**Strength of Materials**

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**Lecture - 31**

**Deflection 1 Moment-Curvature and Load Deflection**

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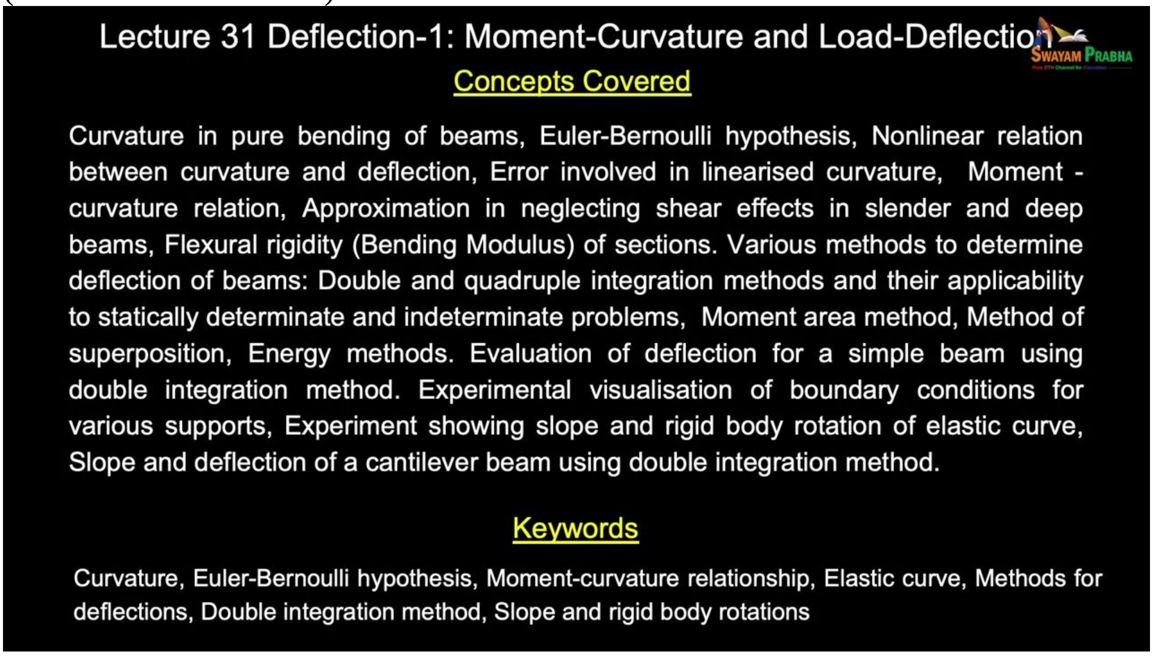


Let us move on to the new chapter on deflection of beams. See, while we studied torsion, we studied torsional stress as well as the twist of the shaft simultaneously. Those concepts were simple, so we could do it in the same chapter. In fact, bending we have done it in two instances; one bending stress where we have looked at the axial stress, then we have also discussed the shear stress and as a separate chapter, we discuss about the deflection. And deflection is fairly simple, provided you understand and recall what you have learnt as idealization of supports in your earlier course. And I have always emphasized experimental knowledge and you should be able to visualize for the given supports, how a beam can bend? That would be the focus of the lecture also today.

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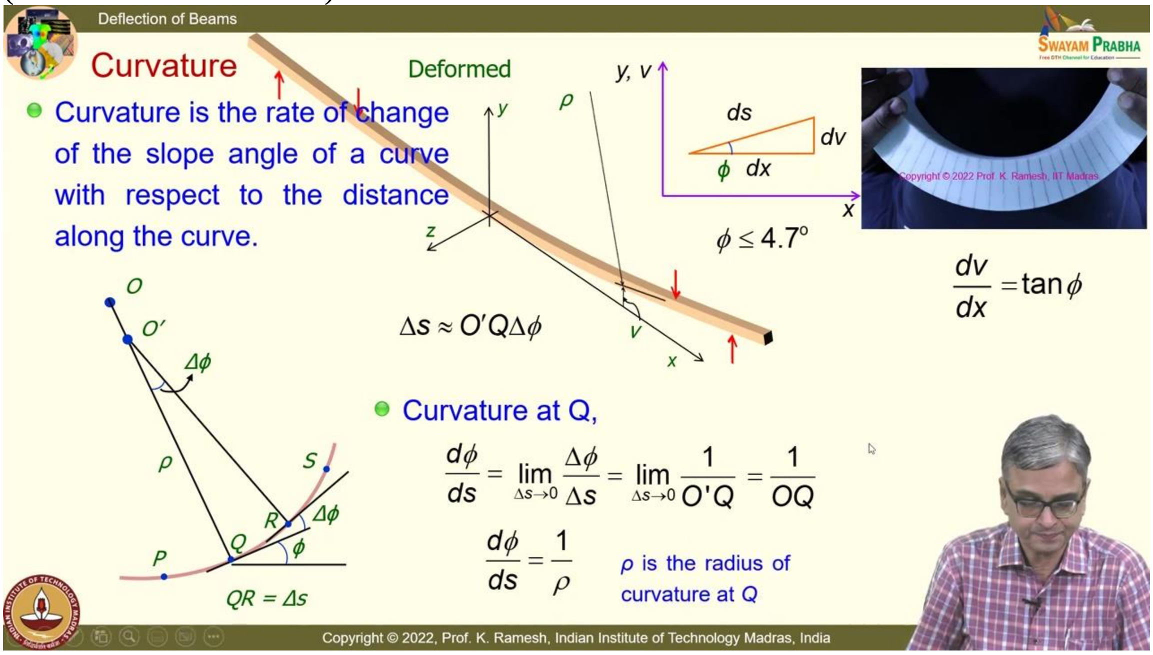


And you know, it is closely related to curvature, that is how we relate the deflection to the bending moment and you have the beam bent. And like we discussed in bending stress, here again all the equations are developed for pure bending. Simply use those equations to any other nature of loading on the beam, look at the bending moment, take out the bending moment value and use it in your equations. So, it is an approximation; we do only engineering analysis.

And once you have the beam subjected to four-point bending, this section experiences pure bending. And you know, you have developed the notion of curvature, you have the radius of curvature rho indicated and the vertical displacement is also labeled as $v$, because we have used $u$ as horizontal displacement, $v$ as the vertical displacement and $u$ and $v$ are functions of $x$ and $y$. And if you draw the tangent at this point of the curve, I can also find out what is the slope angle. I am plotting $x$ versus $y$ as well as $v$. So, I have the line segment $ds$, I have this angle as $\phi$ and this vertical distance as $dv$.

And it is better that we go back and understand, what is curvature? And curvature is nothing but rate of change of the slope angle of a curve with respect to the distance along the curve. So, I take a point $Q$ which is oriented at angle $\phi$ and I take a neighboring point at a very short distance away, I label that as $R$. And if I draw the slope, the difference in the slope is $\Delta\phi$, even the notation is like this. It is actually $\phi+\Delta\phi$ from the horizontal; from the previous one, it is separated by $d\phi$. These are all shown exaggerated for you to easily visualize.

And it is also said, whatever the discussion that we are going to do is valid if the angle $\phi$ is less than 4.7 degrees. You should not lose track, but whatever the deformation I show, they are all large deformation for you to comprehend the nature of deformation clearly. And you can take $QR$ as $ds$, $\Delta s$ and what is curvature at $Q$? This is defined as

$$\lim_{\Delta s \to 0} \frac{\Delta \phi}{\Delta s}$$

which you can also label it as $d\phi/ds$. And we have an understanding what is $\Delta s$? $\Delta s$ is *QR*, from this diagram you can say it is $O'Q\Delta\phi$.

So, I can write this as

$$\lim_{\Delta s \to 0} \frac{1}{O'Q}$$

Because we are talking about $\Delta s$ to zero in the limit, I can also write this as

$$\frac{1}{OQ}$$

And we get the relationship:

$$\frac{d\phi}{ds} = \frac{1}{\rho}$$

Now what we will have to do is, how to relate this $d\phi/ds$ to the deviation *dv* or displacement *v*. Once you establish that, then we will know by looking at the bending of the beam, how to calculate the deflection.

The deflection is going to be represented as the displacement *v*. And mind you, even though we take a three-dimensional beam for demonstration, mathematically we are only looking at the neutral axis. We are only looking at plane of symmetry. We do not recognize that the beam has thickness, we neglect Poisson effect. In fact, we looked at what happens when there is a Poisson effect, what way the beam bends.

We have looked at it in the previous chapter. We said, when we look at small deformation, all of this do not happen. It was only an extremum condition. And $\rho$ is the radius of curvature at *Q*.

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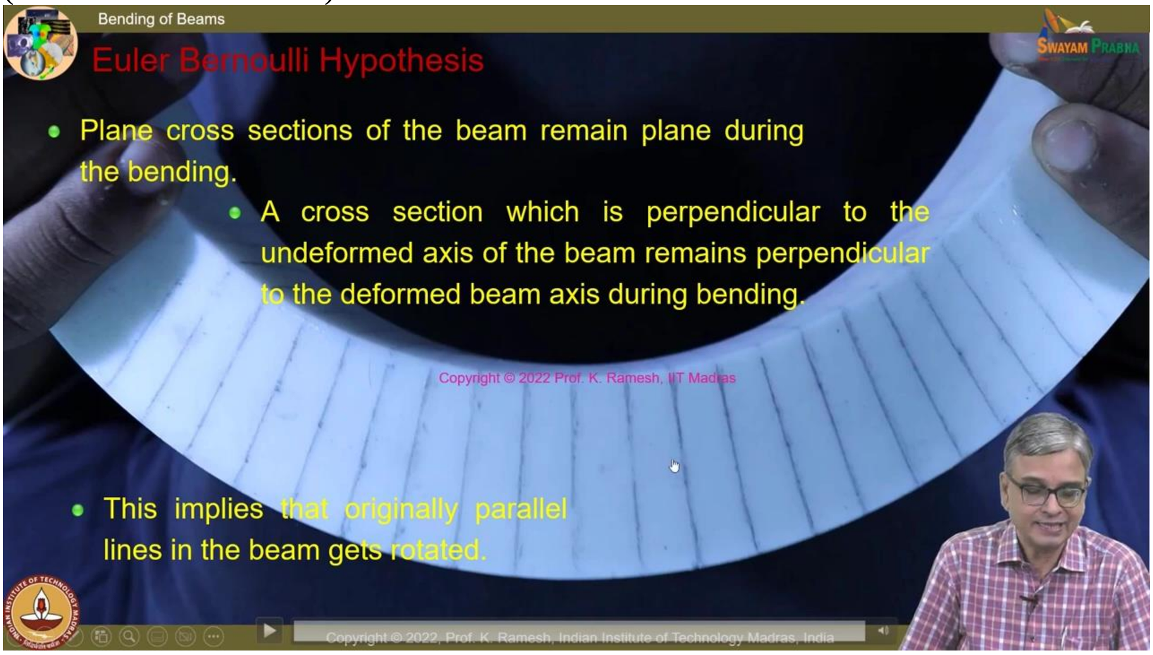


And we go back to Euler Bernoulli hypothesis, just to recapitulate. And I have the beam is bent, I have these lines. So, this line is straight. If you look at these lines, which are drawn here, they are getting rotated. And can you also see visually, that as I go along the length of the beam, the rotation is also becoming larger and larger. And one of the idealizations of Euler and Bernoulli is, plane cross-sections of the beam remain plane during the bending.

In contrast to torsion, we started with the right hypothesis in bending. In torsion, we started with the wrong hypothesis and by experimentation, we proved that hypothesis is wrong. Here, we start with the right hypothesis and the experiment supports the plane section; these lines remain straight, they do not get distorted. A cross-section, which is perpendicular to the undeformed axis of the beam, remains perpendicular to the deformed axis during bending. You know, after relating the curvature to the deflection, we will go back and see by a sketch, the meaning of this statement.

And whatever we have stated here, it implies that originally parallel lines in the beam gets rotated, which is very visible. We have drawn the line; we have the advantage of a soft material. So, we could bend and see for ourselves that these lines get rotated.

