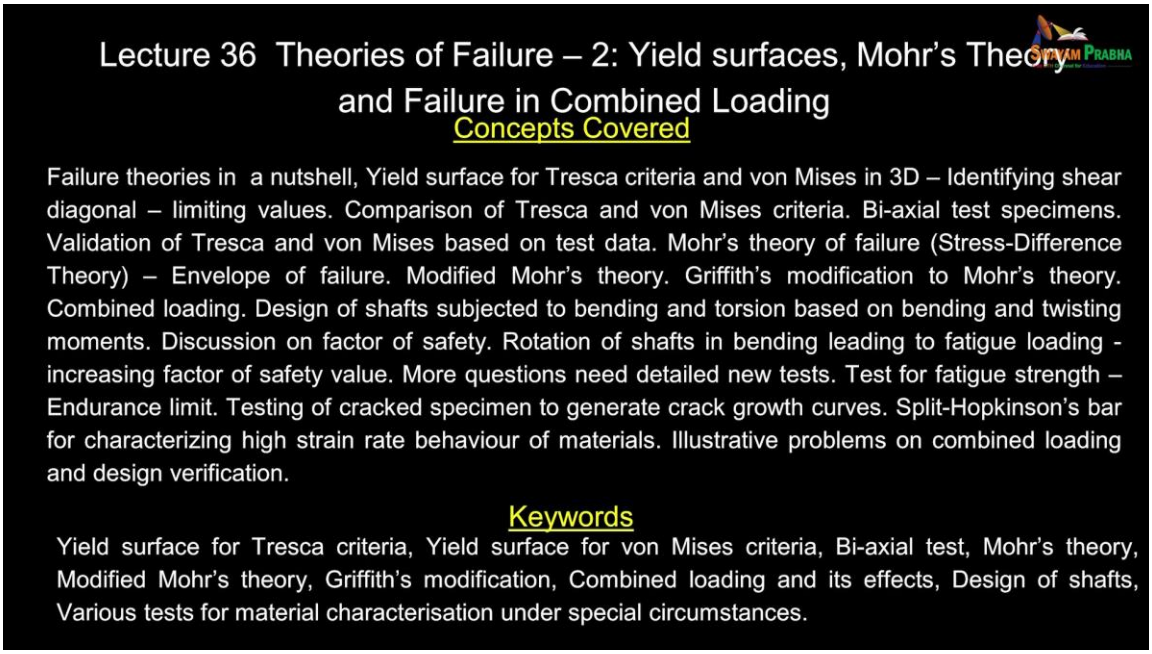
Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

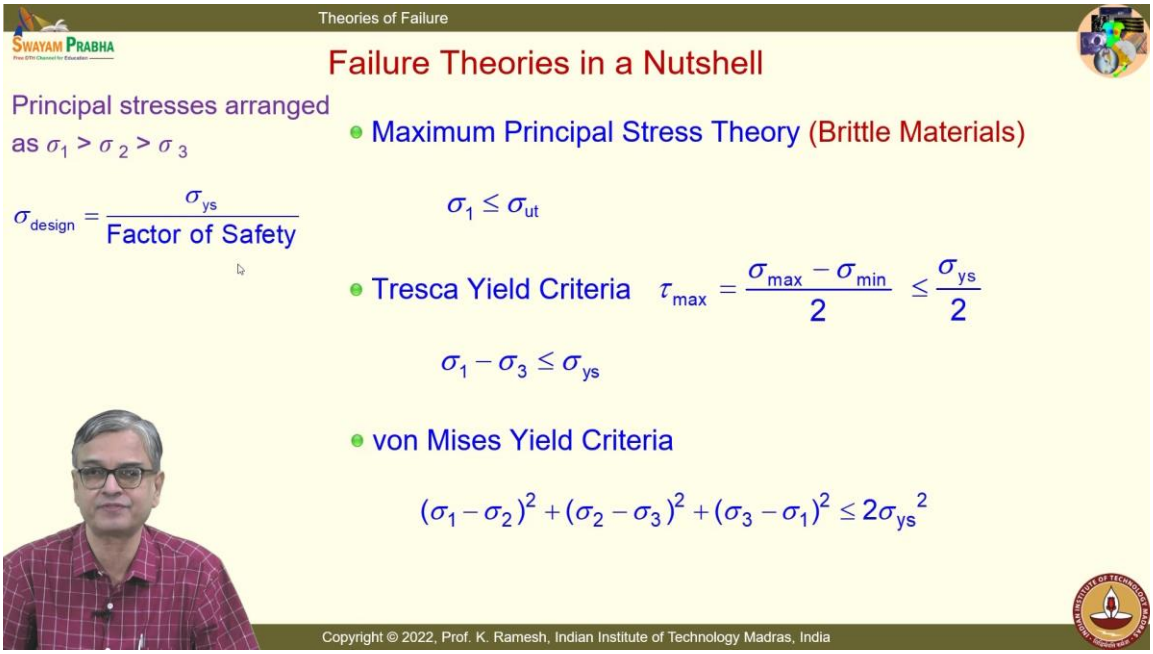
Lecture - 36
Theories of Failure 2 Yield surfaces, Mohr's Theory and Failure in Combined Loading

(Refer Slide Time: 00:20)



Let us continue our discussion with theories of failure. In fact, you should recognize that these theories basically developed from a simple tension test. Later, people have conducted experiments on geometries where they could change both σ_1 and σ_2. Then verified, it satisfies even in a biaxial situation.

(Refer Slide Time: 00:54)



And you know as far as brittle material is concerned, we have looked at maximum principal stress theory. I said once you go to brittle material, the ultimate tensile strength becomes important. There is nothing like yield strength. The material does not give you any warning. It simply fractures. And we will not try to apply maximum principal stress theory to a ductile material, ok.

And one of the very famous theories and it was also developed very early, was Tresca yield criteria. The way you will have to appreciate this theory is, we are monitoring what happens to the shear stress. And it is written as

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

When this reaches a critical value in a tension test; in a tension test, you go only up to σ_{ys} because you are pulling the specimen. You are pulling the specimen and you want to see that when yielding has taken place. You use that information, and you limit the shear stress at that stage to be the permissible one. And one of the aspects you have to keep in mind is, we have arranged the principal stresses as, $\sigma_1 > \sigma_2 > \sigma_3$, algebraically. It is a very important convention that we follow. Then you have rewritten this as $\sigma_1 - \sigma_3 \leq \sigma_{ys}$ is permissible. I can write it less than equal to or greater than or equal to. Less than equal to says it is permissible. Greater than equal to means you should not exceed. The interpretation is slightly different that is all. So, rather than $\sigma_{max} - \sigma_{min}$, if you write it as $\sigma_1 - \sigma_3$, even when both the principal stresses are positive and both the principal stresses are negative, when σ_3 is zero; you will never make a mistake in the calculation.

And we have also seen von Mises yield criteria. And that reduces to

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \leq 2\sigma_{ys}^2$$

And I have also said that von Mises yield criteria comes basically from distortion energy theory. Later, you have this octahedral shear stress that was monitored by Nadai and then he said whatever the results you get from octahedral shear reaching a maximum and distortion energy reaching a maximum, the functional form of equation from the purpose of calculation reduces to the basic equation like this. And it also a verification that the yield criteria has more justification, because you have approached the problem from two different directions, both of these give identical results.

And finally, when you do the design, nobody even goes anywhere near this yield strength. You have what is known as factor of safety. So, you have a design strength. So, all of these equations will change;

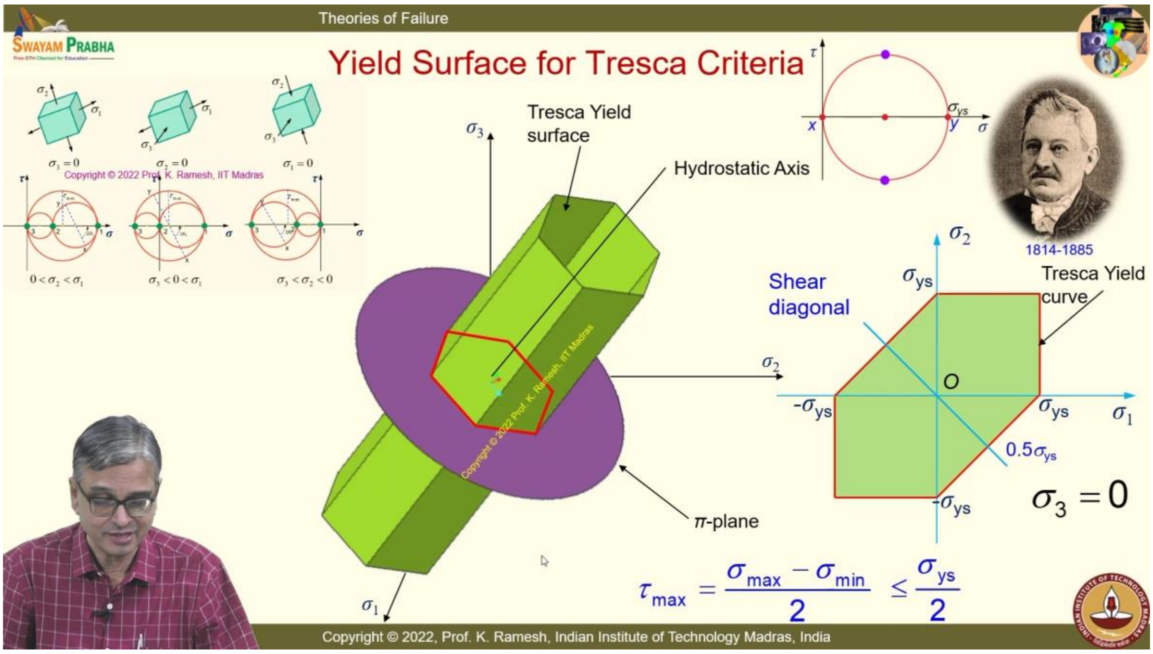
$$\sigma_1 \le \sigma_{\text{design strength}}$$

$$\sigma_1 - \sigma_3 \le \sigma_{\text{design strength}}$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \le 2\sigma^2_{\text{design strength}}$$

And one of the most challenging aspects in design is how to estimate this factor of safety. There are multiple considerations that go into it. We will discuss some of the aspects in the class today. And some other aspects when you do machine element course; design of machine element course, you will have more comprehensive development on how to decide factor of safety.

(Refer Slide Time: 05.18)

