Strength of Materials
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Lecture - 39
Review 2

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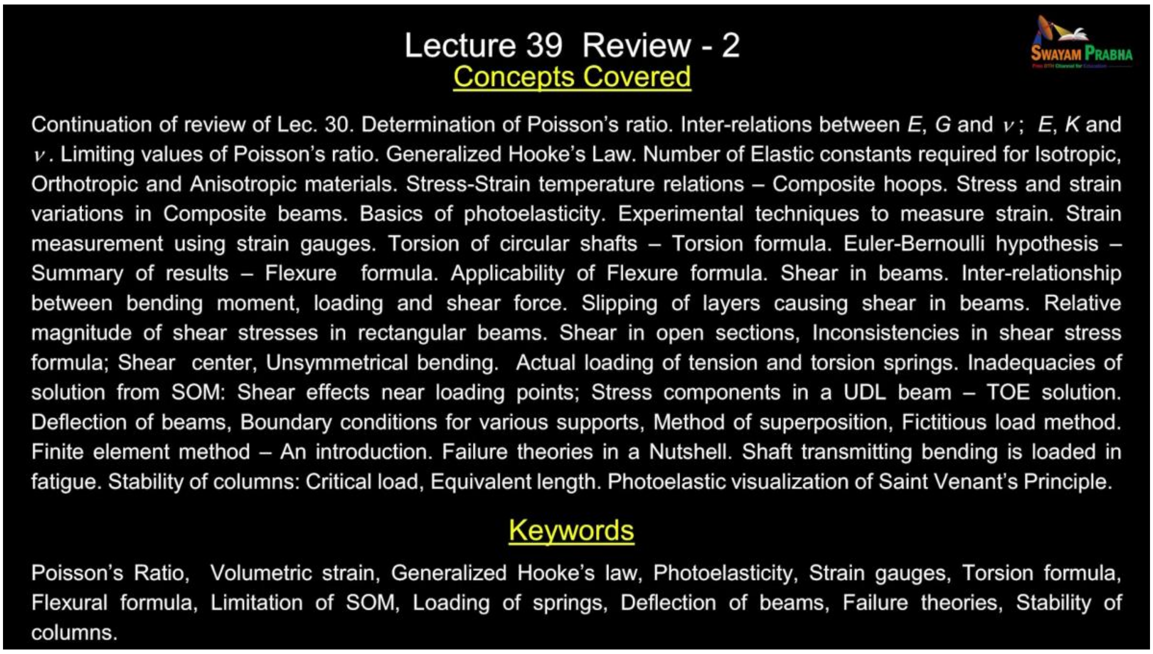

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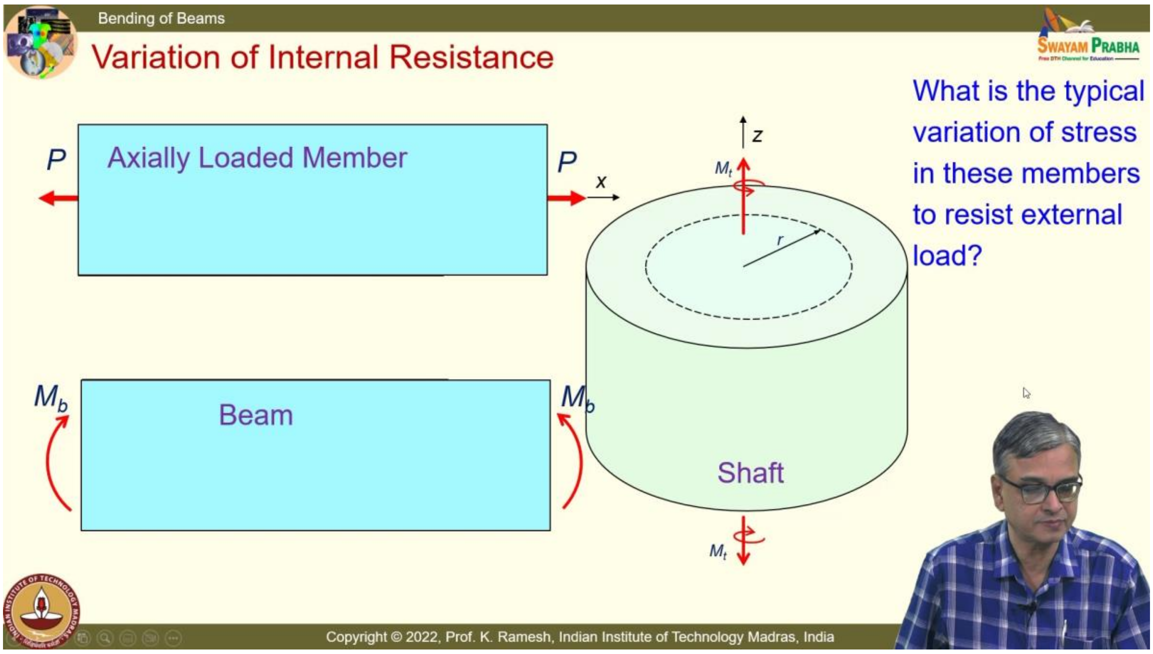

Let us continue our discussion on review of the course. In fact, in lecture 30, I have given review of the course done till that time. And this review lecture, you can take it as a continuation of that. So, if you look at these two review lectures, you have a complete perspective of what the course is all about.

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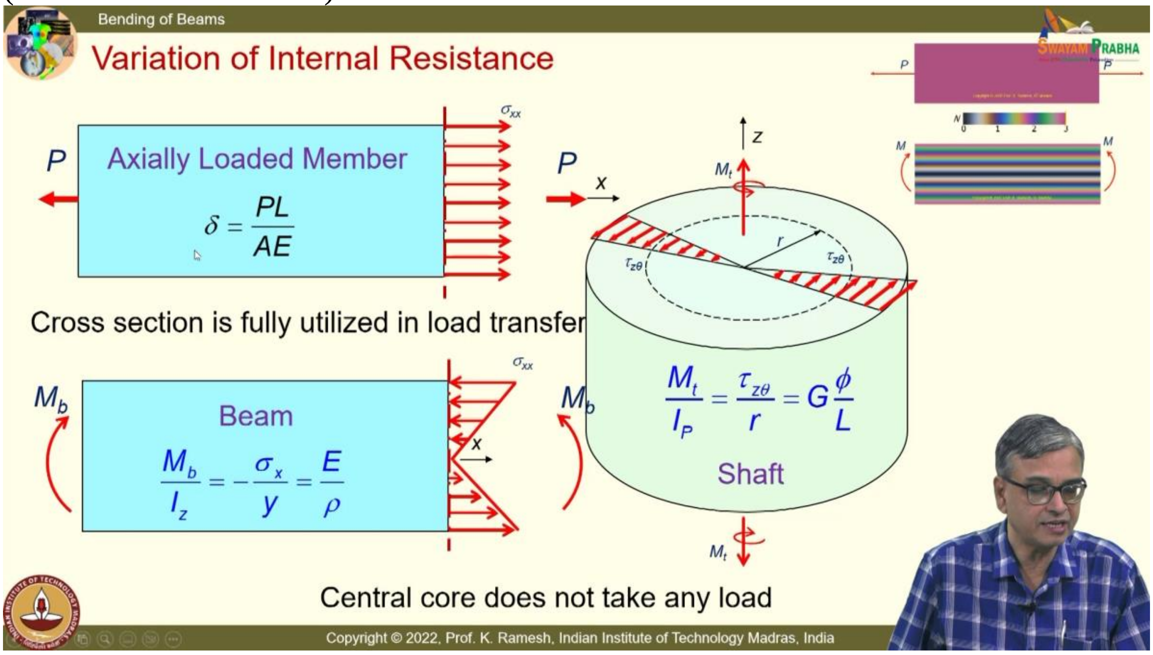