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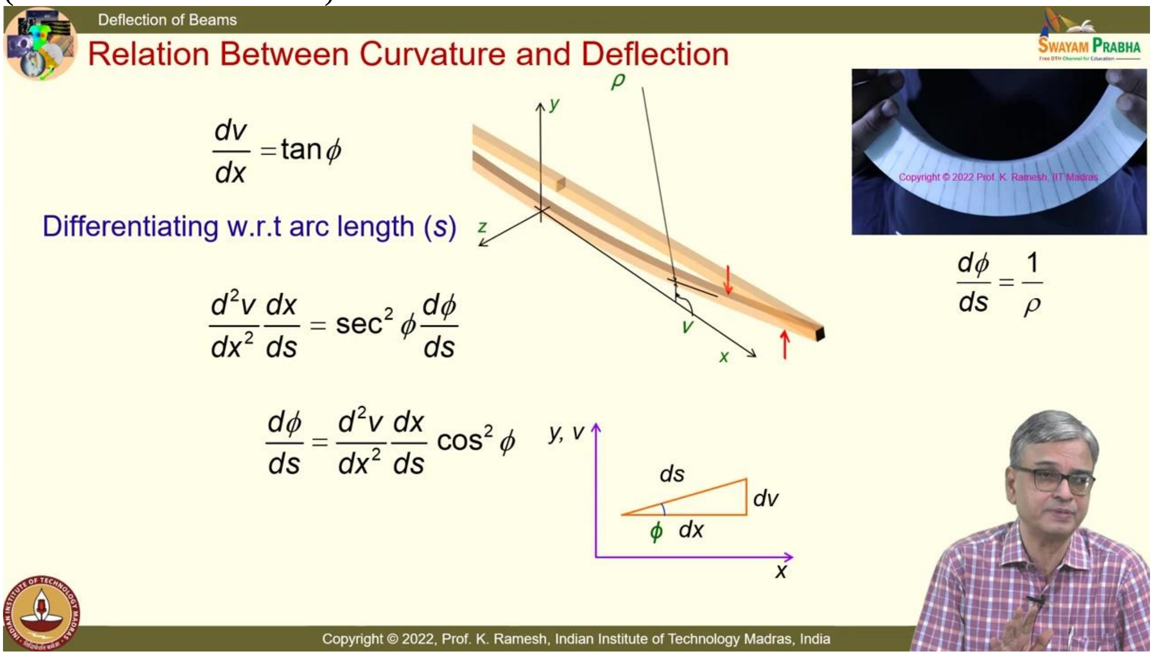


And we have already seen $\frac{dv}{dx} = \tan\phi$; that you get from this diagram. Now, we will have to relate, how to get the value of deflection $v$? We will differentiate this expression with $d/ds$.

So, when I have to differentiate with respect to $d/ds$, I will have to differentiate with respect to $d/dx$ multiplied by $dx/ds$. And $\tan\phi$, you can find out what is the value, it is nothing but $\sec^2\phi$. So, I get this as

$$\frac{d^2v}{dx^2}\frac{dx}{ds} = \sec^2\phi\frac{d\phi}{ds}$$

And let me do the rearrangement of this expression, our interest is to get $d\phi/ds$. So,

$$\frac{d\phi}{ds} = \frac{d^2v}{dx^2}\frac{dx}{ds}\cos^2\phi$$

Can you replace cos squared phi from the expression? When you look at $ds,\ dx,\ dv$, I can also find out $\cos\phi$. $\cos\phi = dx/ds$. I am going to use this for me to replace. And these are all very very simple mathematics. In this chapter, though you learn many methods, the basics are very very simple. There is no great mathematics involved. Very simple exposure which you already have is going to be utilized.

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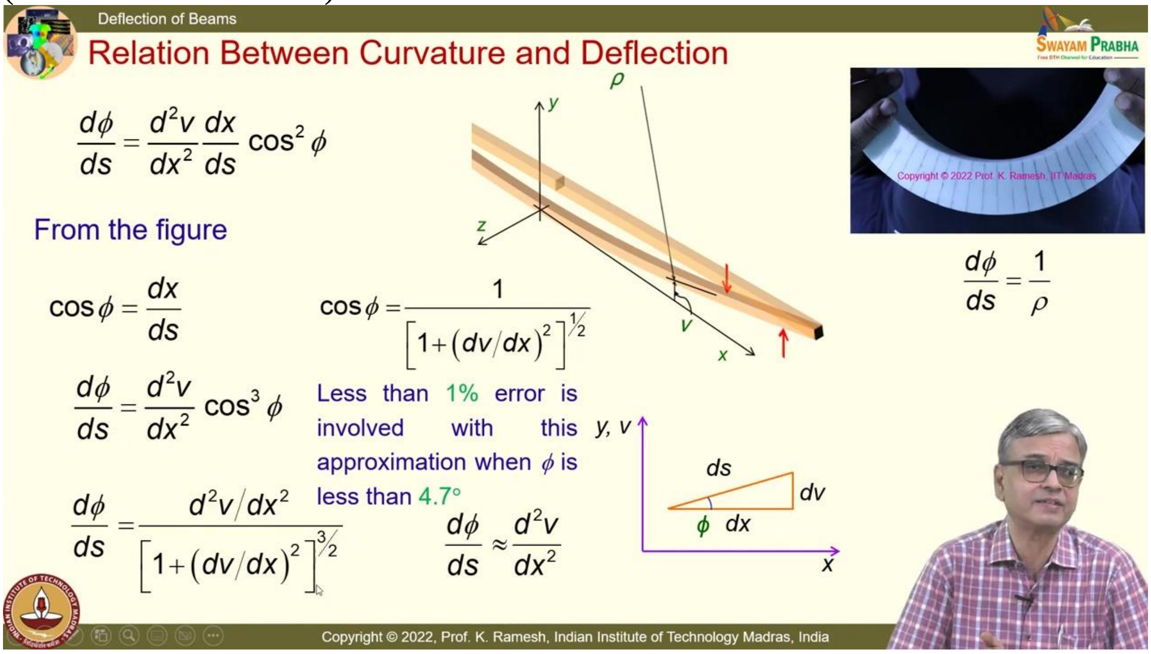


And from the figure, I can write what is $\cos\phi$. So, $\cos\phi = dx/ds$. And so, I can also say that $\frac{dx}{ds}\cos^2\phi$ becomes $\cos^3\phi$.

So, I have

$$\frac{d\phi}{ds}=\frac{d^2v}{dx^2}\cos^3\phi$$

And I would like to replace $\cos\phi$ from the relationship.

$$\cos\phi=\frac{1}{\left[1+(dv/dx)^2\right]^{1/2}}$$

I suppose you are able to simplify and then convince yourself. I have just written what is $\cos\phi$. And once I substitute this, you know I have an expression where $d\phi/ds$ is given as

$$\frac{d\phi}{ds}=\frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{3/2}}$$

And this expression is exact, but it is non-linear. You know we have always been saying that we live in linearized elasticity. That is justifiable not because we want to force it to be linear, because our deformations in actual practice are extremely small.

You must recognize that. So, it is a very valid approximation. It simplifies your mathematics. So, I say $dv/dx$ is very small. So, $\left(dv/dx\right)^2$ is going to be a very very small quantity. So, I say that this is almost like 1; unity.

So, I can say

$$\frac{d\phi}{ds} \approx \frac{d^2v}{dx^2}$$

It is an approximation. It is a deliberate approximation. And we also know

$$\frac{d\phi}{ds} = \frac{1}{\rho}$$

So, I am going to relate what is $1/\rho$ to $d^2v/dx^2$. So, we also recognize that we are doing an approximation. And by doing this approximation, you make only less than 1 % error. It is very very small provided the slope angle is less than 4.7 degrees. See in actual practice, the angle $\phi$ would be less than 0.5 degrees. If you really solve some of the problems, it would not go anywhere near 5 degrees. And so, you will have to recognize that this is an approximation. And all of this you can easily derive. The mathematics involved is straightforward; simple trigonometry.

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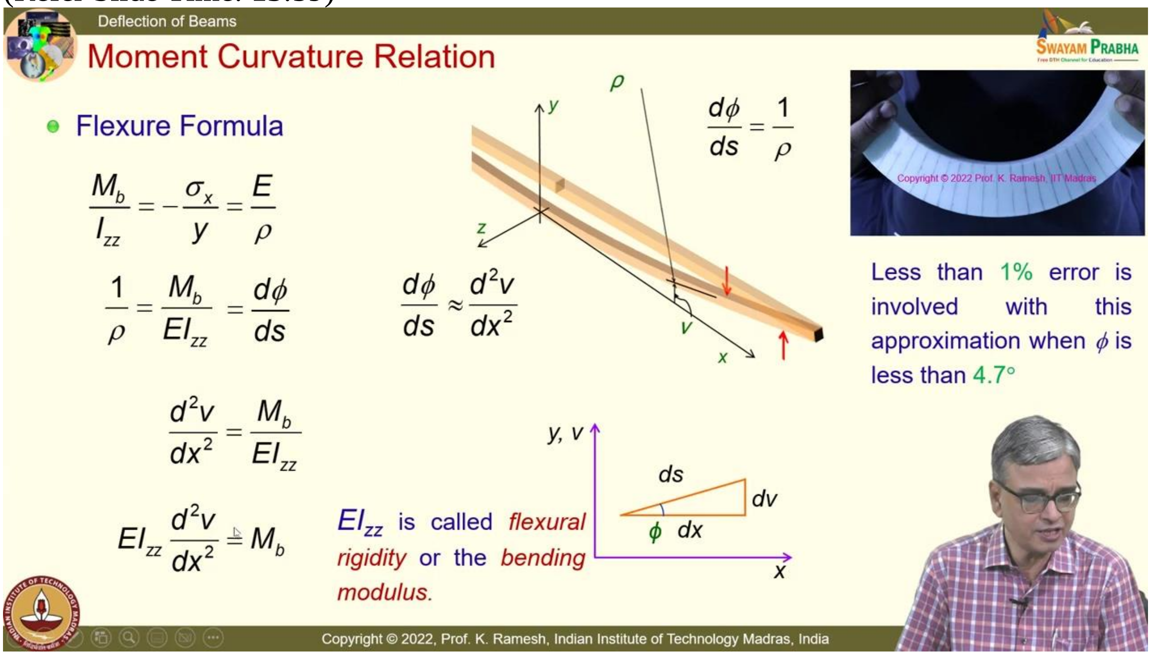


There is nothing great. And you have the flexure formula

$$\frac{M_b}{I_{zz}} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$

So, our interest is, how to relate the bending moment transmitted by the beam to the deflection? We have already seen

$$\frac{d\phi}{ds} \approx \frac{d^2v}{dx^2}$$

And we know

$$\frac{1}{\rho} = \frac{M_b}{EI_{zz}} = \frac{d\phi}{ds}$$

So, now I can establish a relationship between what is $d^2v/dx^2$ and the bending moment. And we have already noted this as flexural rigidity, product of $EI_{zz}$ or bending rigidity. And sometimes you know, since we use axis of the beam as *x*; perpendicular, this one is *z* and this is *y*. People also simply drop off the subscript *zz*, because it is well understood.

We are confining our attention to a class of problems. They may also simply say this as EI, easy to write. Every time when you want to discuss, it becomes easier to write.

And this is famously known as Moment Curvature Relation. Mind you, this is developed for pure bending. Is the idea clear? We develop it for pure bending, but we apply it for a situation where the bending moment is varying. When does the bending moment vary? The moment you have even a constant shear force, like in the case of a cantilever beam or a three-point bent beam, your bending moment varies. That is the simplest variation you can think of.

Even in those cases, I am going to directly use this expression for me to find out the deflection *v*. Do you see any further approximations inherently embedded in this expression? When we apply it for a beam transmitting shear force? Do you see? Have you not totally ignored the effect of shear, when I want to apply this expression to a beam transmitting shear force? Yes or no? Because this expression only worries about what happens because of the bending moment. We have never brought in, what is the effect of shear. We have already seen the shear stress magnitudes are much smaller than that of bending moment, and provided the beam is slender, you can safely ignore the shear effects. Shear effect becomes significant when the depth is larger and comparable to the length of the beam.

