That is the reason why this edge remained straight. That was missing when I have a layered beam.

So, the layered beam is free to move and it has utilized that freedom to come out, slip between the two. And what we would do is, we would see this in an enlarged fashion and you see this animation.

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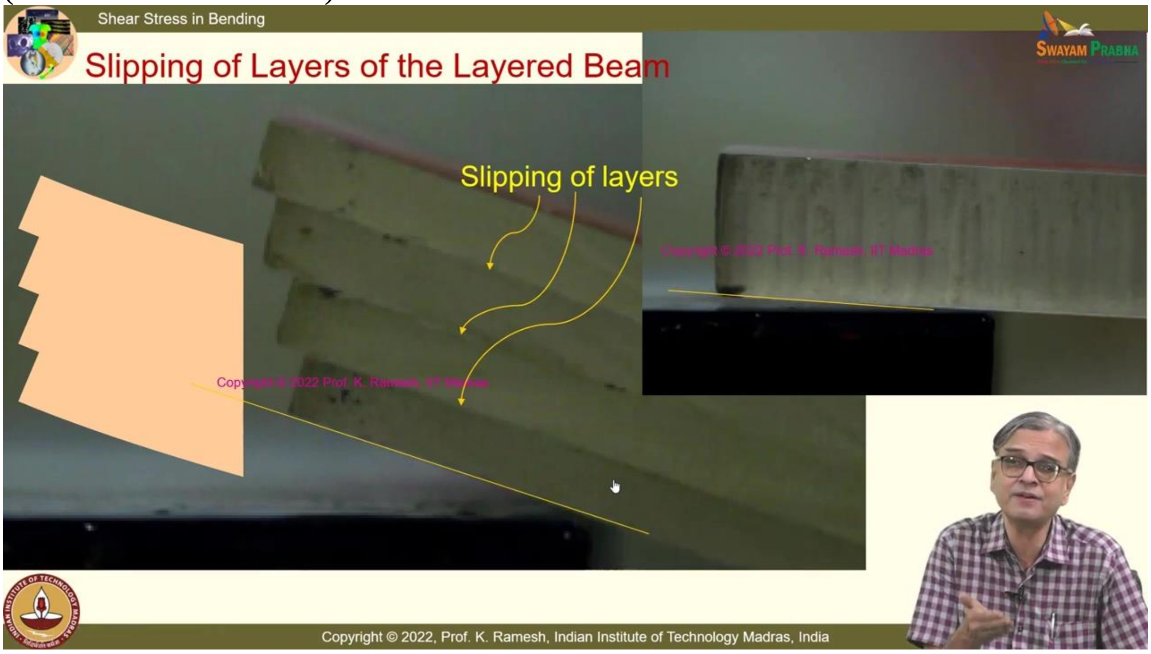

You know, you have to understand this very very clearly. And you know, you see it very clearly that when the experiment is done carefully, you find this has got shifted like this. And you really see the layers have slipped along this boundary.

Make a sketch out of it. I want you to make a sketch. It is a very important observation. I am not sure how many of you have made this observation when you did this experiment and carefully looked at - what happens and why it happens. All are important. See now, what we will do is, we will also see the solid beam, how it gets deflected.

You know, it has gone up. I do not know how many of you have observed. To aid your thinking, I will also draw some extra lines. So, I have drawn a line where the edge is. So, it has moved up like this. Suppose, I repeat the animation, you can see that the beam has moved up slightly.

See, when you perform an experiment, I said, people do the experiment with some anticipation. You do not do the experiment blindfolded. Even before you start doing the experiment, you find out what all things have to be looked at in the experiment because many experiments you may not be able to repeat. If the experiment is dynamic, then you have to be very careful in recording the dynamical behavior. So, you must have a pre-plan, what all things that you need to note.

What is key in this experiment is, you should also look at what happens to the edge. And look at here, this is the horizontal edge; to what extent this beam has got lifted. That is because this had the maximum deflection downwards at the load application point. So, the beam has got deflected like this. When I have like this, it has deflected like this.

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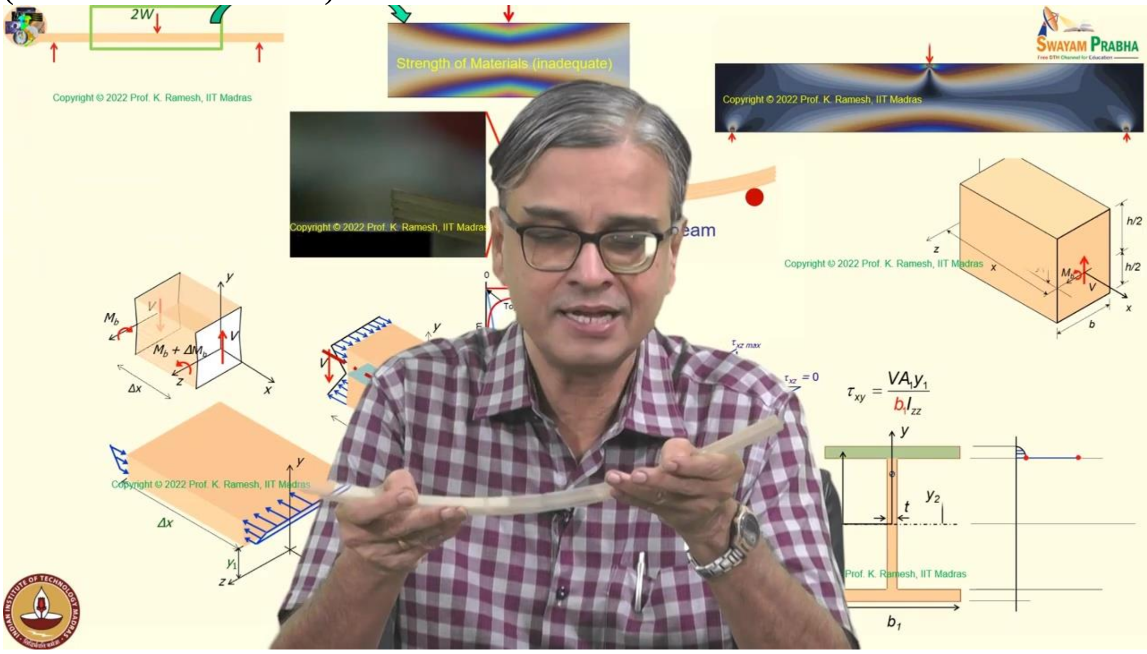

So, you also see that it gets slipped. But this is not a very careful experiment. See, that is the reason why I thought that I would record the video and then blow it up and then show. Is the idea clear? You are convinced that something is happening between the layers. And that we have to quantify and evaluate. And you can see very well, when I have two surfaces having a relative motion like this, you call that as shear. Is it not? Fine. So, if shear stress gets developed, it will try to keep the edge straight like this, fine. And where does this come from?