You know, the main focus of this course is to look at what is the typical variation of stress in an axially loaded member, in a beam subjected to pure bending and a shaft subjected to pure torsion. And you know, we have done this based on inference of photoelastic fringes, at least for axial loading and bending. And when you look at for axial loading, it is uniformly distributed. On the other hand, when I look at it for torsion, this is varying as a triangle; this is the variation of shear stress. And if you move on to the beam, which is transmitting a constant bending moment, here again, you have a triangular variation. But this variation is for a normal stress, which is known as bending stress.

And you know, whole of this course centers around development of this torsion formula and then your bending formula. And for an axially loaded member, the elongation equal to PL/AE is a useful relation, which we will use it in other context of the course also.

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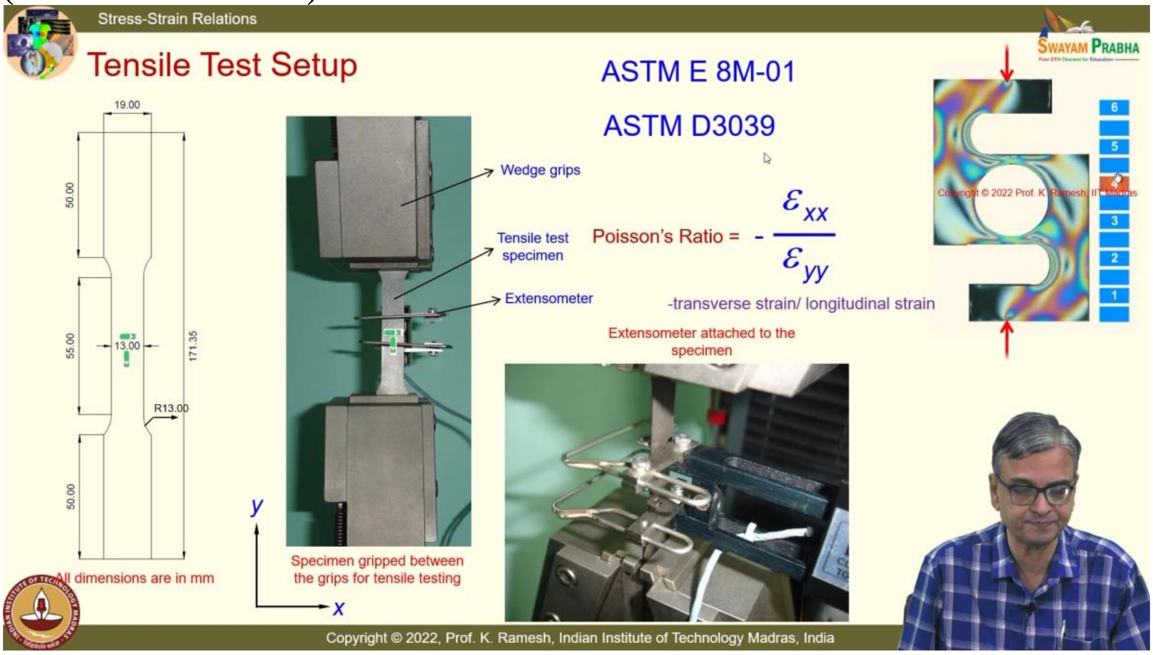

And you know, we have seen that you have a simple tension test to find out the Young's modulus and yield strength. And you can also use the same tension test to find out the Poisson's ratio. So, you have ASTM standards available for this. And one way is to put a strain gauge transverse to the loading. And you measure the strain transverse to the loading and also along the loading direction. And Poisson's ratio is given as minus of transverse strain divided by the longitudinal strain. And you have ASTM standards D3039, which lists out what precautions you need to take to measure Poisson's ratio.

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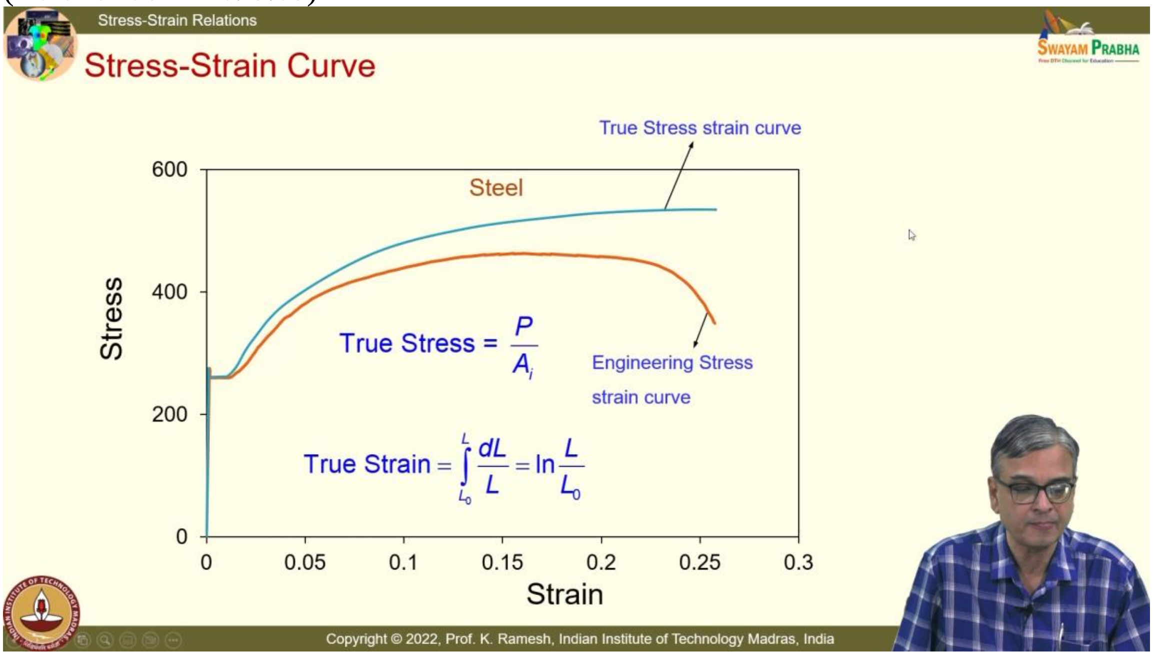

And we have also looked at; there is a drastic reduction of the cross-section when the material reaches the necking point. So, it is better that you consider the variation in the cross-section. And if you plot the true stress as P/A_i , as the current area of cross-section and also define strain as

$$\text{True Strain} = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$

you have a true strain graph. Instead of drooping down after necking, this will increase as until the ultimate tensile strength.

