Strength of Materials
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Lecture - 33
Deflection 3 Method of Superposition and Energy Method

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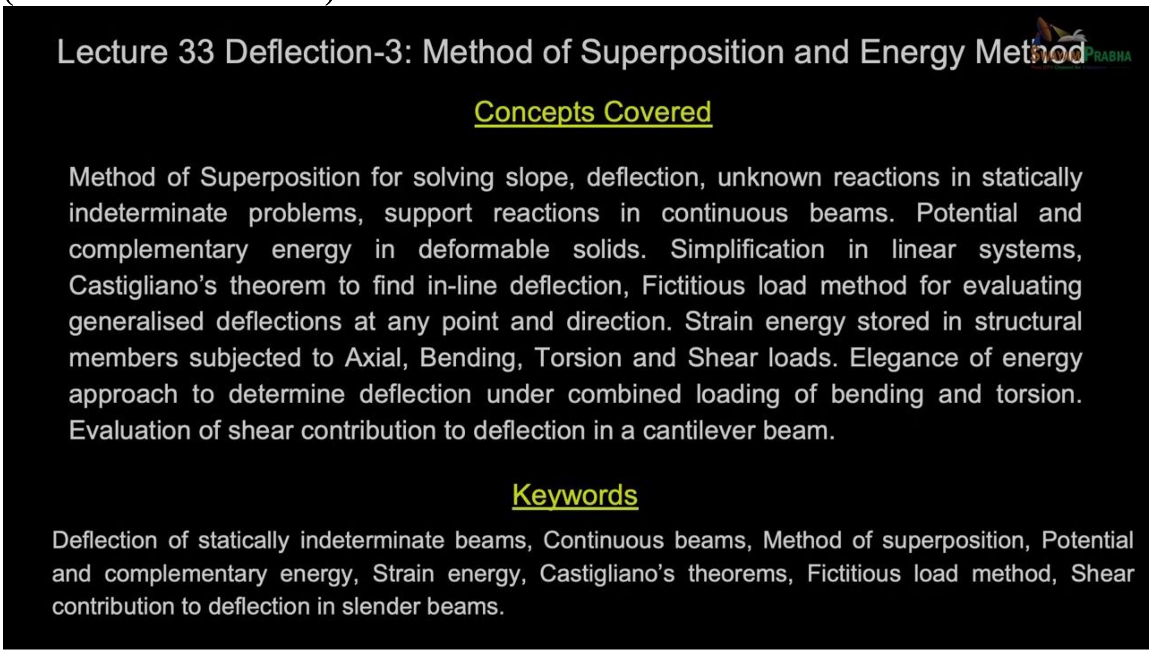

Let us continue our discussion on deflection of beams. You know, I will try to cover some problems from method of superposition followed by the energy method. My main focus will be on discussing how to approach the problem. The mathematical simplification, I leave it as an exercise; that also helps you to ruminate over what is discussed in the class and please do the mathematical simplification yourself.

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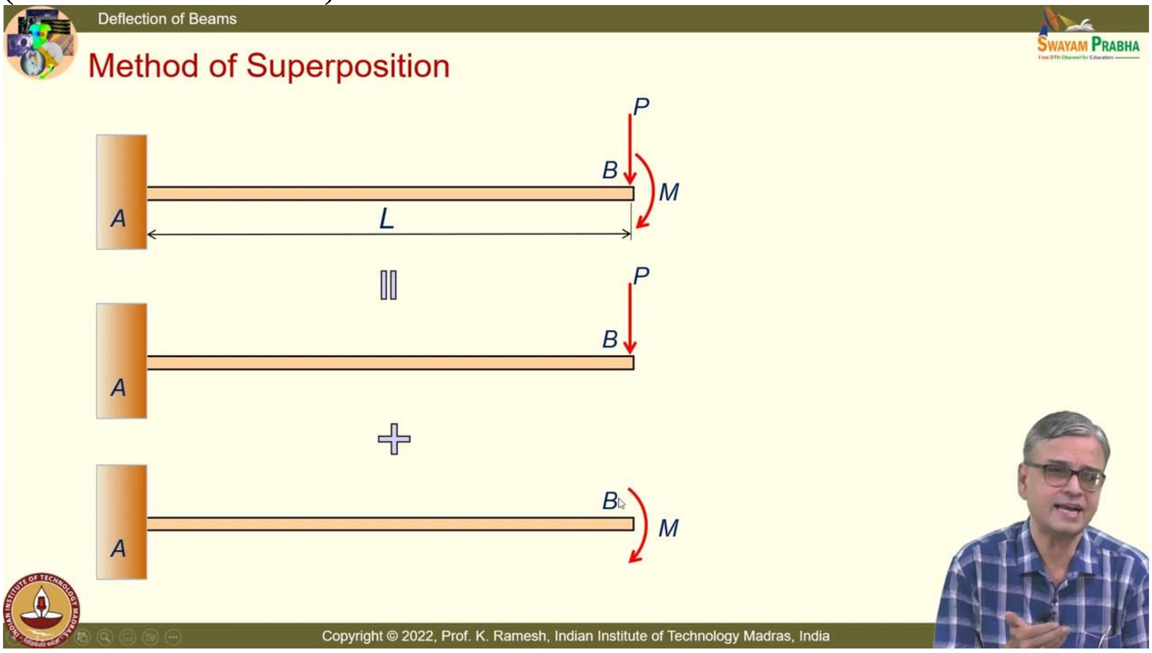

You know, in the last class I asked you to do this as a homework and you know how students operate, so it is better that I discuss it again. We have done by moment area method, what is the end deflection and also the slope at the load application point for both these cases and these two loads are simultaneously applied to the beam, which you can think of as superposition of an end load and a bending moment.

And this is statically determinate problem, fine. You have individual solutions available and you simply add those solutions; whether it is a displacement or a deflection or a slope.

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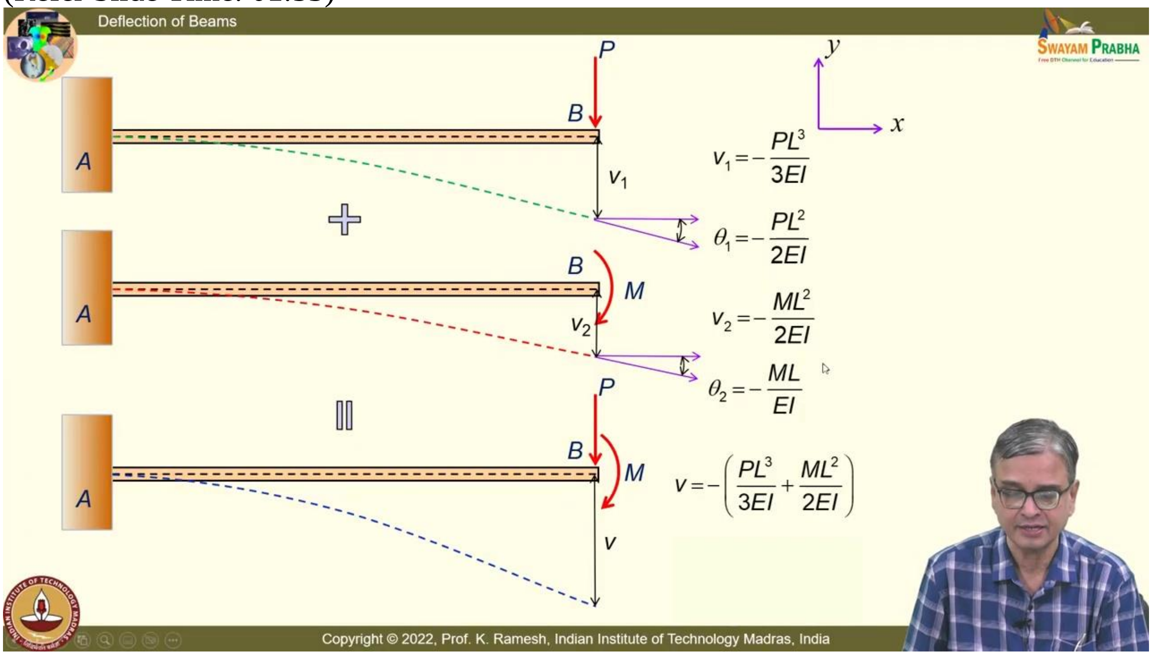

Our focus is to get, what is the combined effect and we have already derived what is v_1.

$$v_1 = -\frac{PL^3}{3EI}$$

I have said that $PL^3/3EI$ is a very very important result. We may be using it repeatedly; it is worth remembering that and you have

$$\theta_1 = -\frac{PL^2}{2EI}$$

We have also seen for a bending moment, what is the value of v_2 and θ_2. So, when I have a combined loading, the principle of superposition says I can add them. For illustration purposes, these deformations or deflections are shown to be very very large.

In reality, they are very small and you can comfortably add them, so that I get what is the tip deflection and also the slope at the tip; simply the arithmetic addition of the values that we have got for individual problems. So,

$$v = -\left(\frac{PL^3}{3EI} + \frac{ML^2}{2EI}\right)$$

See the idea here is, I have also mentioned assigning the sign to these quantities, you have a procedure where you plot the deflection and then label it. That is the best way to do it. If you do a double integration or integration four times, if you follow the sign convention, all the quantities are obtained with the desired sign.

In none of the other methods, you have that luxury. Here I have shown this as minus, but you may not get it from the mathematics as such.