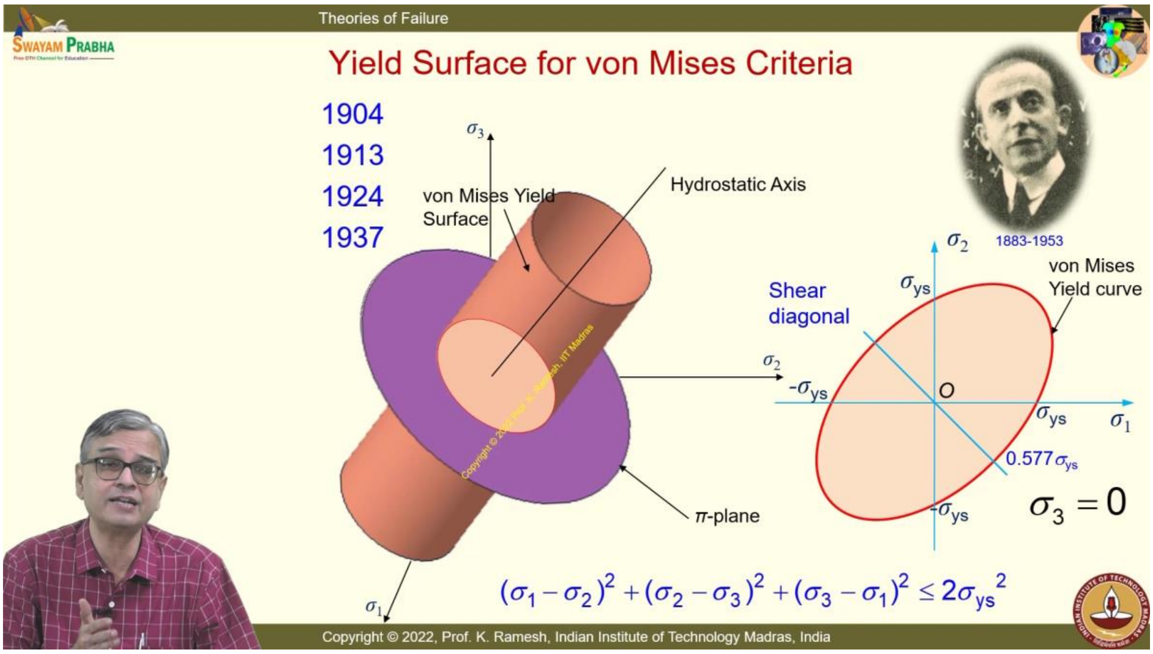


And you should recognize that this was one of the earliest criterion developed in 1864. And I have also been cautioning that when I have this, can I call this as maximum shear stress? You have to look at $\sigma_1 - \sigma_3$, only that gives the maximum shear stress. This becomes particularly important when both the principal stresses are positive or negative, like what you have here. You normally ignore zero, do not ignore zero; zero is very important and that is a contribution by Indian mathematicians. And with all your environment, now you would have zero emissions, ok. So, zero is very, very important and one better way of understanding and implementing Tresca criteria is use that as $\sigma_1 - \sigma_3$, ok. And we have also seen that you can have what is known as a pi plane, which is referred in the principal stress reference σ_1, σ_2 and σ_3 that takes the form of a hexagon, ok. And you know if you want to draw this, you can appreciate I do a simple tension test, I can find out this point. I can find out this because it goes to σ_{ys}. I can also say that I am keeping the specimen horizontal and apply it, I can also get this point. And I can also have a provision wherein I do a biaxial test; you will see the biaxial specimen. I can apply equal values of σ_1 and σ_2, even in that case because σ_3 is zero, from the strength of this, you will again say $\sigma_{ys}/2$ as the maximum shear. So, I can get this point. So, you can imagine how the first quadrant is obtained as a limit from our simple understanding of tension test. And tension test also says something about the magnitude of the shear stress. When σ_{ys} reaches in the tension test, from your Mohr's circle, you can find out what is the maximum shear stress. And once you have a pure shear stress state, you can always look at it as combination of tension and compression. That we have repeatedly seen, we have used it for establishing interrelationship between Young's modulus and shear modulus. And when we wanted to paste a strain gauge on a shaft to determine torsion, we have utilized that knowledge because strain gauge can measure only axial strain, fine. So, the question is, how do I find out the plot in the second and fourth quadrant? One way of doing this is you recognize the shear diagonal. On a shear diagonal, you have this as point σ_{ys}. That means, I can have normal stress as one principal stress as σ_{ys} multiplied by 0.5. Other one is $-0.5\sigma_{ys}$, that represents your pure shear stress state. And all of this comes from your Mohr's circle. Once you look at the Mohr's circle, I have in the tension test it reaches the value σ_{ys}. The maximum shear stress is limited by $\sigma_{ys}/2$. So, if I imagine the principal stresses are not positive, it can be positive or negative combination. The positive and negative combination, the limiting value is only $0.5\sigma_{ys}$ and $-0.5\sigma_{ys}$, when both of them are of equal magnitude. And simply, they have joined it by a line and constructed this Tresca yield curve.

So, in a σ_1-σ_2 plane, you see this as elongated hexagon. When you have this $\sigma_1, \sigma_2, \sigma_3$ reference, where I have seen this as hydrostatic axis which is equally inclined and we have also seen this as a π–plane. The same elongated hexagon appears like a regular hexagon you see here.

See, nature also loves neat geometrical patterns, fine. We will go and see how do you get the von Mises yield criteria in a similar three-dimensional fashion. You know that is very interesting to see. When you see that, you will appreciate the nature is very interesting.

(Refer Slide Time: 10.16)



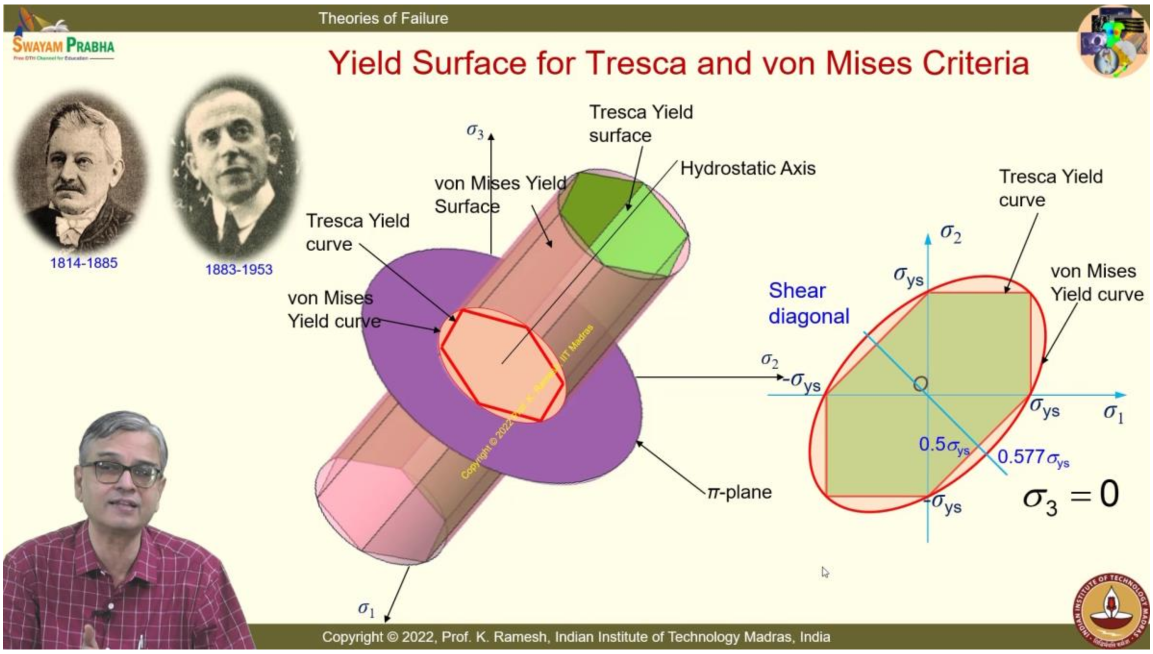
And the moment you get into von Mises, there are multiple contributors. It started at 1904 by Huber, 1913 by von Mises, 1924 better explanation from Hencky and then 1937 you have altogether a different approach by Arpad Nadai, who concentrated on what happens to the octahedral shear stress, ok. So, you have multiple contributors and if you go to historians; see historians always go back and then see even a trace of that knowledge related to what is the modernity, they want to give the credit to him also. And you will be surprised Maxwell also have thought about a theory similar to this. And Maxwell you understand mostly in electrical sciences, ok. So, in those days, it is only physics; all branches of physics come under one umbrella.

And you find this von Mises yield surface is a cylinder like this and then the curve is a circle in the $\sigma_1, \sigma_2, \sigma_3$ plane. But if you put it in σ_1, σ_2 reference axis, it gets elongated, it takes the shape of an ellipse. So, if you plot this, so if your stress state is within this ellipse, no yielding is possible. If it is on the boundary or exceeds, then yielding is definite in a given material.

See all of these people have conjectured based on a simple tension test; you should never lose track of this. You can justify that these theories are useful only when you conduct complicated experiments, plot the experimental data and verify whether it matches with that. We will also see that.

Now, the next question is, I have shown von Mises independently, I have also shown Tresca independently, you have seen the shapes. It is a hexagonal cylinder in the case of Tresca; it is a circular cylinder in the case of von Mises. How do these two compare? That is also very interesting to see. And here also, I have shown the shear diagonal and here what it says is the shear diagonal, the value is higher than what is predicted by Tresca. It says it can go up to $0.577\sigma_{ys}$. So, there is a difference. You have multiple theories mainly because the material behavior is complex.

(Refer Slide Time: 13.12)



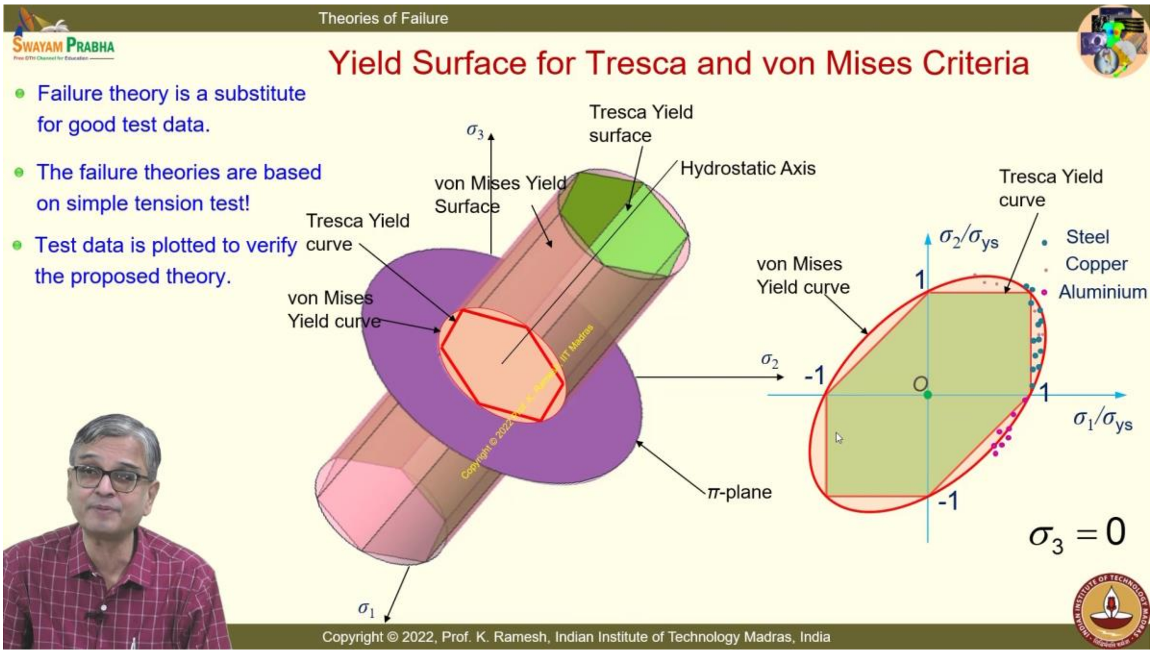
Suppose I plot these two theories in one reference; you have a very interesting geometrical appearance. You have the von Mises cylinder just encloses the Tresca yield surface. So, it is very interesting. So, it is slightly different. And from computational point of view, Tresca was very easy to do. You should imagine if you have a slide rule, how will you compute? You cannot compute $\sigma_1 - \sigma_2$ squared plus so on and so forth as in von Mises. You can definitely find out the difference, ok. So, you should also give credit and it was one of the earliest theories.

So, I have this as a circle enclosing the hexagon. And when you see it in the σ_1, σ_2 reference, when σ_3 is zero, I have this elongated hexagon. This is enclosed by the von Mises ellipse. So, the idea is in a real life situation, when I have complicated loading acting on the system, if you are in a position to find out the principal stresses, the combination of principal stresses lies within this elongated hexagon is predicted by Tresca, so that you will not reach yield. von Mises is slightly relaxing this. So, it allows small increase and within that, if your point lies, you have not reached yield condition. That means you are safe.

See, this is we are looking at the extremum value. When you go to design, you never go near this. Unless you want to have a crash protection, where you want the occupants in the car to be protected, after the accident, you want your crash bar to go out of shape, and you should be in good shape. So, it should absorb maximum amount of energy. There you allow to fail by yielding. In none of the practical applications, you never go anywhere near this.

Now, we will also see the shear diagonal in one diagram. So, you will see the von Mises theory gives this value as $0.577\sigma_{ys}$, whereas Tresca gives this as $0.5\sigma_{ys}$. So, even if you do Tresca analysis, you are conservating. Your failure will not happen. See, these days of optimization, where we are working on space technology, where weight reduction is very important, people want to go and do the analysis to the extent, they just have material to do the intended task. That means your mathematical modeling should be very close to what is anticipated in the actual condition, and you want to optimize; you want to have a light helicopter, isn't it? Initially, they will make a helicopter to fly; then they will try to find out where they can reduce the weight. Same is the situation in the case of landing gears. We have seen photoelastic coating is employed to reduce the weight. So, when you are learning stress analysis, it is not only the maximum stresses that are of concern; even the minimum stresses you want to keep it at a reasonable level so that your weight of the structure comes down. So, it depends on the requirement.