Then I have to consciously bring in the shear effect, which we need special methods to handle it. And this is a very famous expression moment curvature. So, when we apply it for practical problems, it is only an approximation, it ignores the effect of shear deformation of the beams. And we have already seen $EI_{zz}$ as bending rigidity or flexural rigidity or it can also be called as bending modulus. And if you have to calculate the deflection *v*, you have to integrate this equation and apply the boundary conditions, as simple as that.

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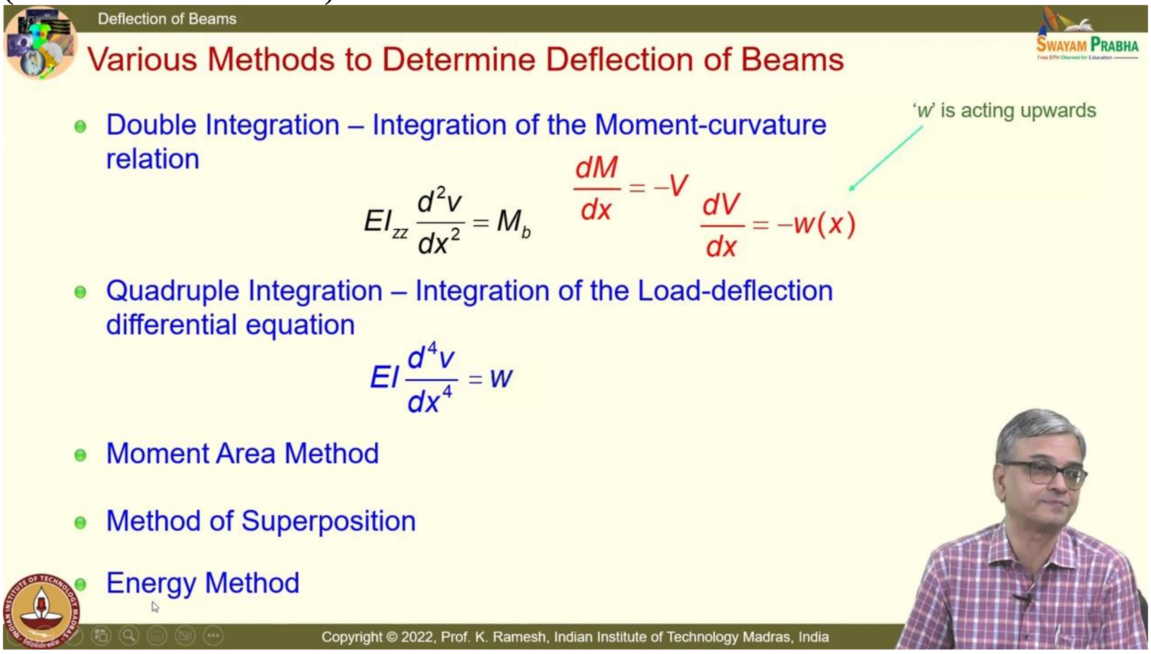


So, you have double integration, integration of the moment curvature relation. So, when you are given

$$EI_{zz}\frac{d^2v}{dx^2} = M_b$$

if you integrate this twice, not differentiate, but integrate this twice and satisfy the boundary conditions. When I integrate it twice, I will have two integration constants and these constants have to be evaluated based on the boundary conditions. And boundary conditions physically you will have something, but mathematically you will idealize it as supports. And these supports are developed in your course on rigid body mechanics.

So, once we understand what happens at these supports, what possibilities exist for the beam to have a deformed shape, the problem is extremely simple to handle. So, you have to have that visualization, what happens at the supports. You have a fixed support, you have a simply supported support, you have a free end, what happens? And we have also looked at certain other interrelationships earlier. We have looked at

$$\frac{dM}{dx} = -V$$

We have also looked at

$\frac{dV}{dx} = -w(x)$, where $w$ is the load that is applied on the beam.

So, when we use these interrelationships, I can also construct another set of equation, which is a load deflection differential equation.

$$EI\frac{d^4v}{dx^4} = w$$

See, I have taken *EI* out considering that I have a homogeneous material, I do not have different materials and *EI* can be taken out. Otherwise, I should have had

$$\frac{d^2}{dx^2}\left(EI_{zz}\frac{d^2v}{dx^2}\right)$$

I cannot take $EI_{zz}$ out. If I have a homogeneous beam, if I have a constant value of moment of inertia, I can take this out. And the advantage here is, I deal directly with the load. It has an advantage. See, if you look at this expression, when you want to do double integration, I need to calculate the bending moment. So far, you have handled only statically determinate problems.

I have also shown what are statically indeterminate problems, wherein you have to bring in what happens to the deflection. Only you bring in the deformation, I get additional information for me to solve. On the other hand, if the load applied on the beam can be expressed as a continuous function of *x*, this is applicable for even statically indeterminate problems. So, this is primarily applicable for statically determinate problems. This is applicable for both statically determinate and indeterminate problems.

And you also have; when you do this integration, you have done the integration mathematically. I can also do the integration graphically. See, in the early development of engineering or any physical sciences, people relied more on graphical methods, because they had no access to computers. So, in order to understand, appreciate interrelationships, they were very comfortable with graphical methods. And particularly, the civil engineers use this Moment Area Method.

And we have always been saying that we are going to use principle of superposition left and right in this course. We have used linearized elasticity and it is an isotropic material. And when you have the advantage, why not use method of superposition? And in this course also, we would emphasize more on method of superposition, rather than this double integration or moment area. The reason is, if you have to apply superposition, you should be able to visualize under the action of the given load, how the beam bends? That visualization is very, very important. As an engineer, it is not that you take an equation and simply blindly do the integration and get the boundary condition satisfied and you say this is the answer.

That is not the way the engineer should function. The engineer should feel where do you have maximum deflection, where do you have maximum slope. That knowledge is very important. So, that you gain more when you do this principle of superposition. And I have already said, when you have this expression, this expression has completely ignored, for a beam transmitting shear stress, the contribution to the deflection from the shear stress, which we could easily accommodate when you go on to the energy method. And we will also solve some other interesting problems when we go to the energy method.

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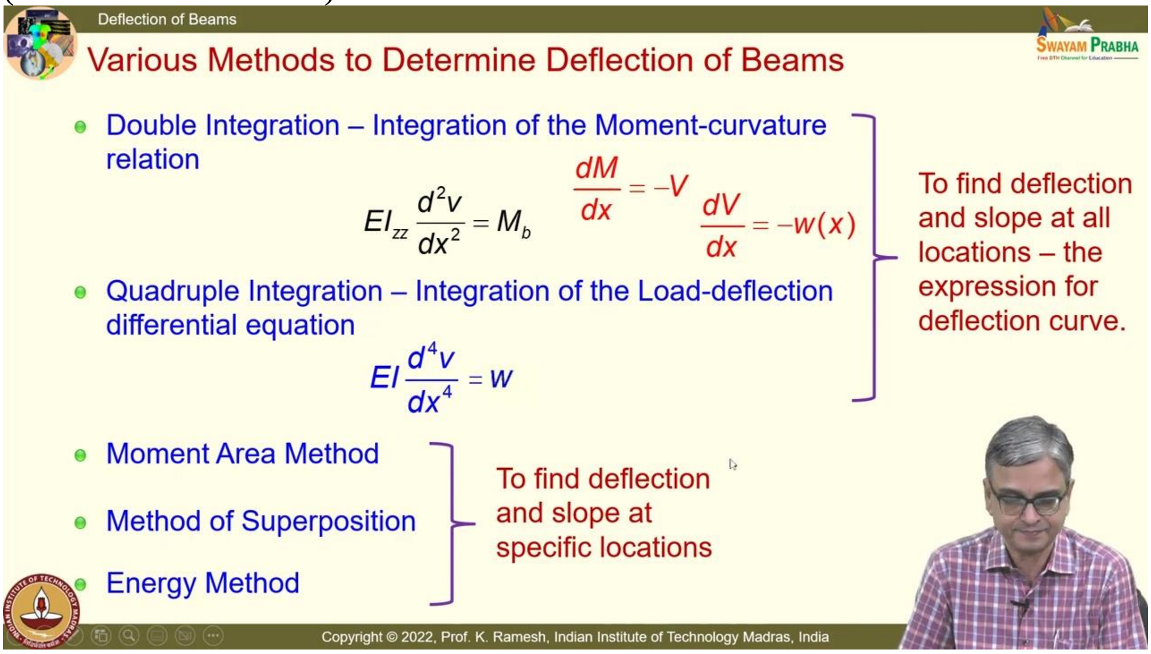


The advantage of this is, you can get the deflection and slope at all locations along the length of the beam. You get that as a mathematical expression. Substitute the position *x*, you get the value of slope, you get the value of deflection. In general, moment area method or method of superposition, energy method, you try to find out at certain specific locations, because when you are doing a design, these values became important at certain locations. So, you also need a quick method, you also need a complete method to get the data.

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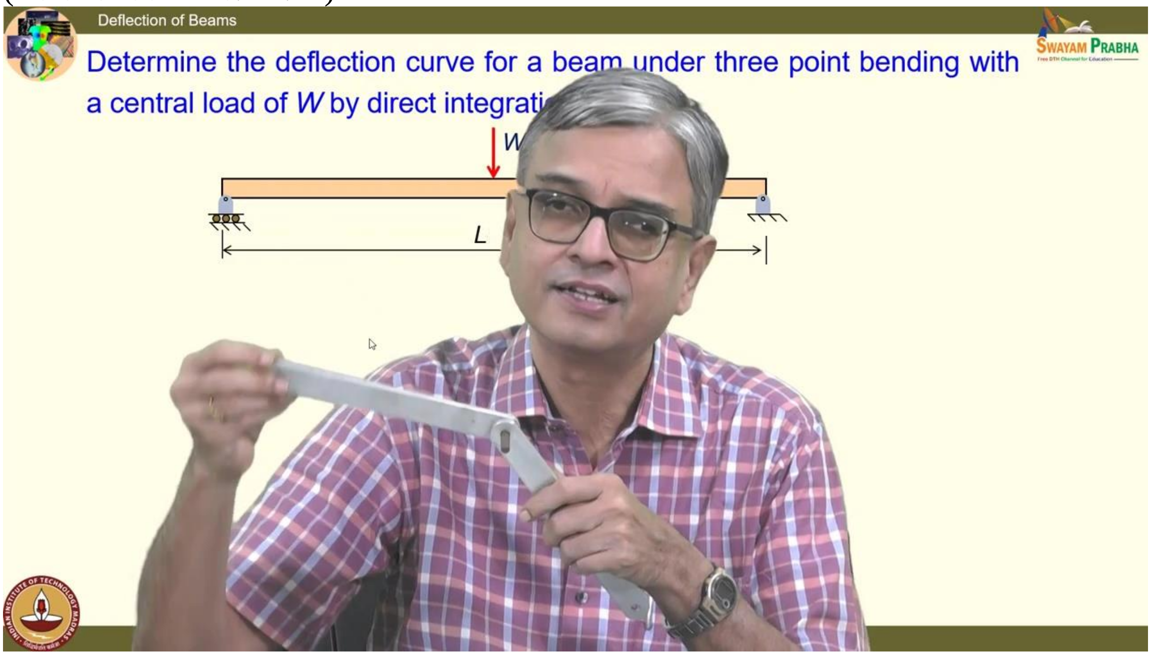

Now, let us take a very simple problem like this. The first question is, how does the beam can bend? What way can I visualize? I have this as a simply supported, this as simply supported and this as a roller support, because when there is a thermal change, you allow free expansion and contraction. And you know what is a pin joint. When you say simply supported, it is a pin joint. What does the pin joint do? It freely allows rotation.

Is the idea clear? The member is allowed to rotate freely. So, that you should visualize. So, what way it will happen? We will see.