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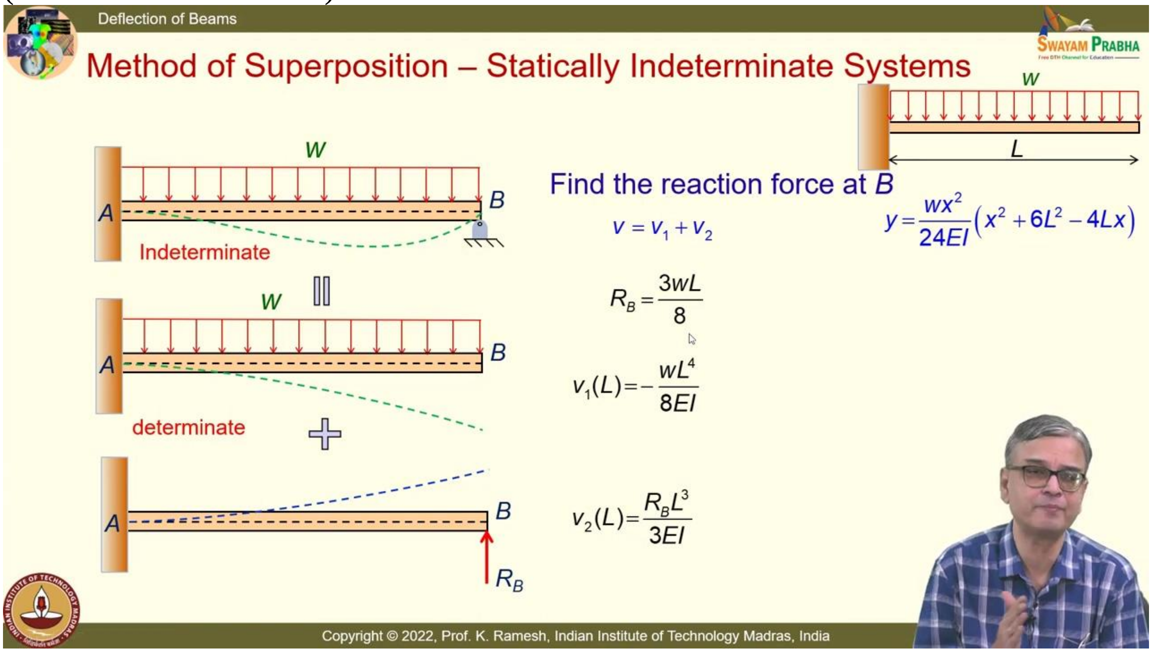

Now, let us look at a statically indeterminate system. I suppose you recognize that this is statically indeterminate. So, the question is, what is the reaction at *B*; which you could not have solved in your earlier course on rigid body mechanics.

Unless I bring in the deformation picture, it is not possible to solve using equations of statics. Now, I will split this as two problems. You should also know how to split it as two problems. It is statically indeterminate problem and you know, you have been trained when I have a support like this, when I have a fixed support, you have in the fixed support, slope is zero, deflection is also zero. So, you know how to sketch this deflected shape, anticipated deflected shape.

What I do is, in the first case, I remove this load; this support, I make this as a statically determinate problem. And if you look at the table, this is what I said, the moment I go into method of superposition, you will be provided with the table and your only exercise is to filter out which solution you should use for your calculations. That exercise you have to do. And our focus is to find out the reaction. So, I have just put the deflection and deflection is put as *y*.

This is what I said, people have used *y* as a symbol or *v* as a symbol and instead of ϕ, they use θ, instead of ρ, they use *R*. So, all of these symbols are seen in the books. So, you should be comfortable by looking at them repeatedly. And you can also sketch what is the deflected pattern when I have a uniformly distributed load. And I can have the second problem.

So, from this it is possible for me to find out the deflection, I call this as v_1, I get this as

$$v_1(L) = -\frac{wL^4}{8EI}$$

And here I remove that distributed load, I replace this as a reaction R_B. And you know this is nothing but a cantilever. And with this load, what way you anticipate the deflected shape? The deflected shape will be like this. This you should be very comfortable to visualize.

Visualization is very very important. So, when I calculate the deflection, I should put the appropriate sign and get the net. But here, the question is to find out the reaction R_B. So, when I have the reaction R_B, I anticipate that there is zero deflection at this point in the actual beam. So, I equate these two deflections by magnitude and I can easily calculate what is the value of the force R_B, which you could not have calculated in your rigid body mechanics. Even though, it is a statically indeterminate problem, method of superposition helps you to do that.

I hope I have the value of R_B; $v_1 + v_2$ should go to zero. So, I get

$$R_B = \frac{3wL}{8}$$

So, a very useful application of method of superposition.

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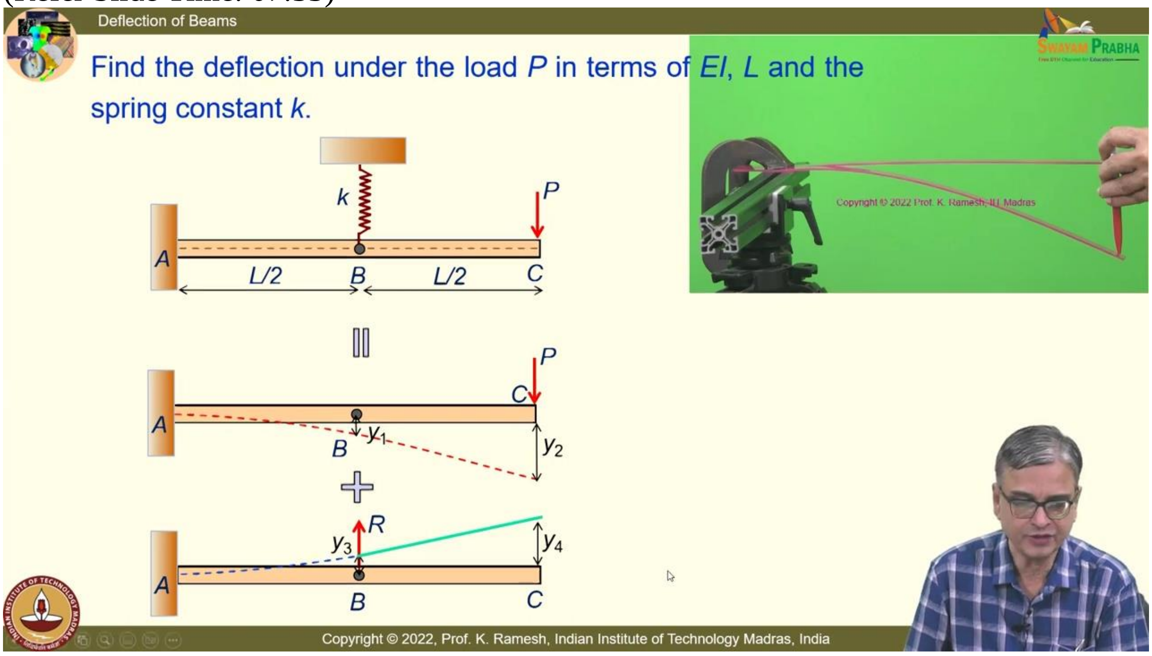

Then we move on to another problem. You know, what is this problem? Can you solve it by rigid body mechanics? Is it statically determinate or indeterminate? First you should recognize that. When a problem is posed to you, you should recognize whether it is statically determinate or indeterminate. If I do not have a spring, it is statically determinate.

If I have a spring, it is statically indeterminate. It is not possible for you to solve using the principles of statics, but you know spring will deform.

So, you can split this as sub-problems and also visualize the deformation picture. So, one problem is just look at the end load, other problem is just look at the spring attached. Spring is going to produce the normal force. The question here is, how do you visualize the deflection there? I have said that these two points are important. So, I have labeled this as y_1 and this as y_2.

And I have the second problem where I replace the spring by a force. And which way should I put the force? All that, you should apply your mind blindly. You should not do. You should understand the physics of the problem. When I put the tip load, when the beam is pushed downwards, the spring will try to pull it up.

You know if you do not look at these issues and do not put the problems properly, then you are solving a different problem. So, you should apply your sense of visualization. So, I will put the reaction like this. Can you attempt to draw the deflected pattern for this? How many of you recollect? What are the nuances that we have discussed? It is very important because you know, in a hurry to solve problem, you miss the nuances. I have load only applied at *B*.

The portion *BC* is unloaded and it is put out upwards and you have this, you can sketch it. What is the way you anticipate the deflected pattern here? Straight line, that is very good. So, I am also showing it by a different line to emphasize that this has the rotation of the slope at this point B and it remains straight. Suppose I want to do it by energy method, when it is rigid, it cannot store energy. That is also you have to use it, which we have seen it beautifully in the experimental one in the previous class; that is also shown there.

And I label this deflection as y_3, label this deflection as y_4. So, if I find out the deflections y_1, y_2, y_3, y_4, my job is done. I am going to discuss how to do this, rest of the mathematics is left to you. That's simplification which you can do. And the first issue is, you know, what is the solution that I should look from the table?

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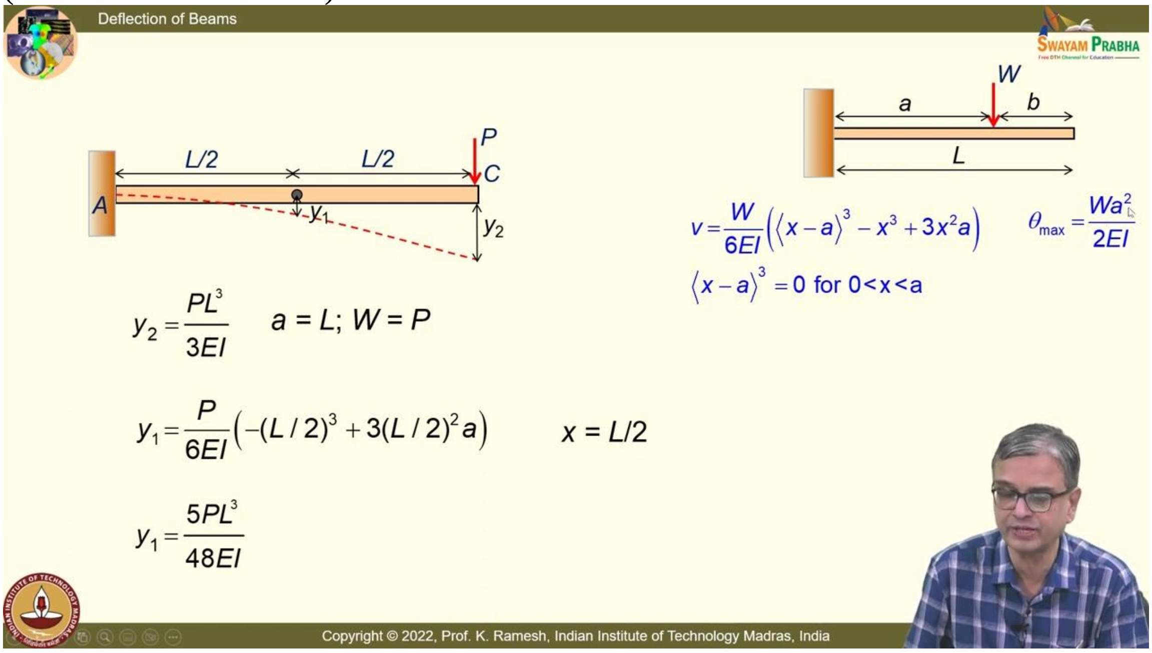

You have a generic solution available for a cantilever with a tip load, not a tip load, but it is at a distance b from the tip.