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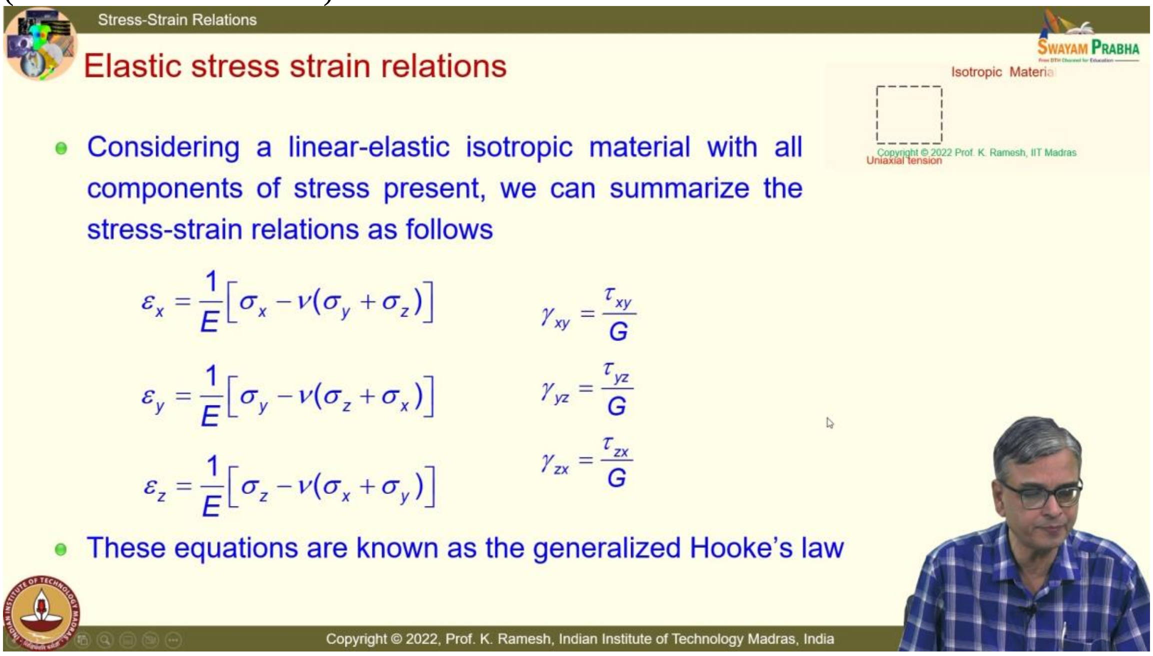

And we have also looked at the stress-strain relations because in a tension test, you apply load only in one direction. When I have all the stress components exist, the normal strains are related to normal stress and shear strain is related to only the respective shear stress.

So, that is the speciality of isotropic material. It makes our life extremely simple. When I move on to an anisotropic material, a normal stress can introduce shear. And a shear stress can introduce normal strain and normal stress. So, it is very complicated to handle.
So, it is also known as generalized Hooke's law in a simplistic sense.

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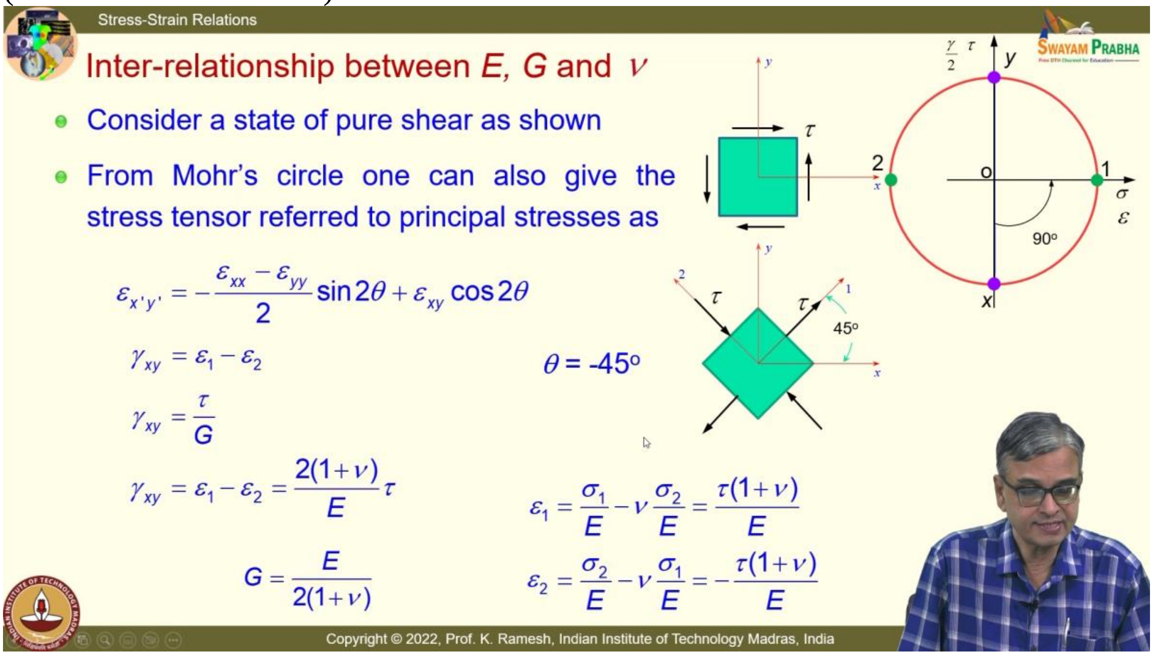

And we have also looked at how many elastic constants that you require to characterize an isotropic material, because we have seen what is Young's modulus *E*, shear modulus *G* and Poisson's ratio ν. And we have also developed the bulk modulus. So, we have four of them discussed. Out of these four, how many are required to characterize an isotropic material? So, if you develop the interrelationship and for interrelationship, what we have looked at is, we have effectively used the Mohr's circle of stress and strain for an isotropic material. If you scale them appropriately, the same principal stress directions are same as principal strain direction. So, you can look at what happens by relooking the shear stress, pure shear stress as combination of a tension and compression and invoking the stress-strain relations. It is possible to establish an interrelationship between Poisson's ratio, Young's modulus and shear modulus.

