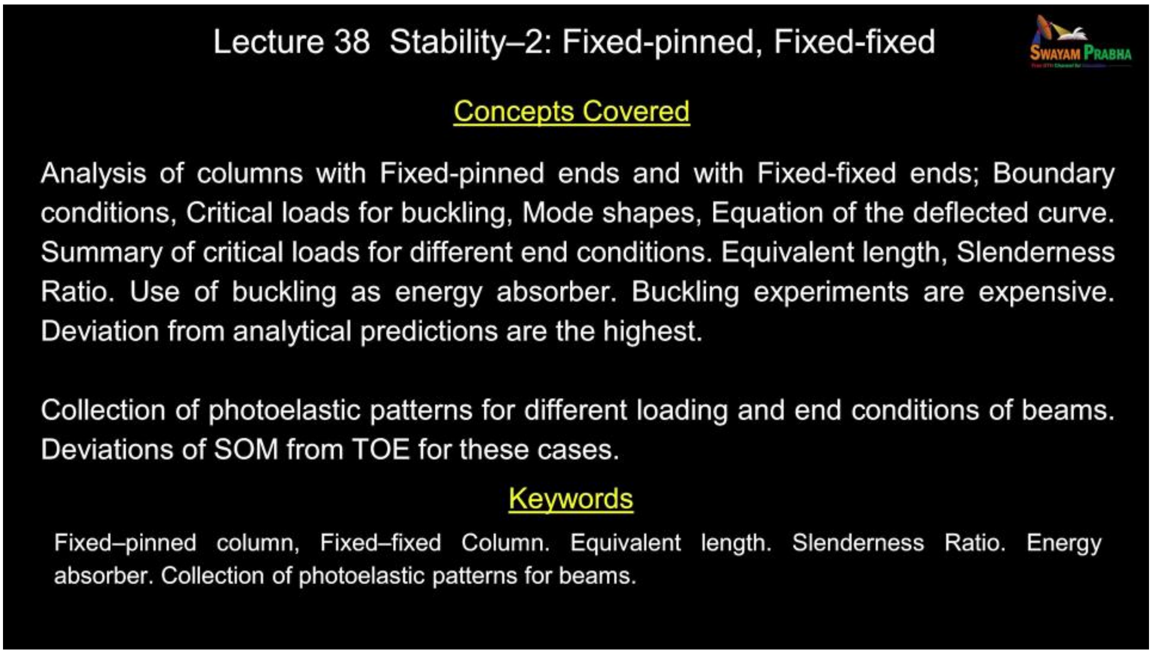
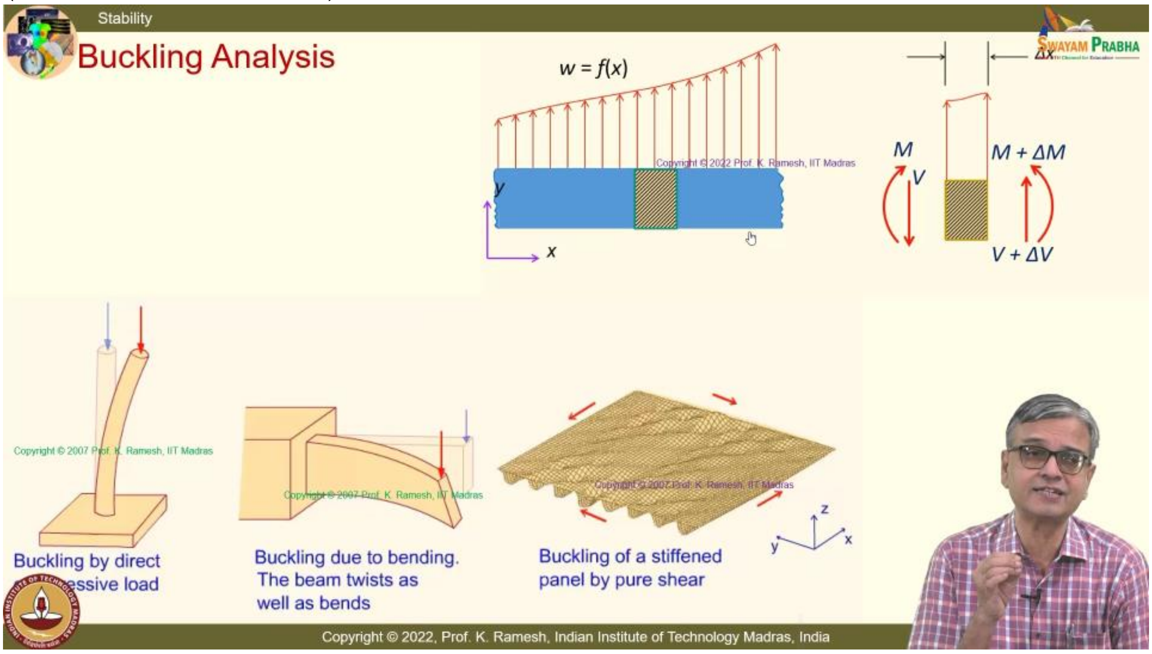
Strength of Materials
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 38
Stability 2 Fixed-pinned, Fixed-fixed

(Refer Slide Time: 00:20)



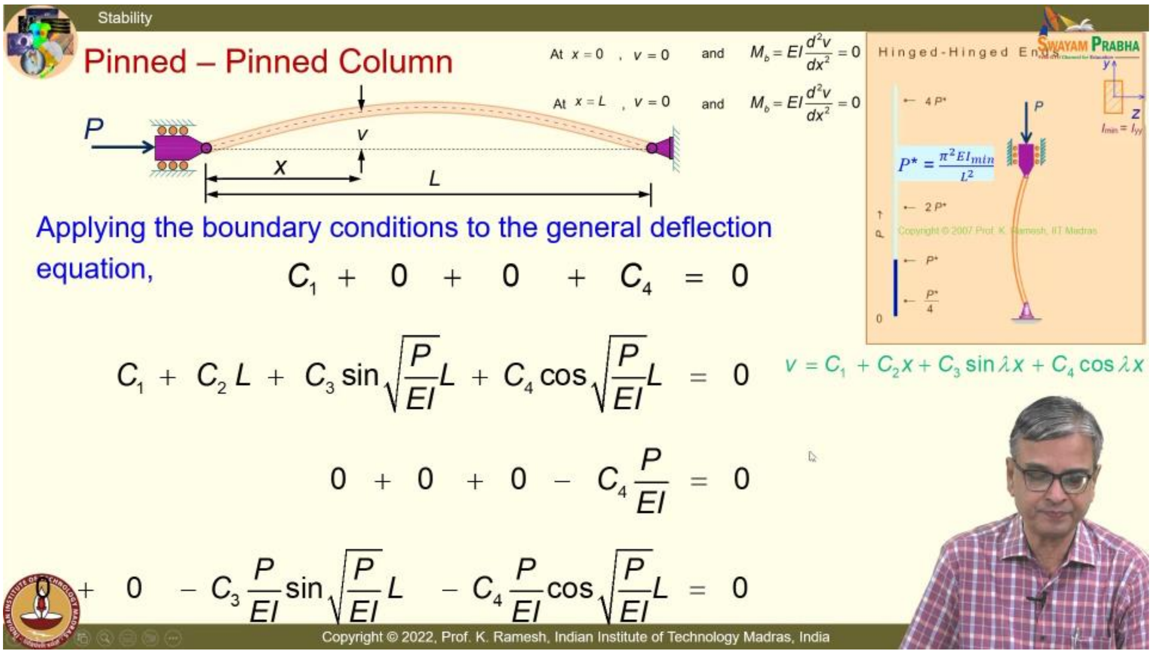
(Refer Slide Time: 00.53)



Let us continue our discussion on stability. You know the whole idea is, when the column reaches the critical load without a warning it buckles. That is where the stability problem becomes very critical and you need to find out the critical load. If you do not know that, then you will have surprises in your structural response. And how do we handle a problem related to stability to start with? Mathematically, even though we have said we are going to live in small deformation, in order to investigate stability, you take the beam as bent, ok. That is very, very important.

And when the load reaches a critical value without a warning it buckles, buckling can happen in a column like this, it can happen in a beam, it can also be precipitated by a shear. And in all these cases what we have done is, we have deliberately taken this as deformed and only when it is deformed, we are in a position to take an element; you get an extra term that I have the compressive load separated by a small displacement Δv. On the other hand, when we analyze a beam for deflection, we have consciously taken this as undeformed configuration; you get the point, that is the fundamental difference. When we did the deflection analysis, we did not consider the beam as deformed, we solved it in undeformed coordinates. On the other hand, when you come for buckling, we consciously take that as a deformed, then only when a position to write the mathematical expression.

(Refer Slide Time: 02.30)



And first, we have looked at the fixed-free, that is the cantilever; then we moved on to pinned-pinned column. And actually pinned-pinned column forms the basis. See, if you look at the critical load, I have also put this as

$$P^* = \frac{\pi^2 EI_{\min}}{L^2}$$

And all the other cases are compared with this P^*. And you know, the boundary conditions are very easy to write, because I have a pinned joint, I have $x = 0,\ v = 0$; at $x = L,\ v = 0$.

And in both the ends, your bending moment is zero, which in turns out to be $EI\frac{d^2v}{dx^2}=0$.

Even though I have written $EI\frac{d^2v}{dx^2}$, you should see that as $\frac{d^2v}{dx^2}$, as simple as that.

At $x=0$, $v=0$ and $M_b=EI\frac{d^2v}{dx^2}=0$

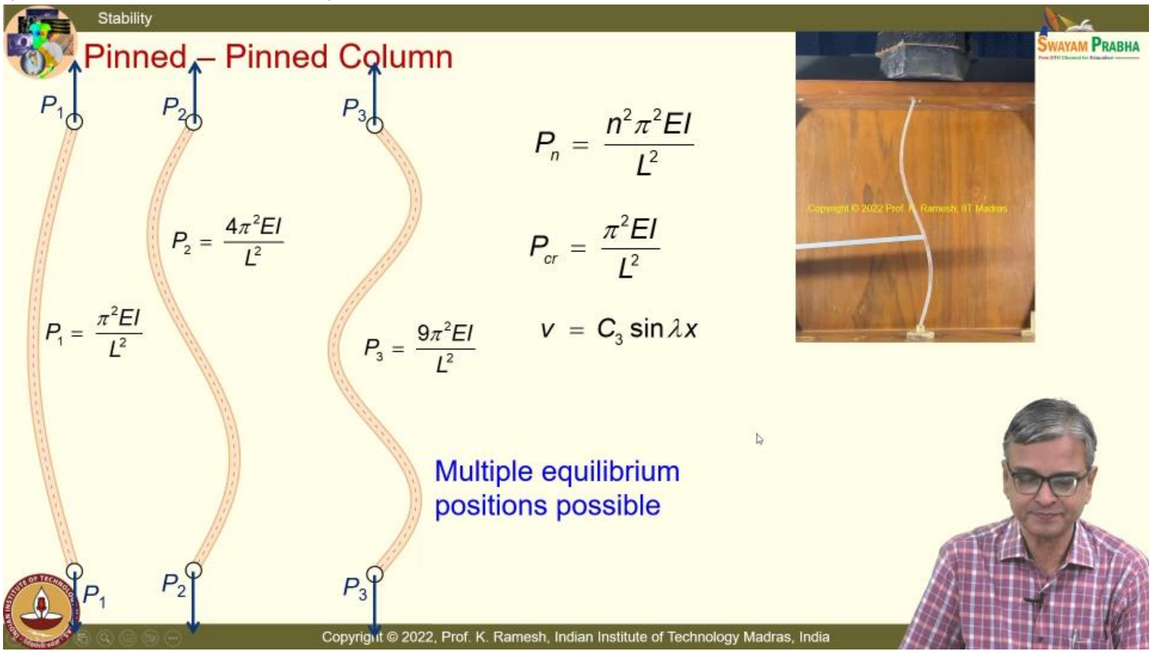
At $x=L$, $v=0$ and $M_b=EI\frac{d^2v}{dx^2}=0$

And what we did was, we got the governing equation, and we did not solve, we directly took the solution from your mathematical analysis. And then you simply take the

$$v=C_1+C_2x+C_3\sin\lambda x+C_4\cos\lambda x$$

I have four constants and I have four boundary conditions. So, we had the enough data to proceed, and we wrote the; when we substitute the boundary conditions, you get these expressions. See, this is not in one order. So, you have to figure out, that is also a good reason why you have to apply your mind to unravel this. For problem to problem, it is slightly different. Intentionally it is done, so that you are alert. And in order to have a non-trivial solution, I need to have this determinant go to zero.