You have the expression for *v*, you also have the expression for the θ. I have that probably it is not coming up. You know I have this as the problem. I have to find out, if I have to find out what is y_2; y_2, can you tell me what is y_2? You do not have to even look up that expression. What is the value of y_2? I have a cantilever with a tip load P, that is all, fine. So, you should; it is a very useful result, $PL^3/3EI$ is a very useful result. I have not even put this symbol; I mean sign here. The idea here is, you know, you will have this and, in the diagram, it is put where is y_2? When I have y_2 listed in the diagram, if I give the magnitude that is also sufficient. I should not make a mix up in my mathematics. And at *x* equal to *L*/2, you can substitute here and you can find out what is the value of y_1.

When I simplify, I will get this. I get this

$$y_1 = \frac{5PL^3}{48EI}$$

So, the first exercise is, you should look at the table which gives you generic solution, which one you need to use it for this given problem; that is the first exercise. And learn how to interpret what is stated in the expression, because it says how to handle this bracket. It is all very very simple; you should be alert; that is all. So, I have calculated y_1 and y_2 in a very simplistic manner.

And I also need the value of theta; at least the next step, we need this; that is given as

$$\theta_{max} = \frac{Wa^2}{2EI}$$

So, you should know how to use this information; that we will see for the spring force, there is spring force which is acting here.

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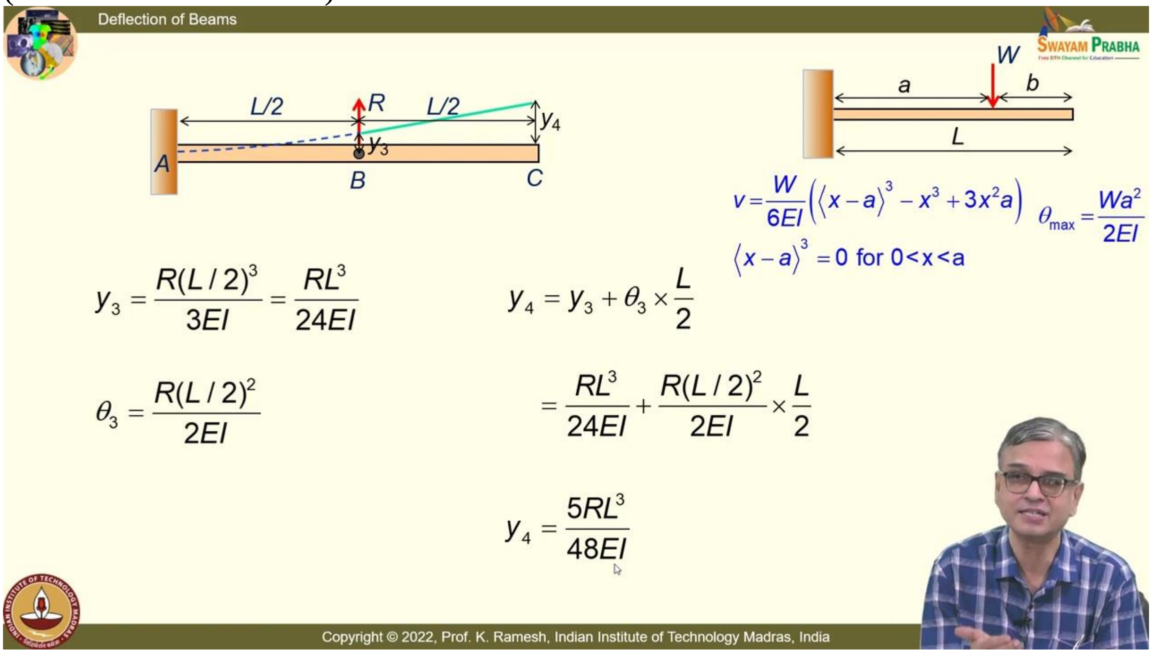

And can you tell me what is the deflection here y_3? You do not have to see anything RL^3; $R(L/2)^3$; that is what you have to do it, the length is $L/2$. So, you have to do that

$$\frac{R(L/2)^3}{3EI}$$

Certain things you know, you have to pick up speed, if you are expected to solve as many problems as possible before you come for the examination.

And we you have to remember $PL^3/3EI$; that is a very very useful result. So, I have

$$\frac{RL^3}{24EI}$$

and how do I find out the deflection at C? You need the slope here and you also need to know the length. So, this will be addition of y_3 plus something. So, I have this as θ_3 is given from this expression $Wa^2/2EI$, a here is $L/2$.

So, $\dfrac{R(L/2)^2}{2EI}$. So, you should know how to handle these equations. It's all very very simple; it is only high school knowledge is required, but you have to be alert and careful. And you have this as a straight line, it is emphasized in the diagram also. So, I have

$$y_4 = y_3 + \theta_3 \times \frac{L}{2}$$

See the $\theta_3 \times L/2$ we are doing it because we are handling very small deformation; that aspect you should remember, fine; that aspect you should remember.

So, I get this as

$$\frac{RL^3}{24EI} + \frac{R(L/2)^2}{2EI} \times \frac{L}{2}$$

So, I get this as

$$\frac{5RL^3}{48EI}$$

Once I determine these deflections, finding out the spring force is doable and also the end deflection.

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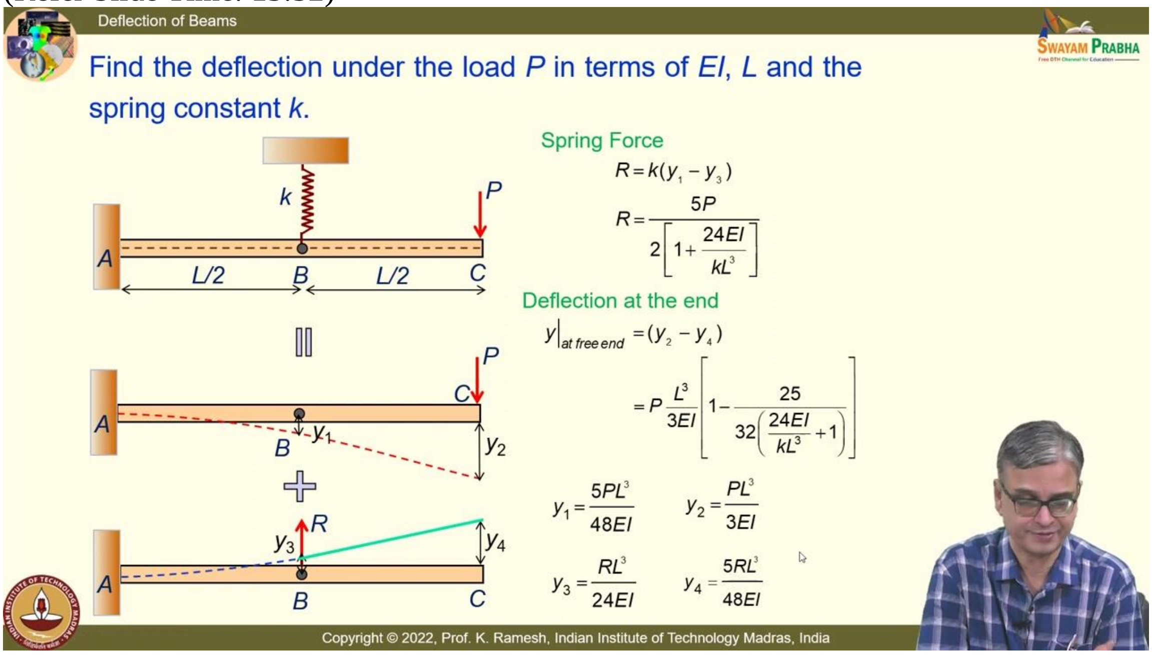

So, that is what I am going to summarize it here. This is looked upon as summation of two problems; one with the end load, another with the spring force; I cautioned that when you look at the spring force, you should put the spring force in the right direction.

If you do not put it in the right direction, you are solving a different problem. So, you should physically respond to what is given in the problem statement. So, I have y_3 and y_4 are available now; $y_3 = RL^3/24EI$; $y_4 = 5RL^3/24EI$ and how do I get the spring force?

I know y_1 and y_3. So, your spring force is nothing but

$$R = k(y_1 - y_3)$$

So, which you can do the simplification and when you do this the spring force comes to be

$$R = \frac{5P}{2\left[1 + \dfrac{24EI}{kL^3}\right]}$$

You may also verify my simplification if there are any typographical errors, please alert me and you can also get the deflection at the end. What is the deflection at the end? What is the deflection at the end? $(y_2 - y_4)$; that is all as simple as that $(y_2 - y_4)$. So, you can draw the deflected shape, assign the sign at the end; that is physically appreciating the problem and getting the magnitudes; that is the better way to handle method of superposition and this turns out to be; please verify my simplification,

$$P\frac{L^3}{3EI}\left[1 - \frac{25}{32\left(\dfrac{24EI}{kL^3} + 1\right)}\right]$$

So, I expect you to do the arithmetic simplifications.

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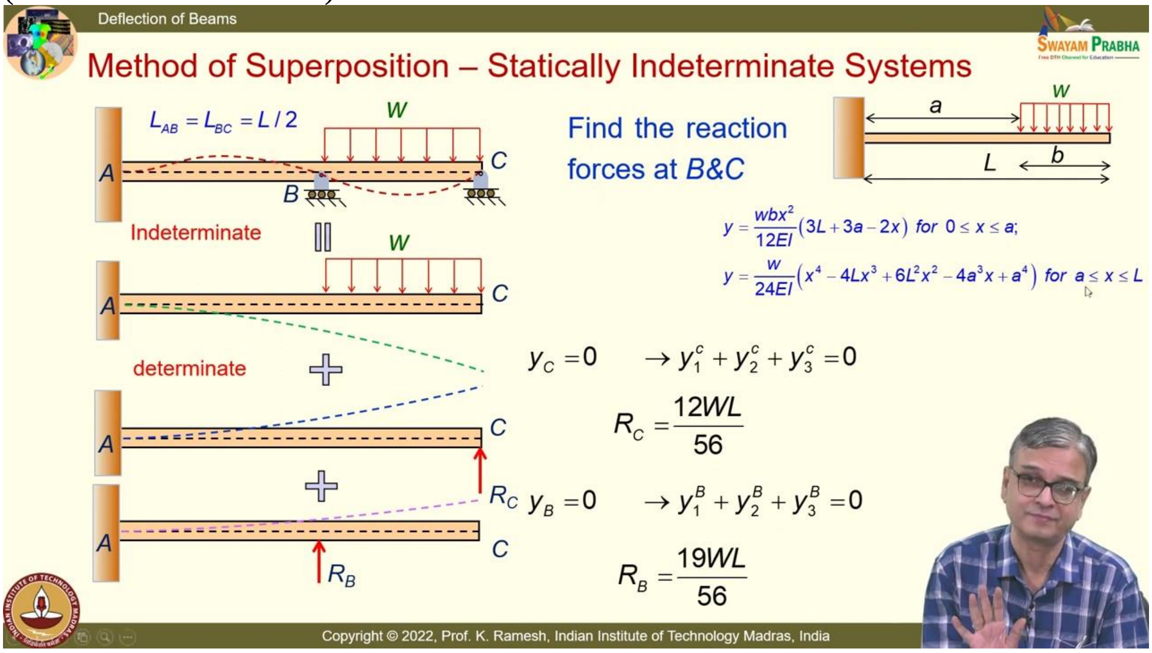

Then we move on to another problem. What is this beam? I have support here, I have support here, I have support here. It is continuously supported and it is statically indeterminate; I have written it in the title itself, but you should be able to recognize that this is statically indeterminate and the question is, find the reaction forces at *B* and *C*. And here, I said that you should be trained on visualizing what is the anticipated deflected shape. You know the supports, fine and I have a distributed load, what do you anticipate as the deflected shape here that you can draw? What is the anticipated shape? It will come down, is not it? I can draw that; I can draw this; that I can draw. The idea to remember is, here you will have a slope because this is not a fixed end.