So, you have this from the strain transformation law and you have the definition of γ_{xy} based on stress transformation and also this Mohr's circle of strain, you can simply write this as $\varepsilon_1 - \varepsilon_2$. And from your stress-strain relation, I have $\gamma_{xy} = \tau/G$. And when you look at from your understanding of principal stress and strain, I can write out independently ε_1 and ε_2. When I substitute it back, I get a very interesting relation

$$G = \frac{E}{2(1+\nu)}$$

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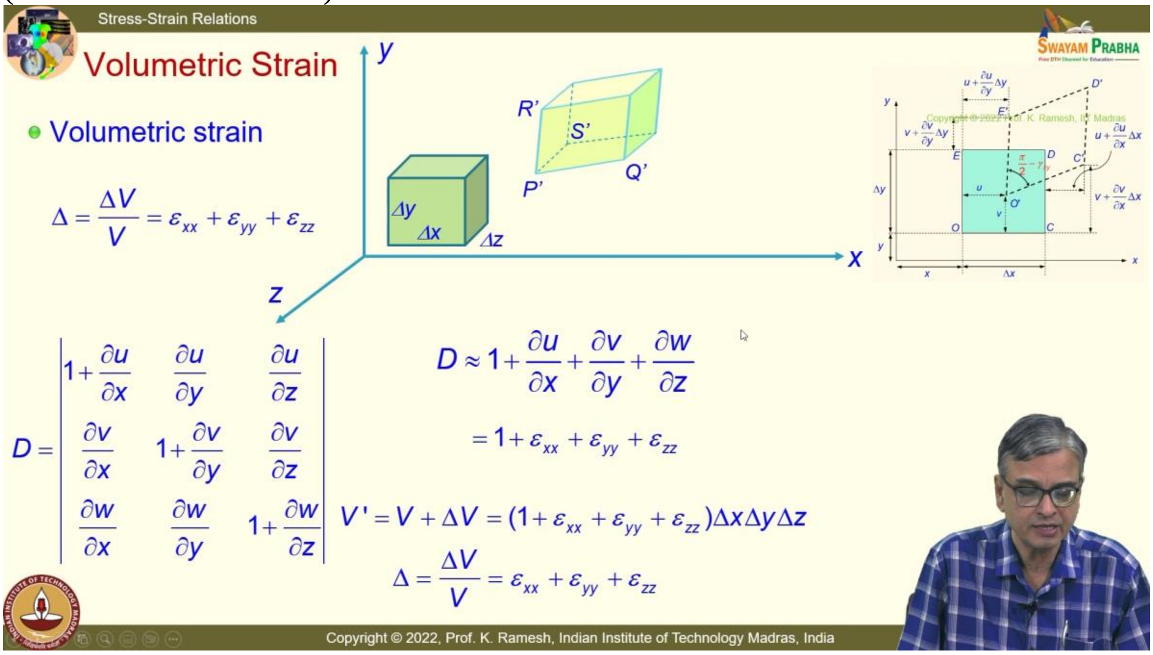

On similar lines, we have also looked at how to find out the interrelationship between bulk modulus, Poisson's ratio and Young's modulus. For that, we developed the concept of volumetric strain that is simply addition of all these normal strains. Volumetric strain is nothing but $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$.

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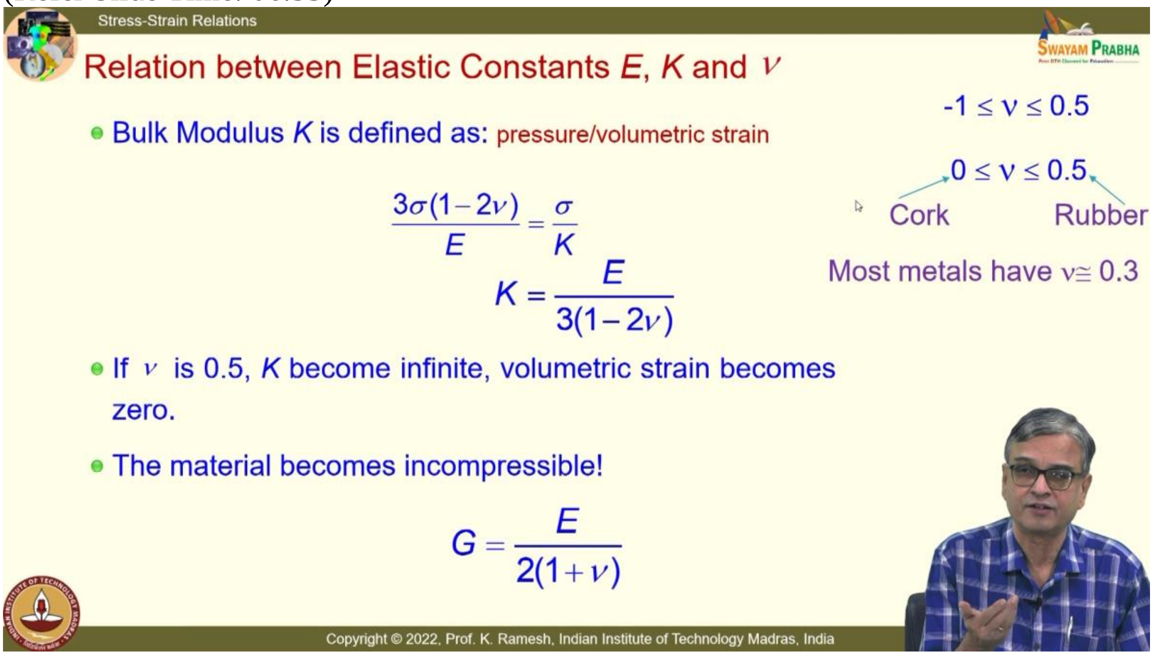

Using this, we have revisited what is bulk modulus. It is given as pressure by volumetric strain and that we got the expression

$$\frac{3\sigma(1-2\nu)}{E} = \frac{\sigma}{K}$$

And from this, you get

$$K = \frac{E}{3(1-2\nu)}$$

So, what you find here is, it is enough you have only two elastic constants to characterize an isotropic material. It is the greatest simplification that we have achieved, ok.

And we have also looked at what are the extremum values of Poisson's ratio. When ν is 0.5, K becomes ∞, volumetric strain becomes zero; the material becomes incompressible. And when ν is –1, you have the other story for G. So, the bounding values for the Poisson's ratio is from –1 to 0.5. And we have seen, cork is a very interesting material. It has a Poisson's ratio is zero, whereas rubber has a Poisson's ratio of 0.5. And we have also seen negative Poisson's ratio comes to advantage when you want to develop a stent for all your arteries and other blood vessels.