(Refer Slide Time: 17.05)

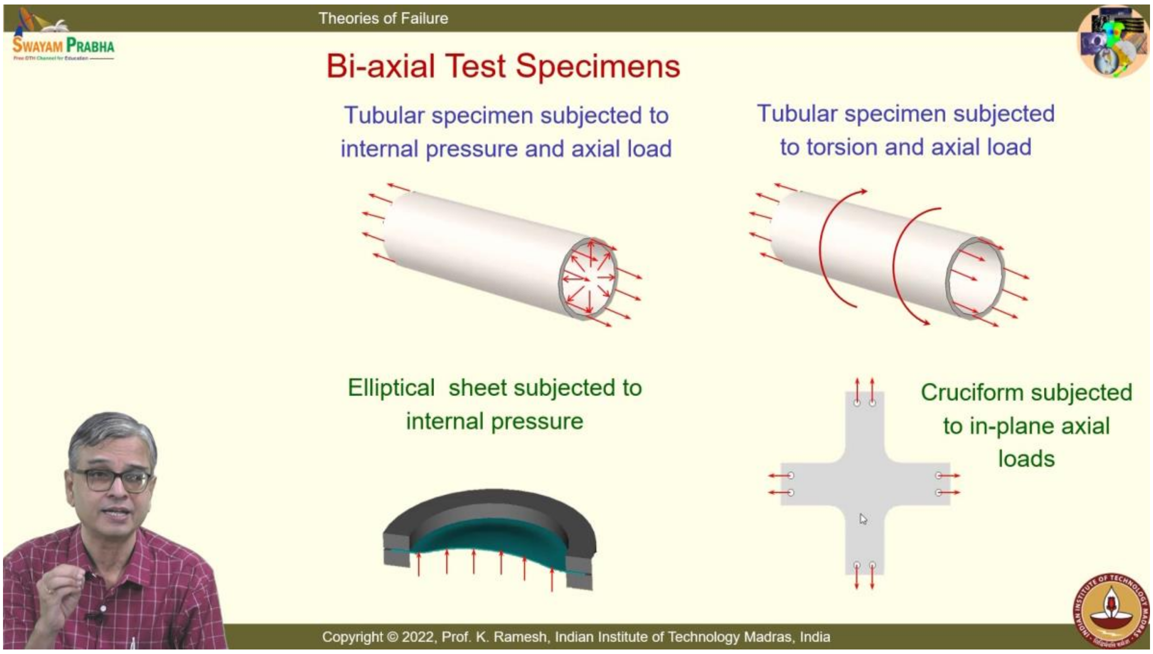


So, remember this failure theory is a substitute for good test data. If I have good test data, I would rely on that. Not only that, you know, in the early development of jet aircrafts, I said that you had big windows and even Queen Elizabeth travelled in that. And the DGCA of US said that these big windows are dangerous and there have been explosive failure of comet aircraft. It happened even from when it was taken, when it was taking off from Calcutta. When they analyzed, they found crack generated from the window and the plane has burst into burst like a bomb in mid air. In order to understand that they had to develop a test rig, where they had water to pressurize and also the wings to be oscillated like what you have in your air turbulence.

So, people have to create expensive test facilities. They are not cheap, ok. And all the failure theories are based on simple tension test. You should never forget that. Test data is plotted to verify the proposed theory. And I have also slightly modified because the test data is reported like this. They put it like σ_1/σ_{ys} , but the shape remains same, and you have steel, and I have steel data like this. So, it is lying between Tresca and von Mises, and at one or two places it has slightly exceeded. See, you have sufficient background developed in the course to find out how to test my specimen, so that I get these stress states. Can you guess? See, I can get what happens on this axis, what happens on this axis from simple tension test. What happens here is combination of your forces in more than two directions. What is the specimen that you can think of? We have discussed that because that is how you have to correlate. Only then the knowledge becomes complete. Think about it; I will show some more material.

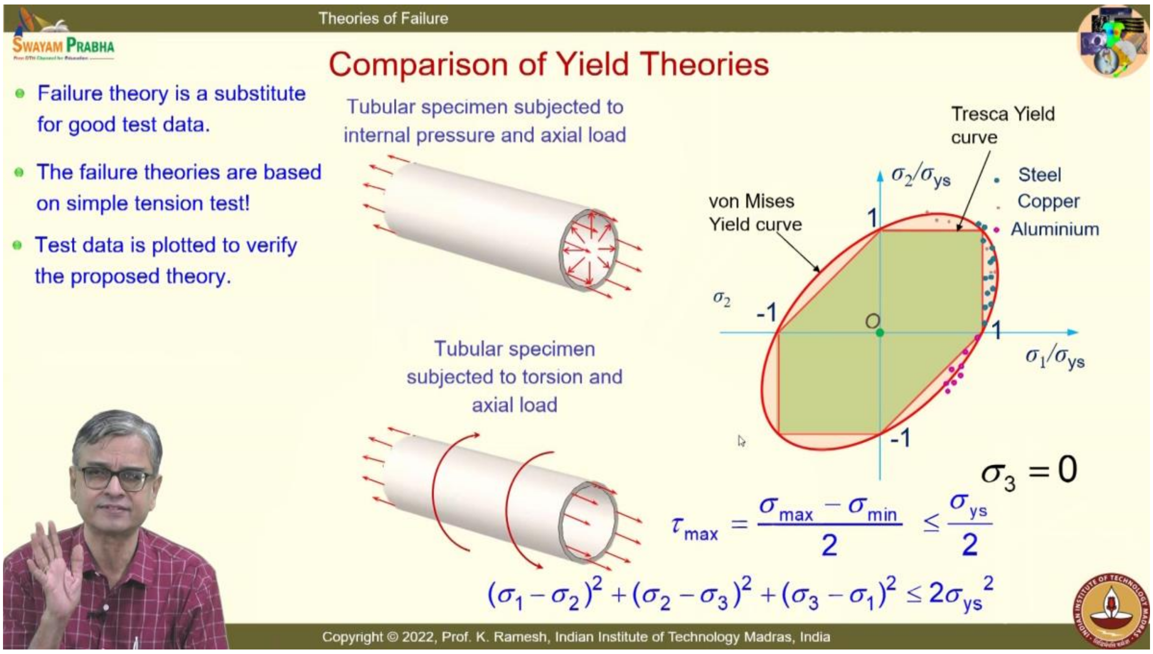
See, the other material that is very well tested is your copper because people want to have this heat exchangers, fine. Once I say heat exchangers, you should know what for copper is used and that also gives you the clue what should be the test. And aluminum alloys are the maximum tested material because that is used in space structures. You also have aluminum and aluminum test data is scattered like this. So, the idea is you need to have complicated specimens to generate this test data. So, what you see here is those tests have reconfirmed utility of both Tresca and von Mises yield criteria. That is the way you have to look at it. You have the greatest advantage that you have proposed these theories based on a simple tension test and is of verified, ok.

(Refer Slide Time: 20.18)



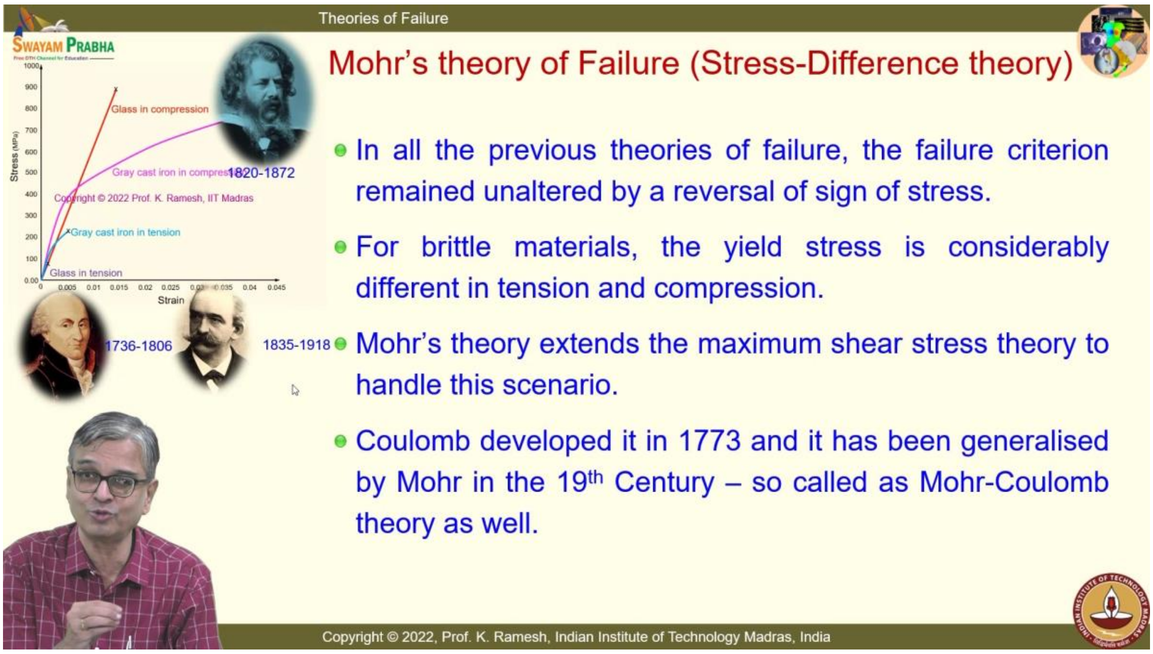
None of you have told me, if you apply internal pressure what happens? It is a true biaxial stress state when the ends are closed. When the ends are closed, I have pr/t and $pr/2t$. Suppose I, in addition apply this axial tension, I have control on modifying σ_1 and σ_2 . See, when I want to perform the test, I want to do it for a variety of combinations. Only when I do it for a variety of combinations, I can pat my back and say that I have exhaustively analyzed; otherwise not. You also have a tubeless specimen subjected to torsion and axial load. And you also have a possibility of using a simple elliptical shield subjected to internal pressure. And the best way to do is a cruciform specimen. I have complete control on what I want to do with σ_1, what I want to do with σ_2. And your test section is the inner core of the specimen. And the design of this fillet, all these are design issues, very subtle point because the specimen should not fail either at the grips or at the fillets. It should fail only in the interior. So, people have also designed new specimens so that you push it and find out whether these theories are valid. The focus is that.

(Refer Slide Time: 22.10)



So, you can feel safe. You know, scientists have not had a dream and then said you follow this theory. It is all tested, verified, reconfirmed by several bright minds over the years. So, you have test data from this. I just want to give you that if you want to get what happens in first quadrant as well as the fourth quadrant. You will have to get some of these tests done very carefully done, very expensive. Then you populate and then satisfy yourself that these theories are good enough approximation for you to carry on with routine design. For any special purposes, people do have final test rigs, very complicated whether it is an aircraft or a car or even your train because now India is developing this Vande Bharat, it was known as train 18. They have to have test rigs to test the complete bogie for several field conditions.