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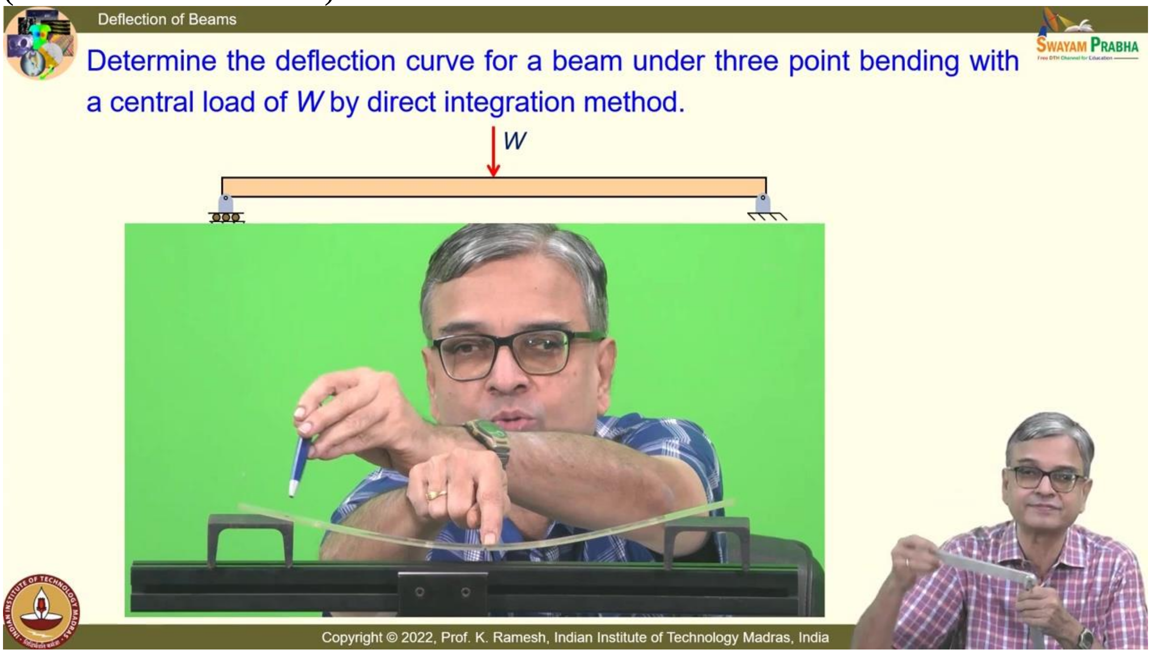


I have the beam supported on simply supported one and applying a concentrated load like this. Can you see how the beam is bent? What happens at the supports? Please make a neat sketch.

You need to visualize this. I said a pin joint can allow a free rotation. So, look at what happens at this end. What happens at this end? This has acquired a slope. Is the idea clear? And you also find, because the loading is symmetric, you find there is a beautiful symmetric curve. Can you write anything mathematically at this point? Because you are going to write certain boundary conditions.

You should know what is the boundary condition. We are going to discuss about what will happen to the deflection *v*. You can say that this rests on the support, so, the deflection is zero here. The deflection is zero here. But because these supports are pin jointed, freely rotate, you can have slope.

Slope is non-zero. Slope can exist. What can you say at the load application point? Slope is zero, but I have deflection. If you understand this, if you look at the beam like this, this chapter is extremely simple. And you should visualize how the beam can bend. That is very very important.

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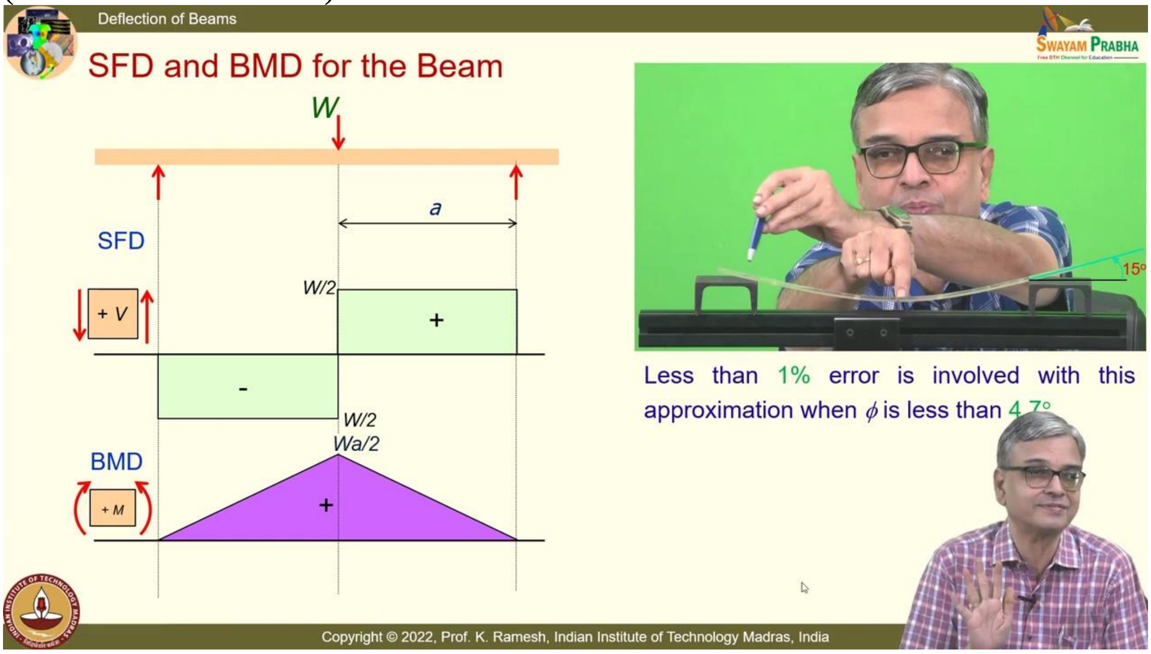


So, we go back to our habit of drawing the shear force and bending moment diagram. You know I am showing this again and again, mainly because you should also pick up speed for these problems by looking at the loading diagram. You should be able to draw the SFD without any intermediate calculations. And we do it from right to left. I have the load here, I go up, I remain constant here and come down by W and then go here and then go up.

Similarly, I can have the bending moment diagram. I have this going like this, fine. And you also have the deflected shape. So, you have all the information that is necessary for you to solve the problem. We have the moment curvature relation. You should write what is the bending moment at a section, apply the boundary condition and get the two integral constants.

And I have also shown the sign convention that we have been using in this course. So, everything is complete about this problem. So, you have to visualize. The idea is, in certain simple problems, I will give you the picture here. So, that should give you a training when loads are given, you should be able to conjecture what could be the deformed shape.

Looking at the support, looking at how the beam is loaded, you should be able to sketch it at least approximately. And you know, I have shown this and I have also measured this angle, fine. And this was turning out to be 15 degrees. I said that these equations are applicable only within 5 degrees as a slope. This is just to reemphasize, when I perform experiment, the deformations are extremely large intentionally for you to visualize how to look at bending as a whole.

Our equations are not going to; not accurate to solve for finding out this large deflection. This is about three times more than this. And I said in practice, if you solve many problems, it would not even reach 0.5 degrees, fine. So, you should have that relative sense of proportion.

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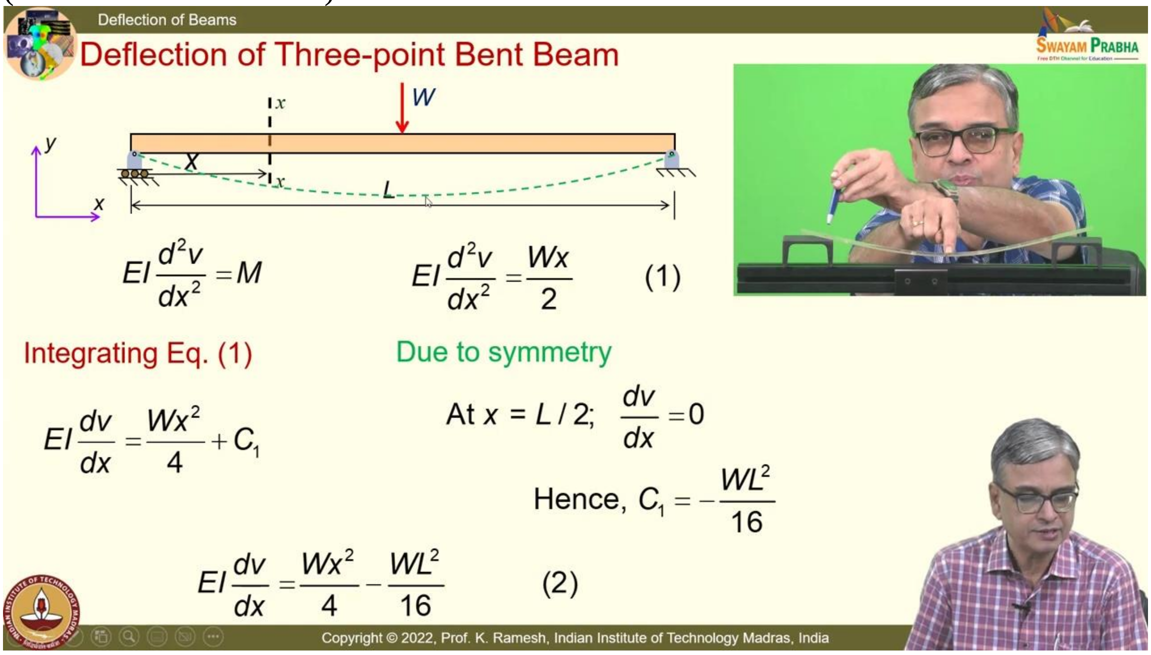


So, can you write what is the value of bending moment at this section *x*-*x*? Please write the moment curvature relation, fill in the bending moment, expression in terms of *x* and then check it with my expression. These things you should be able to write quickly. When you do the double integration, you should write the moment and get it quickly.

$$EI\frac{d^2v}{dx^2}=M$$

I said we drop the subscript because we know what do we mean by moment of inertia here. You should understand in this context, it helps our simplification of writing. Do you have an expression for $M$? You should also get it with the correct sign, it is very important. So, I get this as:

$$EI\frac{d^2v}{dx^2}=\frac{Wx}{2}$$

Is it alright? So, you can easily integrate. So, integrate and substitute the boundary conditions. Integrating equation 1, I get

$$EI\frac{dv}{dx}=\frac{Wx^2}{4}+C_1$$

And what is the boundary condition you can write due to symmetry? At $x$ equal to $L/2$, we have already seen here $dv/dx = 0$. So, you should know how to translate your appreciation of slope as zero mathematically, fine. And once you substitute these, you can find out what is $C_1$? It is very simple and straight forward.

So, I get

$$C_1 = -\frac{WL^2}{16}$$

Just substitute this condition and I get

$$EI\frac{dv}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16}$$

So, I get the variation of slope as a function of $x$ along the length of the beam. Whichever point, if I substitute the value of $x$, I get what is the value of slope. And you can also draw the deflected picture.