How do you represent a fixed end? The slope has to be zero, deflection also has to be zero. So, I can I can sketch what is the portion here. Join these two intelligently; is the idea clear? See you should also; main purpose of getting into principle of superposition is to train you to visualize what happens to the beam even before you start solving it. If you have that anticipation, you can also verify have you done any mistakes in your mathematical development. So, the deflected shape; possible deflected shape is this.

So, the training in method of superposition is to make you visualize. Do not treat this as a problem of simple integration. It has a life, there is a physical problem; you should interpret the physics of the problem as well as possible. So, now I can split this as how many problems? Three problems that is very good. So, I have a problem number 1, I have problem number 2, I have problem number 3; and you should put those reactions appropriately and also filter out, what is the relevant solution that you should take it out from this.

See I need two of that; I need one for the cantilever and also for the in between load. Now you know, after solving a few problems, you find how useful the cantilever solution is; might as well remember it because when you solve several problems, it automatically gets into your blood. So, for these two, we have already seen the solution. So, you know how to get the deflected shapes and also the value of deflection. So, I have this as

$$y_1^c + y_2^c + y_3^c = 0$$

So, that is what happens here. So, that will help me to get me what is R_C; I would expect you to simplify and solve it yourself and get

$$R_C = \frac{12WL}{56}$$

Please correct me if I have made any simplification errors and y_B is zero. So, I will have

$$y_1^B + y_2^B + y_3^B = 0$$

So, this gives me

$$R_B = \frac{19WL}{56}$$

So, you have to recognize how many sub-problems I should have for a given loading and recognize whether it is statically determinate or indeterminate for your understanding; because when you split this, they are all statically determinate problems.

And you have from the table, what solution that you have to cull out; that also requires training and you know to interpret this, you should see what is the definition given. So, handle that very carefully.

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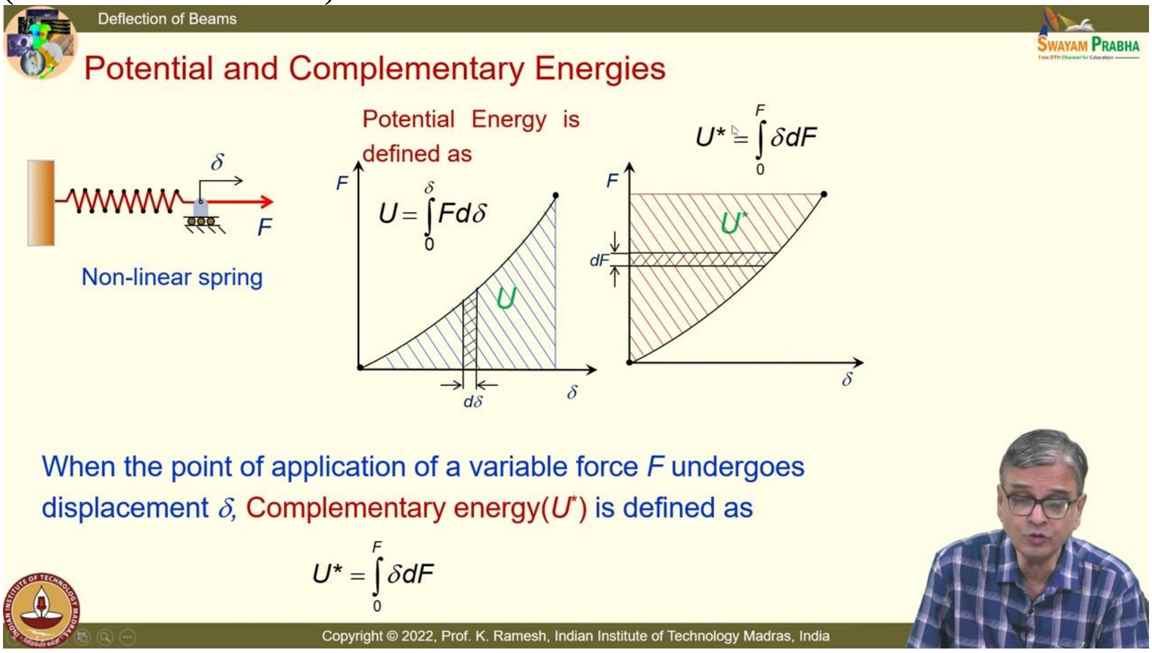

You know, one of the most useful methods in engineering and science development is handing energies. The moment you come to deformable solids; we say the specimen is going to deform that means it has capacity to store the energy. And to make our life simple, we say it is a reversible process, it gives out the energy there are no frictional losses.

Even though frictional losses happen in actual systems, you may try to minimize it and your mathematics becomes much simpler if these are reversible processes. And for illustration you take a spring, but you know very well when I pull a spring, it has a very complex response; we will also solve this problem later. But here what is looked at is, you have an engineering material which deforms similar to a spring. So, I have an axial rod which is pulled out; imagine it like this. And to make our life simple because we want to develop equation in a manner that if I differentiate the energy with respect to the force, I get the deflection; that is what we are going to prove.

We may live on linear systems, but to illustrate the important difference, why I have to work on what is known as complementary energy, we start with a non-linear spring. What is the implication of a non-linear spring? When I draw the force deflection relation, it is not going to be a straight line, it will be some curve. I have the curve like this, is the non-linear spring. And what is the energy stored? And in all these problems, so far, we have discussed, whenever we show a load P, the load P was applied gradually from 0 to P.

It is not that you have an impact load of P. We implicitly understood that these loads are gradually applied and the deflection is seen as the load is increased. So, the area under the curve is going to give you the energy stored. So, you have the potential energy, I have d delta. So, I can write this as; energy is given a symbol capital U. So, do not mix it up with small u, small u is used for displacement; capital U is used for energy. I have this integrated; integrated from

$$U = \int_0^{\delta} F d\delta$$

that is what you will write here, is not it? So, when I do this, I show it pictorially that we are calculating the area under the curve. If I know what is the force deflection relation, I can evaluate this quantity numerically. But this is not convenient for me to use for me to get the deflection. Now I look at, not this energy but I look at the energy in this zone, that is labeled as a complementary potential energy because the spring is non-linear, you will have some other quantity as complementary energy. Is the idea clear? What I have area under this curve or what I have area under this curve will be different; distinctly different, they are not same.

Here I have taken a small element and I have written the integral $Fd\delta$. If I do the same thing here, what way you anticipate? dF will come; that is what is, we are interested in that. When the point of application of variable force F undergoes a displacement delta, complementary energy which is always labeled as U^*; capital U^* is defined as; I have this dF. So, I find out what happens for the entire load application. So, I put this as δdF and if I show this pictorially, I have this as the area and these two are different you have the complementary potential energy area is different.

But this is very nice for me to do it; suppose I differentiate this expression, I differentiate dU^* / dF, I will get the deflection. In this chapter, we are finding out, what is the deflection? If I calculate the energy stored in the body; if I differentiate when I have a non-linear system, I should find out the complementary energy. Can you quickly see what is the simplification in linear systems? Think about it I am going to discuss it. So, this is the complementary energy and you know for sure, U will have some magnitude U^* will have some other magnitude as long as the system is non-linear. Suppose I have the system as linear, what way it will happen?

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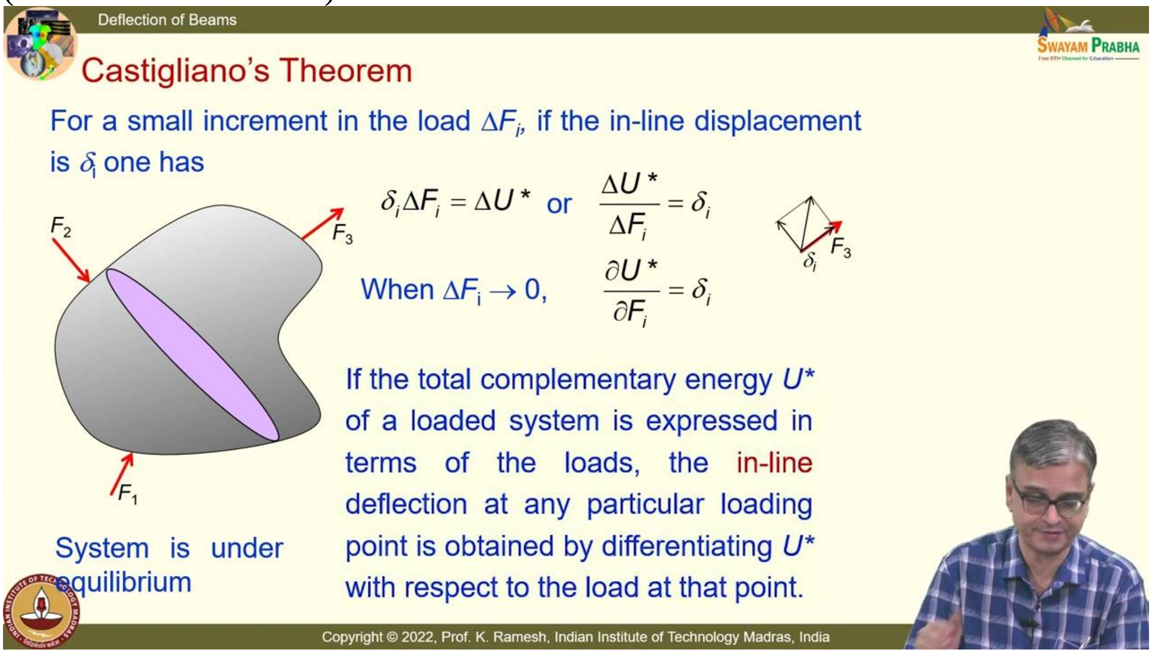

So, you have the Castigliano's theorem; what does the theorem say? For a small increment in the load ΔF_i, this is again a very important statement, if the inline displacement, it is also called work absorbing component is δ_i then one has.