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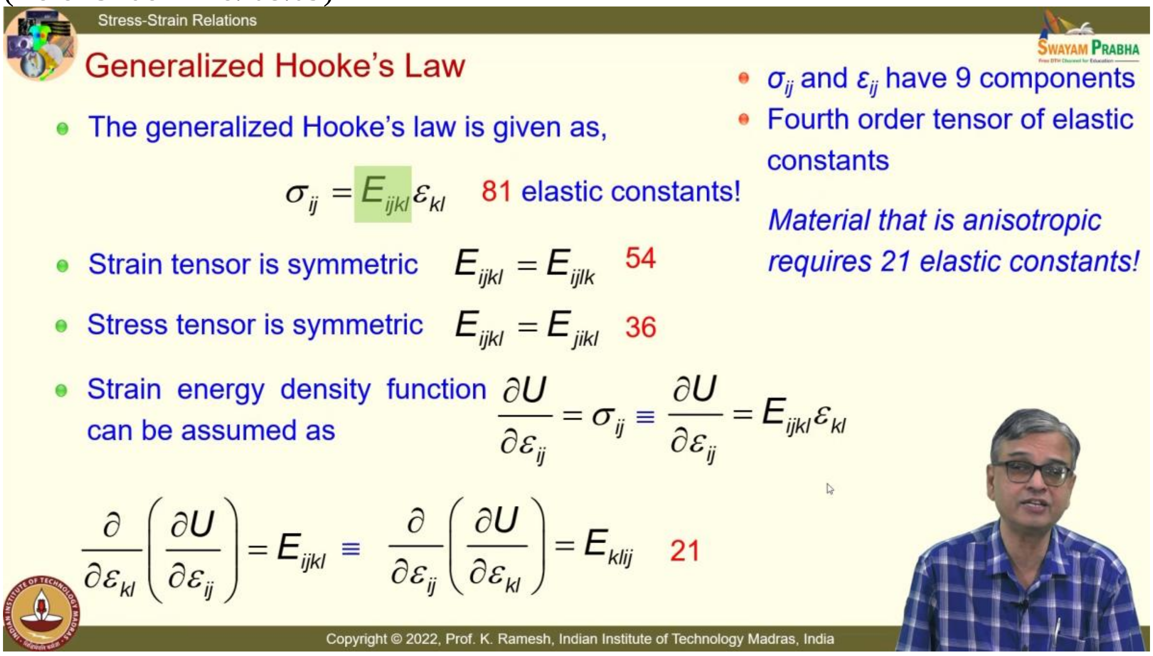

And the generalized Hooke's law for an, if you are considering an isotropic materials, how these are related? Independently, we have 9 stress components and 9 strain components. So, you may think that you may require 81 elastic constants. We have this as

$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl}$$

This is known as elasticity tensor. It is a tensor of rank 4. You may think that you will require 81 elastic constants. But if you look at strain tensor is symmetric, this reduces to 54 elastic constants. If you say stress tensor is symmetric, that reduces to 36 elastic constants. And if you look at strain energy density function and if you differentiate with respect to ε_{ij}, you get σ_{ij}. This is nothing but another statement of Castigliano's theorem. And the order of differentiation does not matter. So, I have E_{ijkl} is E_{klij}. This reduces to 21 elastic constants. So, if you have an anisotropic material, you may require 21 elastic constants to characterize an anisotropic material. Look at; when I have an isotropic material, I need just two of them. So, it makes your life extremely simple. That is one of the reasons why we want to have analysis of isotropic material. Even in situations where it is clearly not isotropic, we make that simplification.

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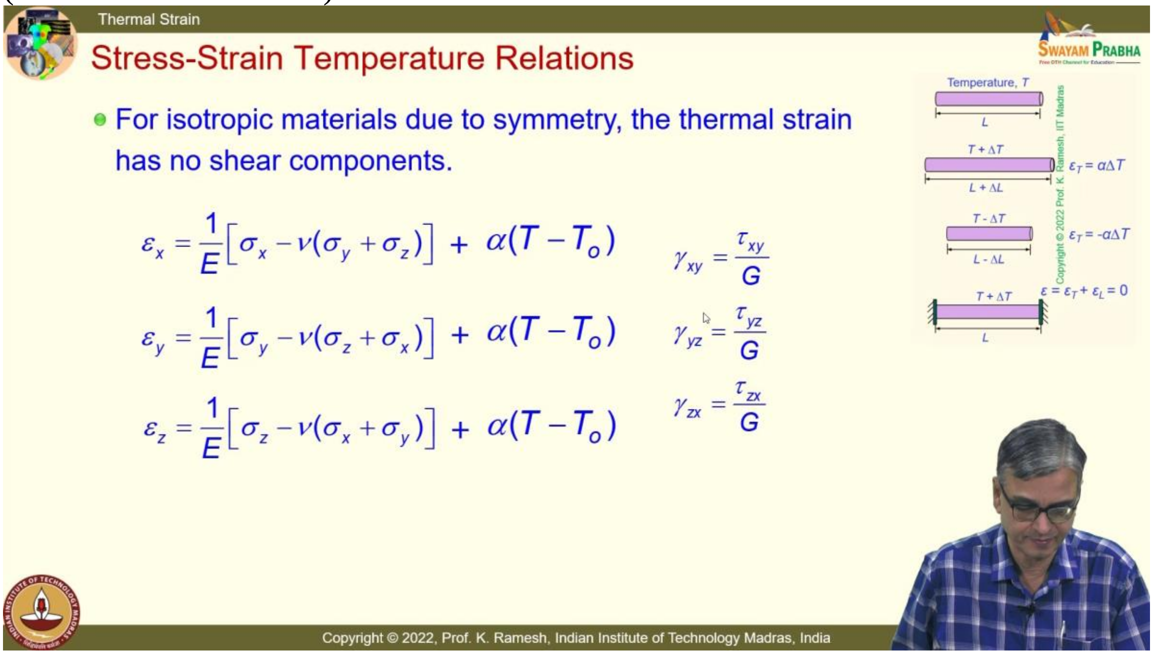

Then we have also looked at stress-strain temperature relations. As long as you do not constrain, you have only thermal strain. The moment I constrain, then what I have is, I also have stresses developed. And if I want to write the thermal effects affect only the normal strain, I have an additional term $\alpha(T - T_0)$, ok. So, it is actually $\alpha\Delta T$. It is not affecting the shear strain in case of isotropic materials. Even though we look at what happens to orthotropic or anisotropic material once in a while, our focus in this course is confined to isotropic materials.