(Refer Slide Time: 04.31)



That gives me an expression, wherein I get the critical load as

$$P_n = \frac{n^2\pi^2EI}{L^2}$$

You have multiple equilibrium positions possible. And when I have $n = 1$, I have the mode 1, which has a buckling shape like this,

$$P_1 = \frac{\pi^2 EI}{L^2}$$

And when I have $n = 2$, I have

$$P_2 = \frac{4\pi^2 EI}{L^2}$$

That means, if I increase the load, you are expected to get that mode shape. In practice, you will have to, you know, do something. This is the point where it crosses zero. So, you have to make this as zero externally. Only then in buckling problems, you will see that. I will also show you in an experiment what we have done.

So, what you find is, as n increases, you have different equilibrium configurations. The column cannot remain straight. That is an understanding that we get out of it. So, the critical load is the least one,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

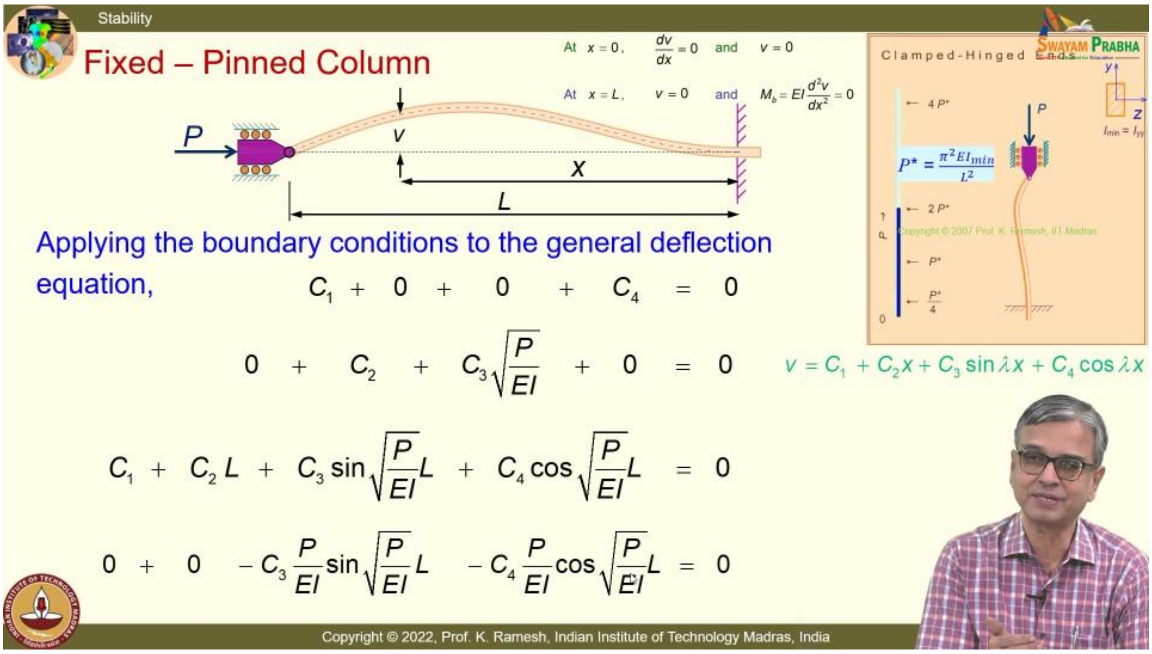
And it is better to recognize I as $I_{\min}$ when I have non-circular cross-section. In a circular cross-section, every diagonal is having same value of moment of inertia and it can have buckling in all the 360 degrees, ok. And once you solve this, you get the displacement as

$$v = C_3 \sin \lambda x$$

And I have also mentioned we are not in a position to calculate the value of C_3. It is indeterminate. It is mainly because we have linearized the governing equation. The moment-curvature relation was non-linear. In deflection, we said we are going to live with small deflection. We can ignore the second- order effects. Same approach we have followed in buckling also. In fact, if you consider the non-linear relationship of moment-curvature, it is possible for you to evaluate for a given load what is the coefficient C_3.

If you look at the buckling problem, what is more important is the value of the critical load. How the buckled shape is of secondary importance because the functionality of the column is disturbed. So, primary importance is to get the critical load. And you know, we also do not have the mathematics to investigate what happened once the critical load is reached because the deflections are very large. And you know, this shows experimentally I can get the mode 2, but I have to do something to do for the inflection point. I have to make that only then I will be in a position to see that. You know, sometimes you will see I have such beautiful curves, you will wonder can a column which looks so strong otherwise can have a shape like this. In order to bring and drive home that point, you know, we have got this experiment done and it has come out beautifully.

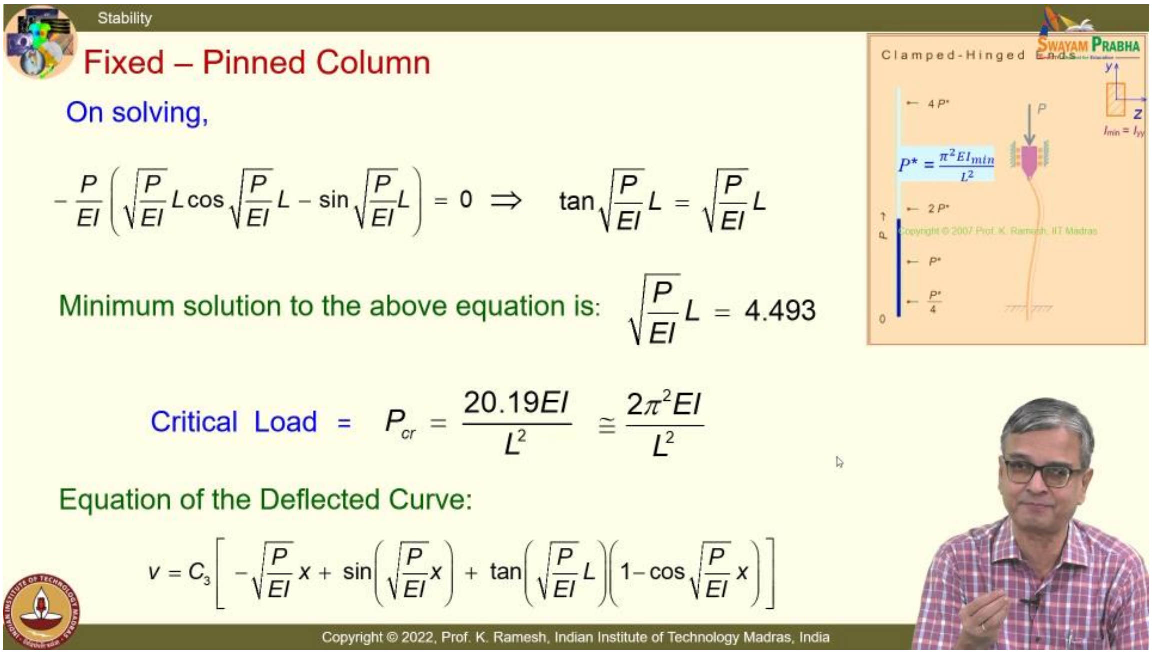
(Refer Slide Time: 08.06)



Next we move on to fixed-pinned column. What is the difference? Only in the boundary conditions. And you know, you have already seen what way to handle a fixed-end, what way to handle a pinned-end. So, it is a question of the equations having a different form that is all and you have to simplify them. I leave that as an exercise, but I will show you the final results. You have the boundary conditions like this and this is the solution we start with. When you substitute it, as I have told you that this is not in an order. So, you figure out whether the equations are correct. That is also an exercise for you to see that you have followed what is done in the class. And as I we have seen earlier for a non-trivial solution, I need to have this determinant go to zero.

See, what happens is we have handled a fixed-free, that is a cantilever. The equation was simple and straightforward. When we looked at pinned-pinned, the equation is also very simple. Once you come to any of fixed-pinned or fixed- fixed, the equations are little more complex and you have to apply your mind to solve it.

(Refer Slide Time: 09.27)



So, what I am going to have this is, I am going to have this to go to zero. You have to recognize this should be translated into

$$\tan\sqrt{\frac{P}{EI}}L = \sqrt{\frac{P}{EI}}L$$

and this needs to be solved. One can solve it easily in a graphical method or analytical also you can solve. And when you solve it, this happens to be a value 4.493 and it is inconvenient to look at as this number. We will also simplify it. So, the critical load is obtained as

$$\text{Critical Load} = P_{cr} = \frac{20.19EI}{L^2} \cong \frac{2\pi^2 EI}{L^2}$$

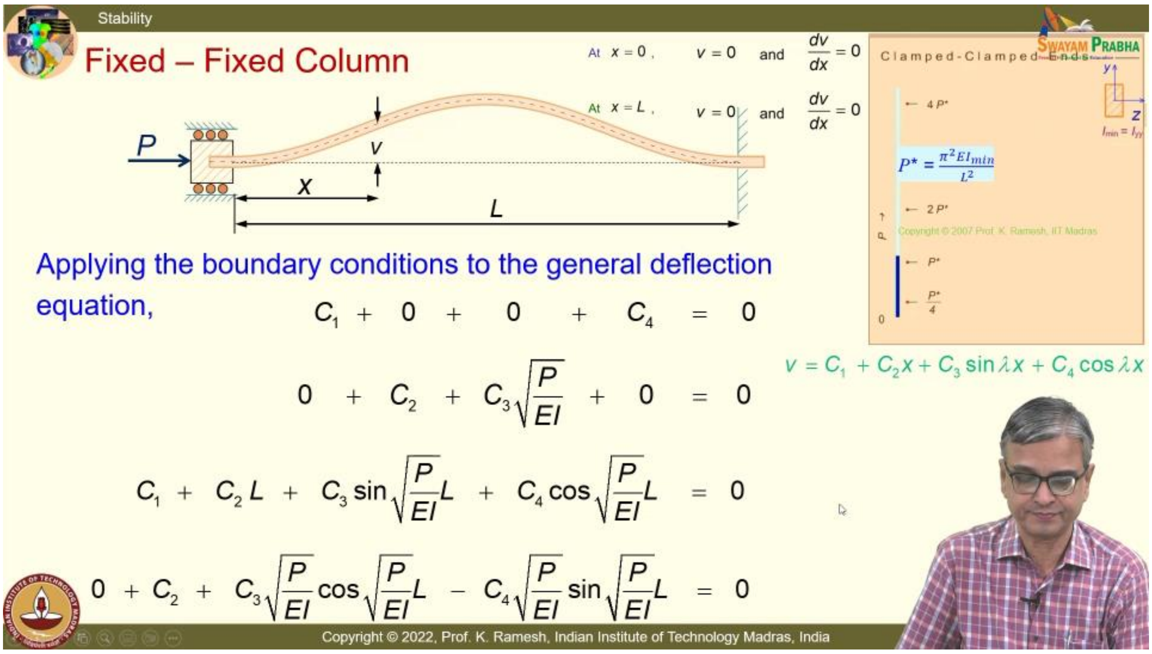
See, I have an ultimate objective. We are going to take pinned-pinned column as the basis. From the length of that column, we will also visualize the length of other end conditions. See, buckling is one problem where the critical load drastically changes because of the end conditions, fine. And in reality, how do we have these end conditions realized? We will also have a discussion on that. And from that point of view, if you want to compare rather than looking at this as 20.19, it is convenient for us to have this as $2\pi^2$, ok. Because $\frac{\pi^2 EI}{L^2}$ is your basic expression.

So, I can say what is the value of the equivalent length of the curve. And once I go to the reflection curve, it is a very long expression. Please make a note of this. I have this as

$$v = C_3\left[-\sqrt{\frac{P}{EI}}x + \sin\left(\sqrt{\frac{P}{EI}}x\right) + \tan\left(\sqrt{\frac{P}{EI}}L\right)\left(1-\cos\sqrt{\frac{P}{EI}}x\right)\right]$$

See, from your notes, you should have all these deflections. Do not make an attempt to remember this. It is not a learning on memorizing your capacity. You have to have a physical appreciation. In all buckling problems, what is very important is the expression for the critical load because deflection, you have to also appreciate that you have a deflected shape of this form. It is of secondary importance because you know the column has lost its ability. That is what we are concerned about it. And here you see, when you compare to the pinned-pinned column, it is able to take twice the load. The critical load means I can increase the load two times. Only then buckling happens. That is the physical appreciation that you should retain in your system. When you look at a pinned-fixed column, you should feel that it is safer than a pinned-pinned column, ok.