So, you have to understand, see when I have the body like this, it will deform; at the point of load application, it can move in a generic path. Which is the one which causes the work or the energy to be stored? Only the work absorbing component that is the component of displacement in the direction of the force. This is a very important statement; in inline displacement. So, I have the force F_3 and I say that it has a deflection in this fashion; this deflection can be resolved into one along the force F_3 and one perpendicular to that and what you have as a displacement here, this is called inline displacement and that would be labeled as δ_i and this will be labeled as F_i, fine.

Only this contributes to energy storage or work done. And what does this theorem say? This theorem says

$$\delta_i \Delta F_i = \Delta U^*$$

and when you say you can also write this as

$$\frac{\Delta U^*}{\Delta F_i} = \delta_i$$

$$\text{When } \Delta F_i \to 0, \qquad \frac{\partial U^*}{\partial F_i} = \delta_i$$

The very powerful theorem which we would keep using it, even in your higher studies. If the total complementary energy U^* of a loaded system is expressed in terms of the loads; that is the key point. I should express my energy stored in the system as an expression of loads, I should get that; the inline deflection at any particular loading point is obtained by differentiating U^* with respect to the load at that point. So, I have to essentially get a mathematical expression, differentiate it then substitute the values; that is how you have to solve the problem when you have the energy method; even if it given as numbers, you must convert that into symbols. So, that you are in a position to differentiate it. Finally, put the numbers.

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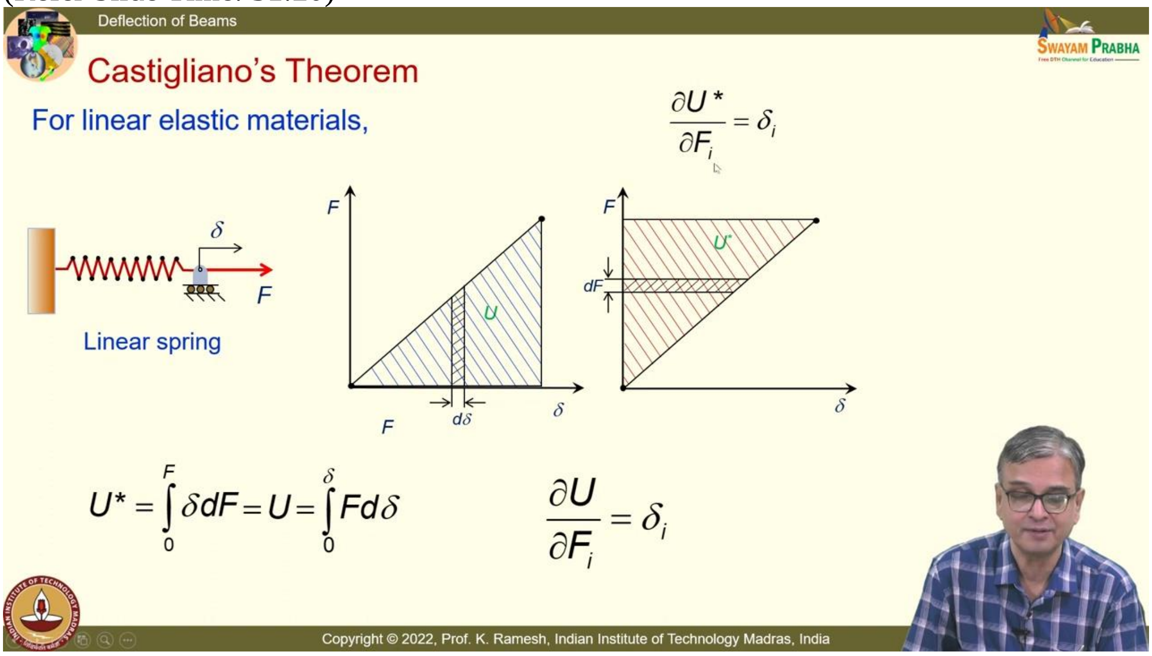

So, what happens in a linear system? Castigliano's theorem, we have developed it deliberately for a non-linear system to drive home the point we need to work with complementary energy you should never forget that. When I take a linear system, it so happens whether it is a; when I have a linear system, I have a line like this. So, automatically, this will be of equal areas. So, I have this as

$$U^* = \int_0^F \delta dF = U = \int_0^\delta F d\delta$$

So, I can calculate the expression in a very comfortable manner and for a linear system, what you have as

$\frac{\partial U^*}{\partial F_i} = \delta_i$ simply reduces to $\frac{\partial U}{\partial F_i} = \delta_i$

So, very powerful theorem, looks very simple; it takes a very small time to develop, but its utility is very significant. Very complex problems, you can solve. I have said, in our development of moment curvature relation, we ignored the effect of shear, isn't it? And I said that I am going to tell you a method by which you will be able to find out what is the shear contribution to the deflection.

We will solve that as a problem. So, many of those seemingly complex problems you can easily handle if you visualize it from the energy point of view, but the development is very simple. Now, what we need to understand is, how to calculate the energy?

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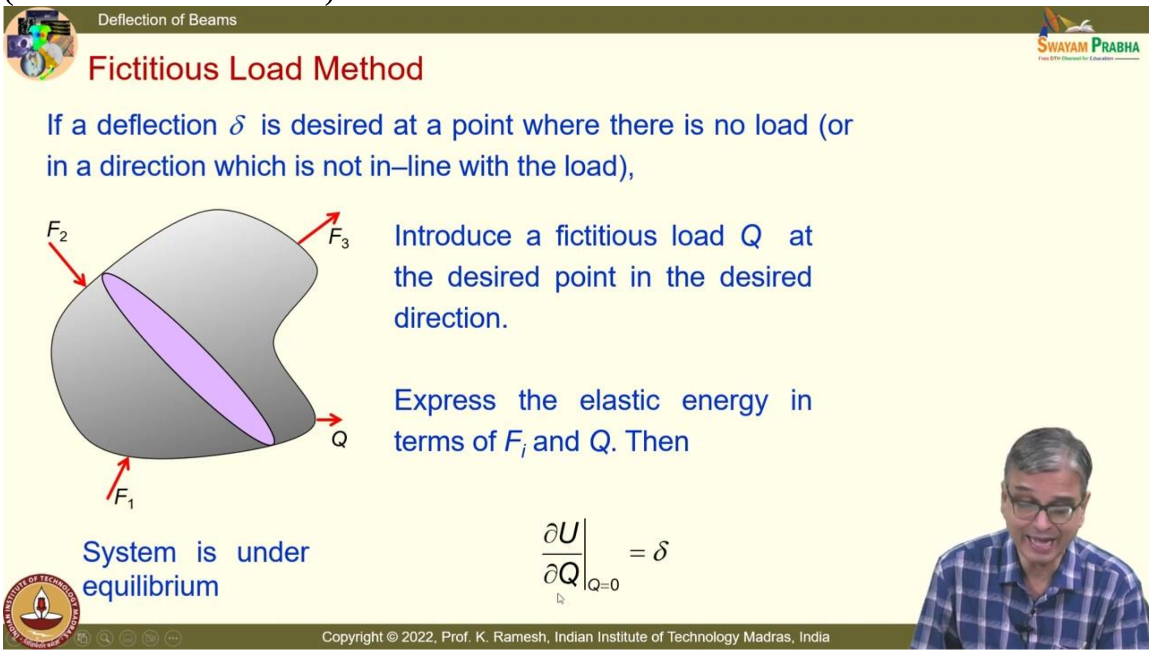

But before that, you know, you may not have the luxury that I want to find out the reflection at this point or this point or some other point. I may not have a load there; one of the requirements of this is, I should have a load acting at that point, fine. And I should differentiate with respect to that, then I will get the inline displacement. The method itself says, it says fictitious load method that means you wherever you want to find out the deflection, you introduce a load.

If a deflection delta is desired at a point where there is no load or in a direction which is not in line with the load, introduce a load and find out the energy because of that Q. There it can be a point here, it can be it can be; that is not in line with this, I want to find out what is perpendicular to that or I can also have in another point. Introduce a fictitious load Q at the desired point in the desired direction. Express the elastic energy in terms of F_i and Q then you have $\partial U / \partial Q$; after you differentiate, you make Q equal to zero, you will get delta. This also we will solve, we will try to solve the problem then you will understand; it is a very powerful technique.

It is a; in a jiffy, you can solve even complex problems, you do not have to do complex integration or anything like that; very simply you can do that. So, now the focus is, how to calculate the energies? Strain energy; we will go and look at that.

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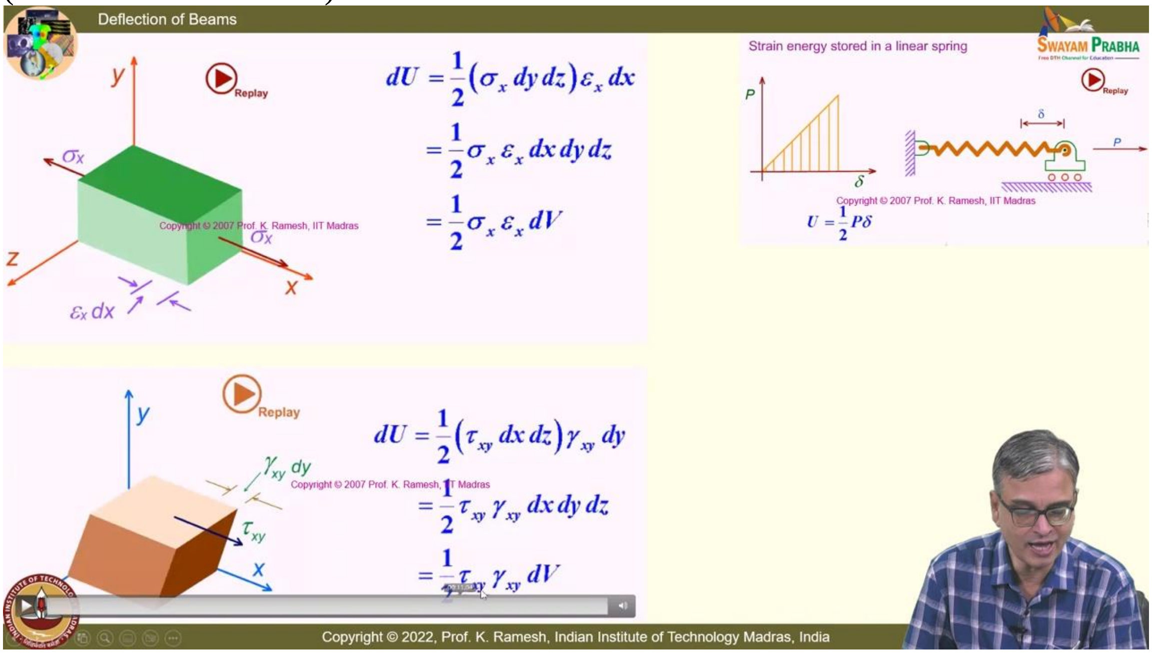

Now here again, though I have shown a spring, imagine that I have a rod made of aluminum or steel which is pulled out and I have a gradual load application. When I have a gradual load application, it is a triangle.