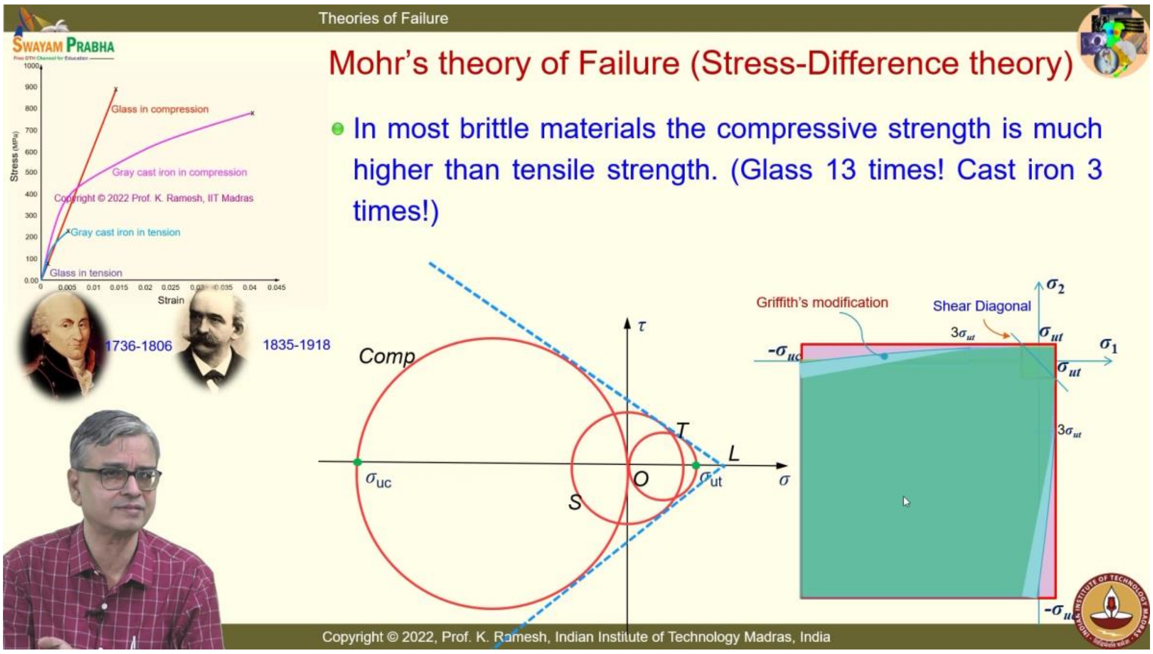
(Refer Slide Time: 23.20)



See, we have always been observing when you have a brittle material, it can have a different compressive strength and a different tension strength. The difference is very large. So, all the previous theories we have looked at, the failure criterion remained unaltered by a reversal of sign of stress. Because whether it is tension or compression, the yield strength in tension and yield strength in compression for most of the ductile materials are very similar. There is no major difference. For brittle material, the yield stresses are considerably different in tension and compression. So, the Mohr's theory extends the maximum shear stress theory to handle this scenario, ok. And if you look at, it was developed by Coulomb in 1773 and it has been generalized by Mohr in the 19th century. So, it is called Mohr-Coulomb theory as well. You call it as Mohr's theory, but if you dig the history, people say Mohr-Coulomb theory. And if you look at after the development of fracture mechanics, Griffith is a very famous name in fracture mechanics. He has also contributed to this theory of failure.

So, there all these failure theories had an history. It is not developed in one day. It is proposed, verified, modified, generalized by several scientists down the line. And you know, when we looked at the maximum principal stress theory, we said it is credited to Rankine. See, Rankine period is between 1822 and 1872. Whereas, you have Mohr's theory of failure in 1773. That means, what Coulomb said was not very clear. It was generalized by Mohr. That is why you call it as Mohr's theory. That happened probably the period later than Rankine. Otherwise, logically you should have given credit for maximum principal stress theory as the theory for the brittle material. People are not comfortable with this. That thinking was there even before Rankine. That is what you learn from historical records.

(Refer Slide Time: 25.43)

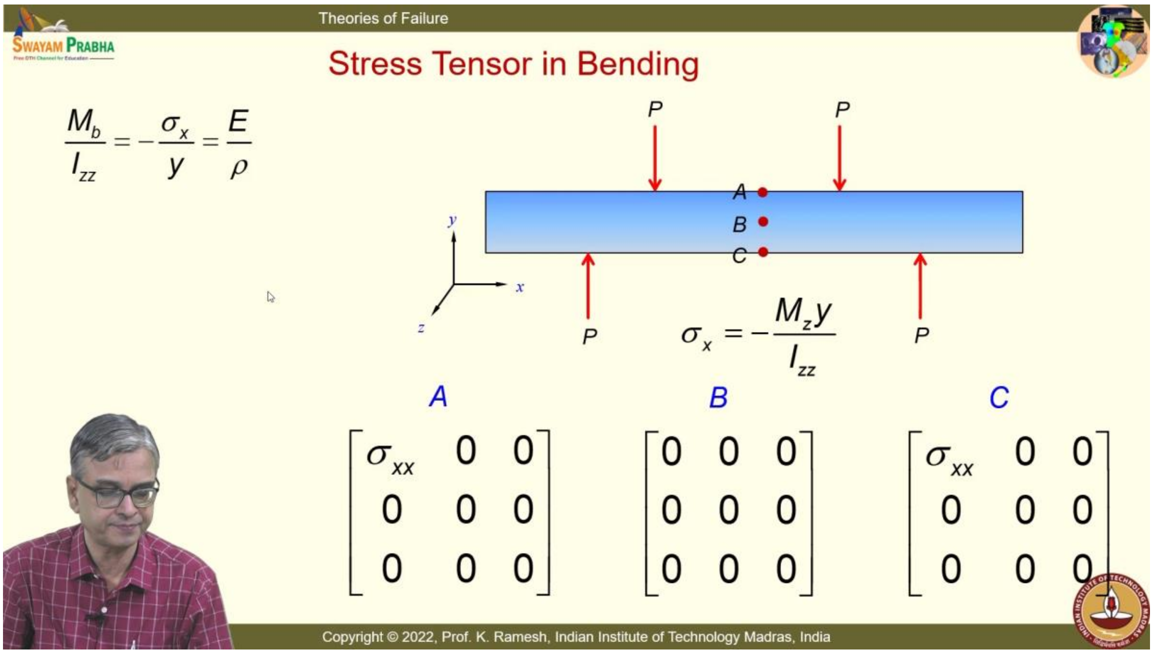


And how do I plot? I want to plot it. I must bring in the difference in the magnitudes. And what is recommended is, you do a test in simple tension and then find out the Mohr's circle. You do the test on the brittle material with pure shear stress state; find out the Mohr's circle. And you find out another test where you have the maximum compression. You only compress the material and draw the Mohr's circle. And the envelope of all these Mohr's circle tells you what is the failure that can happen in a brittle material. For all the previous theories also, you can draw the Mohr's circle and find out what is this envelope. Those envelopes are simple envelopes.

And if I want to plot it like what I have done in the case of Tresca and von Mises in σ_1, σ_2 space. You know I have to draw it in this screen. So, I have to modulate my sizes. If I have a simple maximum normal stress theory, it would be like a square like this. I have in the first quadrant, you have this. And the third quadrant which talks about compressive stress, it will also be of the same size. But your brittle material is not having same as tensile strength. Compressive strength is very, very high; 13 times higher. So, if I extend this, I should have plotted like this. But this was not the theory that is accepted by people. See, we have seen Tresca yield criteria. Following Tresca, one of the modification was you simply join this in the second and fourth quadrant to the σ_{ut} here, draw by a straight line. Some portion is cut off here; some portion is cut off here. And there have been modifications in that, ok. So, what I am going to show is, I am not stopping at σ_{ut} here, but I am stopping at the shear diagonal. I have drawn the shear diagonal. If I stop it at shear diagonal and join this by a straight line, this is called Modified Mohr's theory. The original Mohr's theory is, take this point and join it to σ_{ut} in the σ_1 axis. There is only a small difference. So, instead of there, you join it at shear diagonal; you call this as Modified Mohr theory.

And Griffith has modified it further with the understanding of fracture mechanics. What he said was, you do not stop it at σ_{ut}, you stop it at $3\sigma_{ut}$ and this becomes a curve; it is not a straight line. You should understand that this blue as well as this green area tells what is the Griffith's modification. Because you know, I have put these colors with some transparency to see the layers. So, in the process what happens is, when you put this blue, it does not merge with this green, but it also shows what is the modification by Griffith. So, as long as your stress state lies within this zone, a brittle material will be safe. And brittle material people have worked at, see people had to work with rock; it is brittle material, fine.

(Refer Slide Time: 29.34)

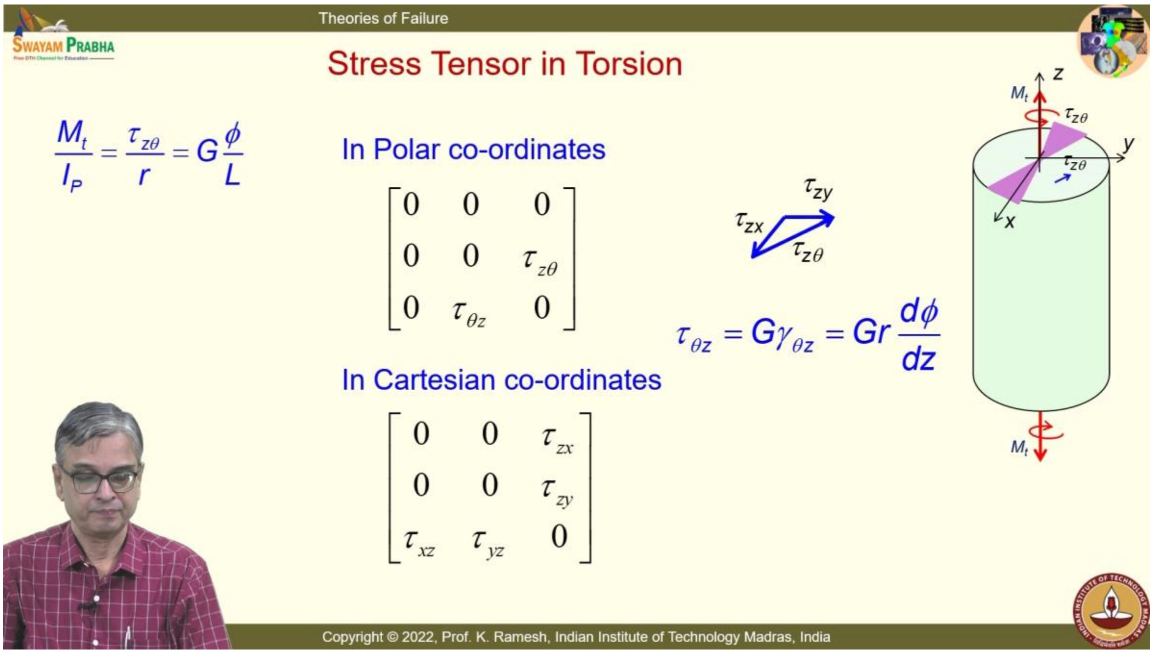


Now let us go back to our learning of bending. We have the famous flexure formula and if you take any of these points, you can find out the stress tensor. That is a training that you have had and you have also developed this

$$\sigma_x = -\frac{M_z y}{I_{zz}}$$

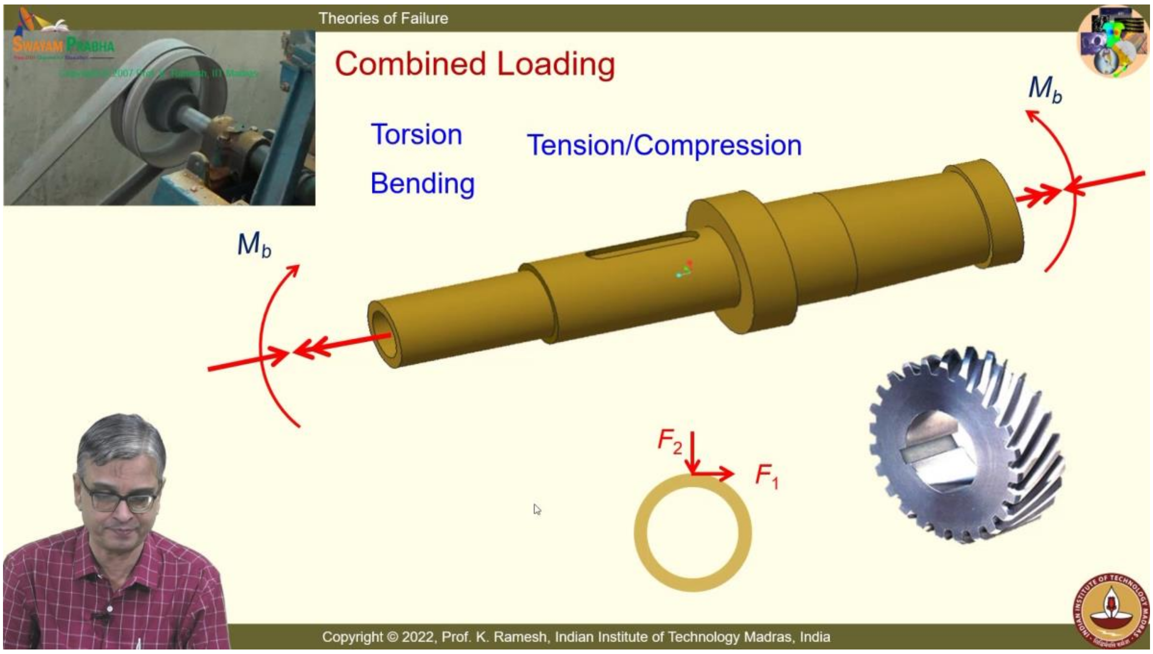
For point A, I can write the stress tensor like this. This embeds the sign. So, I am just writing this as σ_{xx}. The point B, you can recognize that this is in pure bending. Point B has no stress and point C again has this, but this formula has this sign embedded. So, the sign will change depending on whether it is tension or compression.