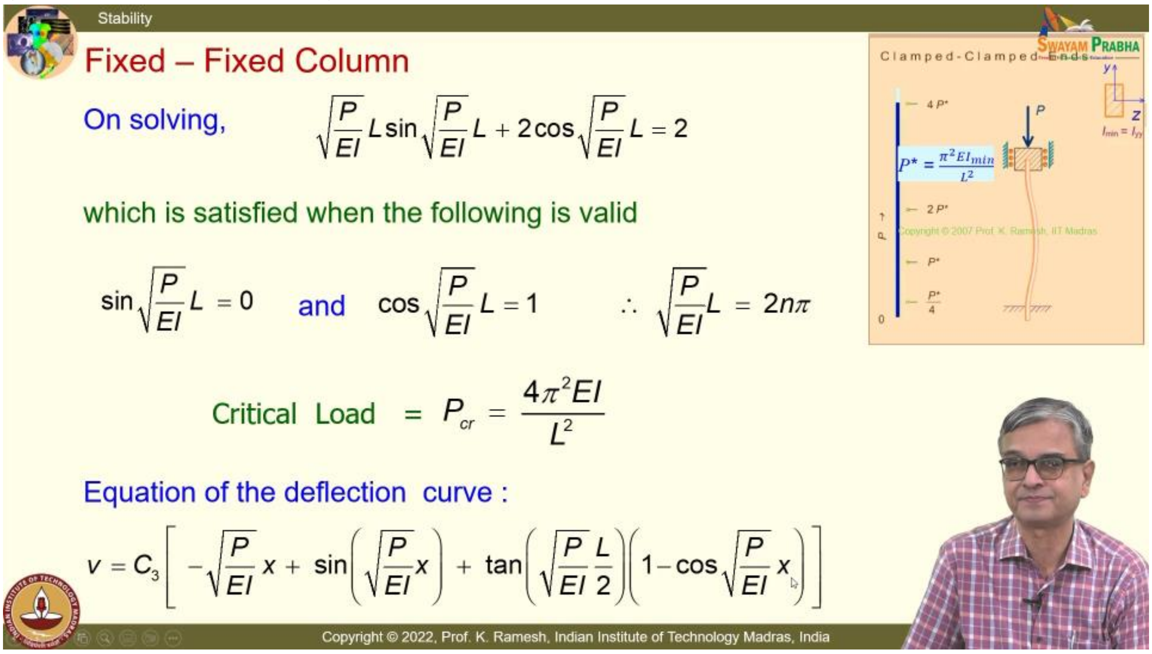
(Refer Slide Time: 13.05)



Then we move on to fixed-fixed column. And once I have a fixed-fixed column, you will all recognize that you will also have slope zero at both the ends. So, the boundary condition is like this. All of this we have looked at even when we discussed the principle of superposition. When we wanted to have a deflection, we have recognized that you have a fixed end, you have a pinned end, you have a free end. We have learnt how to write the boundary condition. So, the boundary condition still remains the same, but the governing equation is different. The governing equation also what way we have obtained, we have deliberately looked at in the deformed configuration. This is the only problem in this course where we write the governing equation by looking at the deformed configuration. So, that is a fundamental difference in the basic analysis here.

So, once I have these boundary conditions applied to this expression v, I have the four equations. Here again the sequence is not in the same way that I have given the boundary condition. So, you figure out and also alert me if there are any typographical errors, fine.

(Refer Slide Time: 14.33)



And for a non-trivial solution, I should have this determinant to go to zero. So, when I do this, I have an expression like this. Tell me what way I should translate this into a condition for me to find out the critical load. See, when I have the determinant, I get the expression that determinant should go to a value two. Can you look at what way we can impose this condition? Can you just read the equation? You all have mathematical ability to recognize this. When will this expression go to two in a simplistic manner? So, this goes to one and this should go to zero. So, I should illustrate the condition as I should have

$$\sin\sqrt{\frac{P}{EI}}L = 0 \quad \text{and} \quad \cos\sqrt{\frac{P}{EI}}L = 1$$

Earlier case, we had $\sin\lambda = 0$. That was what was only one condition. We solved one problem. But now, I should have both of this go to a respective values as indicated. So, this is possible when I have

$$\sqrt{\frac{P}{EI}}L = 2n\pi$$

The difference comes only there. Otherwise, the mathematics is simple, straight forward. Only when you look at the determinant, final expression how do you represent it in a convenient form for you to get the critical load. The focus is to get the critical load.

So, from this, I can easily find out the critical load. So, I have the critical load as

$$\text{Critical Load} = P_{cr} = \frac{4\pi^2 EI}{L^2}$$

So, look at if I have a fixed- fixed condition, the column is very, very strong. Four times the load it can withstand in comparison to a pinned-pinned column. That is what is shown as $4P^*$ here, and P^* is defined as

$$P^* = \frac{\pi^2 EI_{\min}}{L^2}$$

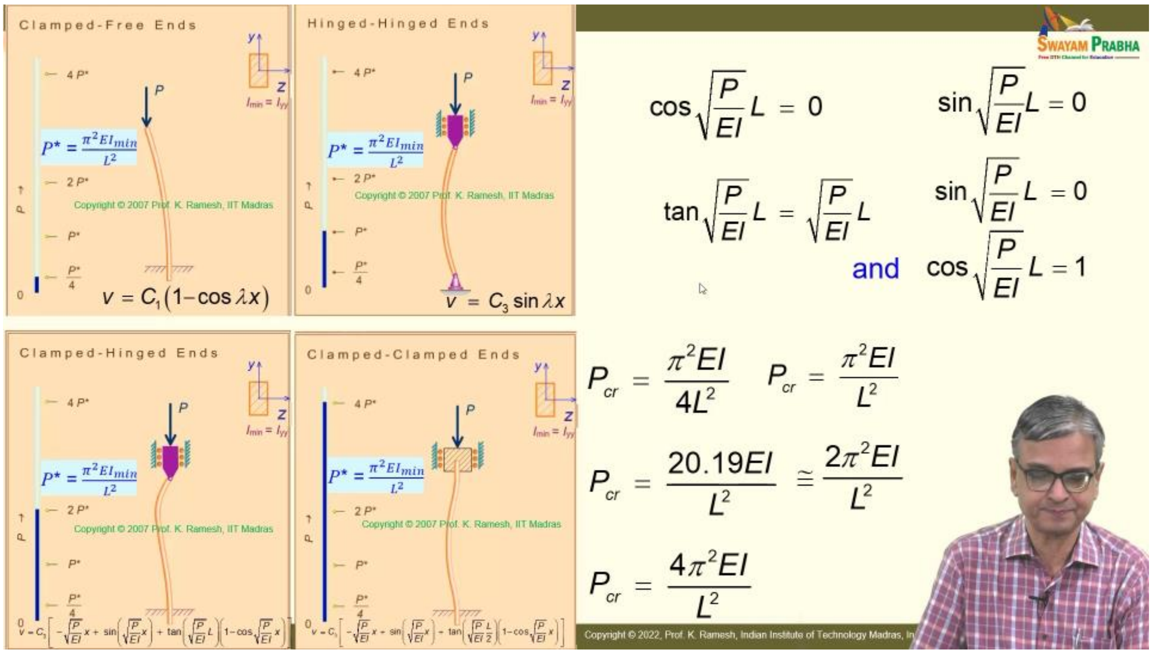
Becau se I have taken a rectangular cross-section.

And here again, you know, if you look at the equation of deflection curve, very similar to what we saw in fixed-pinned and if you look at closely, only this term changes. You write down the complete expression. I have

$$v = C_3\left[-\sqrt{\frac{P}{EI}}\,x + \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + \tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\left(1-\cos\sqrt{\frac{P}{EI}}\,x\right)\right]$$

So, there is only a small difference between what we saw in fixed-pinned and fixed-fixed. And what is significant here is the critical load is very high. That means, you can be little relaxed when I have good end conditions and you know, just observe the animation. The animation is very nicely done. It will drive home the point, you know, how the columns are relatively placed in terms of critical load, fine. You have a visual representation and I am also going to provide an excellent summary.

(Refer Slide Time: 17.41)



You have a visual representation and I am also going to provide an excellent summary. So, in the case of clamped free, the basic expression we had was when the determinant goes to zero,

$$\cos\sqrt{\frac{P}{EI}}L = 0$$

So, that gave me

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

and you could also get the deflected curve as

$$v = C_1(1-\cos\lambda x)$$

When we went to pinned-pinned end, we had

$$\sin\sqrt{\frac{P}{EI}}L = 0$$

This gave me

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

See, in all my expressions, I have just put I. You must recognize that it should be I_{min}, which is done in the diagram, but not in these expressions. So, recognize that you should put this as I_{min}. So, when we moved on to fixed-hinged, I get the governing expression as

$$\tan\sqrt{\frac{P}{EI}}L = \sqrt{\frac{P}{EI}}L,$$

this should be satisfied. From this, I get the critical load, which we rewrite as

$$P_{cr} = \frac{20.19EI}{L^2} \cong \frac{2\pi^2 EI}{L^2}$$

In the last case, the condition was I should have

$$\sin\sqrt{\frac{P}{EI}}L = 0$$

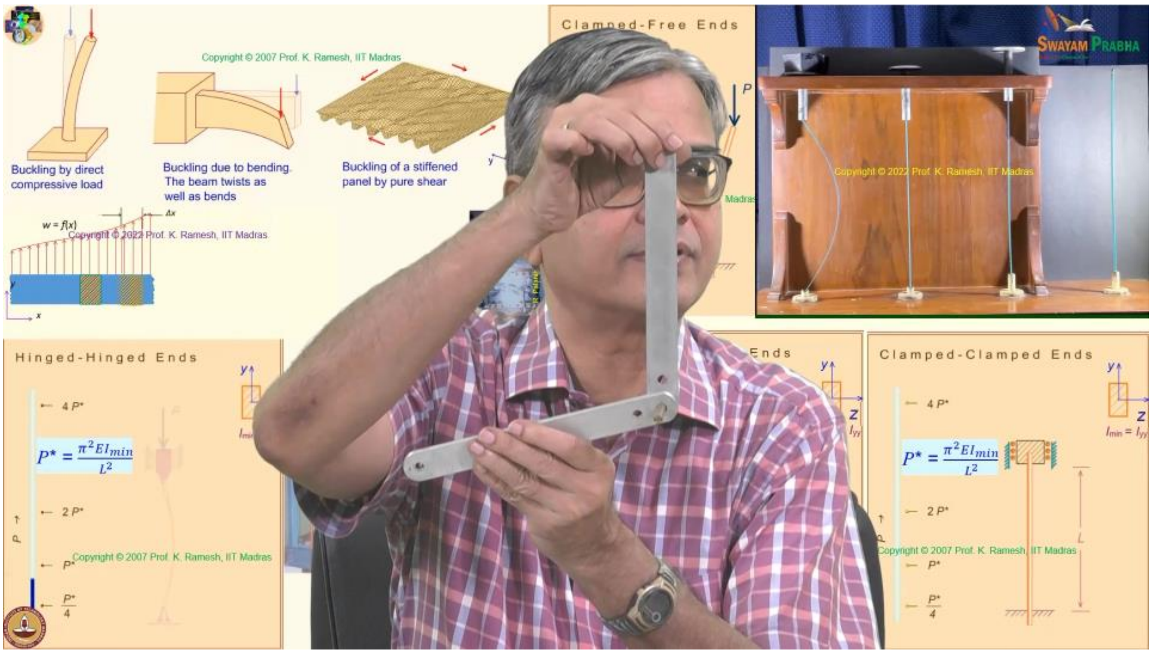
$$\cos\sqrt{\frac{P}{EI}}L = 1$$

So, this is satisfied, I get the critical load as

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

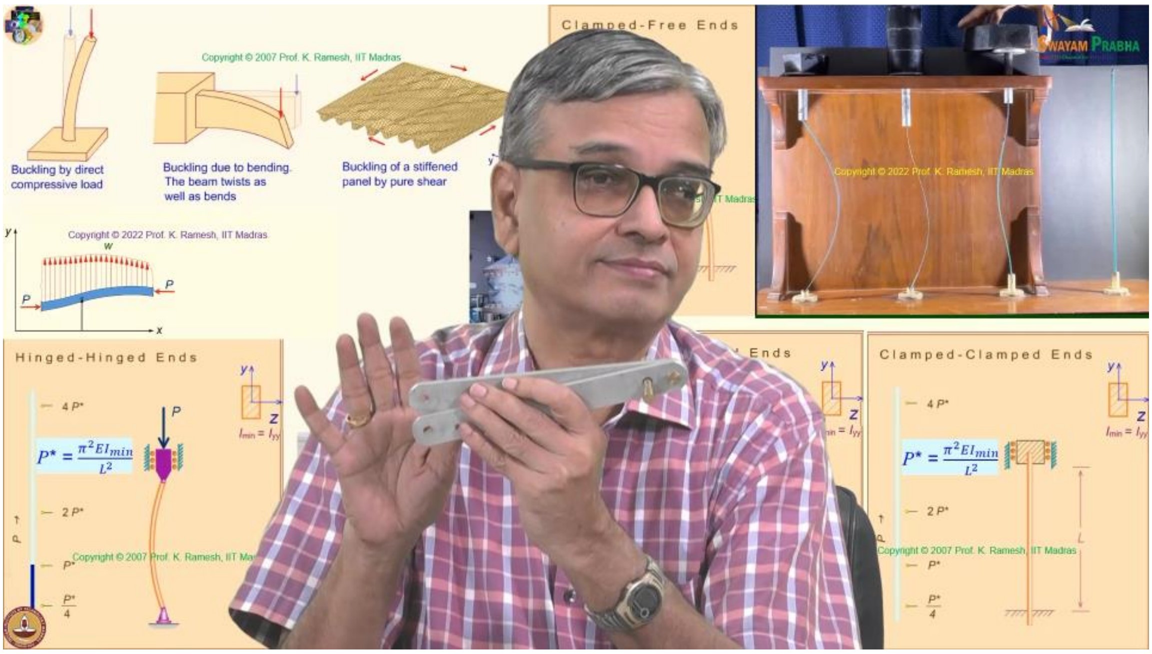
In all these cases, you have the deflection also. It is a very nice summary of what we have discussed so far. At a glance, you get what happens and visually you see what is the shaded region. If the shaded region is the longest one, then you have the strongest column, ok. And it is all compared with respect to the pinned-pinned end.