So, I have this as $\frac{1}{2}P\delta$. So, what I have; force into displacement is work done; because the load is applied gradually, I have this as half. So, you should never forget that. Long time back, you have done virtual work, there the physics is different. Here, I have this load applied gradually.

I have the quantity one half coming into the picture. So, similarly, you know, I can develop the set of equations if I have an axial stress or if I have a shear stress because we have already discussed what happens when a member is subjected to tension, when a member is subjected to torsion, when a member is subjected to bending; you know what are the stresses developed. Once you know what is the area, I can convert the stress into force. So, what this expression says is force into displacement multiplied by one half same thing we will also do it here. So, we will do it for the axial stress. I have this as ε_x and I have this as force $\sigma_x dydz$ gives me the force and $\varepsilon_x dx$ gives me the deflection, the elongation in line displacement; It's one half.

So, when I do this, I get an expression

$$\frac{1}{2}\sigma_x \varepsilon_x dV$$

So, if I go to torsion, find out what is the stress; for torsion, we have to go to shear stress, but if I go to bending, I know the expression for σ_x, I know the expression for ε_x. So, I can calculate what is the energy stored because of a bending moment. Is the idea clear? So, now I go to shear stress; when I apply the shear stress, what happens? You know γ_{xy} is used; γ_{xy} is the way the mathematics got developed. Only when the tensorial transformation came, people understood that you should always use $\frac{1}{2}\gamma_{xy}$ for transformation, but you write this like this.

So, I have the force as $\tau_{xy} dxdz$ and the in-line displacement as $\gamma_{xy} dy$. So, I get this as

$$\frac{1}{2}\tau_{xy}\gamma_{xy} dV$$

So, these are very fundamental expressions now I will go and see what happens in an axial load, what happens in a torsional load, what happens in a bending load.

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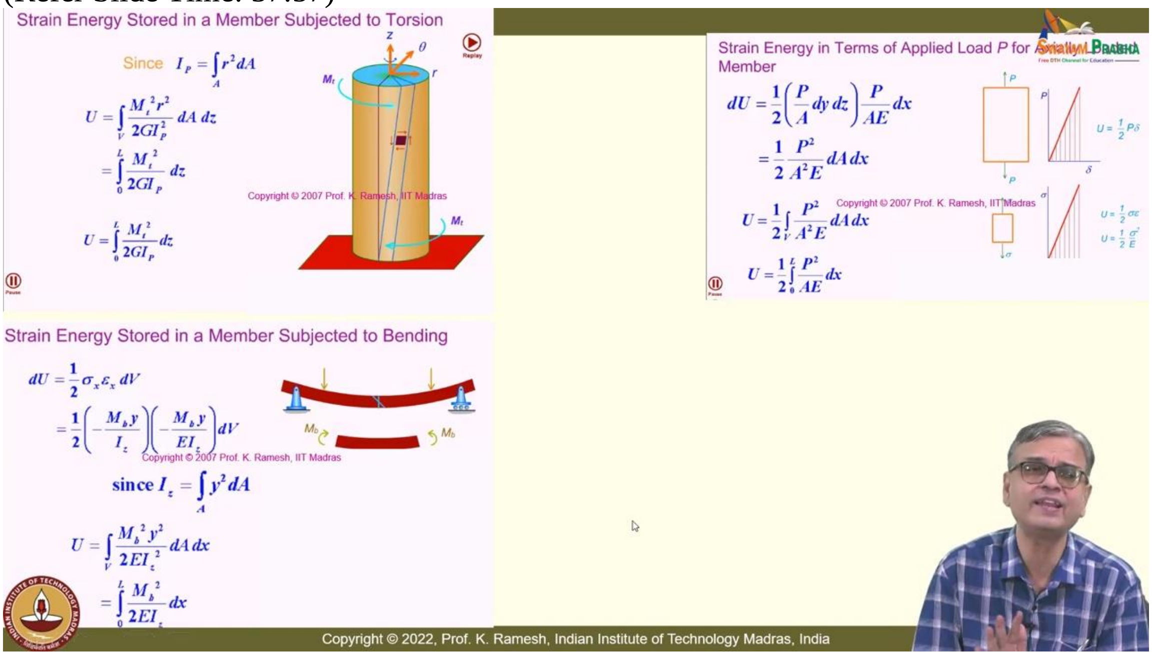

I have this as P/A, σ_x; $\varepsilon_x = P/AE$. So, when I do this, I get this as

$$\frac{1}{2}\frac{P^2}{A^2 E} dA\,dx$$

because I can write the volume as $dAdx$. So, I have the famous expression,

$$U = \frac{1}{2}\int_0^L \frac{P^2}{AE} dx$$

you are talking about a slender member, fine;

Now, let us go to torsion. So, when I go to torsion, I have

$$\tau_{\theta z} = \frac{M_t r}{I_P}$$

$$\gamma_{\theta z} = \frac{M_t r}{GI_P}$$

We have a recipe, how to get the energy when I know the strains and stresses. So, I get the torsion energy as integral

$$U = \int_0^L \frac{M_t^2}{2GI_P} dz$$

Look at the similarity between the two; it is very very similar the expressions are very similar; it is easy to remember also, easy to derive and easy to remember also. Can you anticipate what way you will get it for bending? It is very similar; it is cyclically it is coming.

So, I have this σ_x and ε_x is there and when I say bending I have we are talking about pure bending and then away from the load application points. Away from the load application points, we have this. So, I have σ_x and ε_x. So, I can find out what is the energy stored and when you simplify, this also has an identical picture like this I have this as

$$\int_0^L \frac{M_b^{\ 2}}{2EI_z} dx$$

All these three expressions are very similar. You will appreciate the utility when you solve a practical problem.

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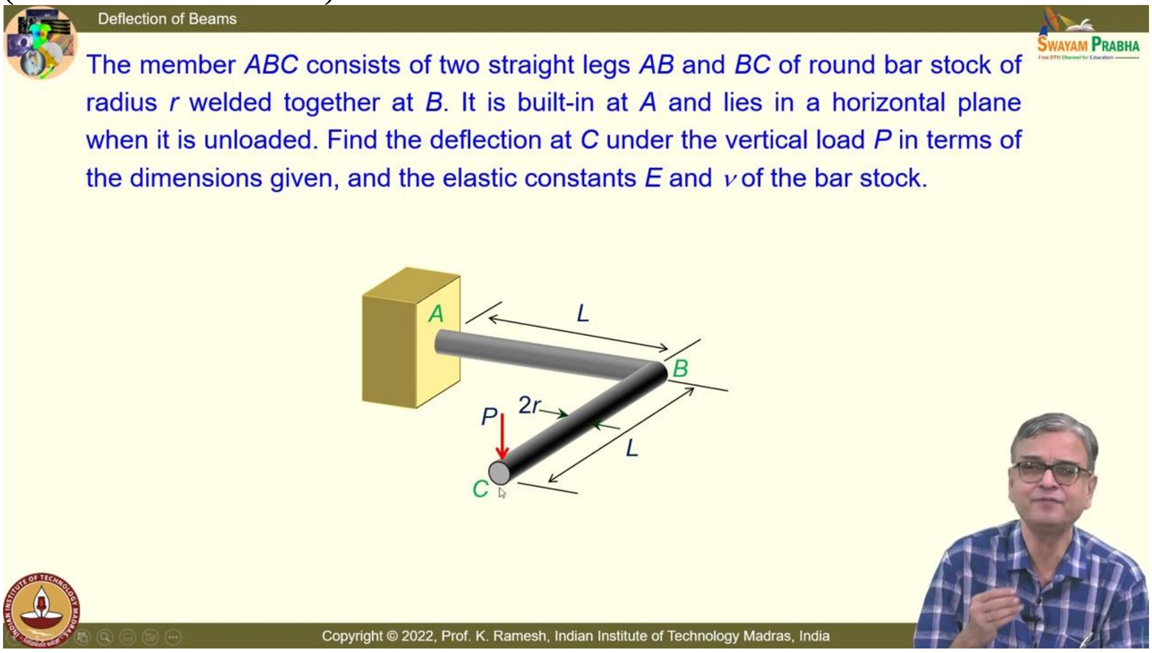

I have a problem where I have a member like this it is like you are having a hand like this somebody is putting a load. So, you should visualize what happens to your shoulder, fine. What happens in this section *AB*? Section *AB* subjected to what? Section *BC* subjected to what? Only torsion. *AB* subjected to torsion as well as bending and *BC* subjected to only bending and we ignore the shear deformation.

We do not want to complicate now. I will solve shear deformation as a separate problem.