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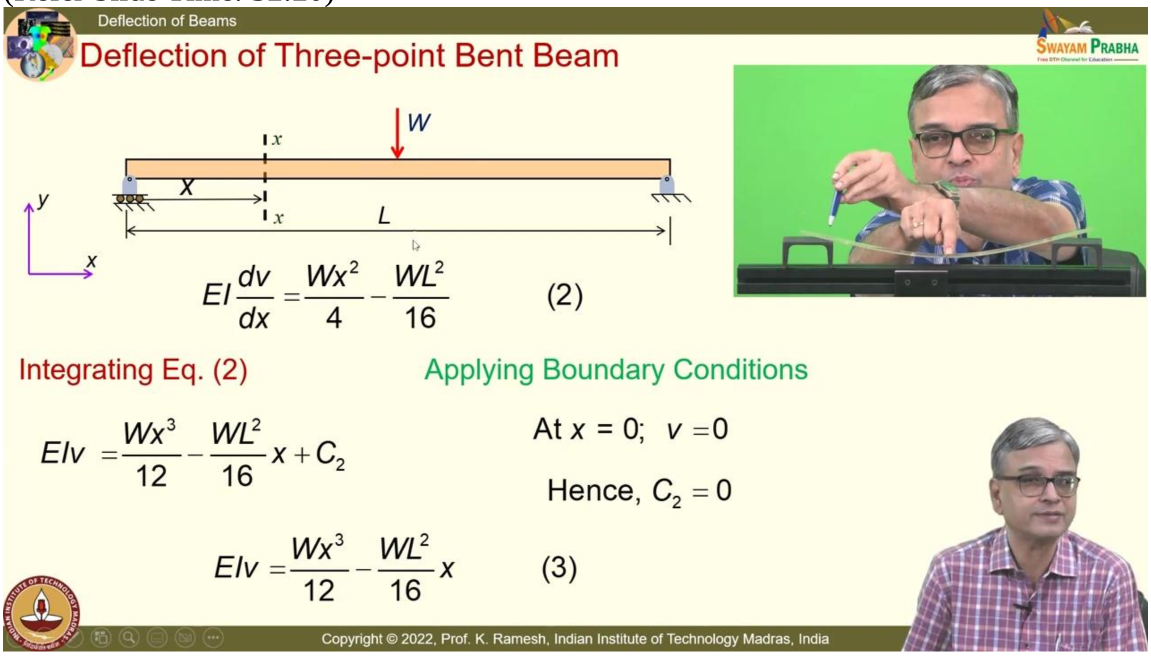


And we will have to integrate it again and find out what is the expression for deflection curve $v$. So, I can integrate this, that is very simple and straight forward. Can you integrate and then check with me? Please do the integration and check with me. It is a very simple and straight forward expressions. I get

$$EIv = \frac{Wx^3}{12} - \frac{WL^2}{16}x + C_2$$

And I have to determine $C_2$ by applying the boundary conditions. And what is the boundary condition that I can choose? Applying the boundary condition $x$ equal to zero, $v$ equal to zero.

So, I can get what is $C_2$? $C_2$ happens to be zero. So, I get the deflection curve

$$EIv = \frac{Wx^3}{12} - \frac{WL^2}{16}x$$

See, I have also been saying that certain quantities you need to remember as part of this course. You know, even after you complete the course, when you have to apply this knowledge in field, you will have to remember this, it makes your life lot more simpler. We will have to find out what is the maximum deflection at the load application point.

That is a very important answer. So, you are going to substitute $x = L/2$, fine.

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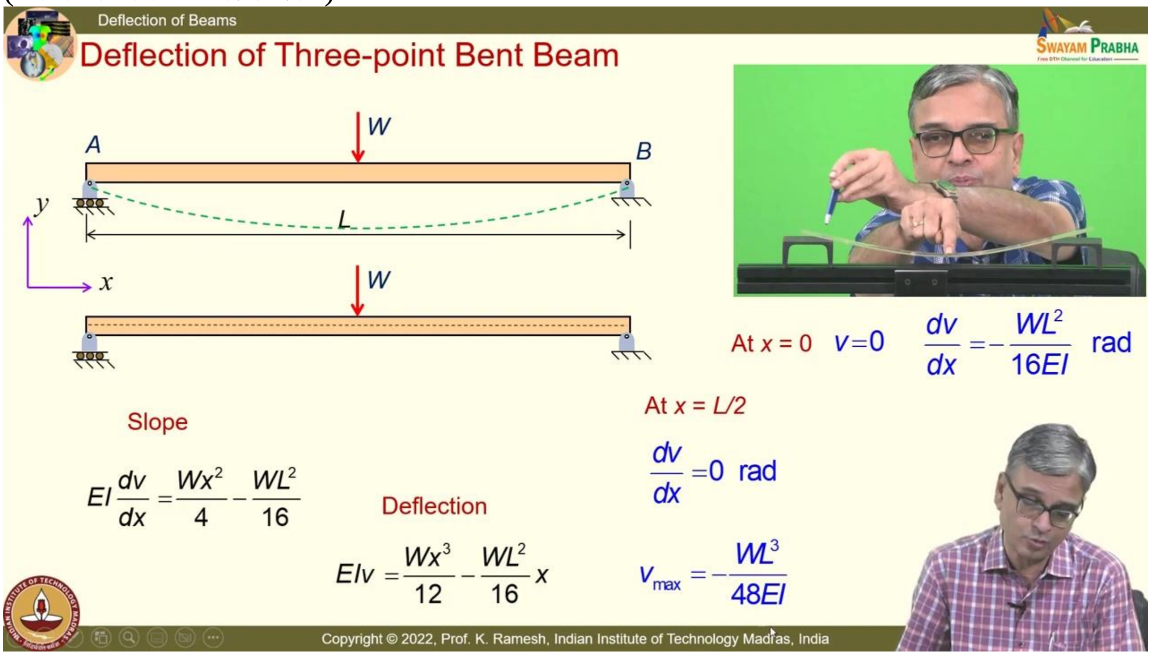


So, I have this and mind you, all these I am doing it just for this line. I recognize the beam as just a line, I apply the mathematics only there, I ignore the thickness. We have found an explanation for that, even though we investigated Poisson's ratio plays havoc, that happens only when the deflections are very very large. We never go near to that situation. So, I have the expression for slope, I have the expression for deflection and find out at important locations what happens.

At $x$ equal to zero, I have the slope, it is given as

$$\frac{dv}{dx} = -\frac{WL^2}{16EI} \text{ rad}$$

and please remember, what you get as slope is expressed only in radians. This is another mistake people commonly make, you will have to convert this radians into degrees appropriately, fine. And you also have $v$ equal to zero at $x$ equal to zero, which is all satisfied by this expression, when $x$ is zero, $v$ is zero. And this reaches a maximum at $x = L/2$, where the slope is zero and the maximum deflection including the sign, it is given

$$v_{\max} = -\frac{WL^3}{48EI}$$

It is a very famous expression. You know what way we remember is, we remember this as

$$\frac{WL^3}{48EI}$$

We do not remember the sign; sign comes automatically when you draw the sketch and take the coordinate system. I have also discussed, even though we have developed the flexure formula with $-\sigma_x/y$, from the application point of view of design, you will simply calculate the magnitude and you say on one side it will be tensile, on other side it will be compressive, if I have a symmetric cross-section about the *z*-axis. If the cross-section is not symmetric, we always make measurement from the neuter axis, whichever is farthest will have the maximum stress, it could be tensile or compressive. So that may happen if I have a *T*-beam; if I have an I beam it is symmetric, if I remove one flange, it becomes only as *T*.

So, *T*-beams will have a different distance from the neutral axis to the one side and the other side of the cross-section. So, you have to appreciate this and we normally remember this is a very famous expression; worth remembering it $WL^3/48EI$.

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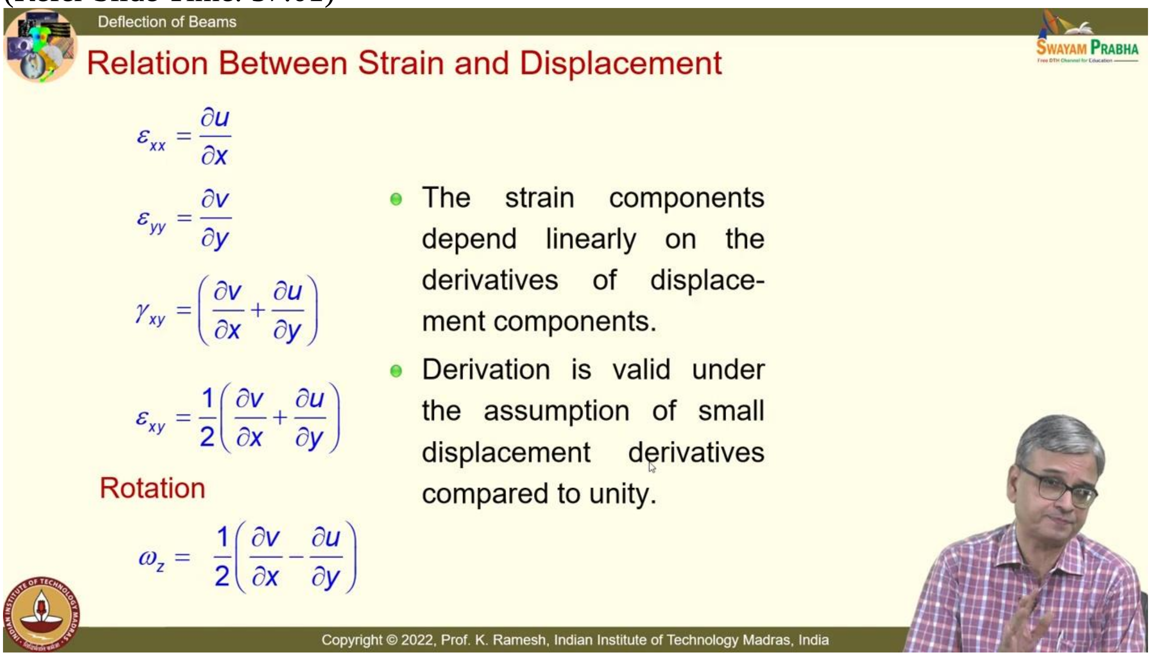


And you know, let us also go back and then see what we have learnt when we discussed strain. When we discussed strain, we have found out that

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

And the tensorial expression of strain is

$$\varepsilon_{xy} = \frac{1}{2}\left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

And we have also discussed rotation. You remember, we have also developed rotation. So, when anything gets deformed, it can have all of these quantities, it can have all of these strain components as well as rotation. Because it all comes from what way the components of displacement $u$ and $v$ varies over the cross-section. And we have also said that the strain components depend linearly on the derivative of displacement components. Derivation is valid under the assumption of small displacement derivatives compared to unity, which you have to always remember.

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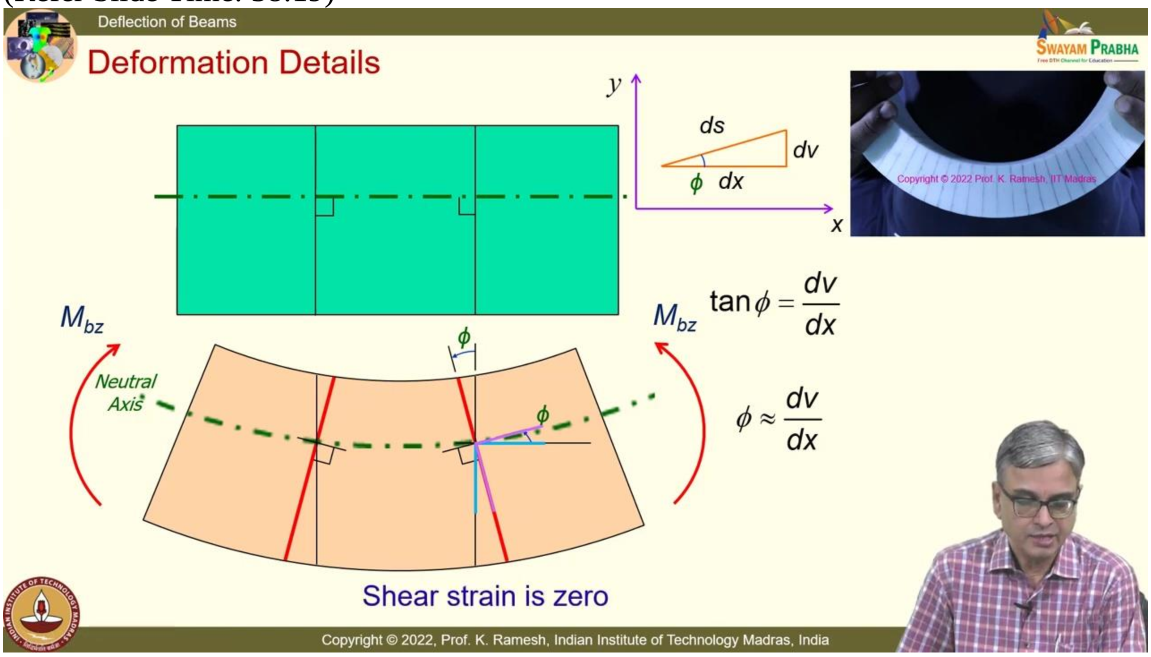


You have to remember this as well as all the demonstrations are large deformation. And I said, you know, when we looked at Euler Bernoulli theory, after deformation also the line remains perpendicular. And you know, I have taken a cross-section which is something like a *T*-beam. See, this is done for a rectangular cross-section, but this diagram is drawn for a *T*-beam, so that the neutral axis is shifted. Because we have always been seeing neutral axis at the center, I do not want to give an impression that centroidal axis is what we are talking about.