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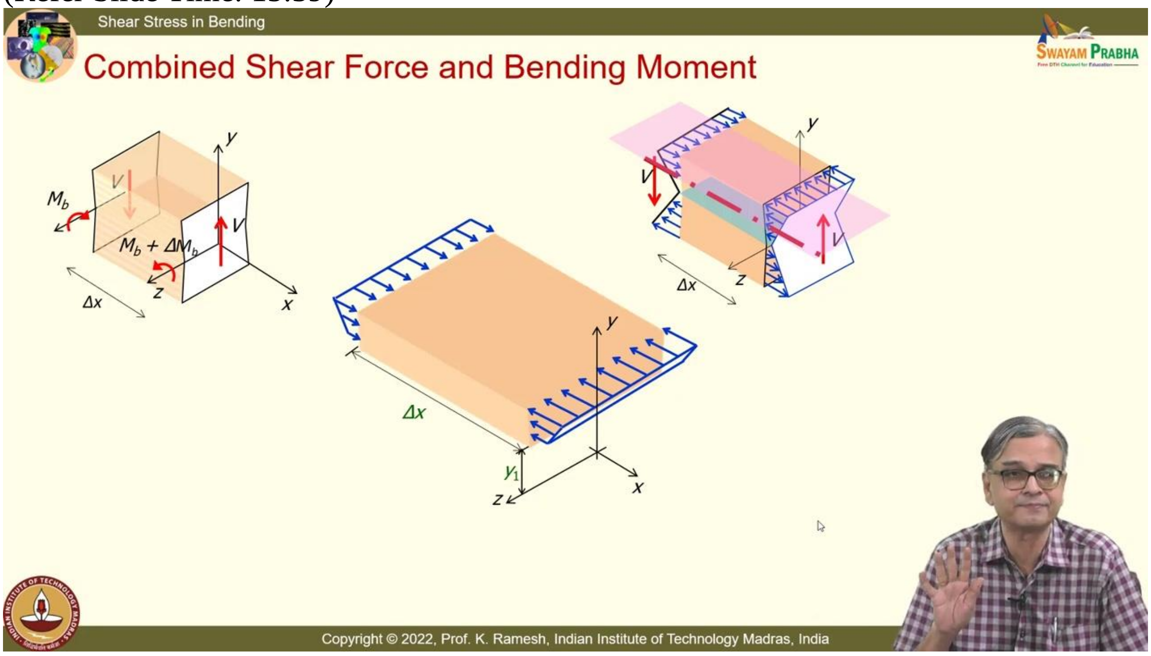

We have already seen that when I have a constant bending moment, the life was very simple. The moment even I introduced a constant shear force, bending moment was varying linearly. Now, we will take up a very generic case. And here, you know, the sign convention you have to follow. That is very important. And we say, on a positive surface, positive shear is shown in the positive direction. And you also have bending moment varying along the *x* axis. And we have shown the variation as M_b here and $M_b + \Delta M_b$. And I have taken anticlockwise moment as positive. You know, sign convention is very important because when we assemble the equation, you have to put them in positive sign for us to get the basic equation and our mathematics will fix the sign automatically.

So, what you find is the bending moment varies. And now you know, if the bending moment varies, we have already said, to find out the stresses due to bending, simply use the bending moment at that cross-section. Use the flexure formula. That freedom we have given, fine. So, if I go and plot what happens on the two surfaces, we know that it is varying linearly. And as the bending moment has increased, so to say, if we take this as an increase, I would have larger stresses acting on this edge, incrementally larger.

Is the idea clear? Now what I am going to do is, you know, the beam as a whole is in a state of equilibrium and it is a deformable solid. For a deformable solid, we have said, every conceivable subsystem should also be in equilibrium. We have to investigate that. So, one of the ways to do it is, let me put a cut.

I also have the shear force transmitted by the beam. So, I will put a cut horizontally that is shown as the pink plane. So, I slice from the top; a particular height and I slice this like this. Is the idea clear? I want to take out that slice. The beam as a whole is in a state of equilibrium. When I take a conceivable subsystem like this, it should also be in a state of equilibrium.

So, let me draw this. And you know, I have always been saying when I want to illustrate, suddenly I will put things very large, so that we see what happens in that section. Never forget, we are handling only slender beams. For illustration, we are doing it. So, whatever we have understood from this, we will translate as forces acting. To make our mathematics clear, we have identified this section what I have put, is at a height of y_1 and I put a horizontal section and the element length is Δx.

So, I have bending stress indicated on this face and bending stress indicated on this face and it is also put for the entire width of the beam. I have this. Please make a neat sketch. That is the reason why I am going slow. These are all conceptual understanding. And if you understand, how we arrive at the shear stress distribution, you will know, when I have complicated cross-section, what way the shear force will show up when I have to calculate; if I have to solder, if I have to join, I will have to find out what way the shear stress shows up. And you have to get the conceptual understanding and we take a horizontal section.

We are very clever. I will ask a counter question later that you will have to do some thinking at home and then come back.

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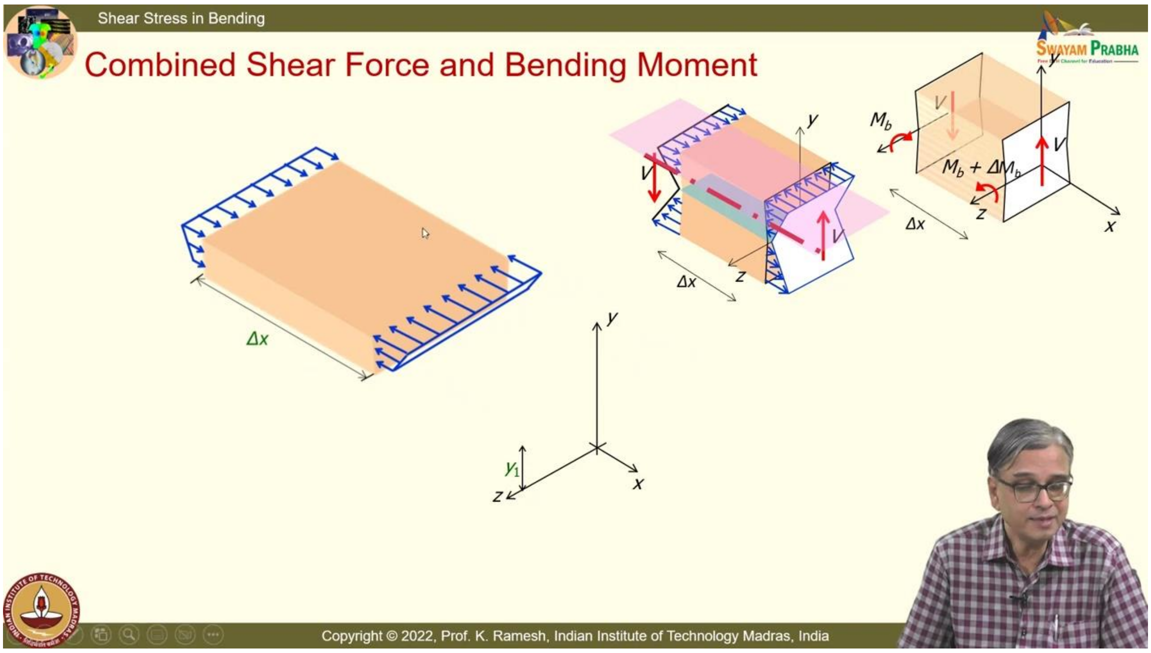

See, the board is not smart. Suppose, I make the board smart, what would happen to this? Do you realize that there is force imbalance in this? My diagram is not complete. We know from the physics of the problem, the beam as a whole was in equilibrium. Suppose, I make the board smart, this element will not remain in place. It will only have a translation, fine. It will keep on going in the direction. I have stopped it at some position. The board has become inert. So, that means, to keep this tied in this place, I should have stresses developed at the bottom of the surface. Is the idea clear? It is a very subtle point.