(Refer Slide Time: 30.24)



And we have also seen stresses in torsion. You have the shear stress varies and this is a tangential and varies as triangular. I can write it in polar coordinates, $\tau_{z\theta}, \tau_{\theta z}$. I can also write it in *xy* coordinates *x-y-z*, then I call the axis as *z*. So, I have this $\tau_{z\theta}$ is resolved into τ_{zy} and τ_{zx}. So, in cartesian coordinates, I have the stress tensor like this.

(Refer Slide Time: 31.03)



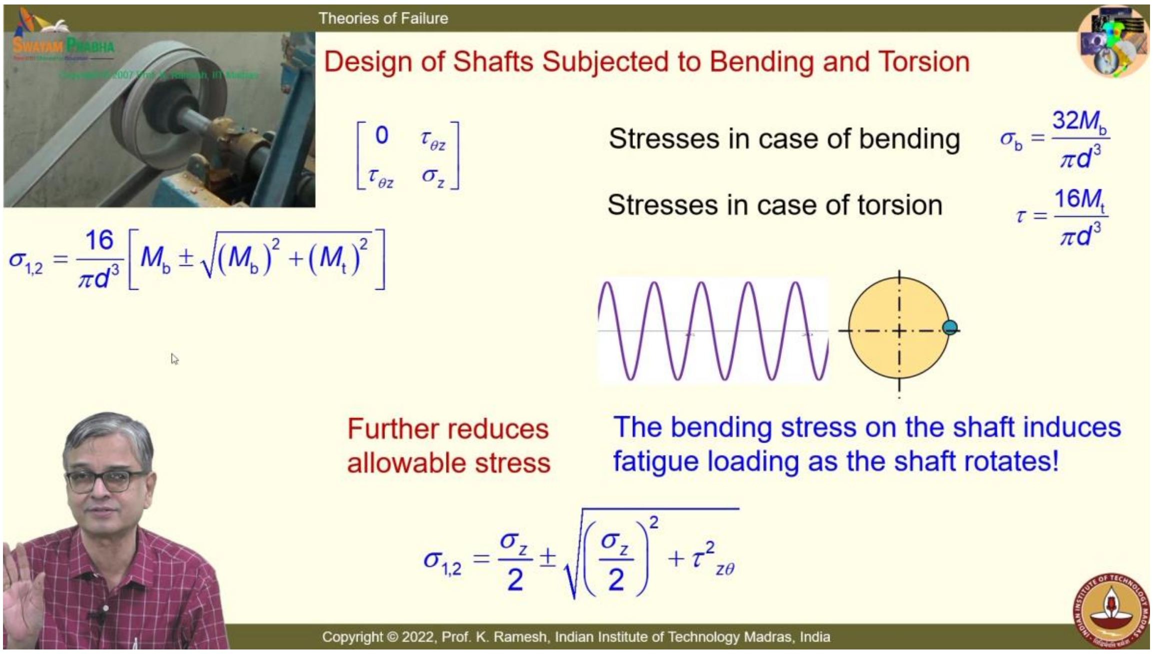
And you know, I have said even a simple atta-chakki problem. Let us see what are the forces that come on the shaft, ok. I have shown a generic shaft. This is not the shaft used in atta-chakki because it is hollow and tapered, and so on. So, when I have a belt drive, you recollect what you have learnt in your rigid body mechanics course. You have tensions

T_1 and T_2 and this essentially introduces only a twisting moment. So, I have a twisting moment acting on the shaft when I have a belt drive.

Suppose I have a spur gear mounted on the shaft. I suppose, you know, when I have a spur gear, you have a pressure angle and the load transferred is along the pressure angle. So, I will have a horizontal component twisting and a vertical component that will add up to bending of the shaft. Or if I look at in the atta-chakki that I have a bearing, it will also give a reaction. So, the shaft will be subjected to bending loads. Torsion and bending loads are very common in a shaft, but we have studied them separately. That is how we can develop the subject.

Suppose on the shaft, I have instead of a spur gear, I have a helical gear. For some purpose, I need a helical gear to take out the power and then operate something else. This helical gear will also have an axial component. So, I have torsion, bending or tension and compression. So, in real life, even a simple down-to-earth problem may have combination of these loads. You never have one unique loading, but while learning the subject, you learn them independently. And we have said, we are working in linear elasticity. So, in a linearized elasticity, I can have principle of superposition, individually compute the stresses and add them up. We have actually solved one problem, ok.

(Refer Slide Time: 33.20)



But there is also something very interesting. Let me see how many of you have a look at it. I have a shaft rotating. I have just now said, the shaft has twisting moment as well as sharing a bending load. And what you have learnt in your course, you know, we have solved one problem where we said that it is prudent to understand the axis as z and then add these stresses comfortably for you to get the combined loading.

So, stress in case of bending is in terms of your bending moment, you have this as

$$\sigma_b = \frac{32M_b}{\pi d^3}$$

Stress in case of torsion as τ equal to

$$\tau = \frac{16M_t}{\pi d^3}$$

You remember, we have solved the femur problem, where I have said how do I superimpose. I change my reference axis in a very convenient fashion. So, if I have a shaft transmitting bending and torsion, any member transmitting bending and torsion, I can get the stress tensor in this form. I can get the magnitudes from the bending moment and twisting moment from these expressions.

Now, I have atta-chakki where this is also rotating. Do you see anything interesting in this situation? When the shaft is rotating, it is subjected to bending. Can you visualize something more interesting happening in a shaft subjected to bending? Long time back, you know, we have taken the example of a jump clip, and then we said when I pull it out, when I pull it out, I cannot fail it. What did I do? Then when I do this, I get this beautifully broken. I have given you the clue. Can you visualize what is happening because the shaft is rotating? You see the animation. You imagine that this is the point on the fiber which is moving. When it moves, do you see that the distance from the neutral axis changes and your bending stress is function of the distance from the neutral axis. So, what you find is because the shaft is rotating, you are not gone and deliberately modified the bending moment like this. It is only rotating, but it is transmitting a bending load. So, your point on the beam gets rotated. So, it experiences a different state which is equivalent to a cyclical loading. What happens? You have said σ_{ys} in your Tresca yield criteria, or your von Mises yield criteria. I can never go near this because I have done repeated loading; Innocently! If you do not recognize that shaft is rotating, you would have missed this. You get the idea. The bending stress on the shaft induces fatigue loading as the shaft rotates. So, what is the consequence of this? It reduces the allowable stress. So, you have to bring in factor of safety which accommodates this. See, if you ask more questions, you will have to do more tests, ok.

Once you go to a design course, you know you will employ this combined loading in a particular fashion. We have already seen when I have a stress tensor like this, your principal stresses are determined like this and σ_θ is zero. So, my principal stresses reduces to a simple fashion

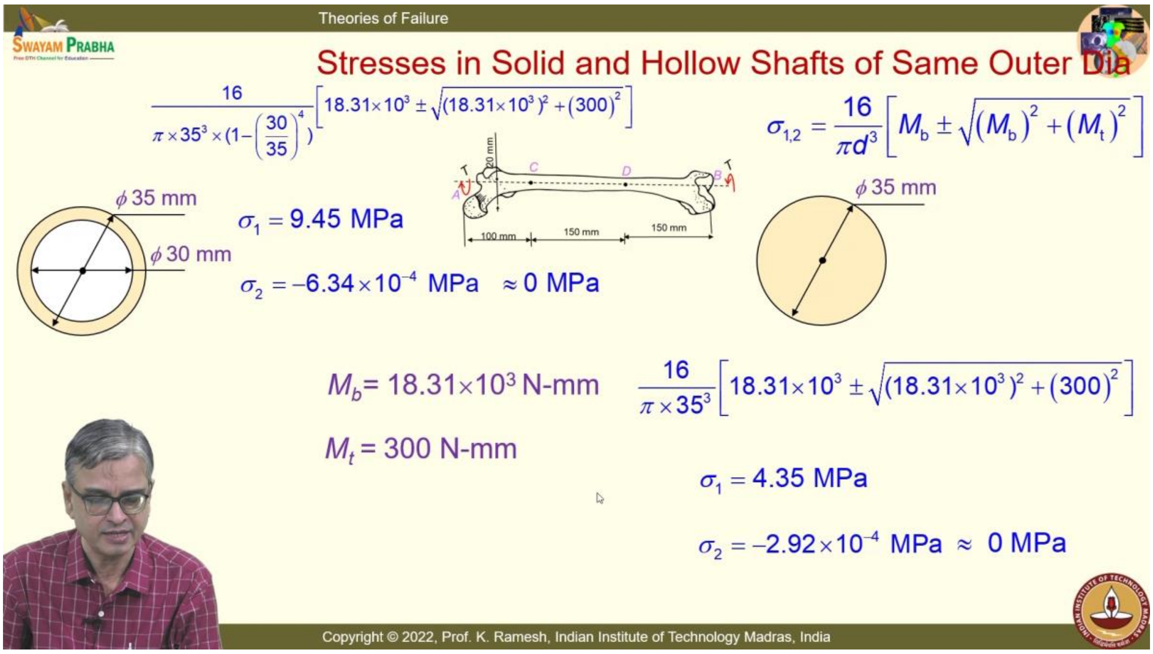
$$\sigma_{1,2} = \frac{\sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z}{2}\right)^2 + \tau^2_{z\theta}}$$

And we have seen all the failure theories use the principal stresses for all the further calculations. Now, because I know expressions for this which we have developed in bending theory and torsional theory; I can directly substitute. So, once you go to the machine element course, they will design shaft based on this expression. You do not have to remember this. You can derive this. The root is you are finding out the principal stresses. You are doing a principal stress superposition and your final expression reduces to

$$\sigma_{1,2} = \frac{16}{\pi d^3}\left[M_b \pm \sqrt{(M_b)^2 + (M_t)^2}\right]$$

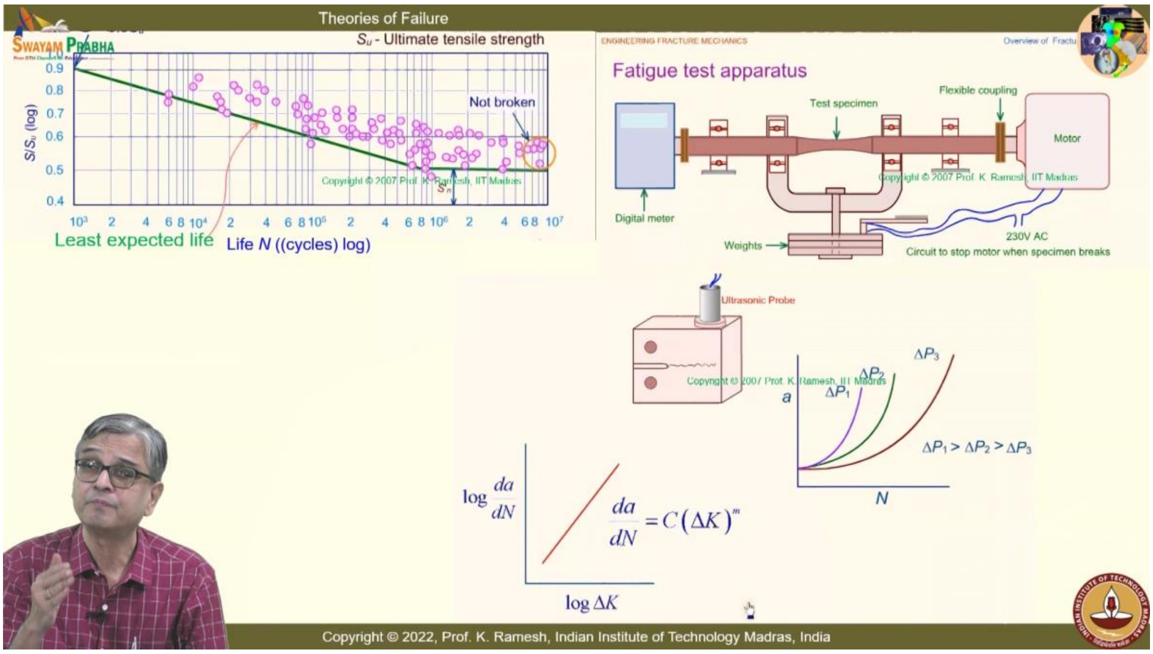
So, you do not recognize this in your design course unless you look at the link, how we have got the superposition of stresses, how we have looked at the principal stresses. This is nothing but the principal stresses which you have developed from your understanding of stress analysis. But people in the field will use this. They do not have to know the theory. They have to know what formula to use. But as engineers, you should know the background.