We are talking about I mean neutral axis. For a homogeneous material both of them coincide, it can remain in the center or it can get shifted. So, when the beam is subjected to constant bending moment, the lines get rotated. And if I draw the tangent and then find out what is this angle, this angle still remains 90 degrees. That is how we have developed the Euler Bernoulli hypothesis.

And the beam that obeys the Euler Bernoulli hypothesis is called as Bernoulli beam. And I have said there is also a Timoshenko beam, which will accommodate the effect of shear. We are confining our attention to Bernoulli beam. You make a sketch and draw this as perpendicular.

So, these lines just get rotated. And let us also investigate what happens to the angle of rotation. We have already seen the slope as angle $\phi$. And then we have also shown this diagram how you have related $\phi$ and the deflection $v$,

$$\tan\phi = \frac{dv}{dx}$$

And we have also said

$$\phi \approx \frac{dv}{dx}$$

So, what you have got the expression as slope is nothing but what you have here, this. And we will also see that this $\phi$ is also the rotation. And as long as you want to investigate the shear, I have to look at what happens to the original rectangle, how this gets changed because of deformation? And the original rectangle remains original rectangle even after the beam is bent. And we have already seen that shear strain is zero, when I have a pure bending. We are discussing only about pure bending. So, this shows shear strain is zero.

And from the geometry of the figure, you can also denote that this is also angle $\phi$.

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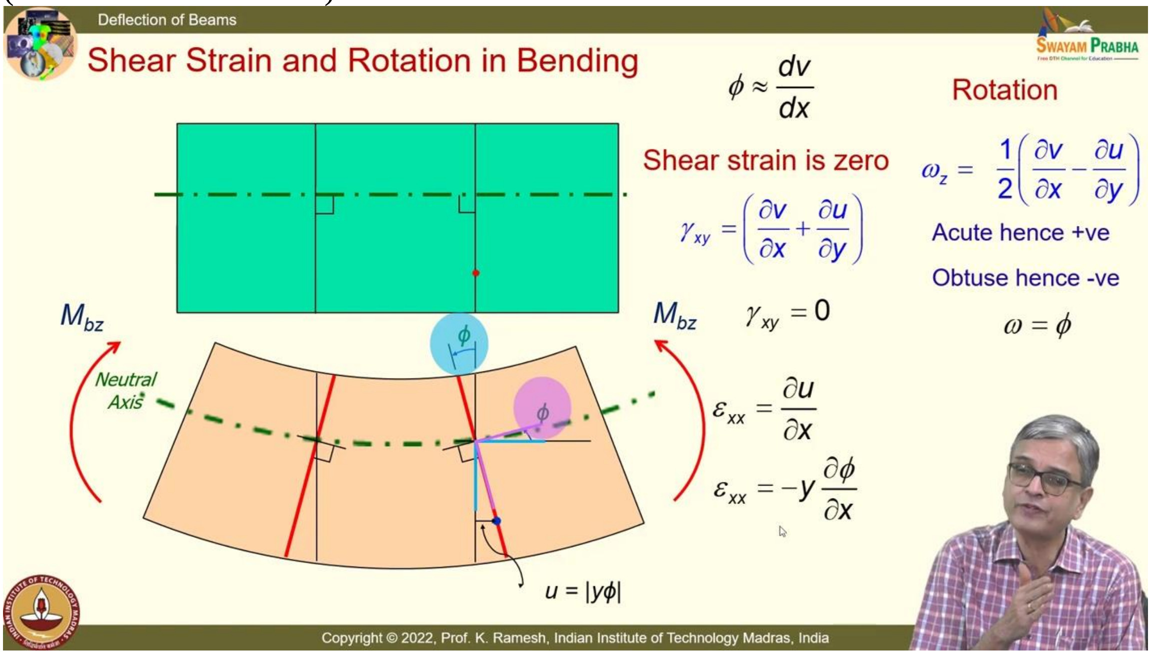


And you know, we will also write what is the shear strain in terms of the displacement components.

$$\gamma_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

And we have seen $\gamma_{xy} = 0$. And we have also seen what is rotation.

Rotation is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

And you have this, I have this as this angle. So, it is this angle will be labeled as positive because it is acute. This angle will be labeled as negative because it is obtuse, the way the deformation takes place. And you can easily see that what you call as rotation is the slope phi from this expression.

If I use $\gamma_{xy}$, they are equal and opposite. If I use this, I will finally get this $\omega = \phi$. So, what you get a slope is nothing but the rotation that takes place along the beam. And you have, suppose I take a point in the undeformed beam; in the deformed beam, this point will be shifted to this. It is rotated like this. And this is nothing but from magnitude point of view, you can write

$$u = |y\phi|$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$

So, I get

$$\varepsilon_{xx} = -y\frac{\partial \phi}{\partial x}$$

What we write is with the sign accommodated. So, it is a smooth transition from what we have learnt as strain displacement relation to what happens in the case of a beam. And I have already shown, when I have shown the beam bent, you saw $\phi$ was changing. You can see the rotation was increasing.

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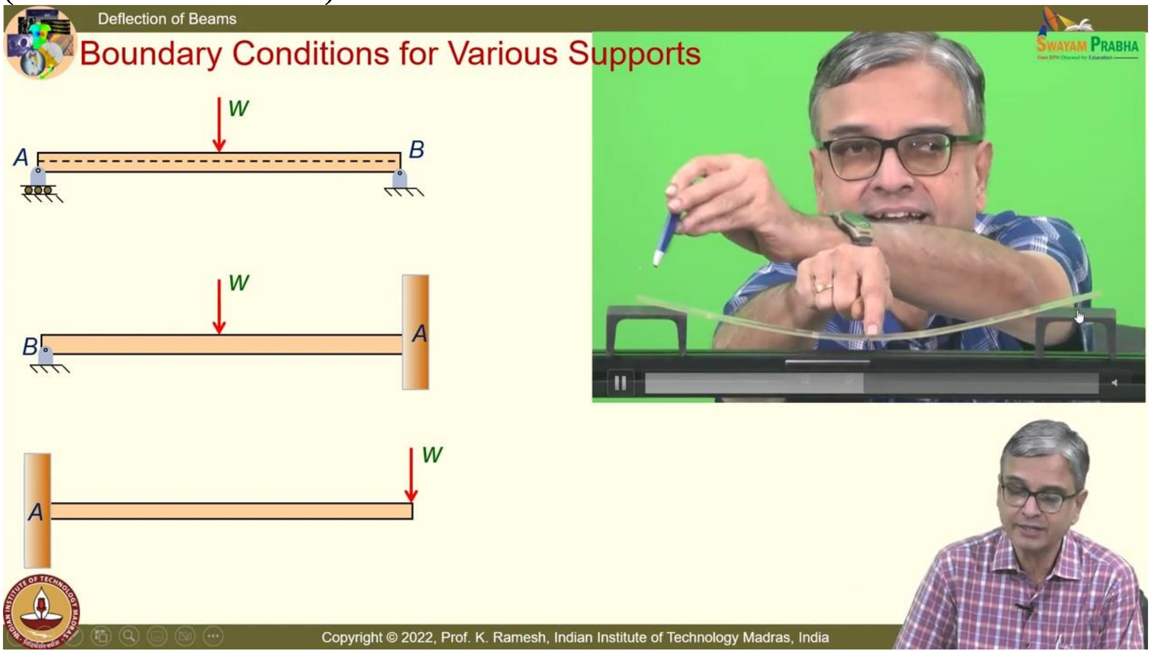


Now, what we will do is, we look at different type of supports. We have already looked at what happens for the simply supported. And we have seen that this is symmetric and you have slope here, you have slope here.

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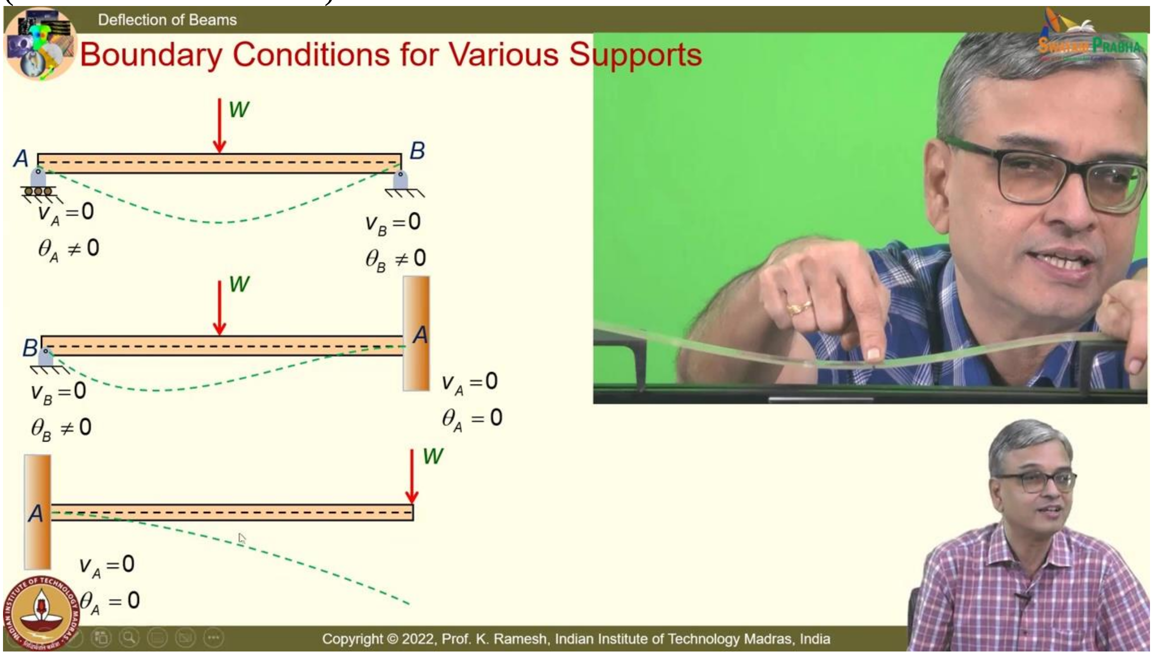


Now, what I am going to do is, I am going to apply a clamping. When I clamp, what happens? Can you observe what happens here? Can you observe what happens to the slope here? Slope is zero.

Slope is zero. You recognize this? Slope is zero here, but what happens to this end? It is not zero. It is not zero. You should recognize that. If you recognize that, deflection is extremely simple. That is all. So, in this case, I have the beam as deflected like this.

And in this case, and you can write,

$$v_A = 0$$
$$\theta_A \neq 0$$

$$v_B = 0$$
$$\theta_B \neq 0$$

Please write down this. These are all the important boundary conditions. You can write from the problem statement, but your notes should have these quantities.

So, you should recognize when I clamp this, I have this slope is zero. But you can have still slope on the other end. Here, I have shown a beam and this is the free end, fine. So, when I have this, I will have a deflection like this.