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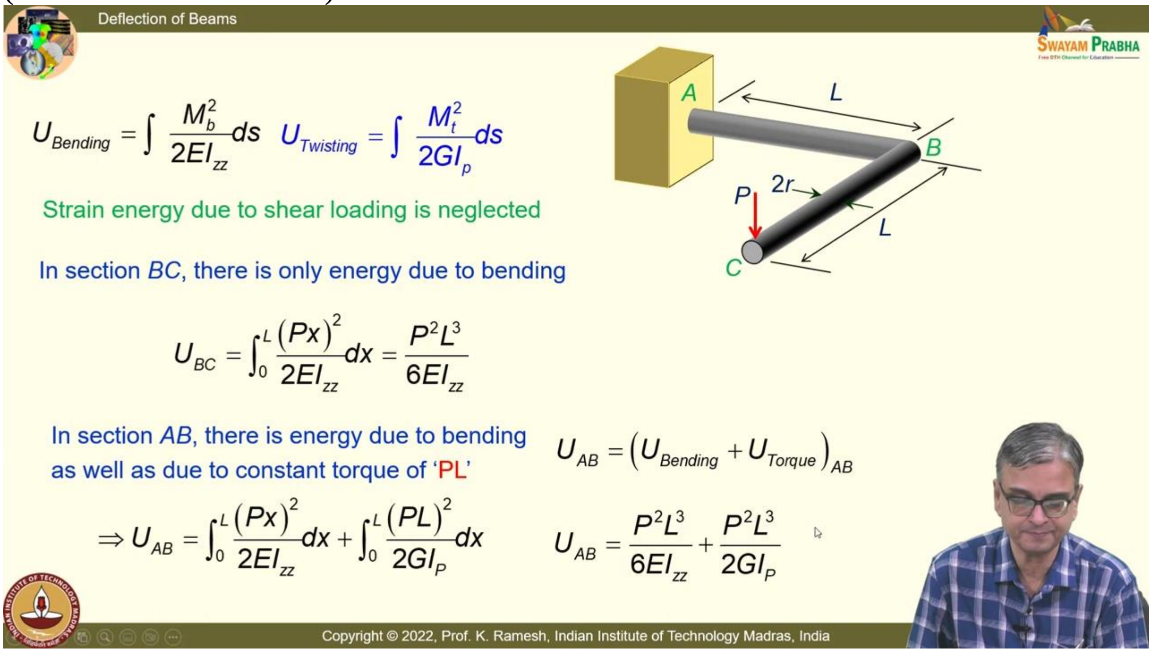

So, you should recognize that I have to calculate the energies for torsion and bending. So, first, I start with the expressions for bending which we have already developed instead of d x I have put this as d s because I have a member like this. So, it represents length along the member and you can find out the expression for bending moment as a function. Strain energy due to shear loading is neglected; that is what I said earlier to start with.

So, in section *BC*, there is only energy due to bending. So, I can write the bending moment is Px. So, I had $\frac{Px^2}{2EI_{zz}}dx$; even the sign does not matter because it gets squared. So,

$$\frac{P^2L^3}{6EI_{zz}}$$

is what I get. In section *AB*, there is energy due to bending as well as due to constant *torque* of *PL*. So, U_{AB} will have contribution from bending energy, contribution from torque. So, I get this as for the bending energy, I do like this and for torsion energy, I get this

$$\frac{PL^2}{2GI_P}dx$$

And you can find out, for a circular cross-section, what is I_{zz} and I_p and you know the interrelationship

$$I_P = 2I_{zz}$$

So, you can simplify it like this mathematically which is done in the next step; here I have put

$$U_{AB} = \frac{P^2L^3}{6EI_{zz}} + \frac{P^2L^3}{2GI_P}$$

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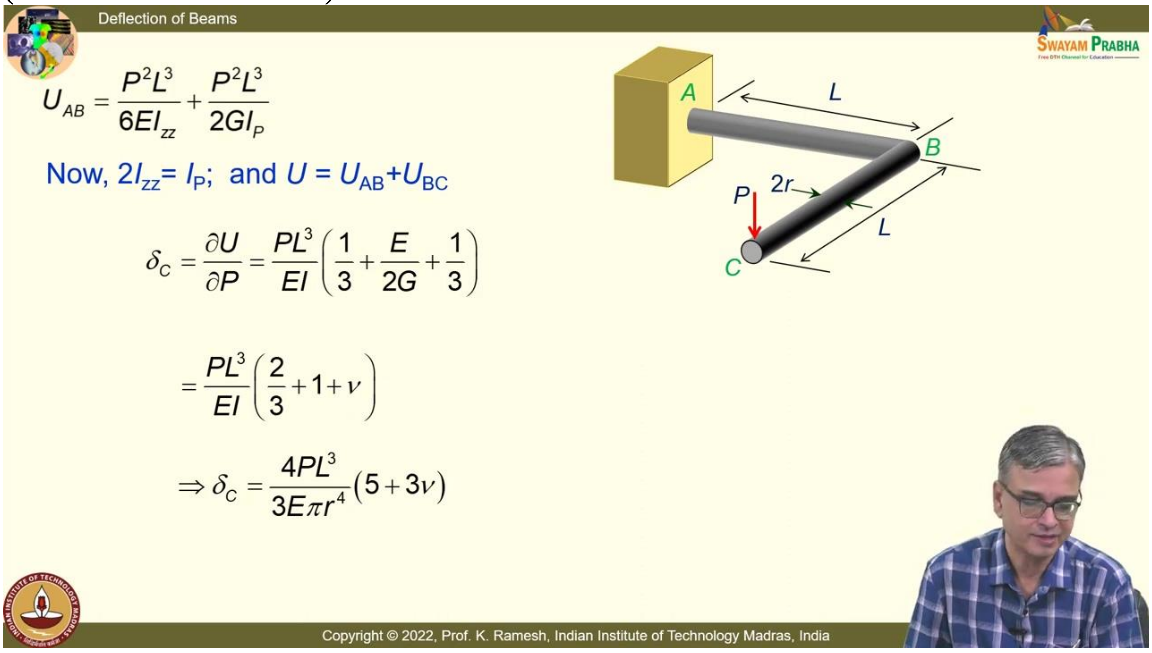

So, you can replace it like this. So, I have this as I want to find out what is the deflection at the point P;

$$\delta_C = \frac{\partial U}{\partial P} = \frac{PL^3}{EI}\left(\frac{1}{3} + \frac{E}{2G} + \frac{1}{3}\right)$$

So, I get this as finally,

$$\delta_C = \frac{4PL^3}{3E\pi r^4}(5 + 3\nu)$$

You try to solve it by other methods, then you will know the advantage and the conceptual appreciation of this; it comes in a very very elegant manner.

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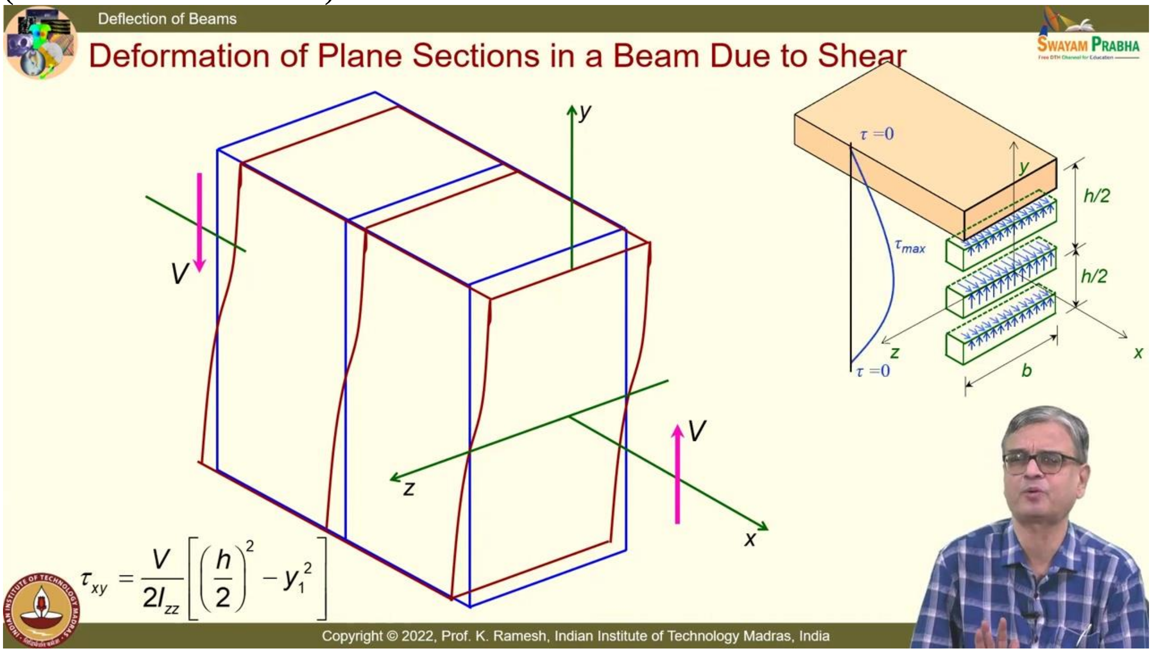

Now, we move on to understanding what happens when you have a shear? It is drawn in a very big picture wherein you see that I have shear deformation and you have the expression for τ_{xy} which we have derived

$$\tau_{xy} = \frac{V}{2I_{zz}}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right]$$

And when you visualize it pictorially, it is like this you have shear stress variation; it is varying parabolically and the interest here is what way this alters the deflection. We will take a very simple problem; we will take a simple problem of cantilever.

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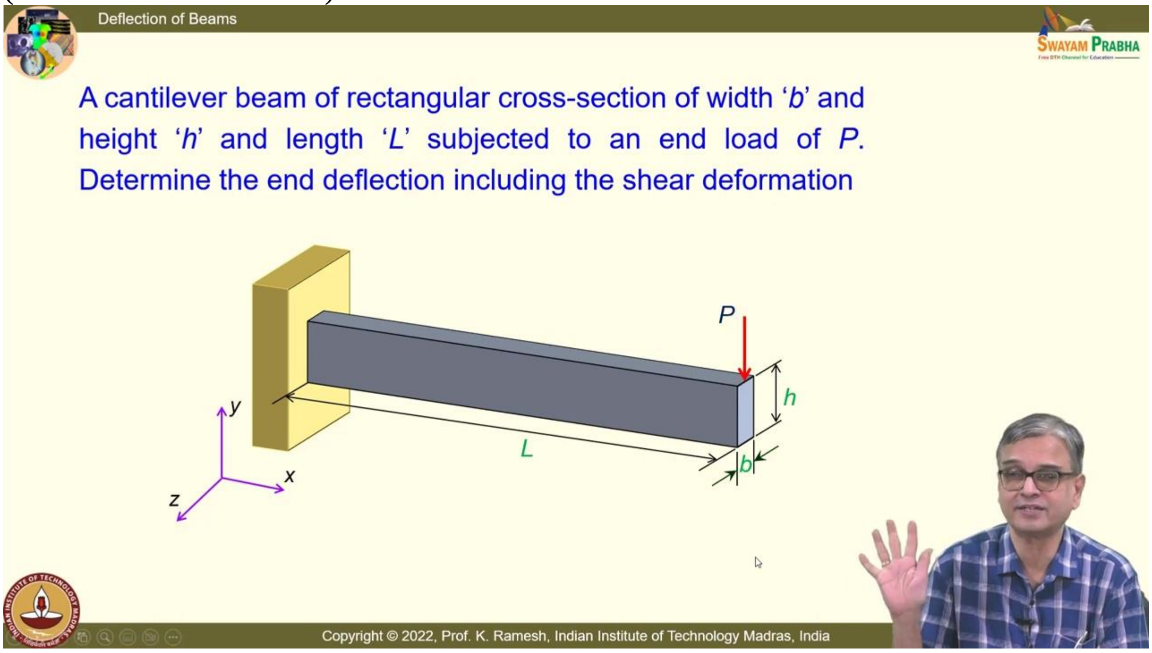

If I articulate the problem differently, you have a very sophisticated instrumentation and find out what is the deflection at this point, it will not equate to $PL^3/3EI$. Whereas, all along we have said it is $PL^3/3EI$; when it is not matching with the experiment do not blame the experiment, your equipments are very old that is why it is showing a deviation. You do a very careful experimentation, very sensitive experimentation you can do, you will find there is a difference, but difference is very very small. Now, the idea is how do I calculate this? With the energy method, you bring in the energy due to shear that is all you have to do. We have all the necessary expressions.