(Refer Slide Time: 38.33)



Now, let me go back to your femur. See femur is basically hollow, but I want to illustrate there is one other concept, ok. I want to ask a question. Suppose I have this as a solid one and then I find out my principal stresses using this; I have some numbers. When I substitute this, I get the σ_1 as 4.35 MPa. I get σ_2 as -2.92×10^{-4} MPa. So, it is almost close to zero. Now let me ask, I want to have a look at the specimen which has outer diameter is 35 but hollow. What we have learnt in the course? We have learnt in the course, if you make the shaft hollow or beam hollow, you can increase the strength. Under what circumstances? Same material or same cross-sectional area. Suppose I ask a question, when I have the outer diameter is 35, I want to have my hollow specimen is also outer diameter is 35. If I put the question differently, I have seen σ_1 as 4.35 MPa. If I calculate the stresses, stresses will be more in a hollow shaft or less in a hollow shaft? Louder! You have to recognize that. You understand the concept correctly. See, femur is hollow, fine. And when I have the femur, I can simply use this expression. I have also given that expression. If I have the ratio of inner diameter to outer diameter put here, the expression gets modified with a factor here, $\left(1-\left(d_{inner}/d_{outer}\right)^4\right)$. I get the σ_1 value as 9.45 MPa. So, understand the concept. Hollow shafts are very strong, but if you drastically reduce the area, it cannot withstand, fine. So, you should understand the concepts developed in the right perspective. This is just an illustration. The same idea you can extend this for all shaft design.

(Refer Slide Time: 40.59)

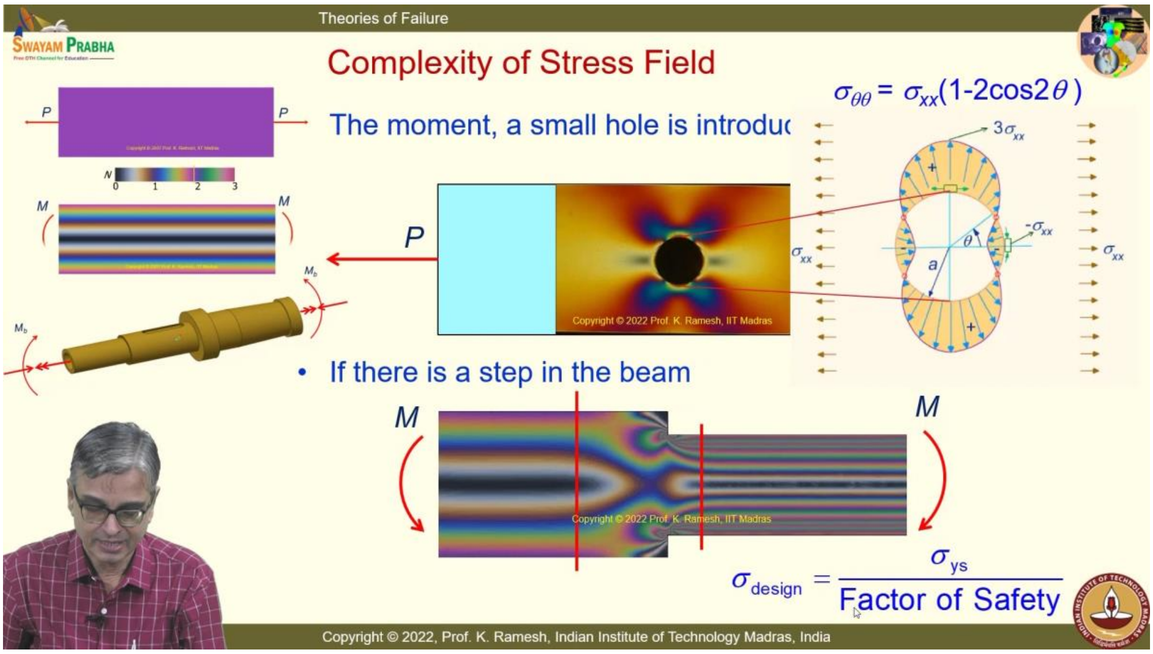


And you know, if you ask a question, if I have repeated loading, the specimen fails, fine. And people have to design a fatigue test apparatus, keep changing the loads and there will be a counter that will say after so many cycles, the specimen will break. That is all you can get from a fatigue test. See, it is much more complicated than a tension test. I need to have hundreds of specimen tested with different surface quality and so on. That is how people have tested. And when I plot this, you know, I have this, which goes as a fatigue loading. Just the shaft rotates causes fatigue loading. Understand this. So, now I plot this for a specimen. I will generate so many points like this from a test. And this shows the least expected life when I apply the repeated loading. And people also had, if I exceed 10^6 cycles, they call this as infinite life. And they have something known as endurance limit. In later courses, you will study this.

So, if I have any repeated loading, I have to determine my factor of safety much below the yield strength. I will have factor of safety number at least two when I have to apply for fatigue loading. It will be greater than two if I have to bring in other aspects. Suppose I ask a question, all the failures, how it generates? You have a crack and crack grows in surface and fractures. If you ask a question, can I monitor how the crack grows? You cannot answer unless you develop another test where I have a specimen with a crack, I have ultrasonic probe to find out how the crack grows. And then I plot crack growth curves. From the crack growth curves, I post process and then; these are known as crack growth curves. And you have da/dN; $\log da/dN$ versus $\log \Delta K$ that becomes a straight line.

And this is known as the Paris law. You know research labs spend enormous amount of money testing the material and no country will give you this. For different applications, you have to develop your own material, you have to develop the testing. In fact, for your LCA development, our NAL laboratory were testing such specimens round the clock for several years. So, if you ask more questions, remember you have to spend more. If the questions do not get answered, simply from your tension test, but people have solved this, ok. So, you have a fatigue loading, you have a crack growth people have solved.

(Refer Slide Time: 43.49)



And we have also seen, I take a specimen with hole, we have just seen that the stress field modifies. You have a flowery pattern. And we also learnt that stress magnitudes are much higher than a simple tension test. I have not said anything more earlier. Now, let us look at what happens. The solution what I am going to show is for a imaginary infinite plate with a small hole, whereas this is a finite plate with a comparable hole. Just an illustration for a infinite plate with a small hole, if you plot the $\sigma_{\theta\theta}$ on the outer boundary, it has a very interesting shape like this. I want you to write this

$$\sigma_{\theta\theta} = \sigma_{xx}(1-2\cos 2\theta)$$

This is a very, very important information. And what I get is, I get this as where I have a density of fringes is very high, I get this as 3 times the stress. Mind you; it is for an infinite plate with a small hole. For a finite plate, it can be much higher than three; it can even be nine times, ok. So, you will always have to do a experimental analysis or a numerical analysis for you to get this stress concentration factor for finite plate.

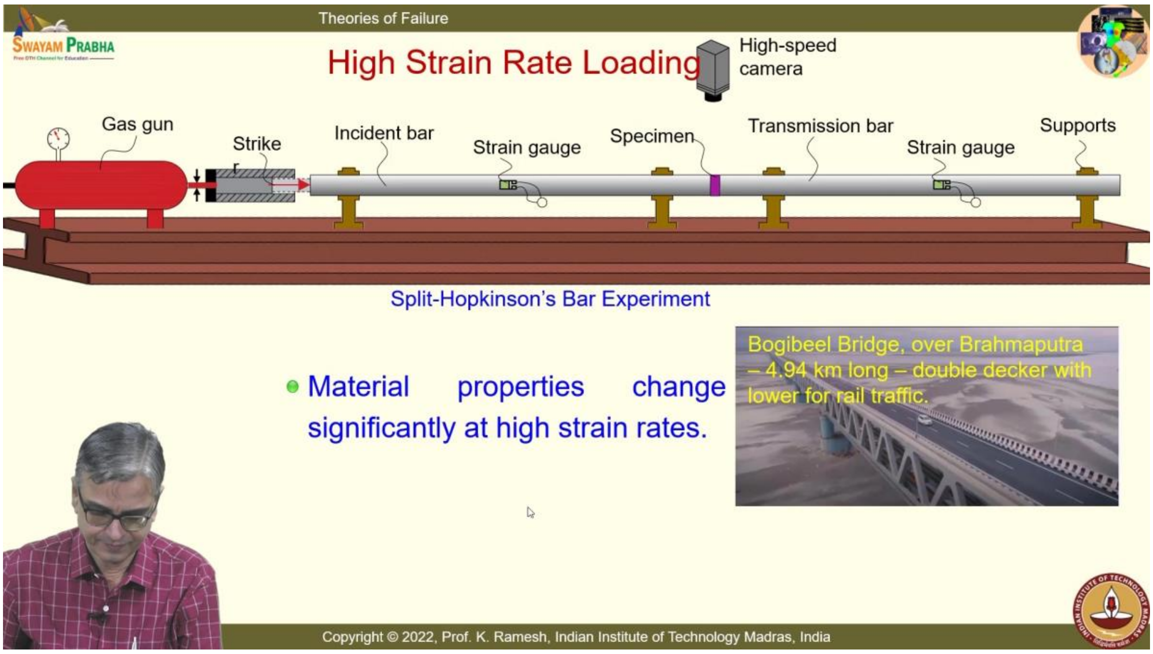
There is also something very interesting. See, I have shown a nice graph, I have also shown the sign change, I have shown something here. How many of you taken by surprise? Do not you think that you should take by surprise? Because I take a specimen, I only pull it. As long as I do not have a hole, everywhere I have only tension. The moment I put a hole, what happens? Stress magnitude becomes larger. Stress field becomes complex. In addition, what is interesting is, I apply tension and what I have here? What I have there? Compression! It is very, very surprising! When you have compression, what is the nuisance? When you are going for optimization. When I have compression, when I go for optimization where I want to make the cross section thinner and thinner, what is that you can anticipate? Buckling! In fact, in fracture mechanics people analyze this. So, do not think a stress riser is a small innocent stress riser, it does many complex things. So, if I have to accommodate a stress riser; and we have also seen when there is a step, because when you have a shaft, if you have to support, I need to put a bearing. I need to put a bearing, I need to locate it. So, I am going to have a stress concentration.

And you know very well that if I analyze it away from the zone, my bending theory is reasonably good. But when you are learning it is fine, but when you are designing and making a component, I must accommodate this in my design. If I do not do that, you will be a failed engineer. Nobody will come back to you for consultation. So, your bookish knowledge is not going to help you. So, how do you accommodate this? The only way you can do this is, do a refined analysis or do a test or take an appropriate value of factor of safety, ok. So, factor of safety has many connotations. So, I have

$$\sigma_{design} = \frac{\sigma_{ys}}{\text{Factor of Safety}}$$

So, it could come from your modeling issues. If I have modeled it instead of a fixed joint, I have made it as a pin joint. I can have issues based on estimation of the load and the type of analysis and the material differences. Suppose somebody is supplying you material because one of the consequences of COVID was supply chain was affected, you are sourcing it from a particular source. Then you change the source, the material property will not be consistent. See, even in your hostel, if they are serving you vada, you have this urad dal. He has to source it from the same source for you to give the same taste. Find some of the good food chains, very particular about what the ingredients from where they source it. Same thing applies for your automobile design also. If they are making components, they will source the material, raw material from the same source and test it every day. It is not like once in a blue moon they test it. Every raw material before it is converted into actual product, they verify the properties are same. Only then they pay it to the supplier. So, very, very important, ok.