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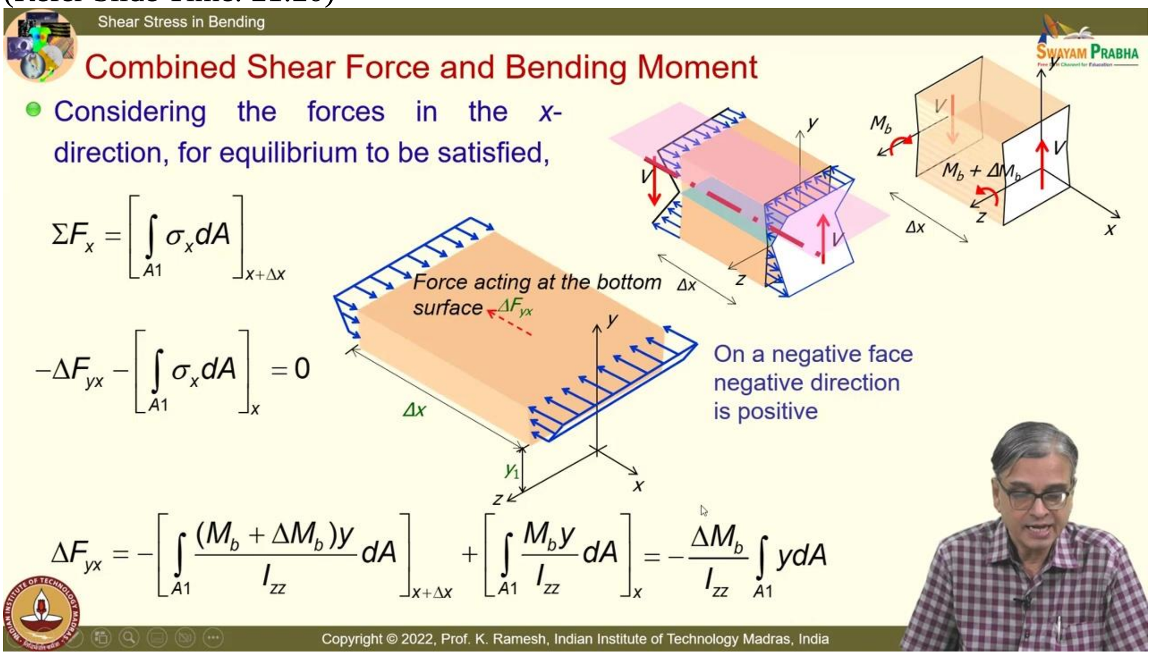

You have to appreciate this. And you will also have to appreciate why I have put this in the negative direction. It has moved this way. I should have put it in the positive direction is what you normally would feel. And you know, we are looking at the negative face.

On a negative face, negative direction is positive. We are developing basic equation. So, unless you have shearing action initiated there, this cannot remain in equilibrium. Once you have understood this, we will go back to our rigid body mechanics; $\Sigma F_x = 0$, $\Sigma M_z = 0$, all that we will do. So, that is how you go back between rigid body mechanics and strength of materials, that is deformable solids.

So, we are actually building a firm foundation. So, you cannot afford to forget concepts in rigid body mechanics and even the concept that we have developed all through the course till today. All that is needed for you to appreciate how the shear stress is developed in the beam. So, considering the forces in the *x*-direction for equilibrium to be satisfied, let us write the expression. See, we are writing stress components and if I know the area, I can find out the force.

So, I want to put $\Sigma F_x = 0$. So, we look at what happens at $x + \Delta x$,

$$\left[\int_{A1} \sigma_x dA\right]_{x+\Delta x}$$

And I call this area as A_1. It is not the complete area of the cross-section. It is the area of the cross-section above the line which is located at y_1 .

This section is; above the pink section is what we are really looking at. And we have understood that there has to be a force on the bottom surface. And when I write the equation of equilibrium, I must put the sign as appropriate. So, this is in the negative direction. So, I will put it as $-\Delta F_{yx}$.

And then this is also, we will have to put what happens at direction *x*. And so, I have this

$$\Sigma F_x = \left[\int_{A1} \sigma_x dA\right]_{x+\Delta x} - \Delta F_{yx} - \left[\int_{A1} \sigma_x dA\right]_x = 0$$

So, I have this as when I want to find out what is ΔF_{yx} . I have this as

$$\Delta F_{yx} = -\left[\int_{A1} \frac{(M_b + \Delta M_b) y}{I_{zz}} dA\right]_{x+\Delta x}$$

because this is what we have said in the idealization. Just use the bending moment at that cross-section.

So, I can find out what is the stresses there. And then multiply it by the area and do the integration from this area A_1 at $x + \Delta x$. And you have force acting on this surface, that is

$$\left[\int_{A1} \frac{M_b y}{I_{zz}} dA\right]_x$$

That gives me ΔF_{yx} as

$$-\frac{\Delta M_b}{I_{zz}} \int_{A1} ydA$$

What is ydA? It is first moment of area, ok. So, what we have actually done is, we have recognized that bending moment varies along the *x*-axis.

And we have utilized the flexure formula using M_b at this cross-section, using $M_b + \Delta M_b$ at this cross-section. And we realized, because the bending moment is different, the stresses are different on this. Consequently, the forces are different. So, in order to keep the element in equilibrium, I should have stresses developed at the bottom surface.

I have shown this as force. Force gives rise to stresses. And then our interest is to find out this. And now, we have got the expression

$$\Delta F_{yx} = -\frac{\Delta M_b}{I_{zz}} \int_{A1} ydA$$

Still, we have not seen that this expression is written in terms of the shear force *V*. And in the limit, what we would do? We would make $\Delta x \to 0$.

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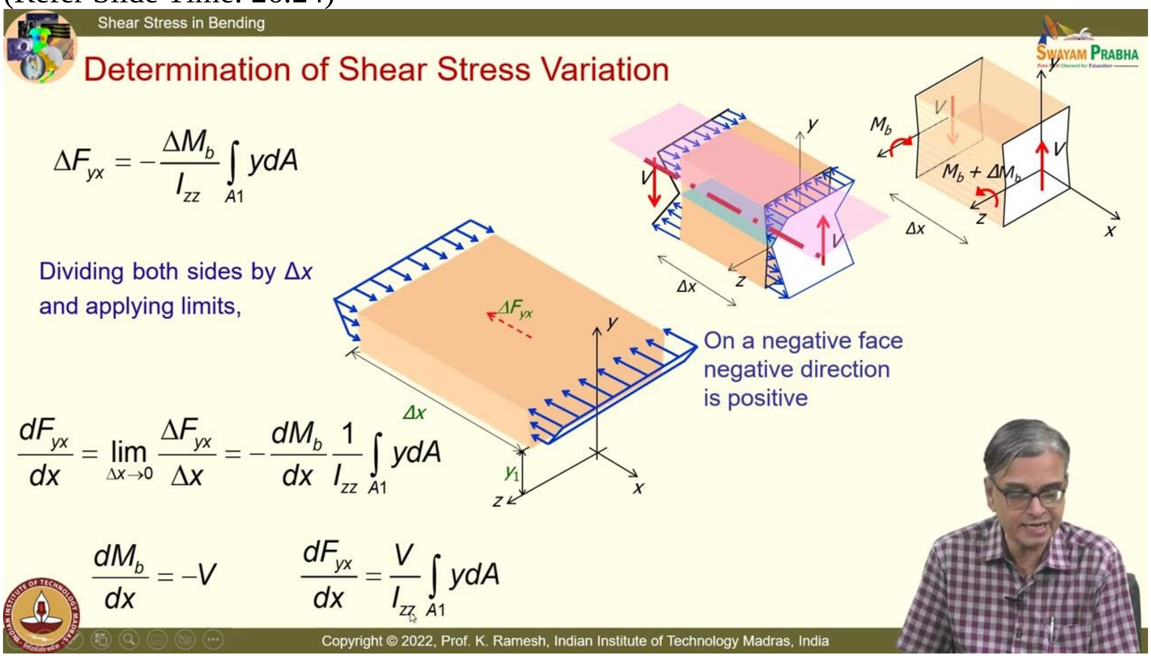

So, that is what we are going to see in the next step. And you divide this by Δx and you put the limits $\Delta x \to 0$. So, I get this as

$$\frac{dF_{yx}}{dx} = \lim_{\Delta x \to 0} \frac{\Delta F_{yx}}{\Delta x} = -\frac{dM_b}{dx} \frac{1}{I_{zz}} \int_{A1} ydA$$

See, I have been able to take out this because I consider the beam has a constant cross-section made of one material. That is also I am conscious about it. And ydA you can easily calculate and we will also replace this by Q. Can you tell me how I can bring in the shear force here? Is there anything that you know from your earlier development of the subject? You can be louder. Equality of cross-shears? Equality of cross-shears, we have discussed to say that this is; equality of cross-shear is one idea because what she is trying to say is if I have stresses developed in the bottom surface, I should have stresses developed in this surface also. So, I essentially have, if I have this as x-plane, I will have τ_{xy} developed and I have equal to cross-shears, whatever the τ_{xy} developed, I will have τ_{yx}. That is fine. But now, I do not want to have this expression having bending moment, I want this expression to have my shear force.