(Refer Slide Time: 19.58)



See, now the question comes, why do we have a pinned-pinned end as a basis? See, if I have a pinned end, it can easily rotate. See, only in your course, problems are posed that this support is pinned- pinned, other one is pinned-free, other one is fixed-free. If you are asked to go and look at the real structure, you will have to idealize depending on the constraint in the problem as one of the supports. See, now what I will do is, I have two holes. I suppose you are able to see two holes. I have put the pin, one pin and I am able to freely rotate. What do you say as fixed end? It should not rotate, fine.

(Refer Slide Time: 20.31)

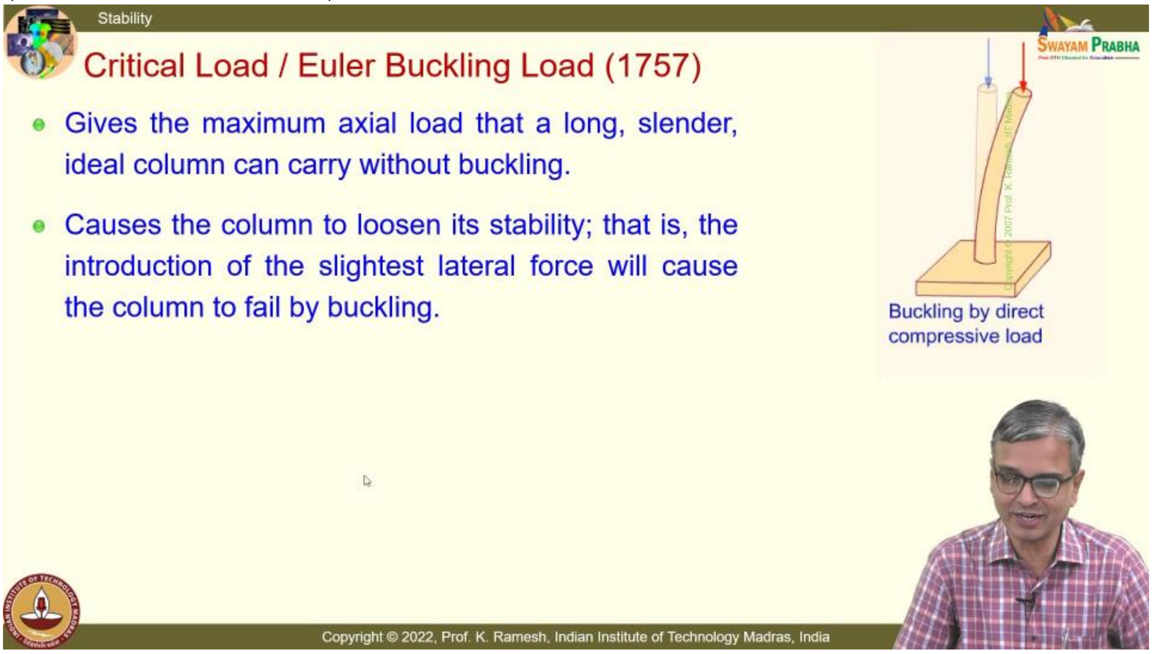


Suppose I put another pin, probably you can see this better. I have two pins and I am trying to rotate. See, this pin is not of the same size as the hole. Suppose I have the pin of the same size, I am able to have some rotation that is because the pin is diameter is smaller than the hole diameter. Suppose I have a properly made pin, even this rotation is not possible. In other words, when I have even two pins, it prevents rotation, ok. That means, if you want to idealize this as fixed end, you are also justified, even though you make a small error. So, in reality, you do not know how the constraints are. We always do the boundary solutions that is, I will have upper bound and lower bound. I will do the analysis as pinned and then get one number. I will also repeat the analysis with fixed. I get another number. So, between these two, I should operate. That is how engineers work on. See, even though in this course, we have seen support conditions, we have also looked at to what extent these supports modify your results.

In fact, while we discussed the deflection problem, we had a truss. In fact, it was a frame. It was fixed at one end, the *I*-beam. We calculated considering that as a pin joint and then calculated what is the deflection. We calculated again the same problem with that as fixed. The difference was about 2.5 or 2.7 percent. Why that was done? See, in your rigid body mechanics, it is sacrosanct; you should know when the symbol is given that is, pin joint, put the boundary conditions appropriately. When it is a fixed joint, put the boundary conditions separately. But all that is indicated by a, you know, we had symbols for that. You will not have symbols attached to practical structures. While learning the course, you need to have those symbols for you to appreciate the different joint conditions. When you come to field practice, there is always a dilemma whether it is a pinned joint or a fixed joint.

And we have also seen in certain applications, the end conditions are not influencing much. Definitely in the case of buckling, it has a significant importance. It is a favorable direction.

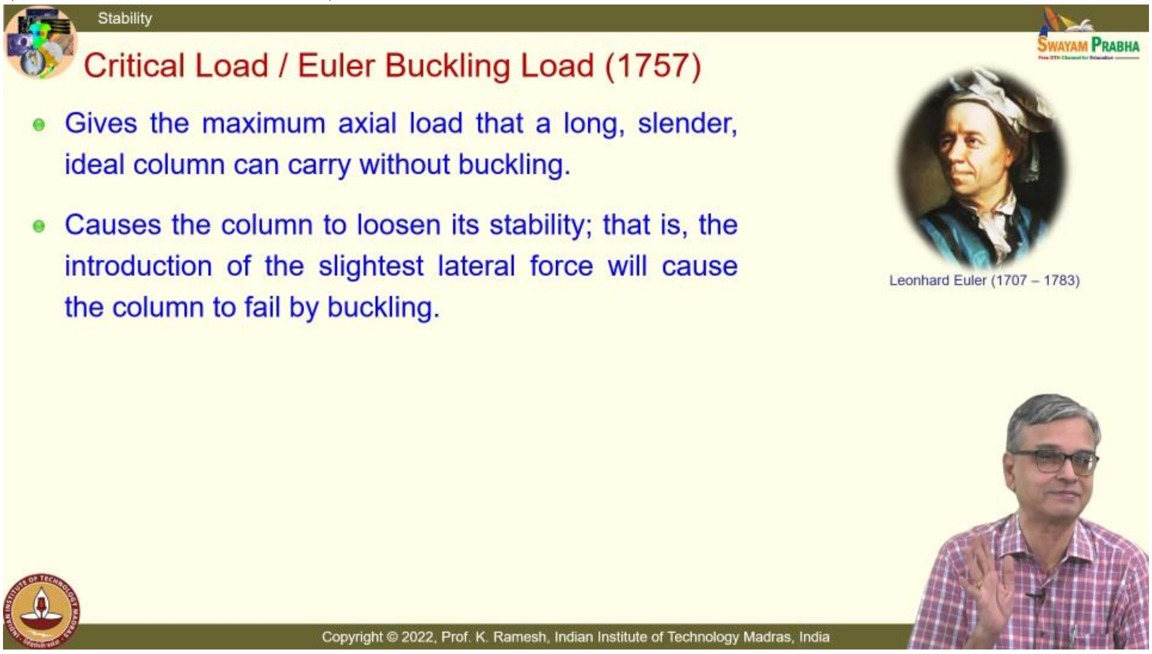
(Refer Slide Time: 23.37)



And you have this, let us go back to this Euler buckling again because you know, when we started this class, you may not have observed certain nuances very closely. So, what I have here is, you look at here, as the load is increased, suddenly it buckles without a warning. That you should recognize. That is why buckling is so important from analysis point of view. And what you find is, it causes the column to loosen its stability. That is, the introduction of the slightest lateral force will cause the column to fail by buckling.

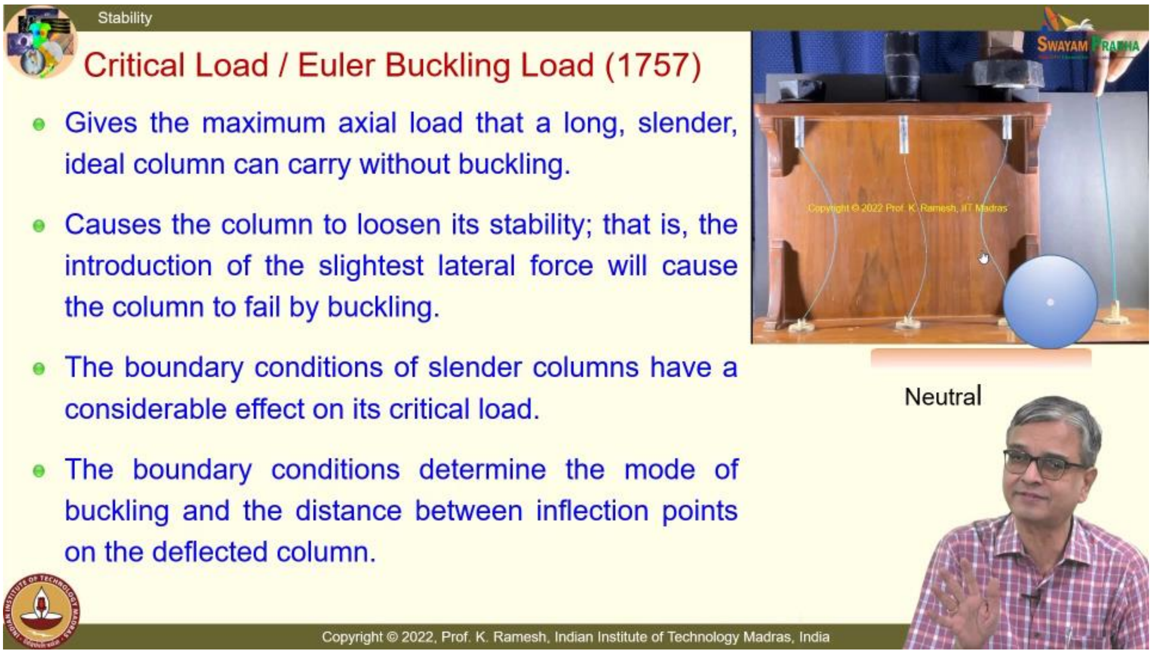
See, I have always been showing experiment and also analytical justification and also numerical justification. And one of the difficult experiments to be conducted are buckling experiments because the problem is, you know, the slightest lateral force can come from imperfections in the material the column is made of, how the column is made of. That is very difficult to model. And we have also said that we have an elastic continuum. That is how we have developed the mathematics. So, among the several experiments, costliest experiments are buckling experiments. Million dollars have to be spent in making the specimen appropriately. And also the deviation from experimental observation and analytical calculation are maximum in buckling experiments. Please understand that, fine. When you do your experiment, you find the deviations are there.

(Refer Slide Time: 25.16)



Do not get annoyed because you cannot make anything perfect because the analytical model wants a perfect specimen. Perfect specimen is extremely expensive. It is not in two, three orders. Its orders of magnitude is very, very high, fine. And we have also said that this is credited to the genius, mathematical genius, Euler. And we have also looked at, he had physical difficulties with all that he was able to do it.