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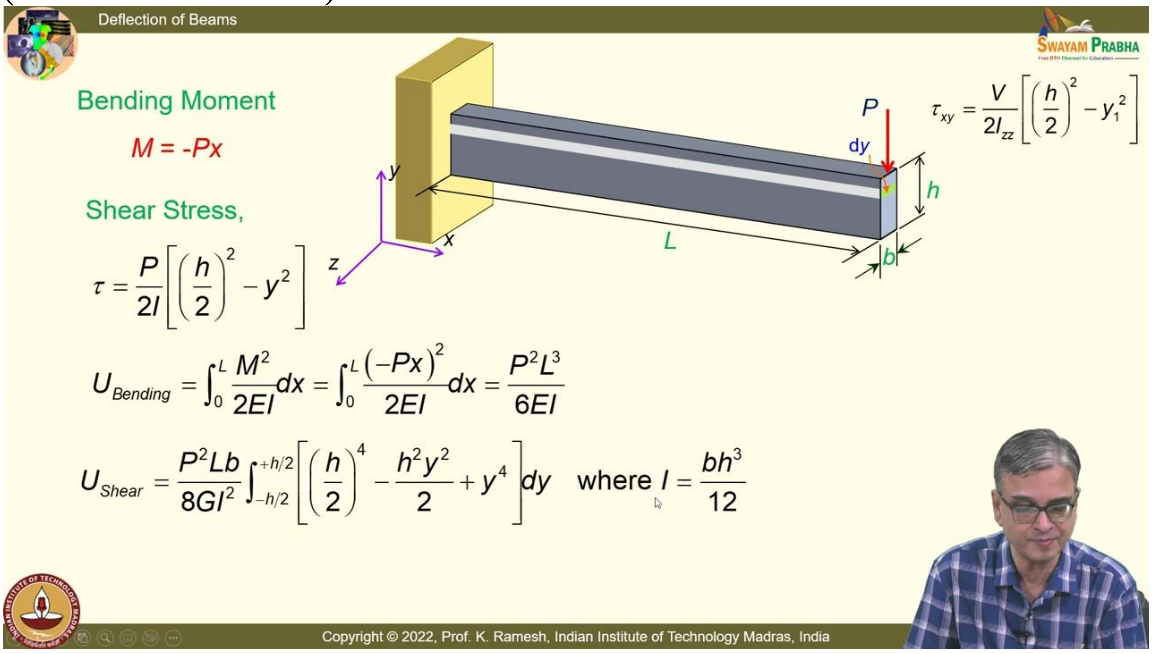

So, I have the bending moment and I have the shear stress; I have the expression like this in the context of this problem, replace it as

$$\tau = \frac{P}{2I}\left[\left(\frac{h}{2}\right)^2 - y^2\right]$$

I have written it in a generic fashion. And

$$U_{Bending} = \int_0^L \frac{M^2}{2EI}dx$$

So, this is what I said, even the sign if I do not put it properly; in the earlier problem, I have not put the sign, it is getting squared. So, it gets absorbed in your mathematics I get this as

$$\frac{P^2L^3}{6EI}$$

that is fine. Now, I have to write for the shear. Shear is $\int \frac{\tau^2}{2G} dV$ which you will have to look at we have already derived this expression.

I have this as $1/2G$, you have to interpret this mathematics properly. See I have *h*, I can see this as $-h/2$ to $+h/2$ and I can also visualize that I have a small this one, dy because the shear stress varies as a function of the distance from the neutral axis. For this entire length, I can have this as $Lbdy$ is your volume and that I integrate from $-h/2$ to $+h/2$. So, I would expect you to do this mathematics I am going to show what is the result that I am getting it. So, on the first stage of simplification I get this;

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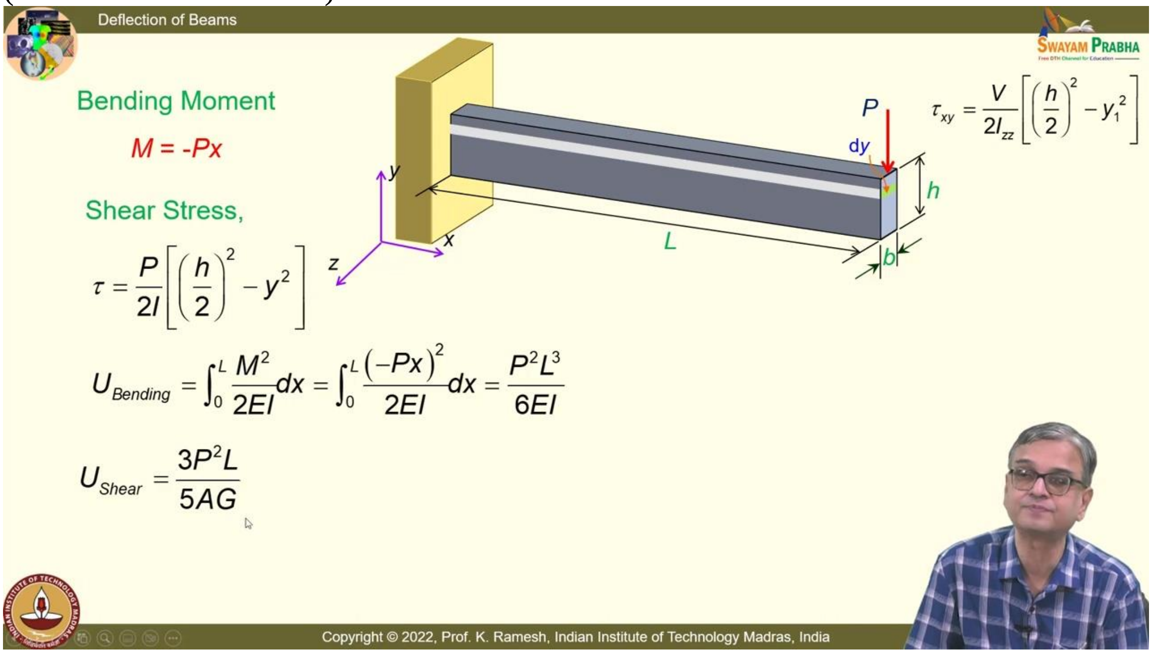

I have not substituted for $I = {bh^3}/{12}$; when I substitute for that also, I even here I have not substituted, but this simplification is done when I finally, substitute, I get this

$$U_{Shear} = \frac{3P^2L}{5AG}$$

So, what you can actually do is you can take a realistic problem supply the values of P, L, A and G for the material, then you will know still I have not got the deflection, deflection is nothing, but ${\partial U}/{\partial P}$ and energy is bending plus shear.

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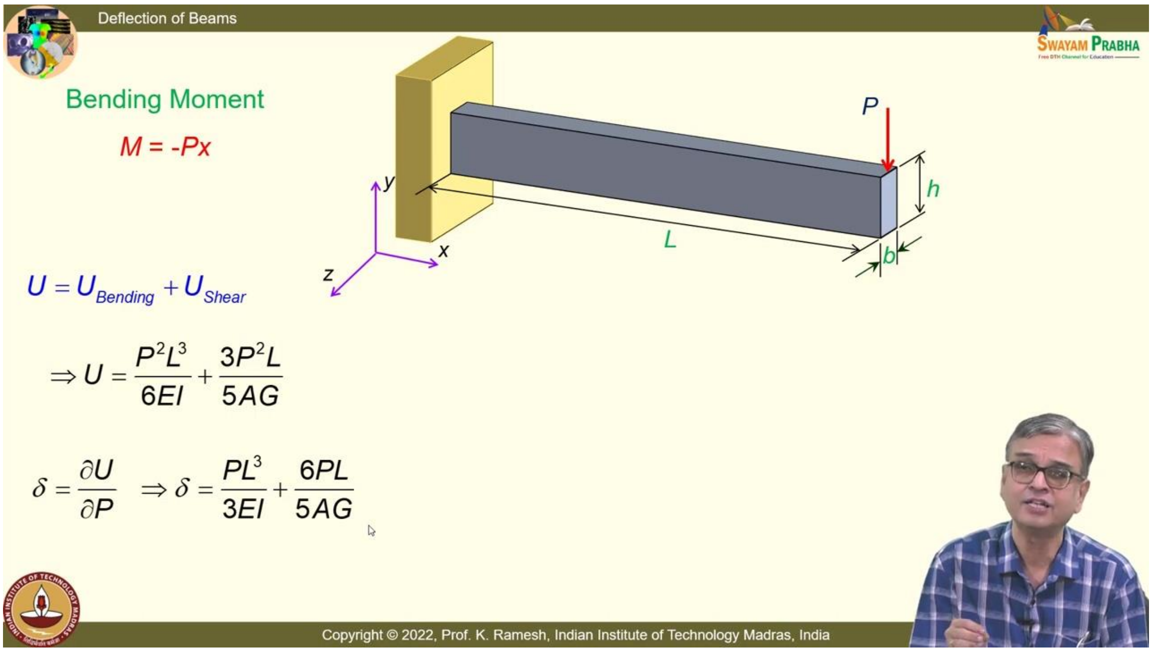

So, I will take the bending plus shear and differentiate it with respect to *P*; *U* is given. So, when I differentiate it with respect to *P*, I get this as

$$\delta = \frac{PL^3}{3EI} + \frac{6PL}{5AG}$$

that is the deviation from your $PL^3/3EI$. So, you can substitute for a given; whatever the problem that we have solved earlier or you give a fictitious distance of *L* and take a slender cross-section, you will find this contribution is extremely small. So, in this class, we have solved a variety of problems by the method of superposition and we have also looked at the powerful method of use of Castigliano's theorem and solving the deflection by using the energy approach. And energy approach is very, very interesting and there are many numerical methods that are developed based on the energy and it is very useful; simple from a mathematical perspective. Thank you.