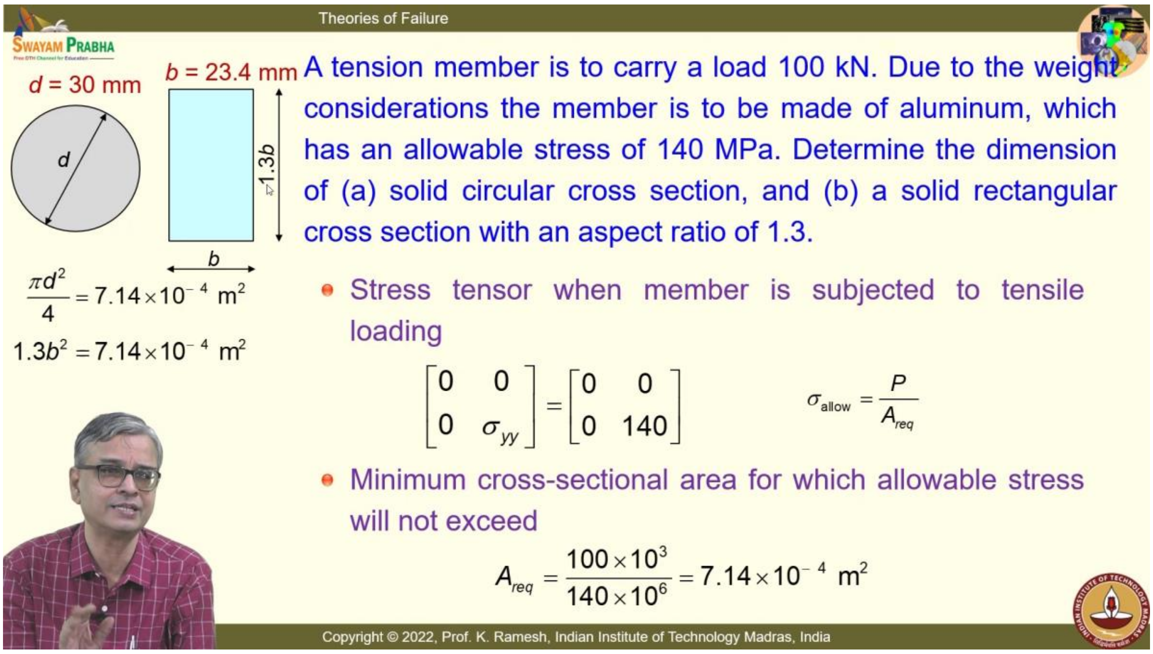
(Refer Slide Time: 49.00)



And you know the modern days you have the issue of explosions. And I said if you ask more questions, you have to do more tests. There is no other go. Now, you have a Bogibeel bridge which is in the Brahmaputra, which connects the northeast; 4.94 km. And this is designed to withstand explosion. And you also have this Chenab railroad bridge which they have recently completed in Jammu Kashmir, ok.

All this you have to design it to withstand earthquakes, to withstand the bomb blast. So, you ask a question, I have a high strain rate. So, you need to have a different test setup. I have a split Hopkinson's bar where I have a gas gun which puts this long bar, and the specimen is very, very small, generates a very high rate of strain. And you have a high-speed camera which records this and you have processing of this data done. So, the material properties change significantly at high strain rates. And also extremum conditions, if I want to work in cryogenics; it is becoming very important in space technology. And if you want to design a gas turbine, efficiency goes up if I work at high temperature. So, I said the tension test what you have done is at room temperature. So, you have to conduct test at high temperature or low temperature, generate test data and material scientists play a very important role. So, all that goes into your design. This forms a basic fundamental and you have the basic mechanics, but when you go to the field, you have to take care of field requirements, ok.

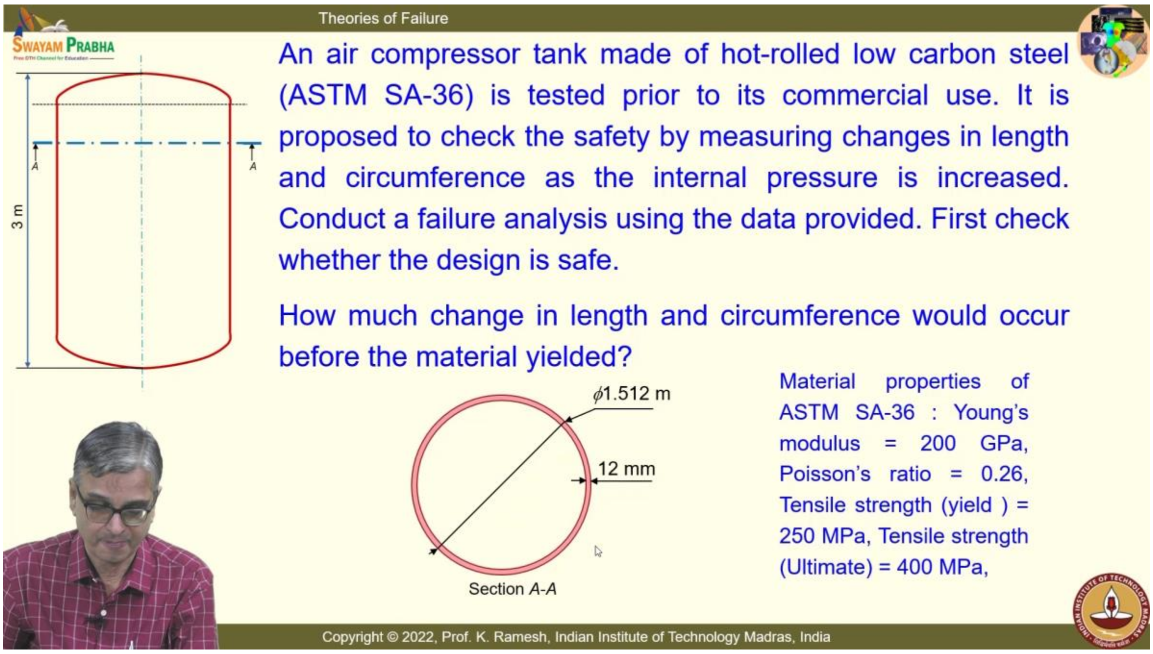
(Refer Slide Time: 50.56)



And we will also see two simple problems. The first problem I will, it is very simple to remind you that if I have a simple tension member, the geometry of a cross-section has no role to play. You have an allowable stress; now I said we are not going to work at yield strength. Somebody has done the requirement analysis and said what should be the factor of safety. So, he has said for this material, you use only 140 MPa. And then you are also given what should be the geometric shape of the rectangular cross-section; aspect ratio is given as 1.3. It is a very simple problem, ok.

I have always said that once you have a problem, learn how to write the stress tensor, ok. It is a tension, so I have the σ_{yy}, the value given is 140. And you have the simple criteria, allowable is force divided by area. And you have an expression, A_{req}. And when you translate into a circle or a rectangle, you are in a position to find out what should be the diameter of the circle or the distance b for a rectangular cross-section. It is a very simple problem. The purpose is to illustrate when I have a axial tension or compression, geometry does not play a role.

(Refer Slide Time: 52.30)

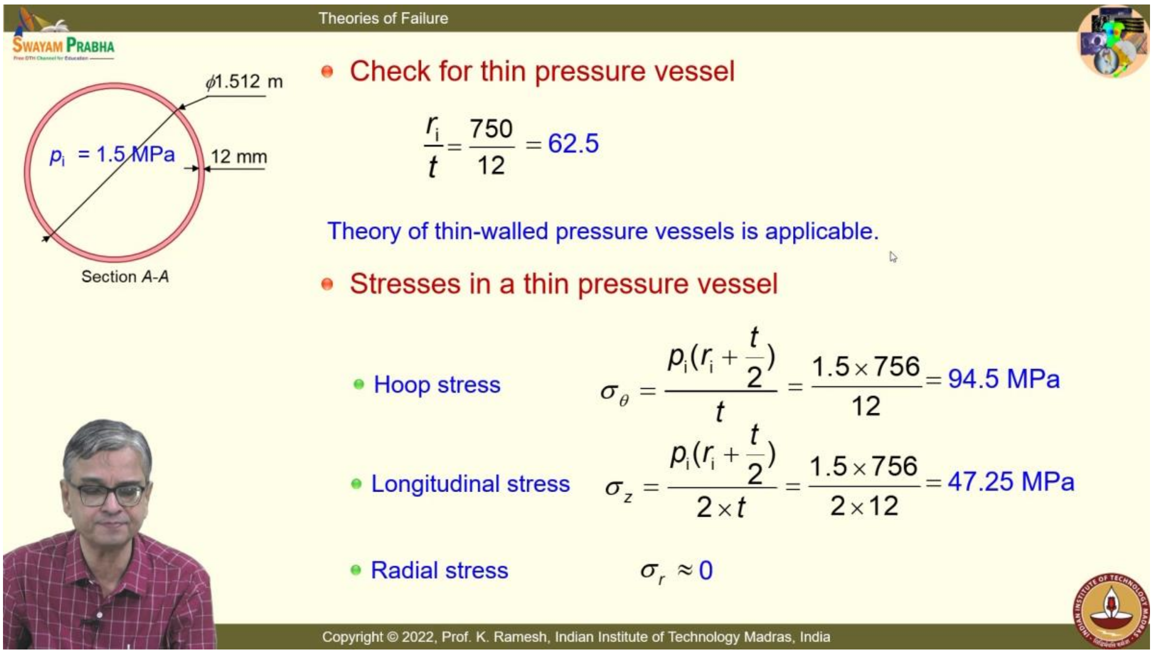


And you know many, many practical problems are compressors. You have your pressure cooker, your Indian cylinder, your aircraft—all of these are pressure vessels. And pressure vessel is one problem where you have true biaxial state of stress. And you have a thin cylinder approximation, you have learnt the ideas when I have a thin cylinder, how this gets stressed. So, here the idea is how do you write the stress tensor and then find out the limiting values.

So, first check is you have to see whether the design is safe. You are given a low carbon steel. Specifications are given Young's modulus, Poisson's ratio and yield strength all of these are given. See when you go to any practical problems, you may collect more data than what is required. See in your school, when somebody is given a question, you will be given there is a clue such data is given, have you used the data that no longer works in engineering. I have given more data here, what are the data I have given? I have given tensile strength; I have also given ultimate tensile strength.

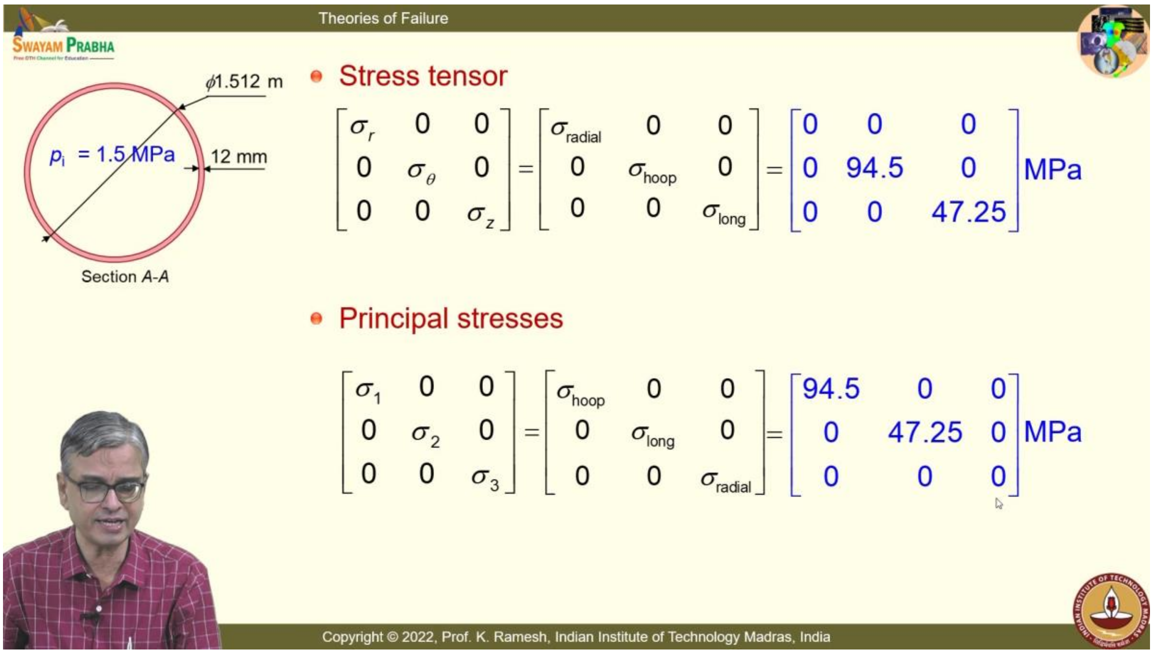
So, it is for you to filter out what is applicable in the problem. So, your earlier training you should use all the data no longer rests here. Some data may be missing also, which you will have to make an intelligent assumption and then do it, that is how engineering is.