You know! I have discussed it in the last class. I have shown you what is the interrelationship between the load applied and the shear force and the bending moment applied and the shear force. So, if I invoke that condition, I will have this as

$$\frac{dM_b}{dx} = -V$$

and then I can write

$$\frac{dF_{yx}}{dx} = \frac{V}{I_{zz}} \int_{A1} ydA$$

See, shear stress distribution in beam is subtle to understand and it also requires all that we have developed as part of the course.

You cannot directly jump and then see how. Maybe you can memorize the final expression. That is not going to add to your understanding and knowledge. Memory is secondary. You have to appreciate how the subject is developed.

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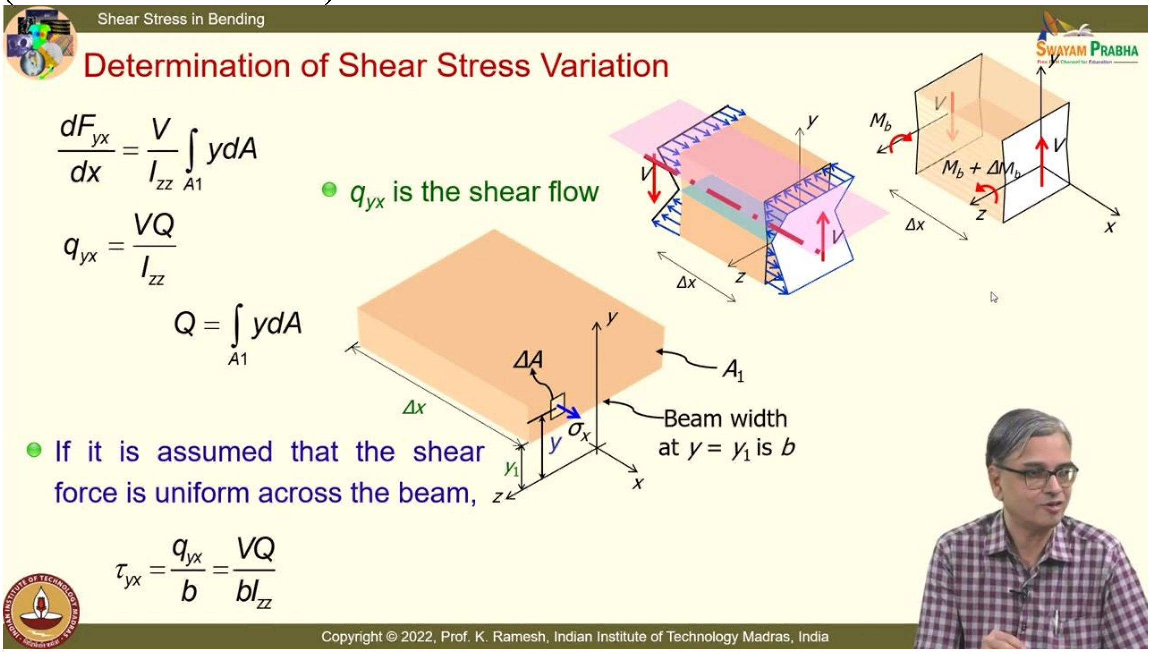

So, I have this small area shown and then I will also write the integral properly. I have this at a distance y and this is area A_1 and we will also see

$$\frac{dF_{yx}}{dx} = \frac{V}{I_{zz}} \int_{A1} ydA$$

ydA is the first moment of area and usually it is given as symbol Q. Q is taken as $\int_{A_1} ydA$.

You do it about the area A_1 and you also have coined another terminology F_{yx} is a force. Force per unit length, you call this as shear flow. Why you call it as shear flow? We make an analogy to fluid flow.

That we would see, when I take an *I*-section and then see how the shear is transmitted. We will say two flows come join and then go like this and separate out. So, this is force per unit length and suppose I know what is the width of the beam at this cross-section. Suppose I take this as b, I can also find out the shear stress. And we also assume that shear force is uniform across the beam. See, all that you can do as part of your development, but final results that we have got should conform to an exact analysis like what you do in theory of elasticity or a carefully done experiment.

It should satisfy both of this. Then, our analysis is correct. So, I have

$$\tau_{yx} = \frac{q_{yx}}{b}$$

that gives me a very important relationship that is equal to

$$\tau_{yx} = \frac{q_{yx}}{b} = \frac{VQ}{bI_{zz}}$$

And you should understand what is the symbolism Q? Q is $\int_{A_1} ydA$. Suppose I want to find out what is the variation of shear stress over the depth of the cross-section, I must find out for a generic cross-section, write the expression and integrate it for the entire this one.

For each cross-section, I should find out. Each cross-section, I should find out what is Q. Suppose I am able to have a simple cross-section, then if I write for a generic cross-section, that itself can be extended to give you the idea for what happens in the complete cross-section. So, shear stress evaluation is very subtle and you have to keep calculating Q for every cross-section you look at. Here we have taken a cross-section which is symmetrical about the x-y plane. The edge is not straight. I have taken a small deviation in the edge. Just to give an idea, we are only insisting on symmetry along a plane.

The shape is immaterial in our theoretical development. But when we do the problems, we will take that as a rectangular cross-section or an I cross-section or a circular cross-section for which you can easily calculate the mathematics.

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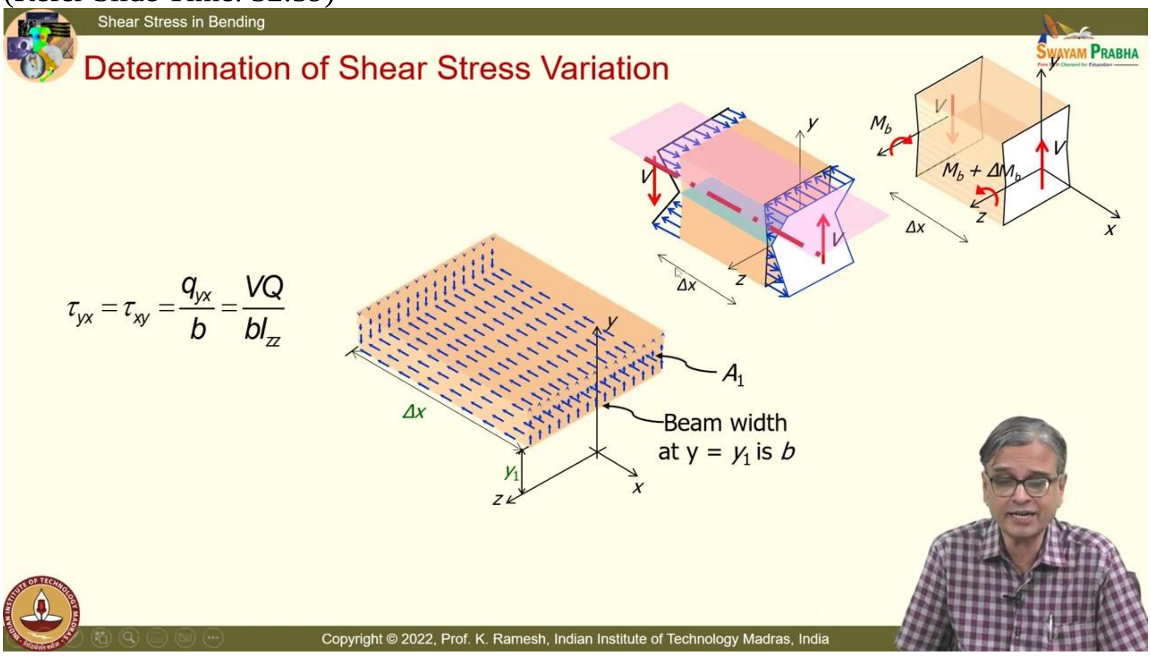

And we will also illustrate what way the flow happens. And here you understand and bring in, on the top surface, can I have any shear stress? Even though this cross-section transmits shear force, we have said the shear force distribution should be such, it should go to zero. Can you tell me whether it goes to zero on the basis of this expression? Can anybody see, how it goes to zero on the top layer? What happens to Q on the top layer? Is the question clear? See, I have taken y_1.