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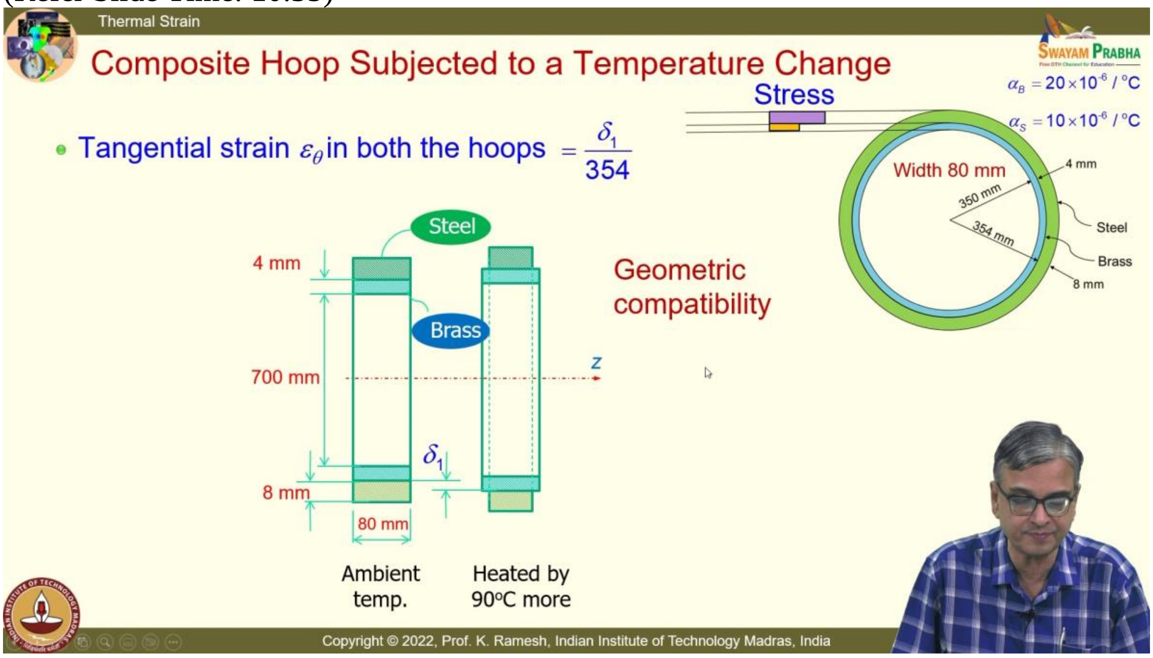

And you know, we have also looked at what happens to a hoop subjected to a temperature change. And then you know, we brought in geometric compatibility. We said that tangential strain should be identical in the interface. And we also determined the stresses and we have also plotted. See, across this, you know, you maintained compatibility of strain at this interface. But when I plot stress, stress can be discontinuous. And in the case of a hoop, we have simply said, it is like, you know, you have a circular one, it is opened up and then you are actually applying an axial tension. So, it is supporting only a constant stress that you know as pr/t. So, when you plot the stress variation, the variation would be like this.

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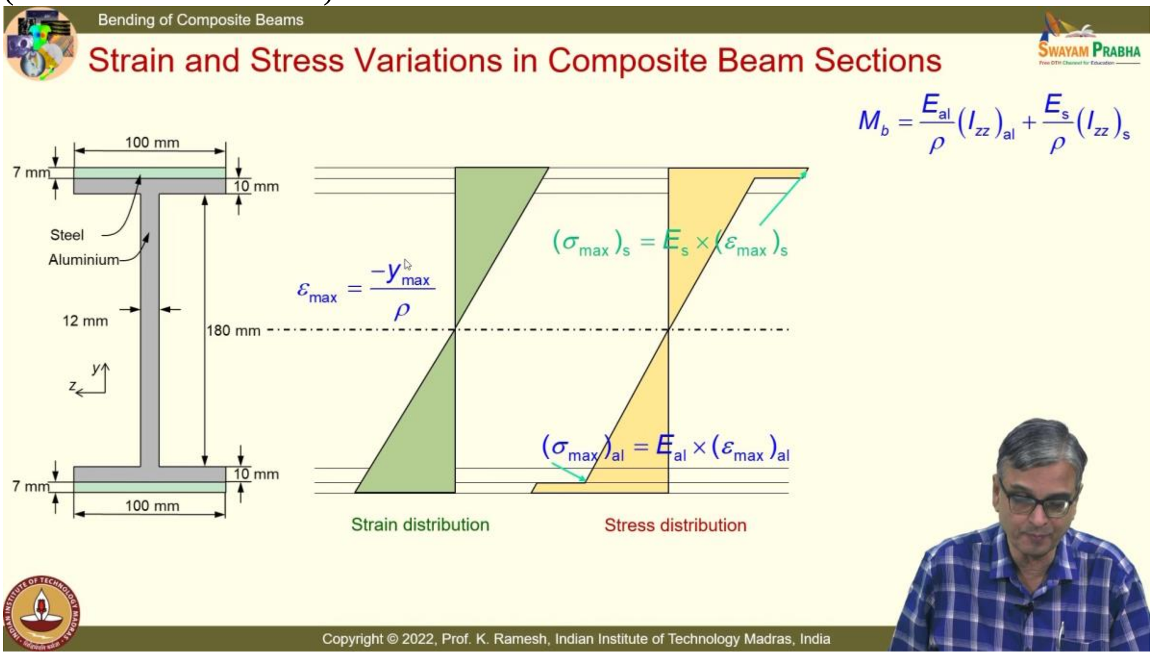

And we have also looked at if you have to strengthen a beam made of soft material, you can put a steel on top of it; top of aluminium; top as well as the bottom, so that you maintain the symmetry. And when we plot the strain variation, the strain variation is linear. However, when I plot the stress variation, stress variation will have a step at the interface. And the other subtle point is, because I am looking at bending, even though the thickness are small, there would be a linear variation because you have the factor y coming in the strain expression as well as stress expression. It is not going to be constant. In the case of a hoop, it is constant. In the case of a bending, it varies linearly, even though thickness is small. It is a subtle point for you to note.

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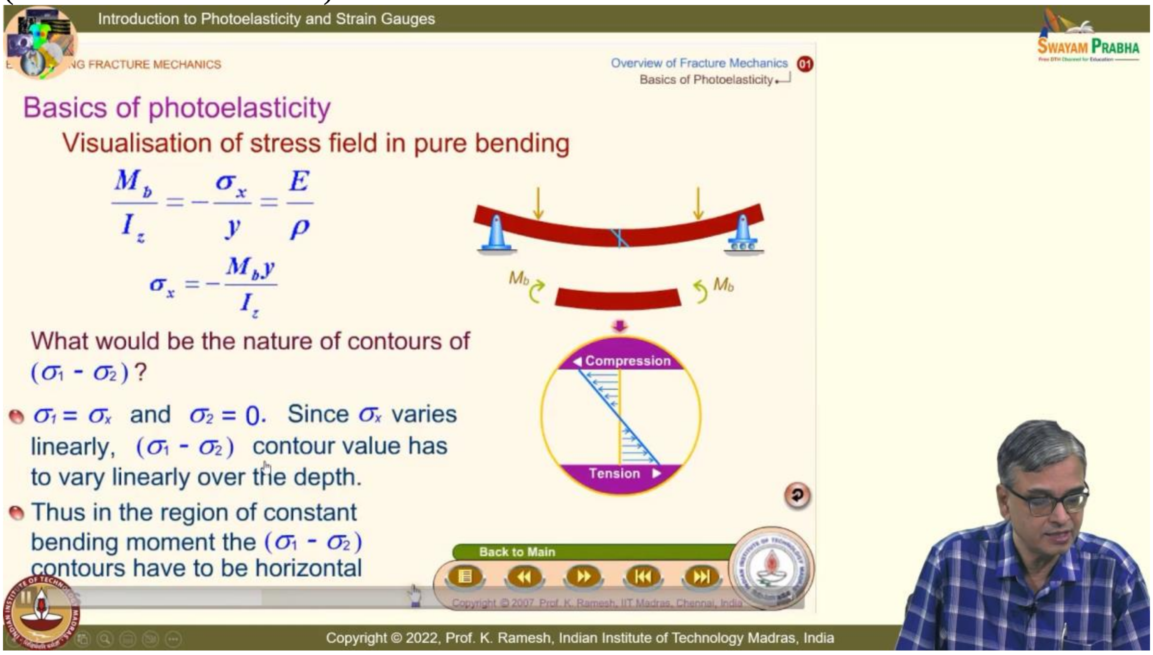

And you know, we have also looked at basics of photoelasticity. Since now you have the knowledge of stress-strain and also principal stresses, what would be the nature of contours of $\sigma_1 - \sigma_2$? We have σ_1 σ_x, σ_2 as zero in one section of the beam. Since σ_x varies linearly, $\sigma_1 - \sigma_2$ contour values always remains positive.