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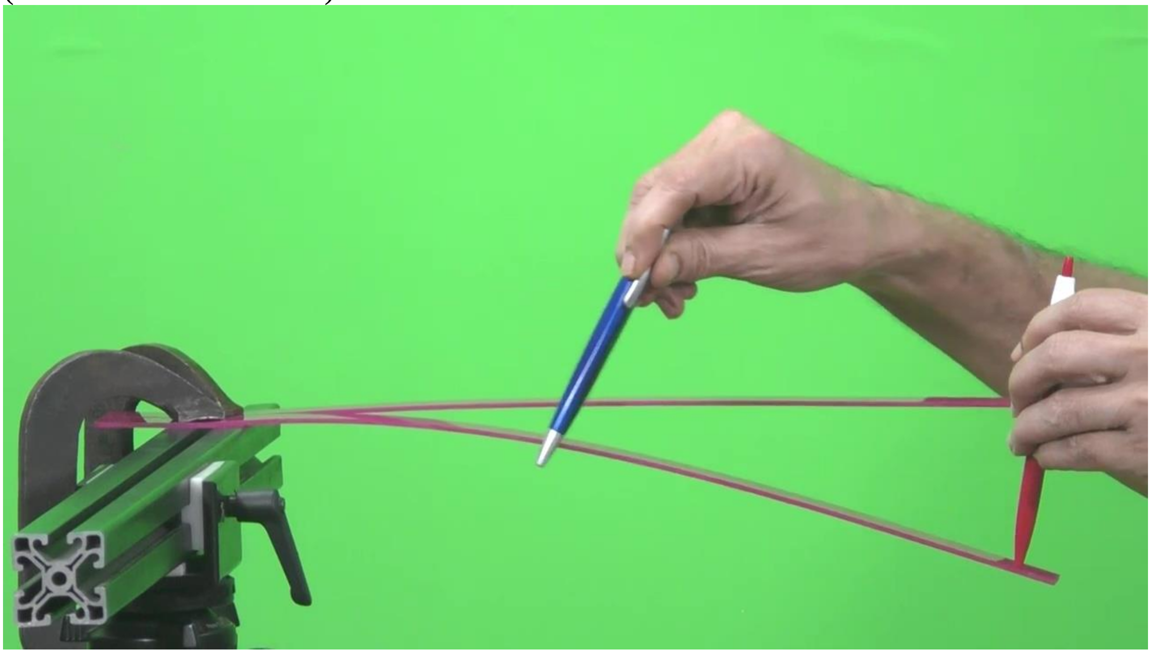

And I will also try to show an experiment. See, I have two cantilever beams. I have it with a purpose. And what I am going to do is, I am going to apply a load, end load here. And when I apply the end load, along the length of the beam; now we have learned that this has strain developed because of bending and it also has rotation continuously varying. Is the idea clear? When I have the beam bent like this, I have like this.

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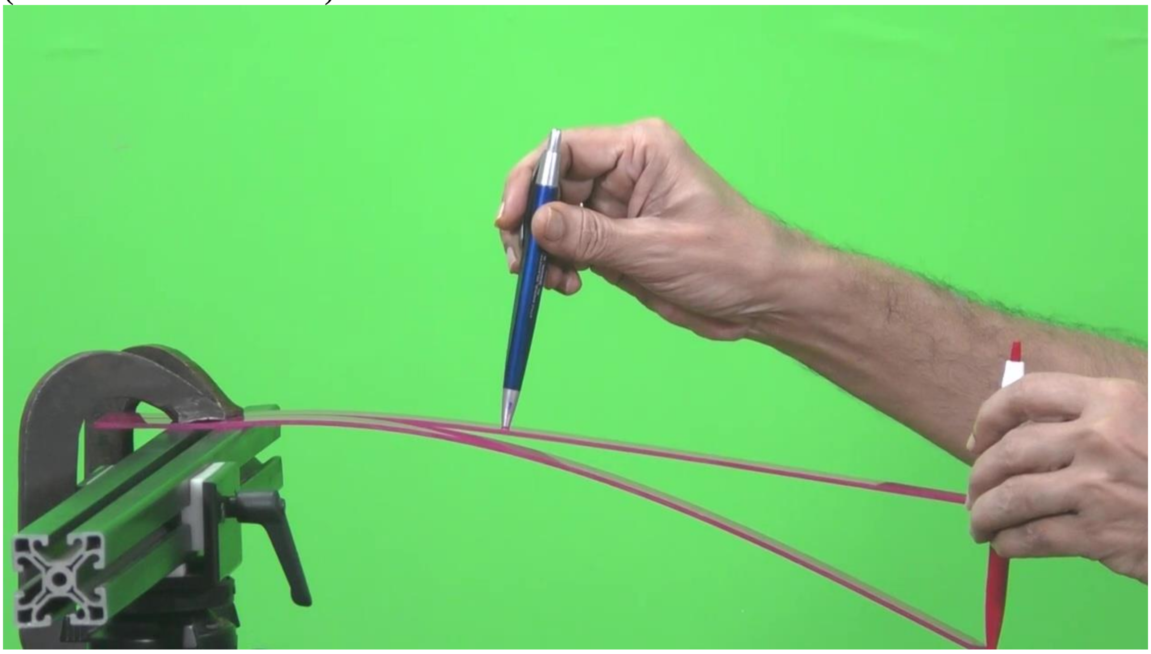

Now I take this beam and I apply load somewhere in between. Technically, I have no load on this section. When I apply the load, I want people to recognize, what is the deflection up to the load application point from the fixed end? And what happens after the load application point to the free end in the second beam? So, this section remains straight, but rotated. That you have to recognize. I have this load applied here.

I apply the load here. Up to the load application point in the second beam, the beam has variation of slope as well as strain. In the rest of the portion, what I have is, this is a straight rigid body. That you have to recognize. If you recognize, you know, your job of handling deflection is extremely simple. And now, you know, I can also have the conditions. I can only write

$$v_A = 0$$
$$\theta_A = 0$$

I cannot say anything beyond that. On the free end, it can have slope as well as deflection.

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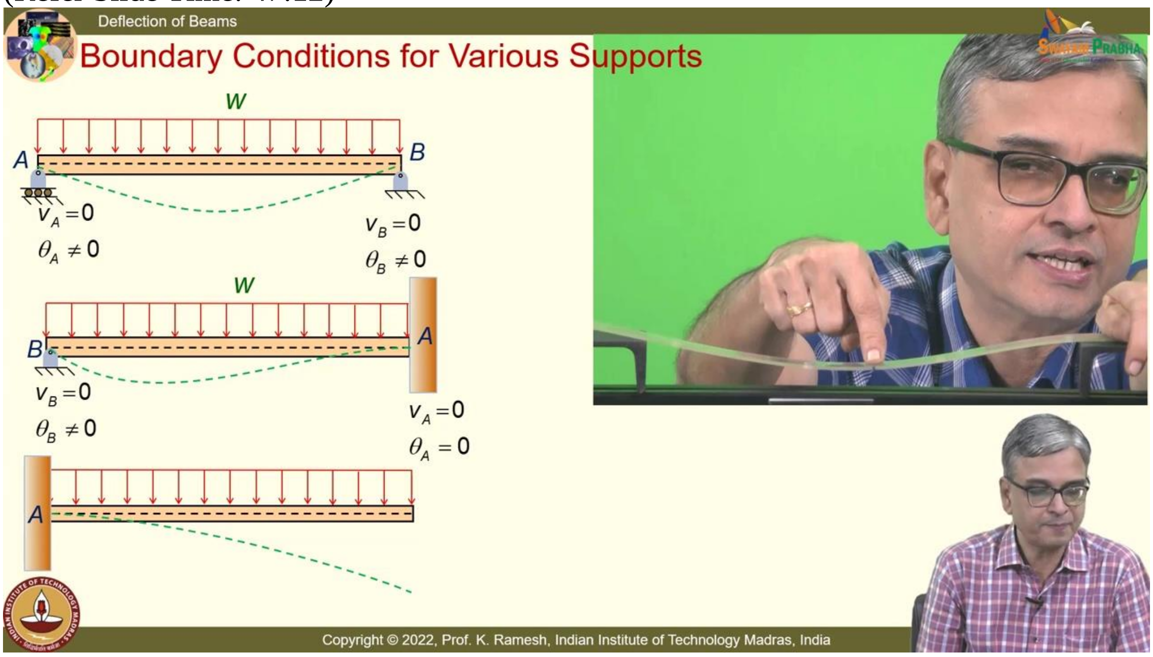


Suppose, I have instead of concentrated load, I have a distributed load. The distributed load disturbs the beam very lightly. So, you will have a similar displacement, but the magnitudes can be smaller. So, the idea is, you should not get worried about the nature of load. You should worry about how to handle the support. So, if you handle the supports and join them in between from your visualization, you will have the deflection of the beam plotted reasonably well. So, I will have this as like this and you have this like this.

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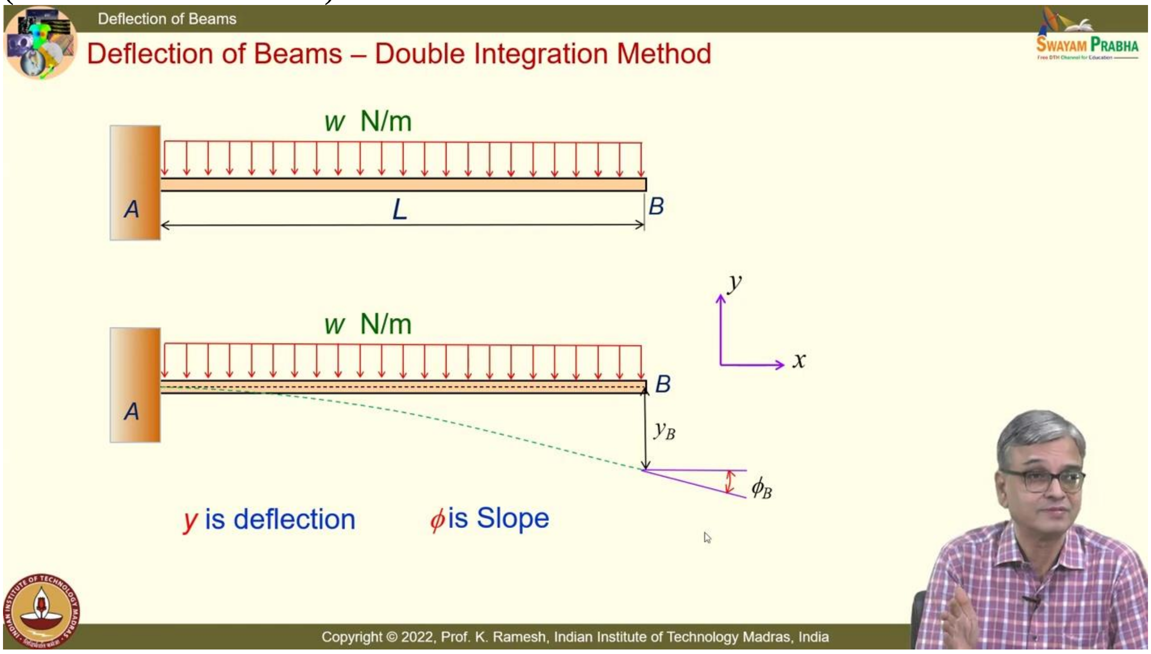


Now, what we will do is, we will solve one more problem. I have the distributed load, which you can easily do. My anticipation is, you should solve faster than me. Take a typical cross-section and then, $EI_{zz}\dfrac{d^2v}{dx^2}$ equal to; you write the bending moment with sign, integrate it twice, utilize the boundary conditions and get the constants, integration constants.

We have already seen how the beam bends. I have deflection. See, some books call this as *y*, some books call this as *v*. So, you should be comfortable with any of the symbolism, if you know the diagram. So, at the free end, I have a deflection as well as a slope. And we have already seen, slope is nothing but what we learnt as rotation in our strain displacement relation.