I will increase the y_1 to middle of the section. I will increase y_1 to the top of the section. When I put it at the top of the section, what is the value of *Q*? *Q* is zero. You get the point. I start from that.

So, there is no area to start with. The area builds up only when I have some sufficient thickness. So, what we conjectured based on free surface is also mathematically seen. So, our mathematical development is correct. Now, I put the shear flow from this. I show this as a bigger arrow because my *Q* is larger here. For the next one, from the illustration point of view, how do you expect the arrow head to be? Is it going to be smaller or bigger? Because when I show this, I can show it with few arrows as an illustration.

When I put the arrow of one size here, what would be the arrow that I should put next? Is it going to be smaller or bigger? Is the question clear? That means, I have found out what is the shear stress at this cross-section. Now, I have moved up. You can even say that I have moved up by this distance, the height of the arrow.

There *Q* becomes smaller. When *Q* becomes smaller, your shear stress also becomes smaller. So, I will have a small arrow and when I go to the top, it is almost close to zero. And what happens on the bottom surface? I will have a shear flow like this. Please make a sketch because these are all conceptual appreciation. See, conceptual appreciation is what is going to stay with you because tomorrow, if you take up a problem which is challenging your mind should think and simplify the problem. To aid your thinking process, I go slow, show what are all the interactions that happen and then appreciate that it satisfies all that what you have developed earlier. That gives the comfort that the logical step that we are taking is in the right direction. See, if I had taken the problem as a boundary value problem and solve the differential equation, follow all aspects of mathematics, that is good enough to see that you have got the solution. But surprisingly, even though I say theory of elasticity gives you exact solution, the exact solution available are practically for a very few problems. That is the reason why in strength of materials, we put lot of emphasis on physical appreciation and thought experiment, looking at the experiment, simplify the problem, so that you get the idea of how the system is responding.

So, the procedure what we have taken is not fully mathematical. We use mathematics in between and in order to verify that our steps are correct, we look at the experiment whether our statements are right, then we say the next step. Final solution, we compare it with theory of elasticity. Fortunately, it exists for a rectangular cross-section, fine.

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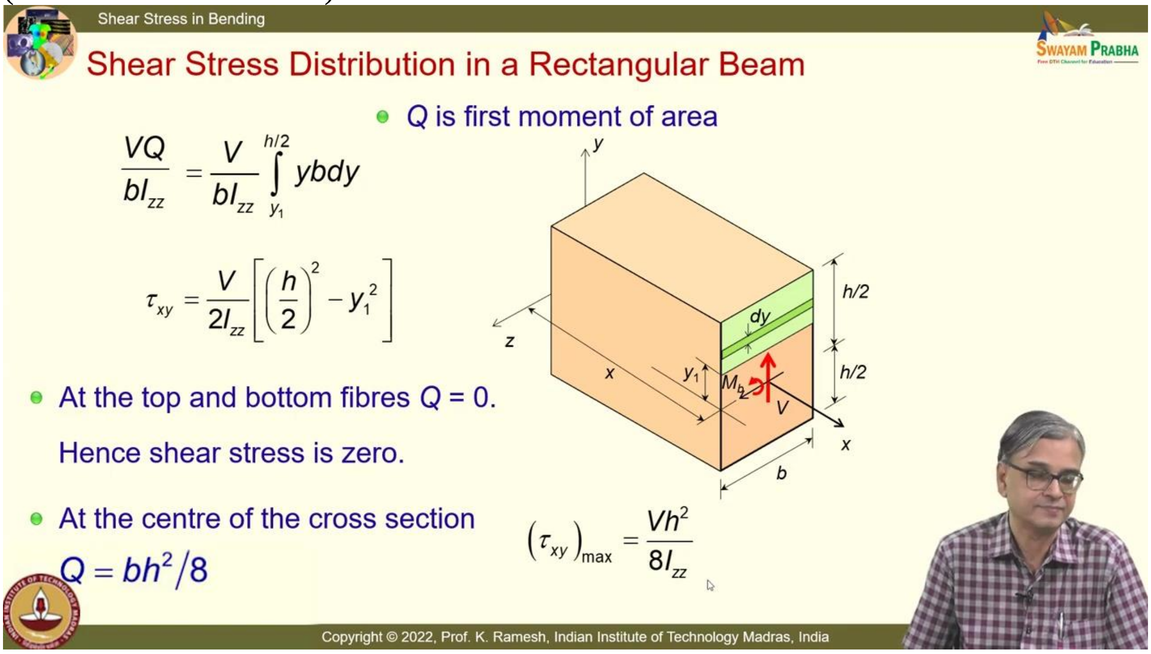

Now, let us do it for a rectangular cross-section, how the shear stress distribution is. Here, you can find out *Q*. You know, you can easily find out *Q* because it is a very simple cross-section. Please make a neat sketch. And if I have to find out *Q*, I will write it in a generic fashion because if the cross-section is not rectangle like this, then I should do the integration properly. So, I take a small strip. This strip has a height of *dy*.

So, I will write the expression for a generic cross-section

$$\frac{VQ}{bI_{zz}} = \frac{V}{bI_{zz}} \int_{y_1}^{h/2} ybdy$$

and I want to have the area. Area is given as bdy. So, this I have written it for a representative cross-section at y_1. Because the cross-section is simple, the same expression by substituting the limits appropriately, you can calculate *Q* easily. That is what I said earlier. If I have a simple cross-section, if you write a generic expression like this, this is enough to get *Q* at any cross-section of interest. And what is the cross-section of interest? The center of the cross-section, centroidal section is important because centroidal section is also a neutral surface.

And I have made an argument when we look at the cantilever that you may have the shear stress reaching maximum at the centroidal plane. So, I have this as

$$\tau_{xy} = \frac{V}{2I_{zz}} \left[\left(\frac{h}{2} \right)^2 - y_1^2 \right]$$

and I have to put the limits appropriately.

I have gone from y_1 to $h/2$. At the top and bottom fibers, Q equal to zero. So, shear stress is zero. Now, what happens at the center of the cross-section? Q becomes

$$Q = bh^2/8$$

because y is zero. And you get the

$$\left(\tau_{xy}\right)_{\max} = \frac{Vh^2}{8I_{zz}}$$

And what is the nature of variation? See, in the case of uniaxial tension, the stress transmitted was uniform across the cross-section. In the case of twist, we found the shear stress was varying linearly. In the case of bending, the bending stress was varying linearly. There is a difference. This is a function of? It is a parabolic function. You get the idea.