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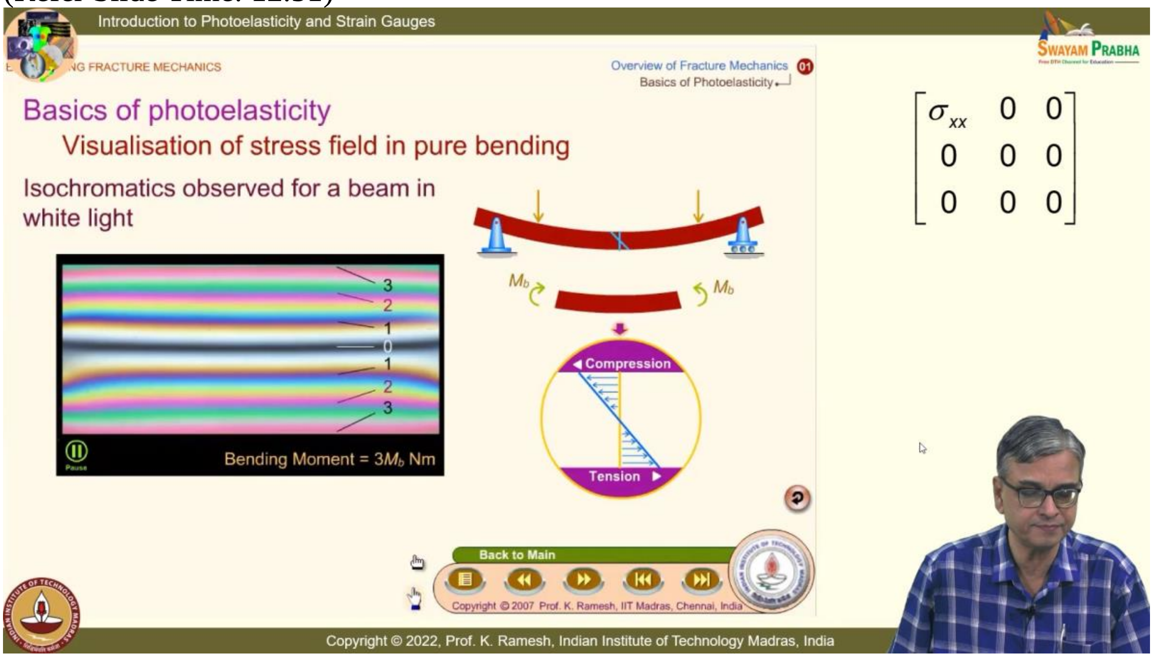

And if you plot them analytically, you get essentially parallel lines, which is also verified by your photoelastic experiment. So, this is another indirect validation of photoelasticity giving contours of $\sigma_1 - \sigma_2$. So, you have beautiful color variation. And we have also seen use of photoelasticity in several other applications. And your stress tensor is shown.

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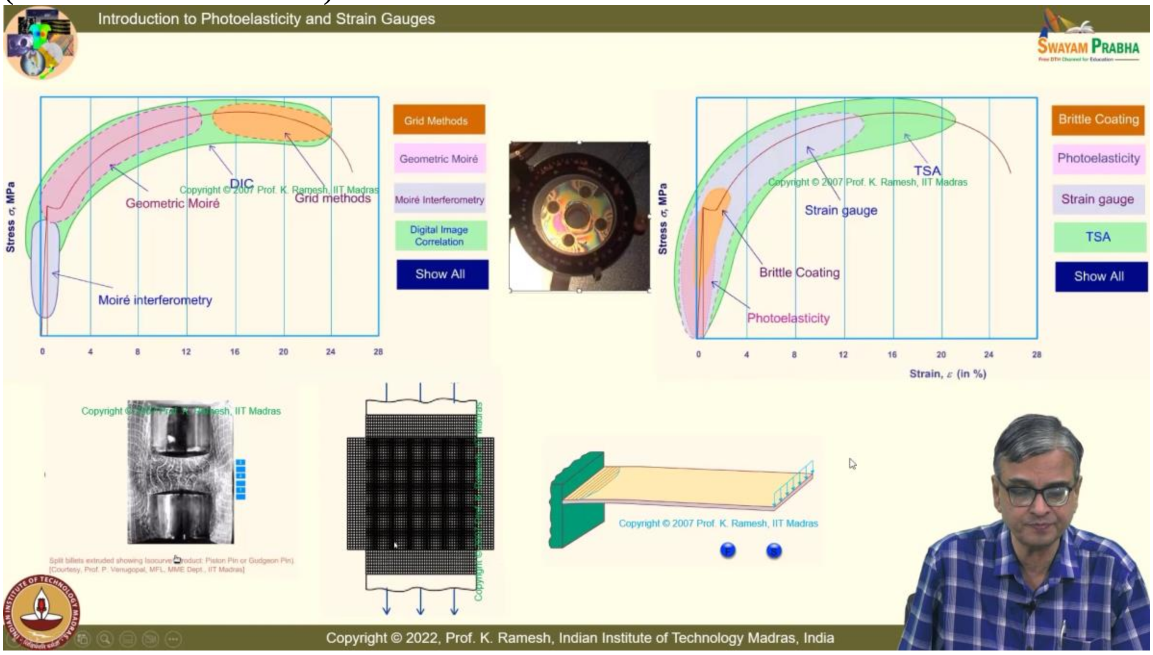

And you know, we have also looked at how to measure strain. There are multiple methods. The moment you go for measurement, you know, the resolution and range of each of the technique you have to look at. Depending on the capacity of the technique, it has a resolution possible, like you have a distance measurement by a scale or a vernier caliper. So, you should choose an appropriate technique that would meet your requirement. If you are working on this range, I can go only for moiré interferometry. If I work in this range, which is like a gudgeon pin, I can go for a grid method.

And you can also apply photoelastic coating to reveal the strain patterns. And you have the moiré, which also gives you displacements. And you have a brittle coating, where normally you do not want cracks. Here crack is the information; crack provides. The material fails in a brittle fashion. So, you get contours that are when you draw the tangent to that, you get the principal stress direction. And you know, a general purpose analysis is possible with a strain gauge. But for special application, you need to go for appropriate experimental technique for strain measurement.