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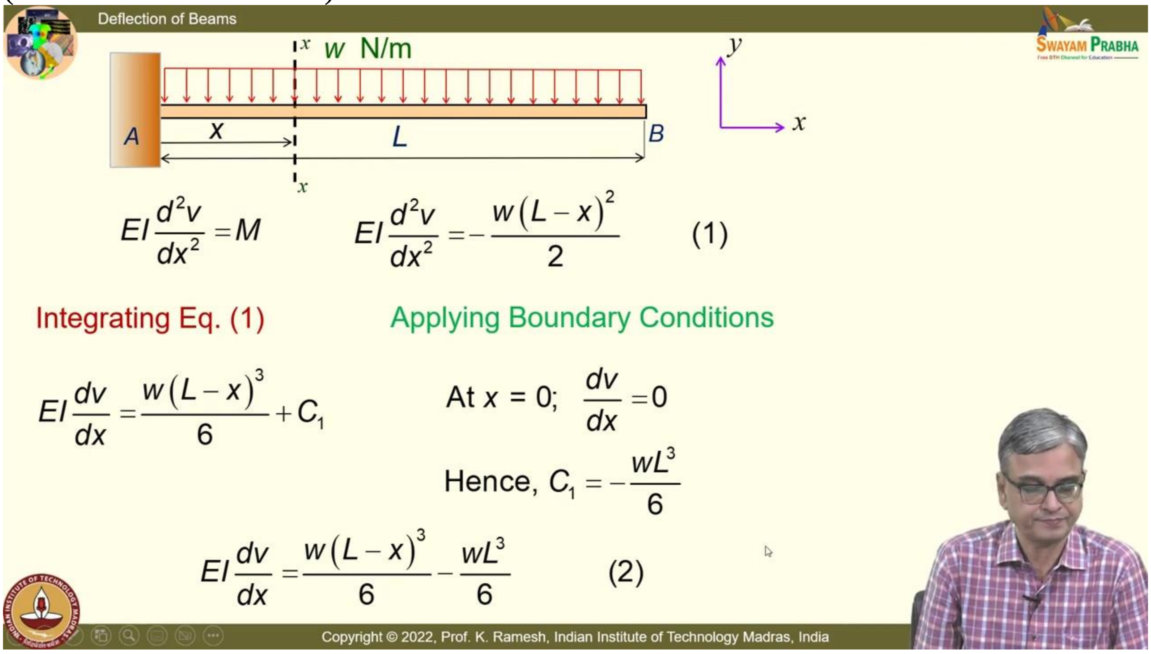


So, you should write the expression. You have the expression for *M* with sign. I want you to write it with sign. So, it is easier for me to write like this,

$$EI\frac{d^2v}{dx^2}=-\frac{w(L-x)^2}{2}$$

Once I have written this correctly, this is a crucial step. If you make a mistake, rest of your problem is going to be troublesome. If you write this correctly, then the integration is very very simple. This all you have done in even in your high schools. So, I have this

$$EI\frac{dv}{dx}=\frac{w(L-x)^3}{6}+C_1$$

and

$$\text{At } x = 0;\quad \frac{dv}{dx}=0$$

This gives me

$$C_1=-\frac{wL^3}{6}$$

And I have

$$EI\frac{dv}{dx}=\frac{w(L-x)^3}{6}-\frac{wL^3}{6}$$

I can do the integration second time.

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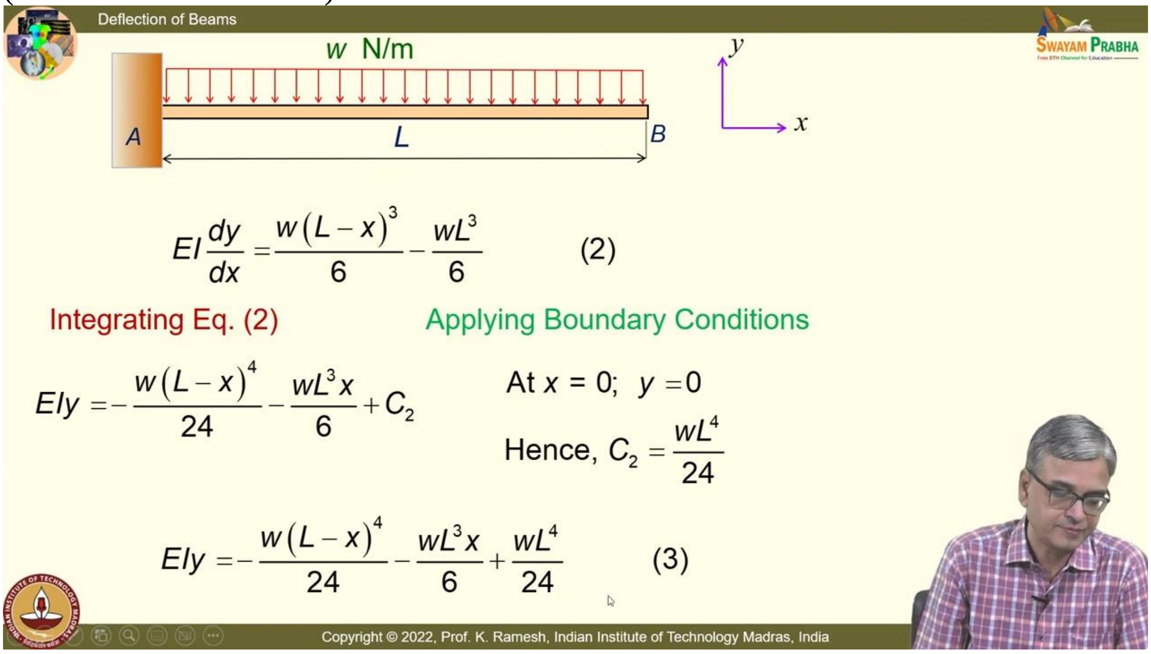


I think I have written this as $dy/dx$ here. So, you have all these expressions. People call it as $y$, people call it as $v$. So, you should be comfortable with both symbolisms.

And I get the value of

$$C_2=\frac{wL^4}{24}$$

So, I get

$$EIy=-\frac{w(L-x)^4}{24}-\frac{wL^3x}{6}+\frac{wL^4}{24}$$

So, the idea is double integration is simple, straight forward, when I have simple loadings. When I have multiple loadings, then I have to do the integration separately for each of the segments. You also have a mathematical development called singularity functions, which we would not get into it primarily because it becomes only a mathematical exercise if you do solve problems related to deflection.

As an engineer, you should have lot of visualization. So, it is better that we adopt other methods, where it gives you more comprehensive appreciation of what deflection is.

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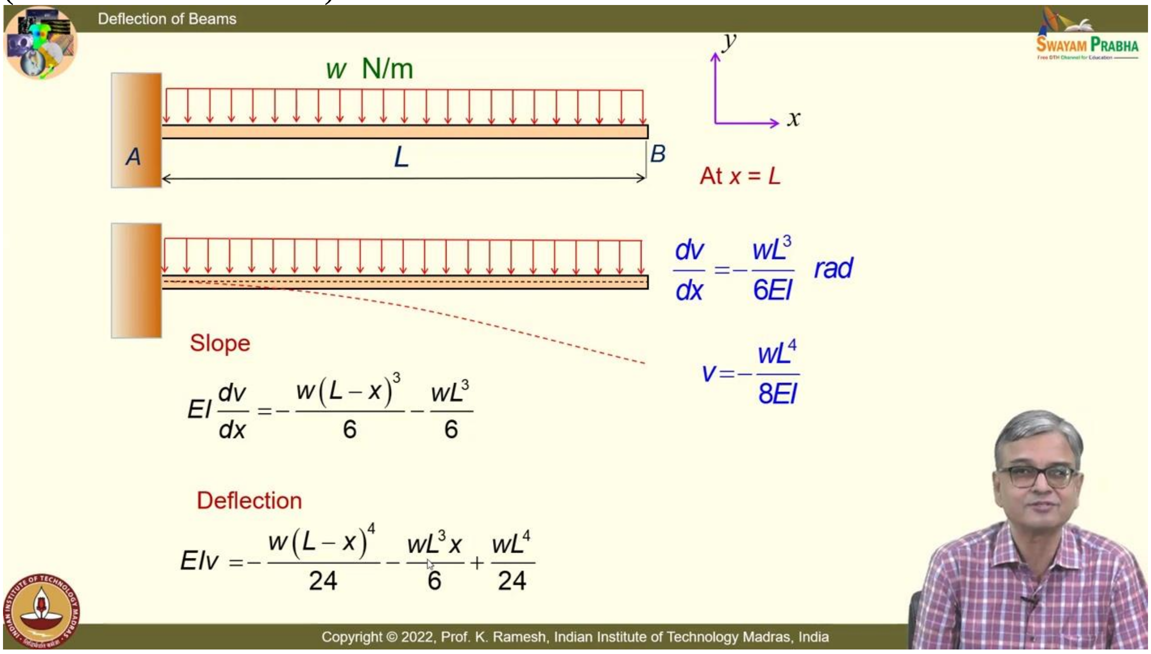


And I can also find out what are the maximum values at the free end. And you know this we do not remember. I said $WL^3/48EI$ is a very famous answer. This we do not remember. You have to do the calculation and get it.

$$\frac{dv}{dx} = -\frac{wL^3}{6EI}$$

in radians; I am emphasizing this. Never forget this. This is not in degrees. And

$$v = -\frac{wL^4}{8EI}$$

And once I have such solution available, you can have a table. Use the table for method of superposition. The table will be supplied in your quizzes and exams. Do not worry about it.

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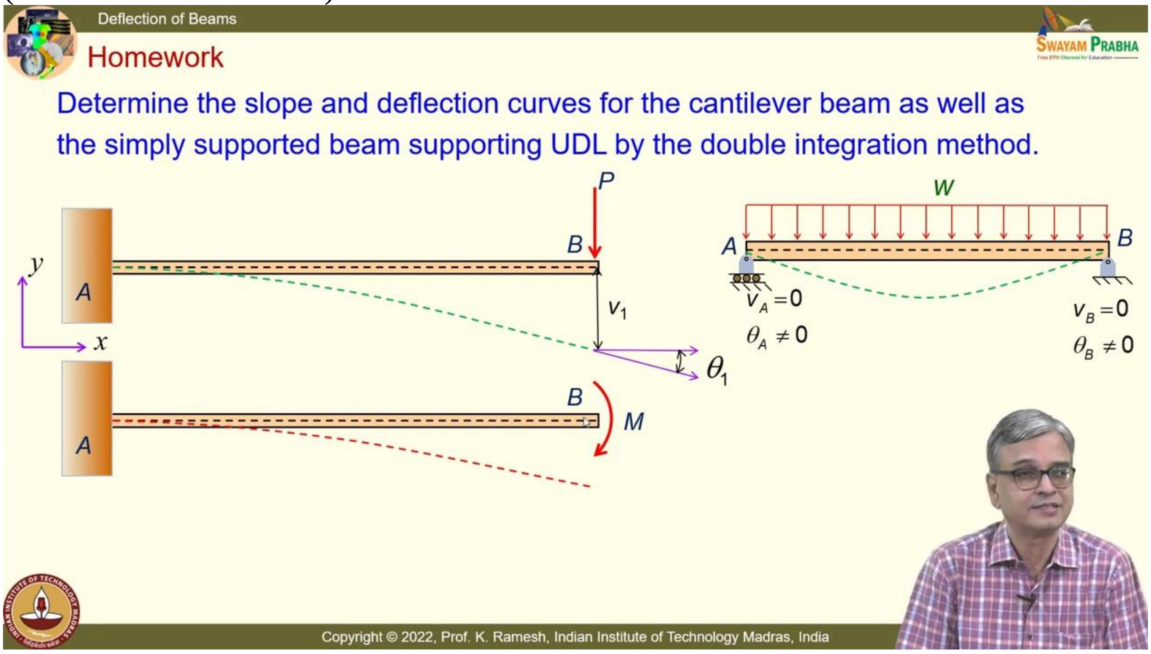


And I want you to do some homework. You know the purpose is your learning will be better if you do the homework. So, I want you to apply the double integration method for the UDL, simply supported. The boundary conditions are given. And a cantilever with an end load and also a cantilever with an end bending moment.

And we normally remember what is the tip deflection in a cantilever. Like I said, in a three-point bent beam, what is the deflection at the center. Similarly, this is again a very famous expression. What is the tip deflection? So, certain quantities you need to remember. It is also better.

It will make you very comfortable when you have to deal with beam deflection or beam design. So, in this class, we have looked at what is deflection of a beam. We have looked at; you have a deflection as well as a slope. And we have seen slope is nothing but the rotation what we have learnt in strain displacement relations. And here again, you develop the basic equation only for pure bending. We extend it for beams transmitting shear force, neglecting the influence of shear or you have infinite shear rigidity, whichever way you look at it.

We are justified in doing it because shear effects are normally very small. And I said, we also have special methods by which you can find out what is the deflection because of shear force. That we will postpone it for the time being. But we will definitely do as part of this course. Thank you.