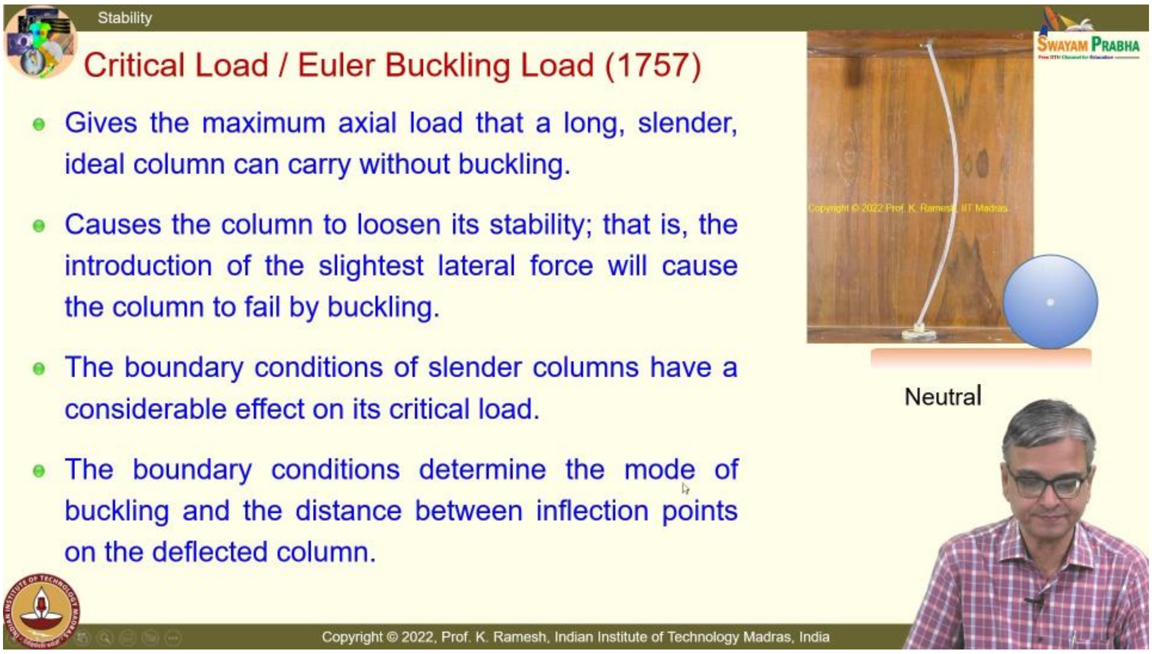
(Refer Slide Time: 25.53)



So, you look at the deflected shape. And I have already said, these are experiments indicative of the relative merits of the capacity to withstand the critical load because you can see the weights put. See, if I have to perform the experiment in an idealistic manner to get the critical load, I should have provision to vary the load, I mean, in a manner continuously. The loading system, what I have shown is not alright because I have discrete loads. These discrete loads may miss the critical load, fine. And what we have also seen is, even if you go beyond the critical load, you will see a deflected pattern so beautifully in your experiment; your analysis may be difficult. That is different! And I have also used this to show another aspect. You know, we have discussed that what you have as these positions are neutral equilibrium positions, fine.

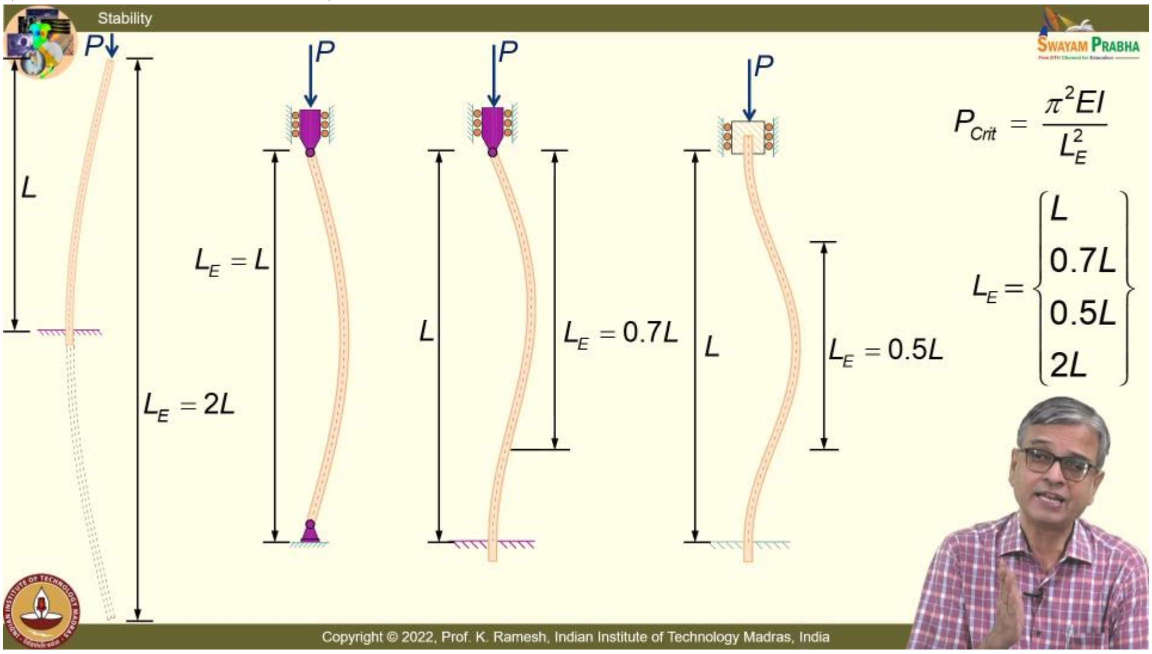
And when I redo this, you can recognize this has already buckled. This load is higher than this. Here you can see in a subtle fashion, and you can see that in a definite fashion here. I have one equilibrium position, I apply load; I have another equilibrium position; I apply another load, I have another equilibrium position. So, these are all neutral equilibrium positions, but transition from one to another is sudden. So, it loses its stability, ok.

(Refer Slide Time: 27.40)



And I said whether buckling is useful or not, people have used it in your toys, people also have used it in electrical switches where you have snap buckling is used for the making positive contact.

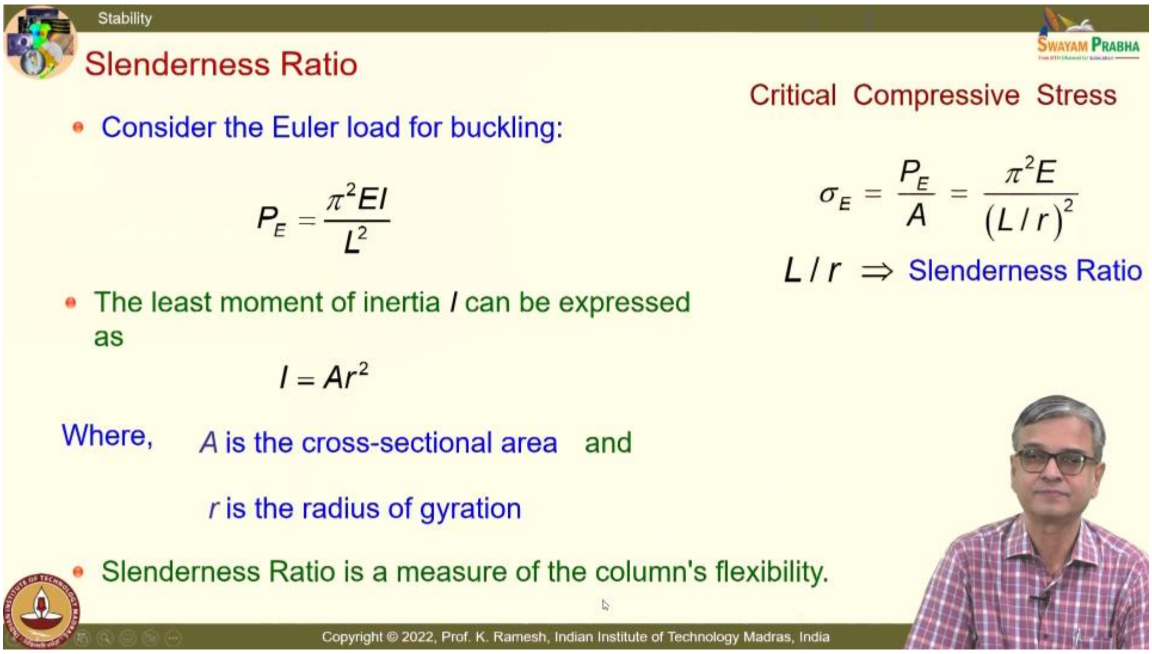
(Refer Slide Time: 27.54)



I am now going to discuss the concept of equivalent length, ok. So, I take the pinned-pinned column as the basis. I call this as $L_E = L$. I can also visualize, see if I have a longer column, it is dangerous. A shorter column is better. That is the way you have to look at it. Longer the column, its tendency to buckle is more; Its tendency to buckle is more. If I go to fixed-pinned, we have already got the expression. Now, we are going to have a relook from a different perspective. I call this as $L_E = 0.7L$, ok. And if I have a fixed-fixed, it behaves like half the length of the pinned-pinned column, ok.

Suppose I have a cantilever, it behaves differently. It behaves as if it is twice the length of this column. That means it can have only the least load of critical load. This will have the highest critical load possible. So, another way of looking at a column problem is, look this in comparison to a pinned-pinned column. What way you can visualize the length of the other columns. So, from the point of view of critical load, the boundary conditions are very significant. In other problems, what we have looked at the boundary conditions, you can get the lower bound and upper bound solution. You can also do the same thing for buckling also because physical problems do not have the symbol attached. You have to go and find out from the constraints to idealize and true fixed conditions are always difficult to simulate.

(Refer Slide Time: 30.00)

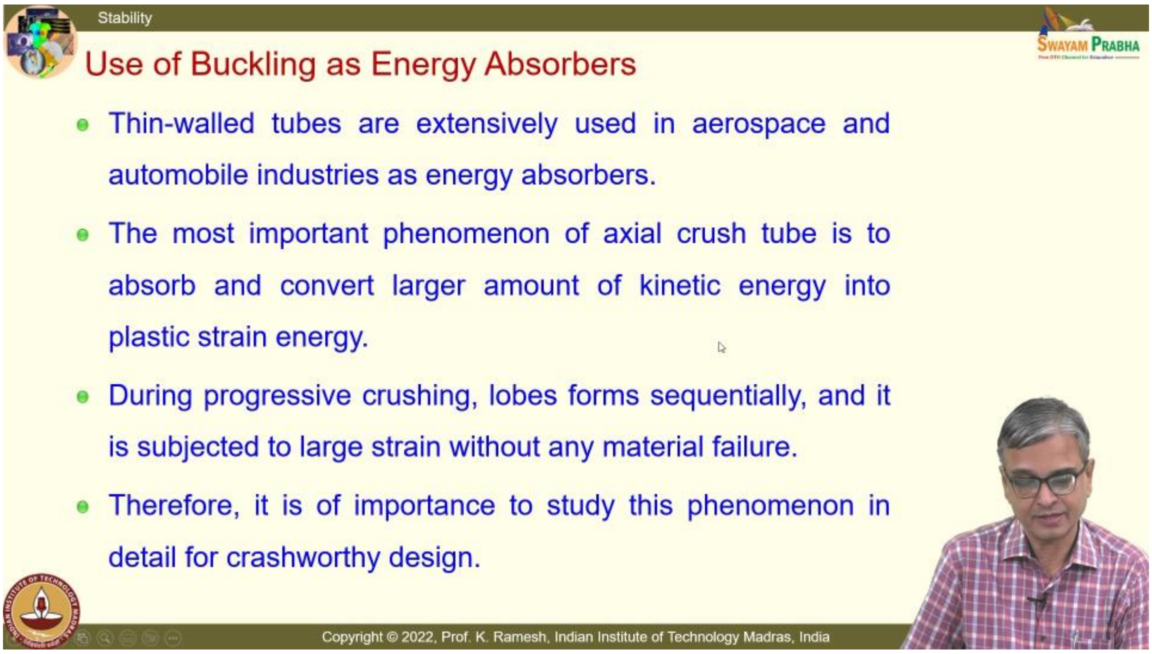


And you also have another concept called slenderness ratio in column buckling. And what you do is, you write the moment of inertia as $I = Ar^2$, where r is the radius of gyration. So, instead of looking at that as a critical load, I can also get that as critical stress

$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L/r)^2}$$

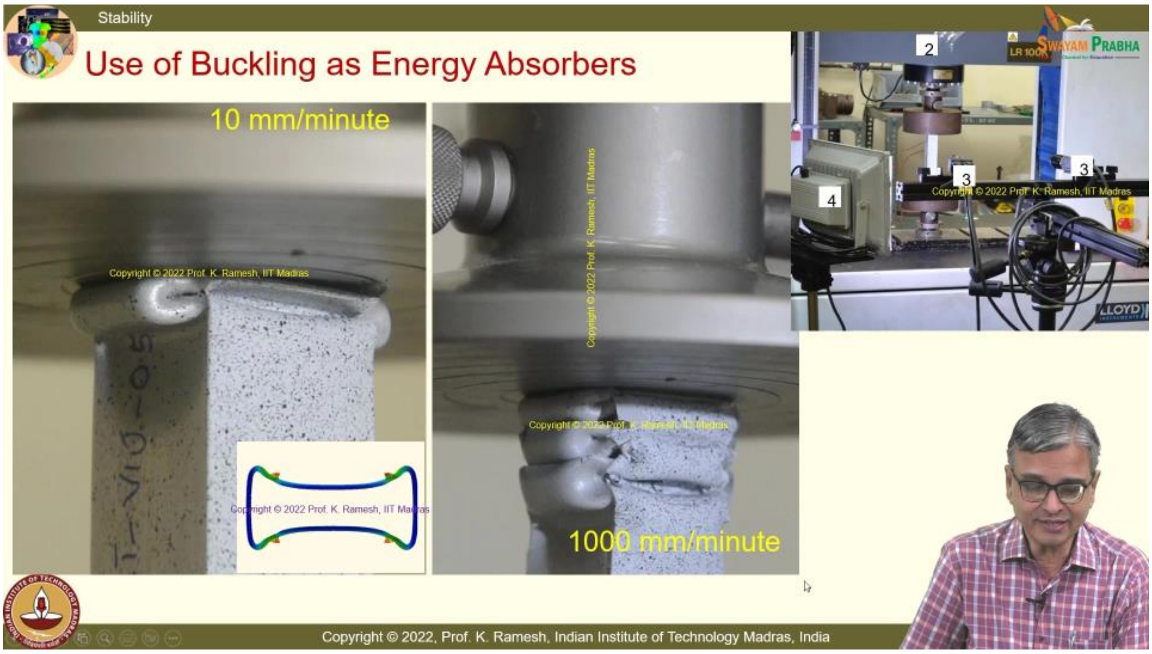
So, L/r is a slenderness ratio and if L is larger and larger, the column is more flexible. So, that is what is said. Slenderness ratio is a measure of the columns flexibility. You do not want that to be flexible. In a structural application, you do not want that to be flexible. You want that to be strong and I have to withstand the load, fine.