(Refer Slide Time: 54.07)



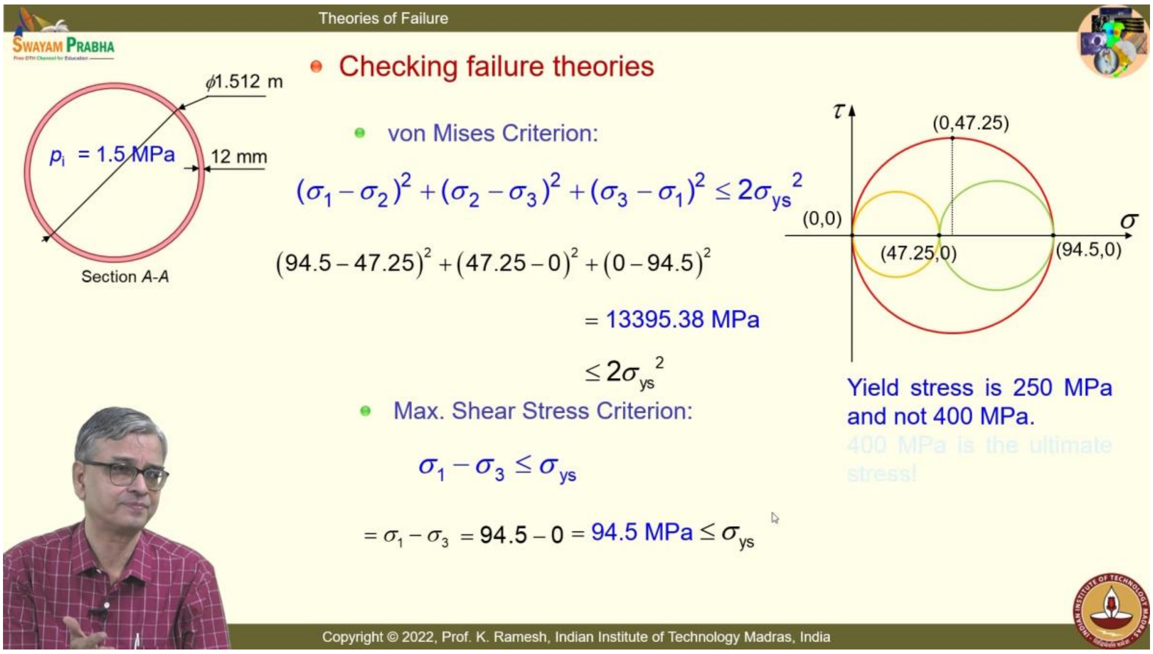
So, first is you have to check for a thin pressure vessel. It is obviously a thin pressure vessel. So, I have this greater than 10; it is about 62.5. And you have a thin pressure vessel is applicable and you know the stresses in a thin pressure vessel. I have a hoop stress. See, usually what they do is, when they do the calculation, they do the calculation for the median line, ok. So, that is what is illustrated in this; pr/t is the basic expression. Instead of inner diameter, you put this as $r_i + \frac{t}{2}$, that is one convention that generally people use. So, I have this and I can find out the longitudinal stress, even without the calculation, you can say it is one-half of this because it is pr/t and $pr/2t$. And we have labeled this as σ_θ and σ_z. And your radial stress is only the internal pressure, we normally say that it is very, very small compared to these numbers, so we take that as zero.

(Refer Slide Time: 55.15)



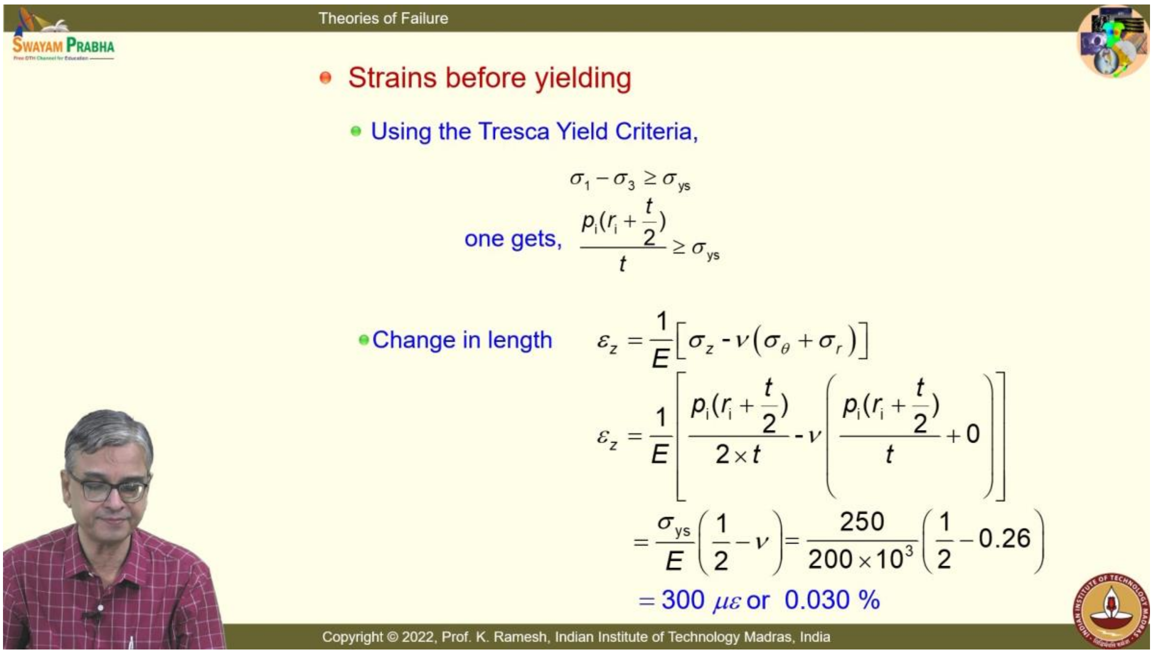
And you learn how to write the stress tensor and when I have the stress tensor it is like this, fortunately all of them are the diagonal. Now you know from the design perspective, we want to look at as algebraically maximum, algebraically the least. So, how do I write this as principal stresses? Can I have this as radial hoop and long? It is not the right this one, but you have this as hoop stress is the maximum. So this becomes your σ_1; this becomes your σ_2; this becomes your σ_3; then you simply apply your failure theories, ok.

(Refer Slide Time: 55.59)



So, the computations are very simple. So, I have the von Mises criteria; you write the von Mises criteria in the basic form, substitute these numbers. And you are in a position to calculate this and check whether you should use 250 or 400. In this case either way it does not matter, but number wise, but concept wise you have to use the yield strength appropriately. And you have the maximum shear stress criterion. And here you know you will have to recognize that if you look at only pr/t and $pr/2t$, you will get only this Mohr circle. You will be tempted to use this as maximum shear stress; please do not fall a trap to that. You must recognize the existence of zero. You must recognize that you should draw the larger circle, but you will never make a mistake if you say it is $\sigma_1 - \sigma_3$, ok. When I put this $\sigma_1 - \sigma_3$, I get the correct expressions and you can find out that this satisfies the yield criteria. So, the material is safe. See, in the field what is it the person will immediately see. He will only see geometrical changes. So, that is the spirit behind the next part, what are the strains before yielding.

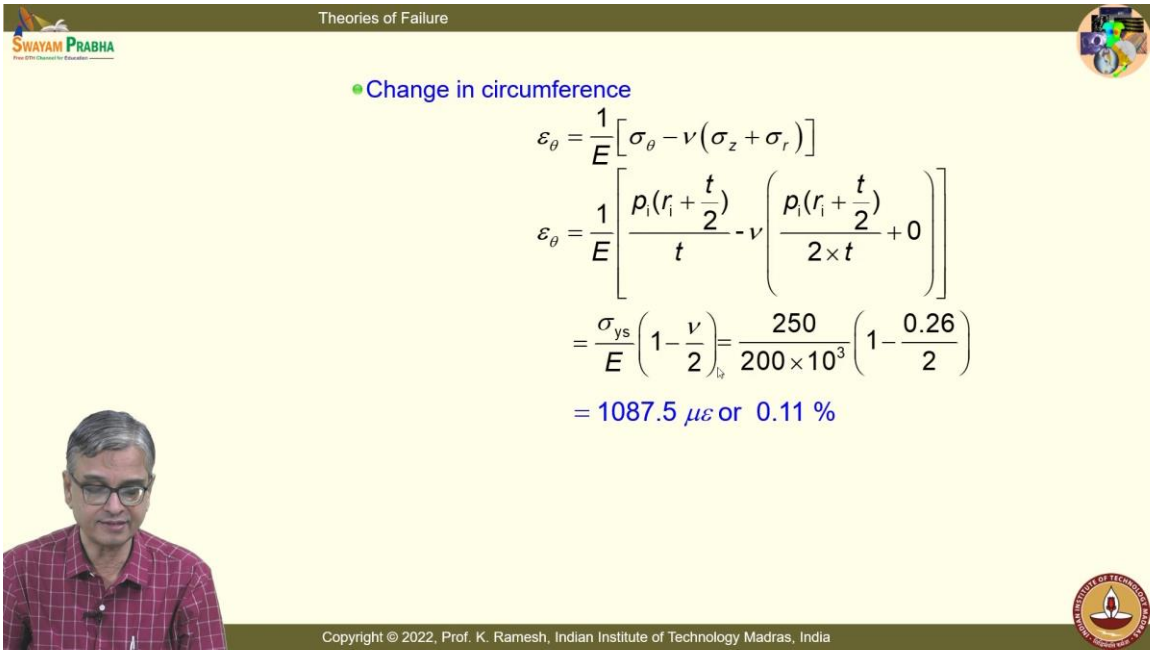
(Refer Slide Time: 57.28)



In order to make our life simple, we simply take the Tresca yield criteria; it is again an engineering decision, my calculations are very simple, and it is very close to Tresca; von Mises yield criteria.

So, I look at this, so now I write the expression for strain. So, here write the strain expression completely, do not simply say σ_z/E; these are all common mistakes people do. You should recognize I have σ_θ and σ_r, you have the Poisson ratio coming into the picture. So, find out what is the strain in the *z* direction, ok. That you can easily compute, and this is about 300 με.

(Refer Slide Time: 58.19)



And you can also find out the circumferential change. So, you have the strain stress relations; use the relations completely. So, in that sense you know every concept that we have developed or reviewed, that is the idea behind this theories of failure. You get to know how to use the concepts, so I get this as 1087.5 $\mu\varepsilon$.

So, in this class, we have looked at threadbare the nuances of theories of failure and we have also had a bird's eye view on what are the complications that you face in an actual design. Even a simple shaft if it is rotating, you have a surprise that this has a fatigue loading. The moment you have fatigue loading, the allowable strength drastically reduces at least by a factor 2 and when you have a stress concentration, it increases the stresses.

You can also have a very complex stress magnitude; you can also have surprise compression. So, your factor of safety is not a simple idea; it is a very, very important idea that you have to understand how to choose appropriate factor of safety. And we have also seen that when you have a high strain rate loading, material properties change. In extremum conditions material properties change, for all of this you should do test and any test is expensive. Thank you.