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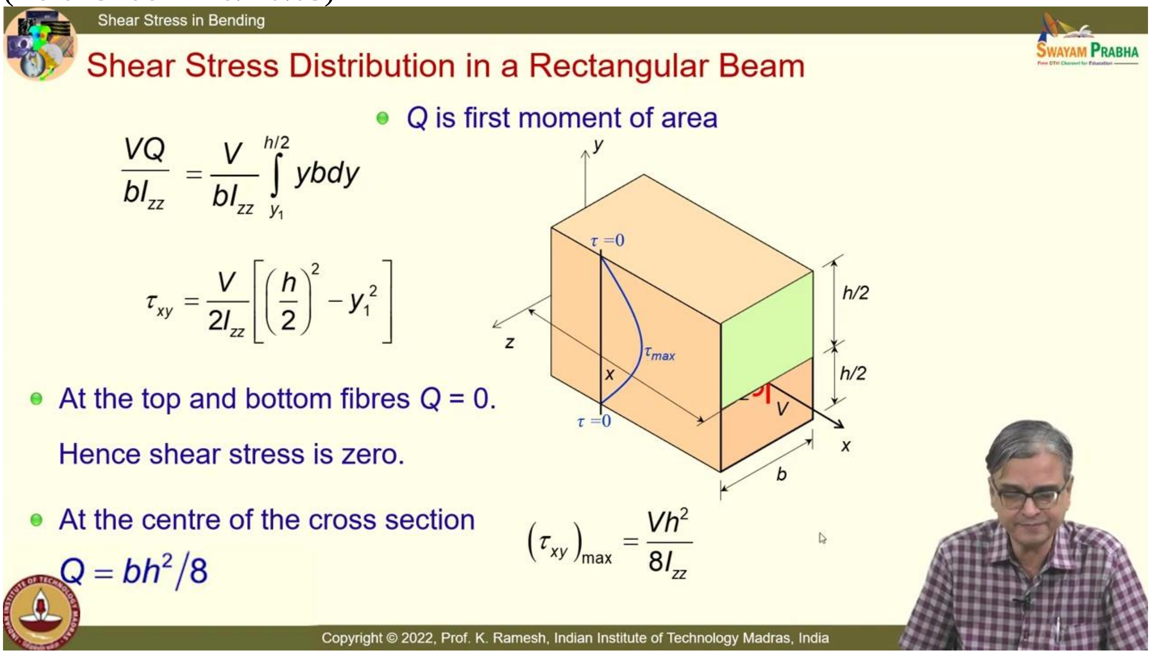

So, it is non-linear. And when I do it for the center, that is what is shown. And I have this as a parabola. So, this is non-linear and on the top surface, it is zero and the bottom surface, it is zero and reaches the maximum at the centroidal plane, which we have also seen long time back when we discussed the free surface.

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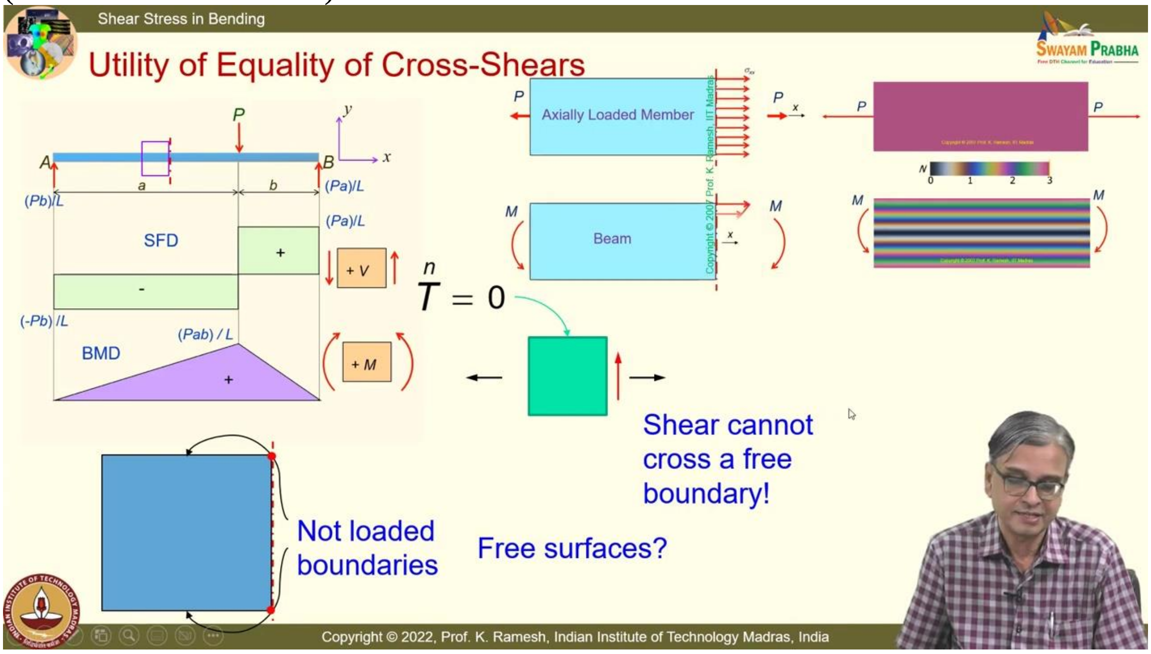

I took a generic three-point bending. And there, the argument what we put forth was, I take a section and these are not loaded boundaries. And I said, I can find out what happens at this top point and bottom point. And our logic was, it was a free surface, there is no shear acting on that. And we said that this shear force is not acting because it is a free surface. So, all the other shear stresses go to zero. And we also raised a question, can it support the horizontal force in this plane? All that is happening in the cantilever beam as well as these three-point bent beam. We have seen, shear stress is zero at the top point and at the bottom point. And at the top point and the bottom point, you have the maximum normal stress. So, this conforms to that understanding.

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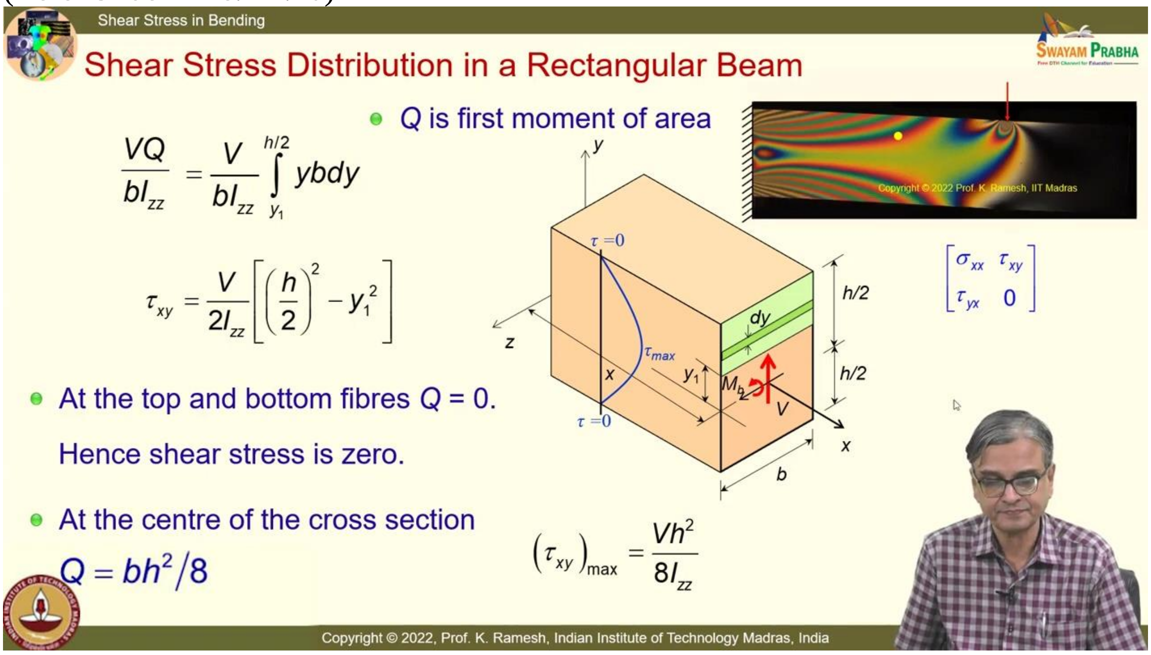

And I have also said, I have a cantilever beam and I asked you to observe what happens to the centroidal axis, whether it is remaining in black color? It was not black color; it has acquired a color. See, experiment, whether the value is small or big, it is truth. Right now, what we have discussed is, what is the variation of shear stress? We have not commented about what is the magnitude of shear stress in comparison to bending stress.