(Refer Slide Time: 30.50)



And are there other applications? You know, I have always gone beyond the basics of the course to motivate you to see what all you can learn in future. See, use of buckling as energy absorbers. Thin-walled tubes are extensively used in aerospace and automobile industries as energy absorbers. The most important phenomenon of axial crush tube is to absorb and convert larger amount of kinetic energy into plastic strain energy. During progressive crushing, lobes form sequentially, and it is subjected to large strain without any material failure, because the idea is to protect the occupants. See, you also have metallic foams. That is another way of softening your crash victims, ok. Another way is to use thin-walled tubes. So, it is of important to study this phenomenon in detail for crash worthy design.

(Refer Slide Time: 32.00)

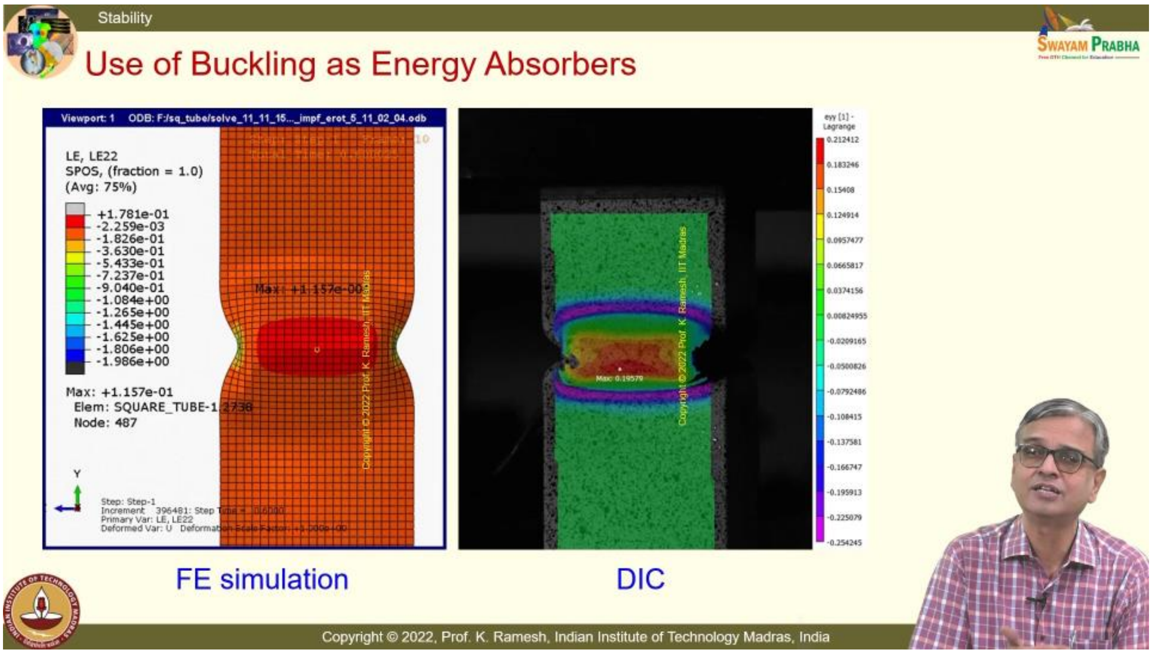


And you see this beautiful animation. You see how this is crushing. This is done by one of my students who did his masters in IIT Madras. It is an excellent piece of experimental and numerical analysis and you could quickly see; what are these dots? Do not say defects! DIC! See, I have said you have to be current students when digital image correlation is the rage now. Once finite element was a rage, everybody was hanging on to it. In experimental mechanics, everybody wants to learn digital image correlation, and this is an excellent candidate for doing digital image correlation, because I am going to have ultra-high strain; strains of 30 percent and DIC is highly applicable for large deformation problems. And you should guess from this experimental setup. I have two cameras and that is the reason why I brought this.

I want to sensitize you that you have this and I also have another way of looking at it. This is from the numerical work which shows how the square tube has buckled and this is plastic buckling. See, material has not separated and this is what? This is happening at 10 mm/min. Now, what I am going to do is, I am going to increase my speed of rate of loading and I have said rate of loading is different and look at how beautifully the patterns develop, ok. Do not you think this is very, very interesting? Isn't? So, that means, the crash tube is doing its job very well in absorbing more and more energy. So, you can be protected like engineers, future engineers, you have to develop all of this, fine, for passenger car protection.

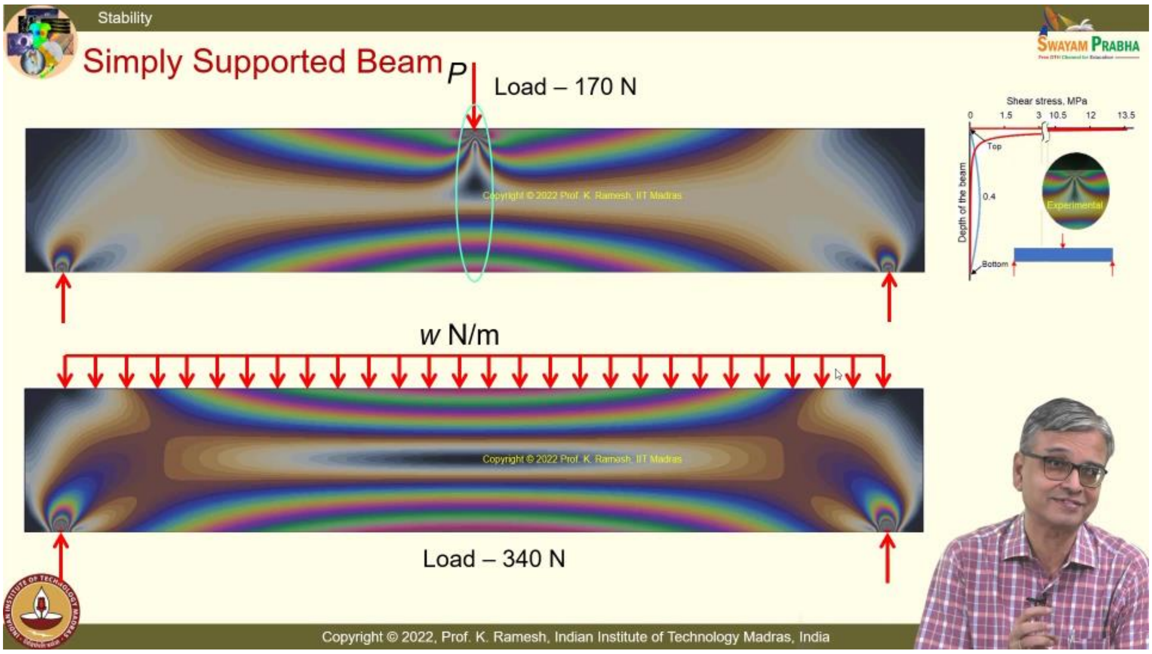
I think you can see the animation again. This is so interesting to see. See, what is peeling off is the paint that is put for, you know, image correlation. Image correlation because you know, the paint should be adhered to the surface very well because it is an aluminum tube, some places it is too smooth.

(Refer Slide Time: 34.27)



So, it is not adhered and not only this, see my students have always compared numerical as well as experiment. Very few labs across the world do these comparisons. Only when you compare, you burn your fingers and you understand your modeling correctly and look at here, this is the non-linear finite element analysis, and this is a digital image correlation. Look at visually the patterns are quite identical, and this is a very, very complex problem. It is a non-linear finite element, it is not a simple problem, simple problem to handle with, ok. So, this should give you motivation. What you have learnt as strength of materials only is basics. They are very, very important, fundamentals are very important, and you have to graduate and solve very challenging and complex problems of today. So, that requires your training in a numerical technique as well as an experimental technique, ok.

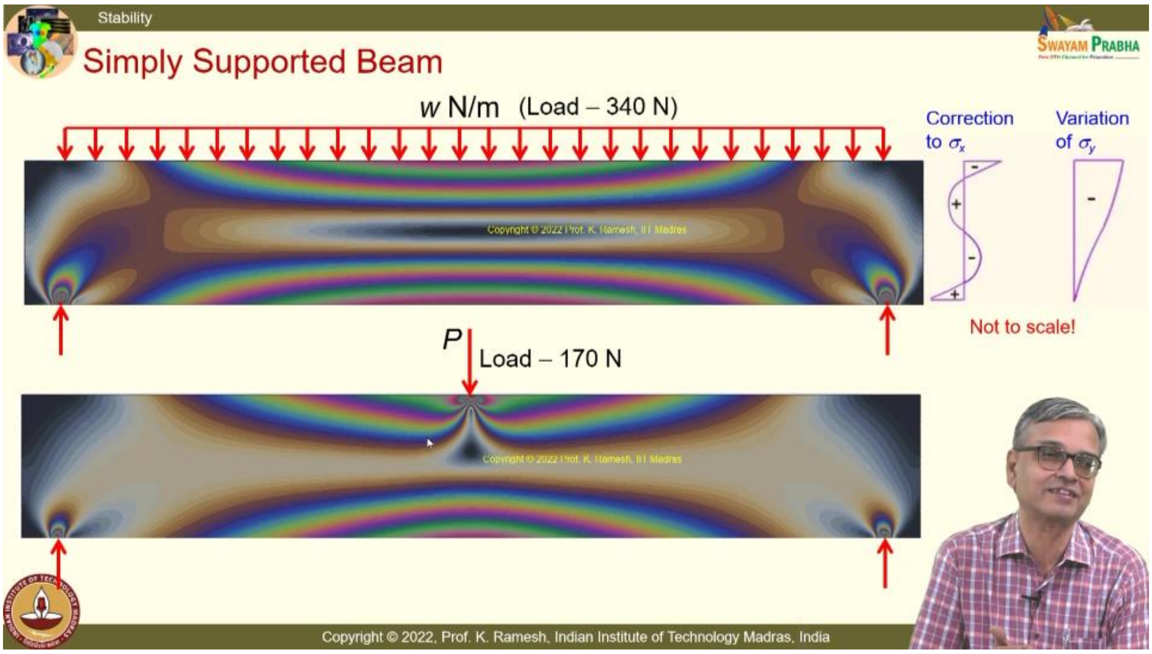
(Refer Slide Time: 35.54)



Then in this course, you know, we have also looked at extensive use of photoelasticity to understand the concepts related to stress. That advantage even the developers did not have; they had done crude experiments. This is the subject which developed based on experiential knowledge, it is not theoretically done. And wherever possible, I have emphasized where there is a deviation and when you are looking at shear stress, there are many places where you can ignore, but you cannot ignore near the load application point. Near the load application point, your strength of material result of parabolic variation is observed. The shear variation is very, very high just below the boundary, but on the top surface it is still zero because it has to satisfy the free surface requirement. And these stresses are comparable to your bending stress. So, failure can happen unless you strengthen this. When you are doing the design, strength of material helps you away from the zone, away from the load application and end conditions, you have solution from strength of materials. But even that was it right?

We have also looked at; you can actually take a photograph if you want, because you know, they have very nice, beautiful patterns, you do not get them anywhere, you do not get them anywhere. That is the reason why I thought that I should have it here. I have this uniformly distributed load. When uniformly distributed load strength of materials, was it right in all respects? I have also pointed out where it was deviating. We will see that. We will see that.

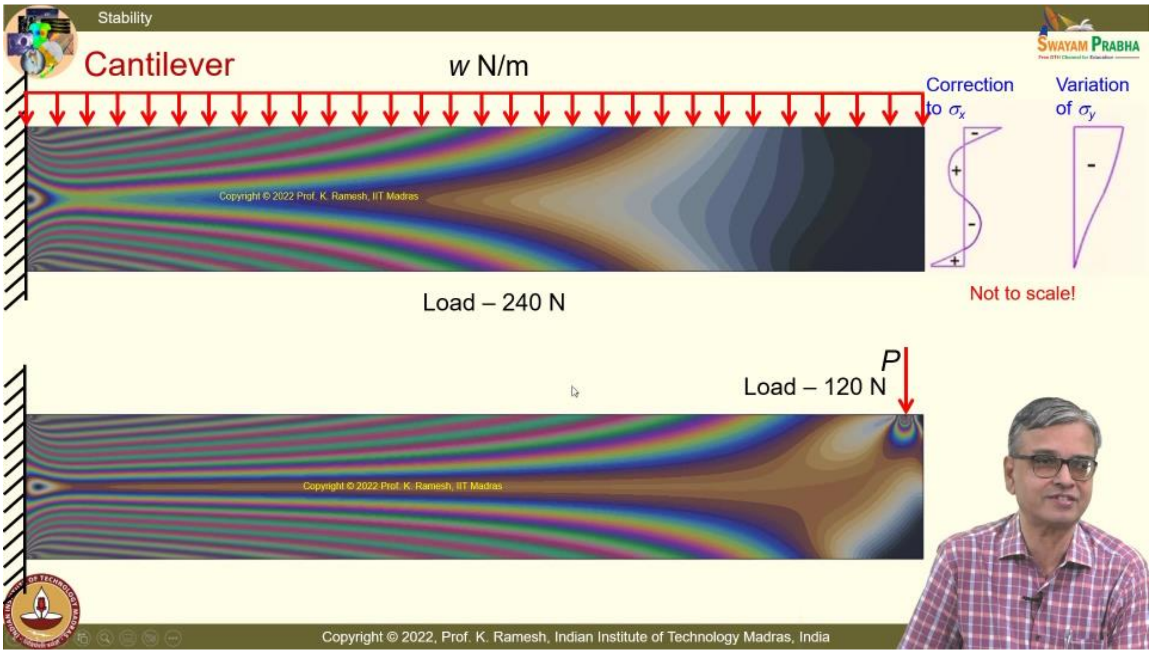
(Refer Slide Time: 37.14)



What are the two quantities I showed? It is again very important. This is correction to σ_x, it is no longer varying linearly, it is non-linear. And you also have in addition, there is sigma y. Can you see from the pattern existence of σ_y ? Or your eyes are sensitive enough, tuned to that. You look at what happens at the bottom, you look at what happens at the top. Do you find there is asymmetry here? The fringes are well developed here; fringes are not fully developed, ok. You see, beyond this red some slight shade of color coming out. The width is broader, the width is shorter, ok. You know, you have to be sensitive with the experiments reveal truth, please understand. You may make certain simplifications for the purpose of your analysis. You may say curvature is linear, experiment recognize this as non-linear. You may say shear stress is small, experiment recognizes even that small shear stress and throughout the experimental pattern in tune to that. There are no approximations in an experiment. So, you should do a careful experiment and interpret every aspect of the experimental analysis. Do not ignore! Do not come out and say the experiment is 30 years old. So, I see asymmetry and then walk away. Like you know, in your history books what each king has done, he has planted trees on either side of the road. Same answer for every king. Have you not done that in your history in the schools? And whenever you find deviation from experiment and your mathematical analysis, one of the blame is on the equipment, not on the performer. The performer has to be careful and systematic in performing experiment. These are the usual ways people wriggle out of real challenges, ok.

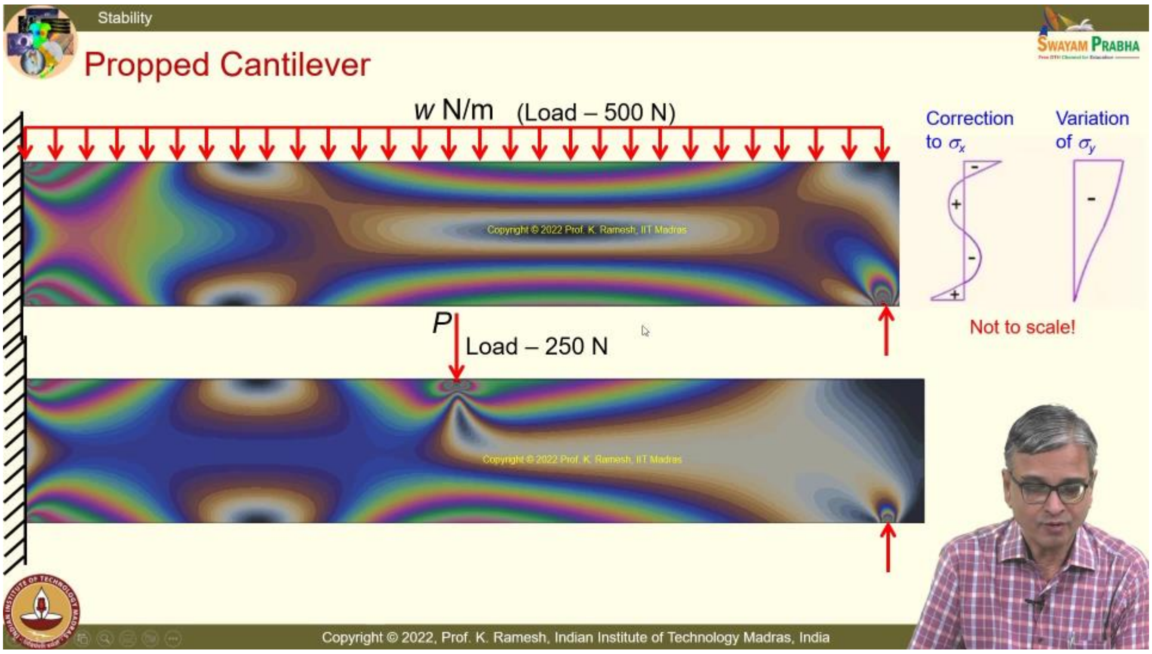
And I have said only in buckling experiments, even when you do a careful experiment, the deviations are very high because you cannot get a perfect material. You cannot have a perfect cylinder. You cannot have a perfect sphere free of defects. So, they are all very, very expensive. We have seen this for simply supported beam.

(Refer Slide Time: 39.47)



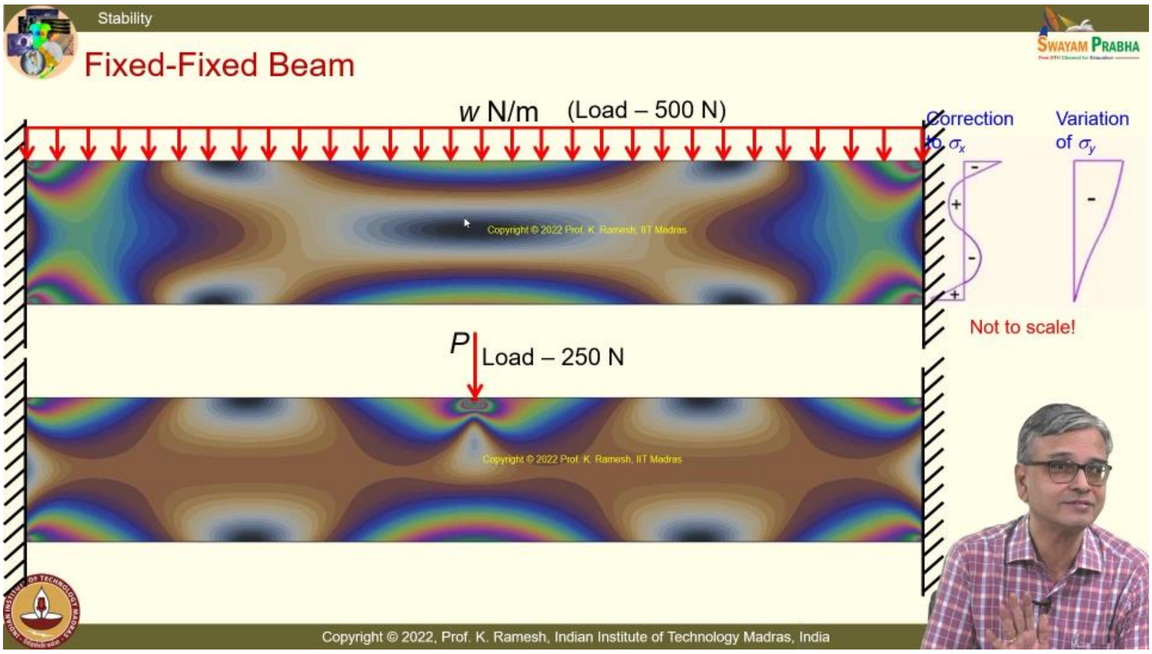
We have also done it for a cantilever. So, when I have a cantilever with a distributed load, you should anticipate with a pinch of salt, strength of material is reasonable for you to accept, but theory of elasticity will always say other quantities. But they are second-order effects. I have also put not to scale. That means, this is very, very small, but you should recognize that there are deviations. And when I have a tip load, I have the fringe pattern like this. Now, let me change the conditions, fine.

(Refer Slide Time: 40.27)



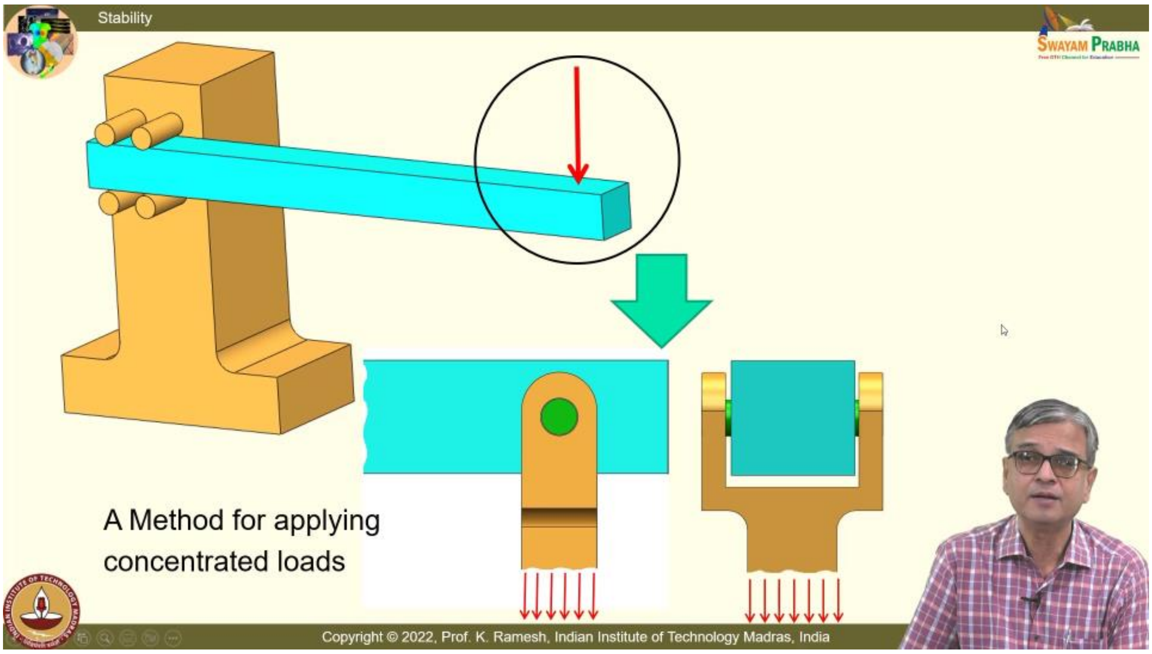
Suppose I have this as a propped cantilever. That means, one side is fixed. Look at the change in the fringe patterns. For all of this, what do you use? You use your flexural formula. From that point of view, flexural formula is a success, ok. But look at all the nuances in the small changes are captured in the experiment and you find beautiful patterns, ok. See, if you look at this is like a cantilever, this is like a fixed beam, ok. You see combination of both because when I change the end conditions, it also gives you an appreciation of why we want to look at for different end conditions. From a mathematical analysis, we should understand what is happening. In a physical system, if you look at these end conditions, you have to apply the engineering acumen to model it. Only then you become a good engineer. If you keep solving problems, somebody has coined it as fixed support, somebody has coined as pin support, is not going to make it as a good engineer. You should know what are the limitations. If you come across the limitation, how to wriggle out of it. So, look at and get the lower bound result and upper bound solution.