The focus is only on the variation. And I have asked you to look at two aspects. One is the centroidal axis is having a color and second one is, this fringe is not a straight fringe, but it is having a curvature. Can you tell me what is the state of stress at a point like this? Because the beam is transmitting both shear force as well as bending moment, what is the state of stress at this point? Can you tell me? You understand? What are the stress components? See, flexure formula gives you bending stress. And then, we have evaluated even shear stress.

I have taken a generic point here, fine. It is something like some point in the; it is below the surface. See, you are looking the side view. If you look at from the cross-sectional point of view, it is some point just below like this.

You can even be taken as y_1. So, I will have shear stress here. I will also have bending stress. So, if I put that stress tensor is

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & 0 \end{bmatrix}$$

Is the idea clear? And what you have here is, your σ_{xx} is varying linearly. And your bending moment also in a cantilever beam varies linearly from this loading point to the fixed end. But I have this rather than being a straight line, it is curved. Where does that curvature come from? You know photoelasticity plots only $\sigma_1 - \sigma_2$ and you are putting τ_{xy}; τ_{xy} is non-linear.

So, your variation of $\sigma_1 - \sigma_2$ is not linear over the length of the beam for a horizontal line. So, you have this as a curvature.

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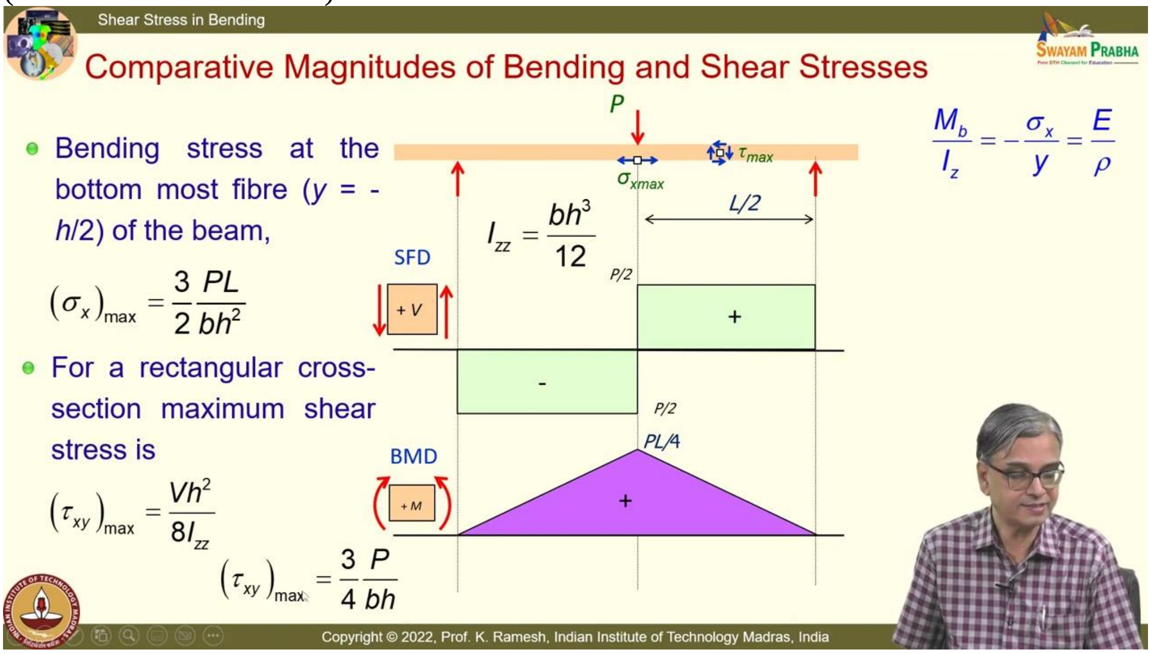

Now, let us look at relative magnitudes of bending and shear stress. You know, you have a rectangular cross-section and you have shear force and bending moment diagram. And where do you have the maximum bending stress? You have the flexural formula here.

I have the maximum bending moment is $PL/4$ that happens here. And I can have this as a maximum tensile stress. You can easily calculate the I for this is $bh^3/12$. So, if I do this, I get the

$$(\sigma_x)_{max} = \frac{3}{2}\frac{PL}{bh^2}$$

And you also know the expression for τ_{xy} for a rectangular cross-section, the maximum shear stress is

$$(\tau_{xy})_{max} = \frac{Vh^2}{8I_{zz}}$$

When I substitute I_{zz}, I get this as

$$(\tau_{xy})_{max} = \frac{3}{4}\frac{P}{bh}$$

So, this gives a comparison. That comparison is listed in the next slide.

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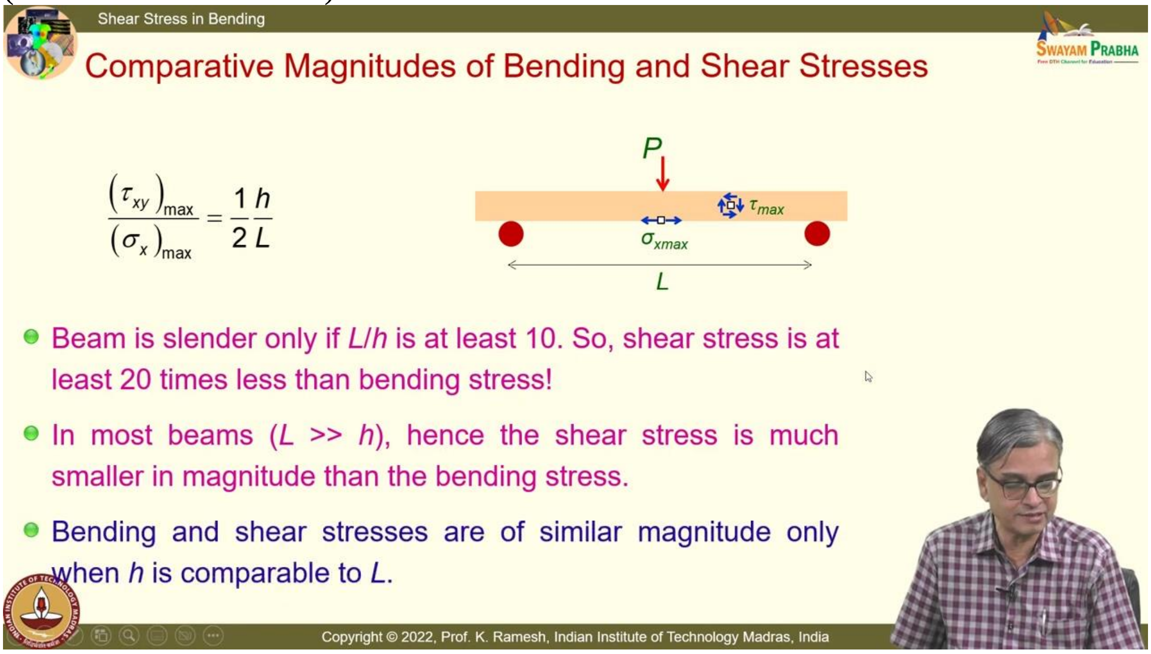

And the ratio is $h/_{2L}$. And you know, we want to have a slender beam. The depth should be much smaller to the length of the cross-section.