(Refer Slide Time: 41.37)



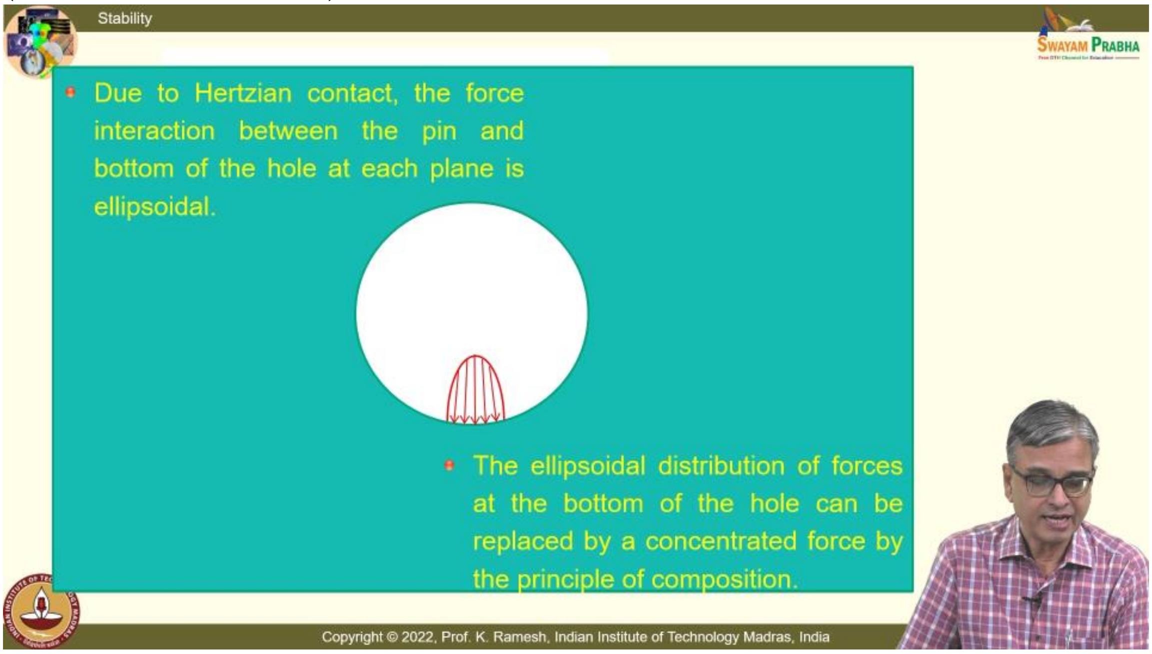
Suppose I have this as fixed-fixed. In all these cases, you will have this variation because when I have a distributed load, I am compressing it deliberately and you will see the symmetry. Asymmetry is seen here. I could see orange color here. I could not see that orange color here. Is the idea clear? So, the experiment recognizes the presence of σ_y just because analytically you cannot handle that. We have simplified it because the error introduced is small. We have ignored it. As a designer, what way we could accommodate this? Go and have a careful factor of safety. Is the idea clear? So, we did it as practicing engineers at some part of the analysis. We do not ignore it. If you ignore it, you will have surprises like bridge collapses after few days of inauguration. So, these are all very beautiful patterns that you have, and the end conditions are changed.

(Refer Slide Time: 42.55)



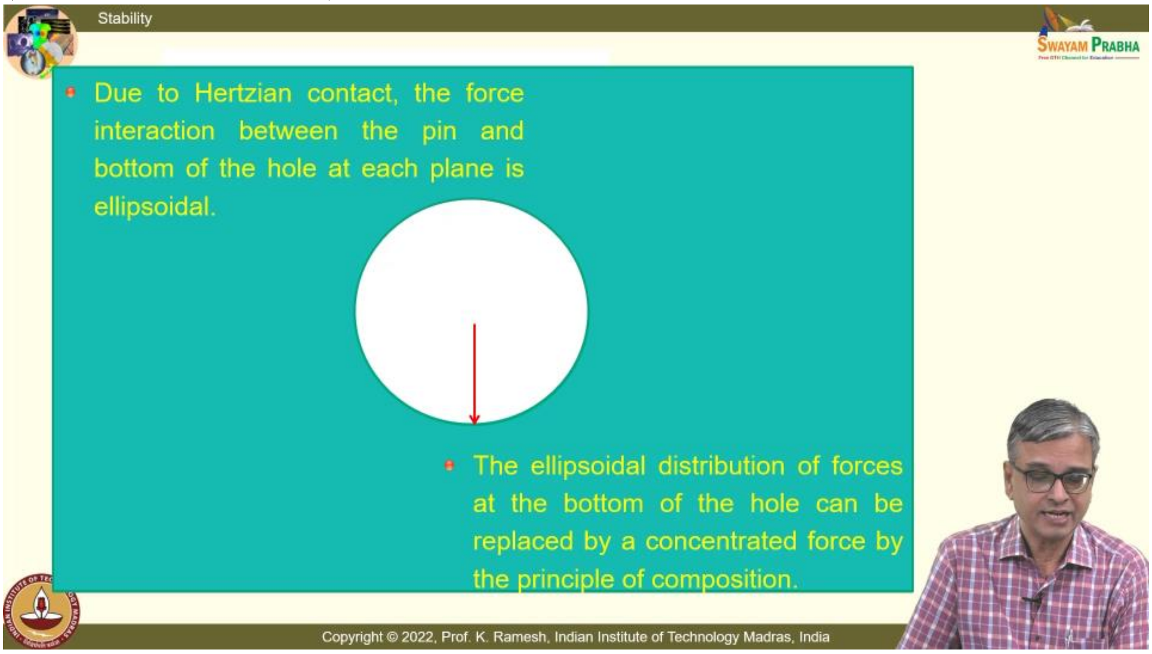
I have also emphasized, you know, I said that I can have a fixed end, but just by two supports. And you know, I also felt that some of you have to go and definitely see my engineering mechanics, the discussion on force systems, equilibrium and also move the force from one point to another point. You have to master it. If you master that, then you have a better grip of the subject because I say in a simplistic manner, a load is applied. How this load is applied? If you look at, I have to take a pin and then pull it like this. If you look at from a side view, it will be like a pin resting on a hole. And then if you go and look at it, we make several modeling steps. All that is taught in your rigid body mechanics. If some of you have not looked at it carefully, please go and have a careful look at it.

(Refer Slide Time: 43.49)



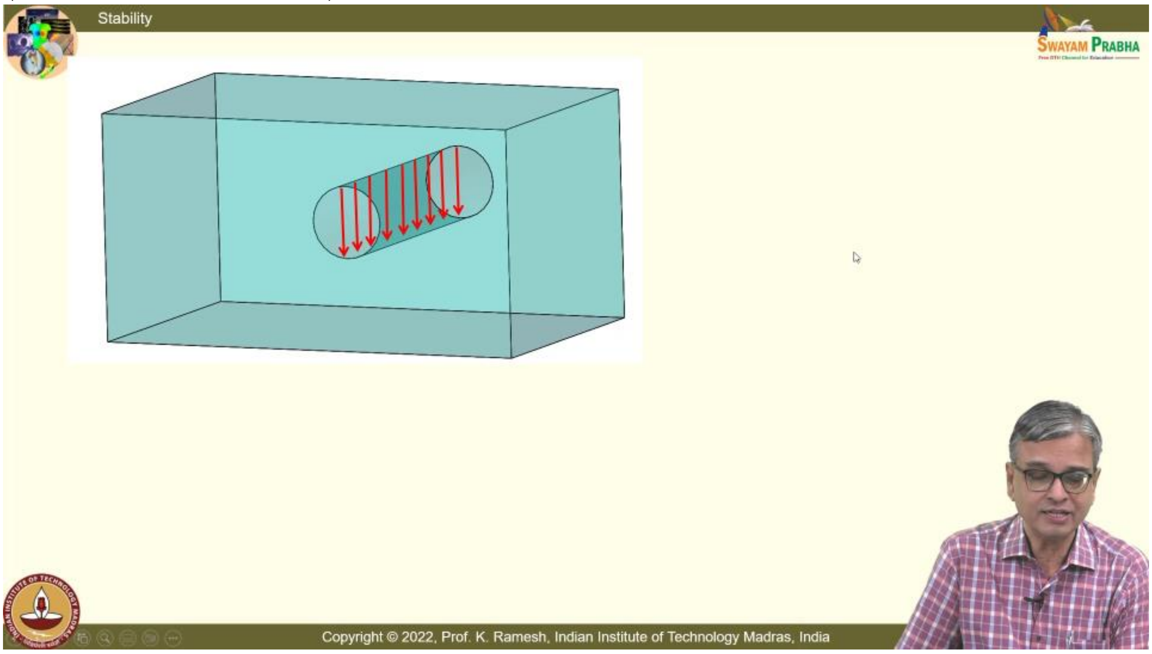
So, when I have a pin, you always have deformation. Even though in rigid body mechanics, we say it is rigid, you have deformation, you have force interaction.

(Refer Slide Time: 44.19)



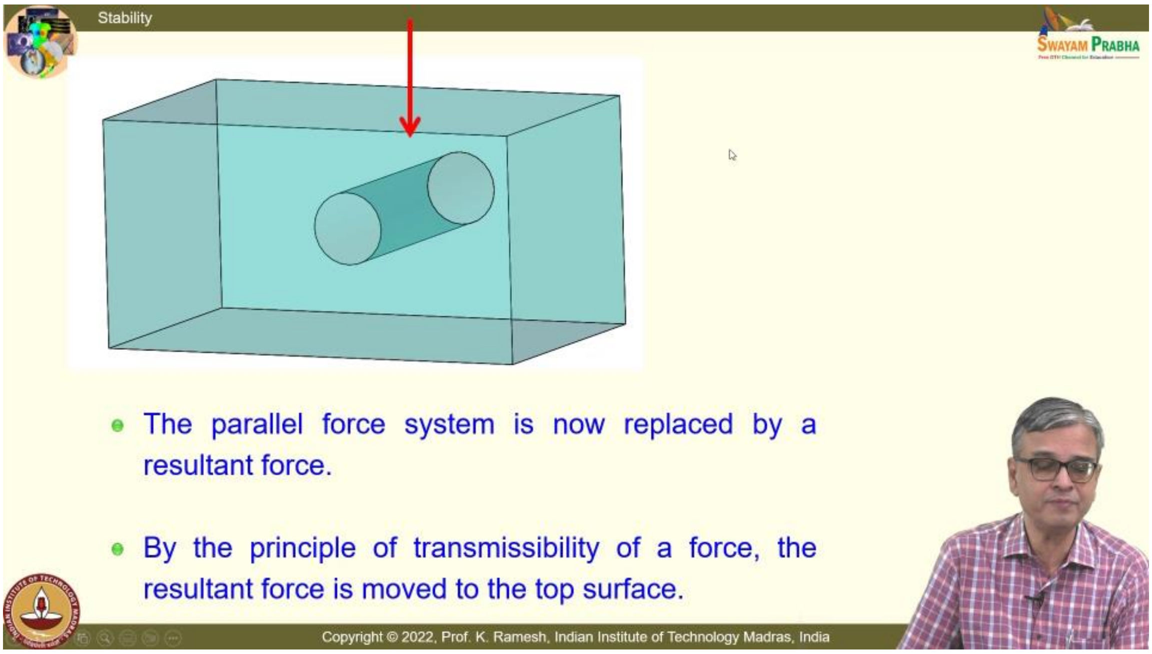
And this force interaction is elliptical distribution. This is done by Hertz. And you model this as by a resultant as a single concentrated force.

(Refer Slide Time: 44.21)



And then you replicate this concentrated force as along the line. Then take an equivalent value, then use principle of transmissibility and move it to that point.

(Refer Slide Time: 44.28)



So, a simple problem where you have a end load which is put like this, when you physically translate, it has many, many things aspect added to that. So, transmissibility of force, you can move the force along its line of action. When you want to move the force from one point to another point, you have to recognize you have a force as well as a couple. If you understand this, your bending is simple, torsion is simple and engineering analysis is very, very simple.

So, in this class, we have looked at stability in a broader perspective. We have looked at different end conditions affecting the critical load. We have also looked at equivalent column length and discussed about slenderness ratio. And also, we had threadbare discussion, how do you physically approach a problem? The physical systems do not have the end conditions marked. As an engineer, you have to go depending on the constraints, idealize it as a pin joint or a fixed joint. And fixed joints are very difficult to build in reality. And if you have a dilemma, always do two analysis, get lower bound solution as well as upper bound solution. Then I have also said, in order to motivate you to take interest in solid mechanics, there are very interesting challenging problems that are beyond this course, which really makes human life much safer in the new emerging challenges. Thank you.