Normally, you have this as 10. So, shear stress is at least 20 times smaller than the bending stress. Isn't it surprising? You know, many books what they do is, they give you the picture like this.

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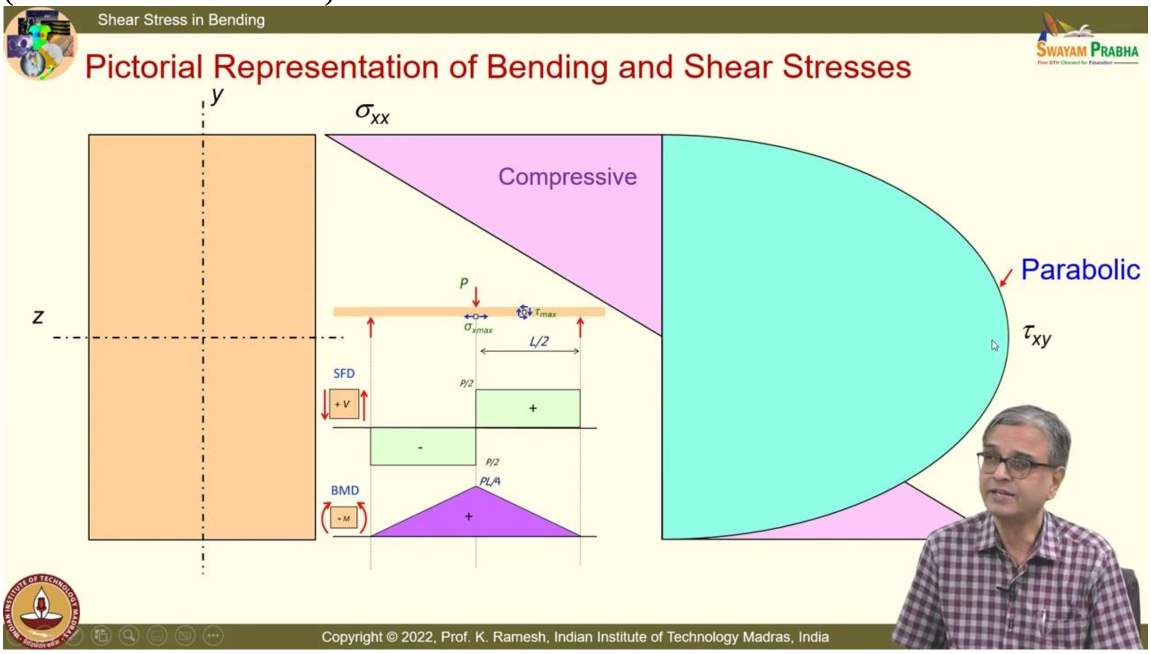

They do not go and emphasize, what are the relative magnitudes. I have drawn these stresses in a very big picture for me to illustrate the point. So, this is for the case of a rectangular cross-section subjected to three-point bending.

That means, it has a constant shear force and a variable bending moment. And suppose, I plot to some scale the stresses, the way the beam is bent, I have the top surface is compressive, bottom surface is tensile. Is the idea clear? And many books simply put the shear stress variation. The focus is only on showing that this is a parabola, fine.

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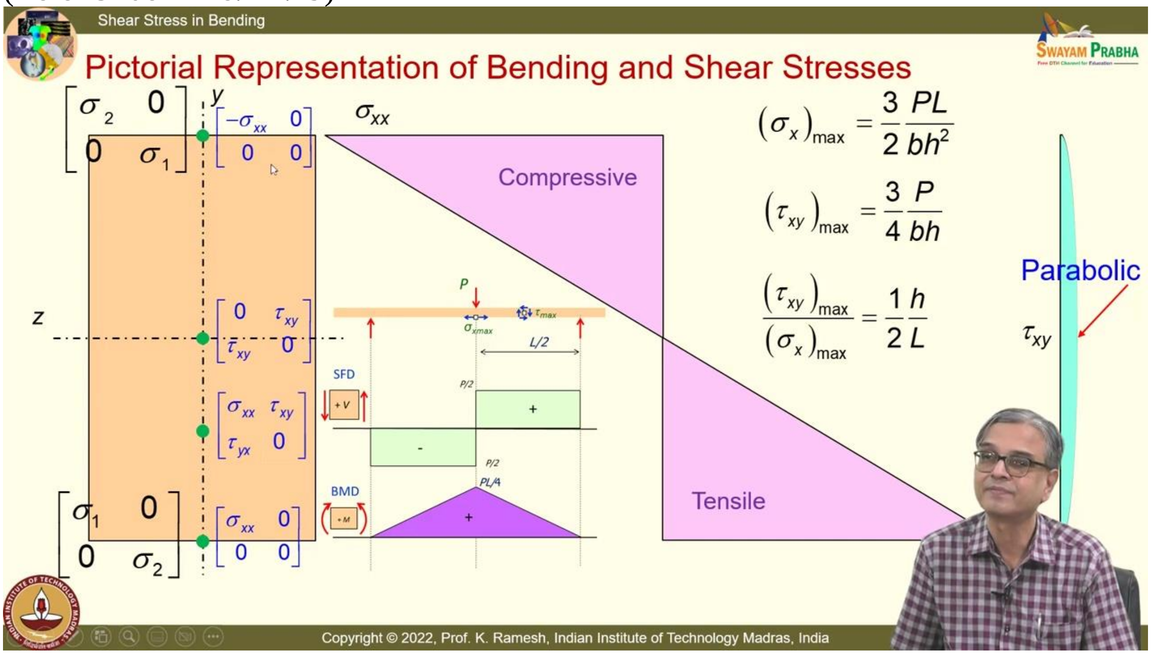

And you get duped that shear stress magnitude is comparable to tensile stress magnitude. That is what you carry. When you look at a picture, the same thing I discussed when you are looking at stress strain curve in the tension test. I focus that you have to give importance to the level of strain. On similar lines, suppose I plot to scale, what is the shear stress, hardly anything.

Look at this and look at this. In many of our studies, we will consider shear stress magnitude small, we will ignore it. Even in your deflection, we will ignore in many of the problems. There is justification. When it is important, you have to worry about it. That we will come to it because we have still not discussed what happens near the load application point.

And there are also many things that you can learn from this, fine. I want you to make a neat sketch of this. This is a very, very important slide. It gives you lot of conceptual understanding. Can you tell me what is the stress tensor here? I want you to develop. See by flexure formula, you get only component of stress.

You should know how to write this as a stress tensor;

$$\sigma_{ij} = \begin{bmatrix} -\sigma_{xx} & 0 \\ 0 & 0 \end{bmatrix}$$

And can you tell me what is the stress tensor here? It is tensile. So, it is

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & 0 \\ 0 & 0 \end{bmatrix}$$

What is happening at the center?

$$\sigma_{ij} = \begin{bmatrix} 0 & \tau_{xy} \\ \tau_{xy} & 0 \end{bmatrix}$$

What is happening at in between point? It is

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & 0 \end{bmatrix}$$

In terms of principal stresses, I will have this as σ_1, σ_2 for this point, no problem, because this is done algebraically larger and one is labeled as σ_1.

Algebraically smaller is put as σ_2. Suppose, I come to the top surface, can I write like this? I cannot write. This becomes σ_2 and this becomes σ_1 because this is compressive. So, it is a very, very important conceptual slide. So, in this class, you know, we have looked at how the shear stress is developed. It is a very, very important and subtle concept. We have looked at an actual experiment and understood if I have multiple layers of the beam, we could identify there is slipping happening that indicates what is the shear stress possibility.

Then, we have also learned how to calculate the shear stress. We have found that it is zero on the top and bottom surfaces, reaches a maximum at the centroidal plane or at the neutral surface, whichever way we have to look at for problem to problem. Thank you.