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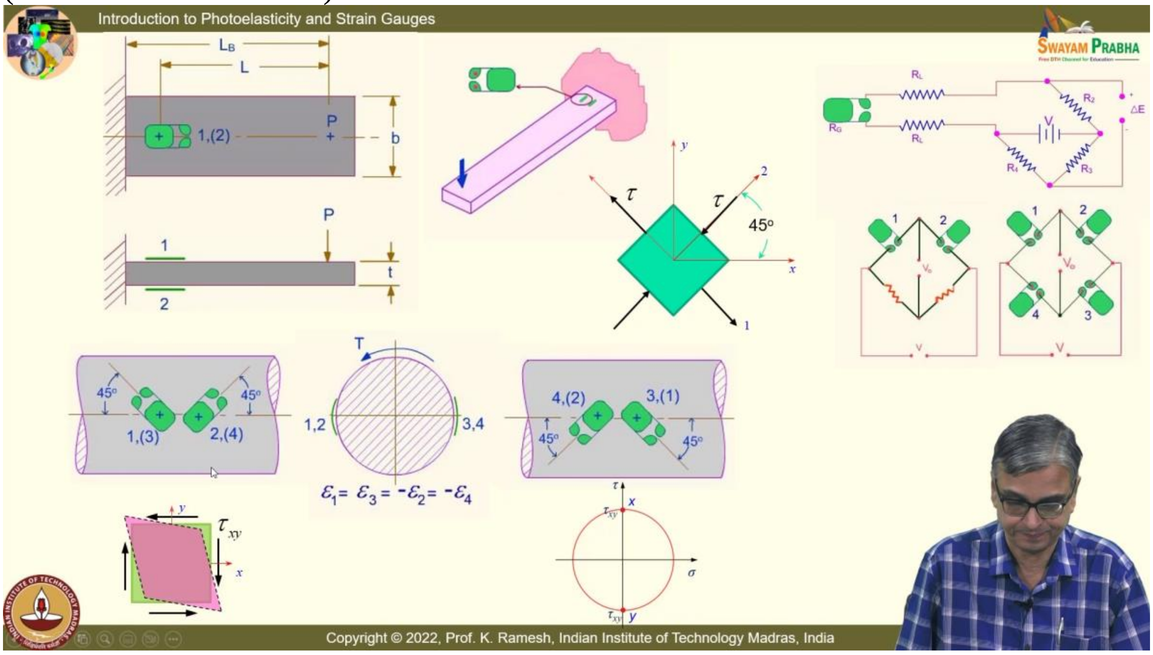

And you know, one of the discussion in the case of strain gauge is how do I connect the strain gauge in a Wheatstone bridge? If I do not connect them properly, I may get wrong results. In transducer applications, you want to maximize the signal. So, you know how a cantilever beam behaves. Because you know bending now, when I apply this load, this is subjected to tension and this is subjected to compression. So, these are strain in opposite direction. Connect them in adjacent arms so that strain magnitudes get add up, ok. So, you amplify the signal in a transducer applications. I can also do the amplification by putting two strain gauges on this. But if I do not connect them properly, suppose I connect them like this; what is the result you will get? You will get twice the signal or zero signal? I will get only zero signal. So, you will also have to handle the Wheatstone bridge appropriately.

And we have also looked at how to extend this for torsion because strain gauge by itself can measure only axial strain, fine. And we have understood what way shear stress can be looked at as combination of tension and compression. And using that and looking at the Mohr's circle, you can have the justification. Using this, we have identified that strain gauges have to be aligned at 45 degrees to the axis and connect them appropriately in the Wheatstone bridge, so that I can quadruple the signal. That is the advantage. So, I can measure torque with much more accuracy when I use that as a torque meter, ok. You have torque wrenches which measure the torque. So, you have definite applications. How do I put them? This is very important; how I have pasted it on the shaft.

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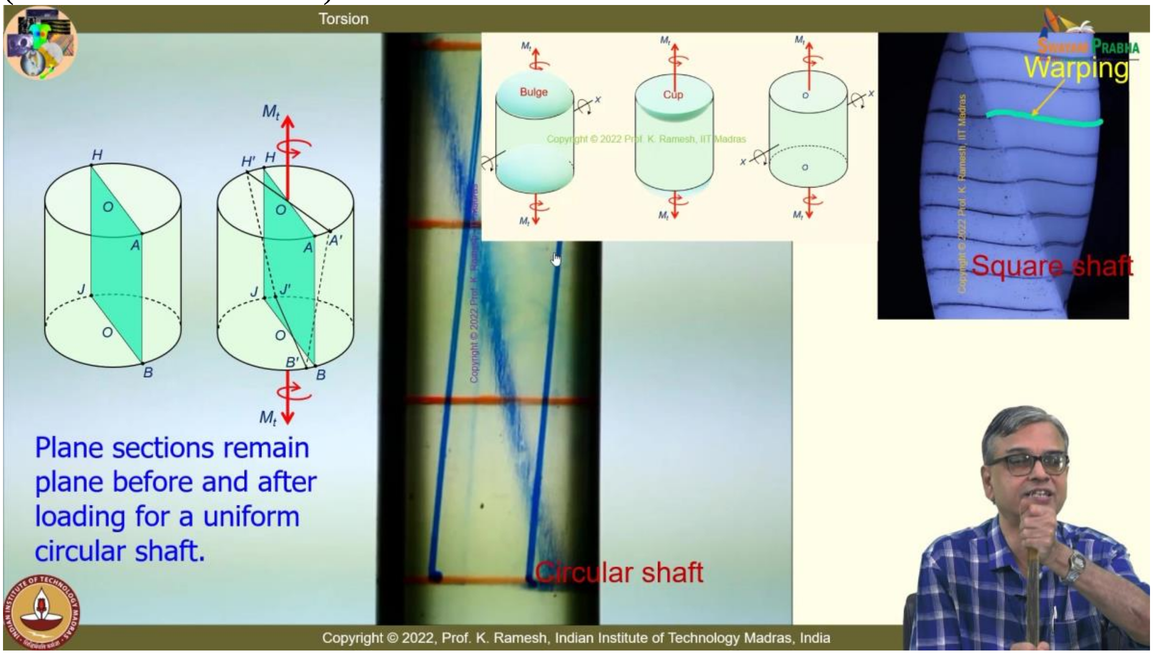

Then we moved on to torsion. We have looked at circular cross-section. To understand circular cross-section in a simplistic manner, we have also looked at a square shaft. In a square shaft, you have a warping which is absent in a circular shaft. That made your life lot more simpler. That is the reason why in this course, we choose the cross-section and the loading so that plane sections remain plane before and after loading. We postpone torsion of circular cross; non-circular cross-section to the next level course. It is not that it is not solved. It is solvable, and Saint-Venant is the first person to solve it, ok. So, plane sections remain plane before and after loading. We have done it by a thought experiment, ok.

And then you can also see it visibly when you take a simple circular shaft, draw the lines and then twist it.

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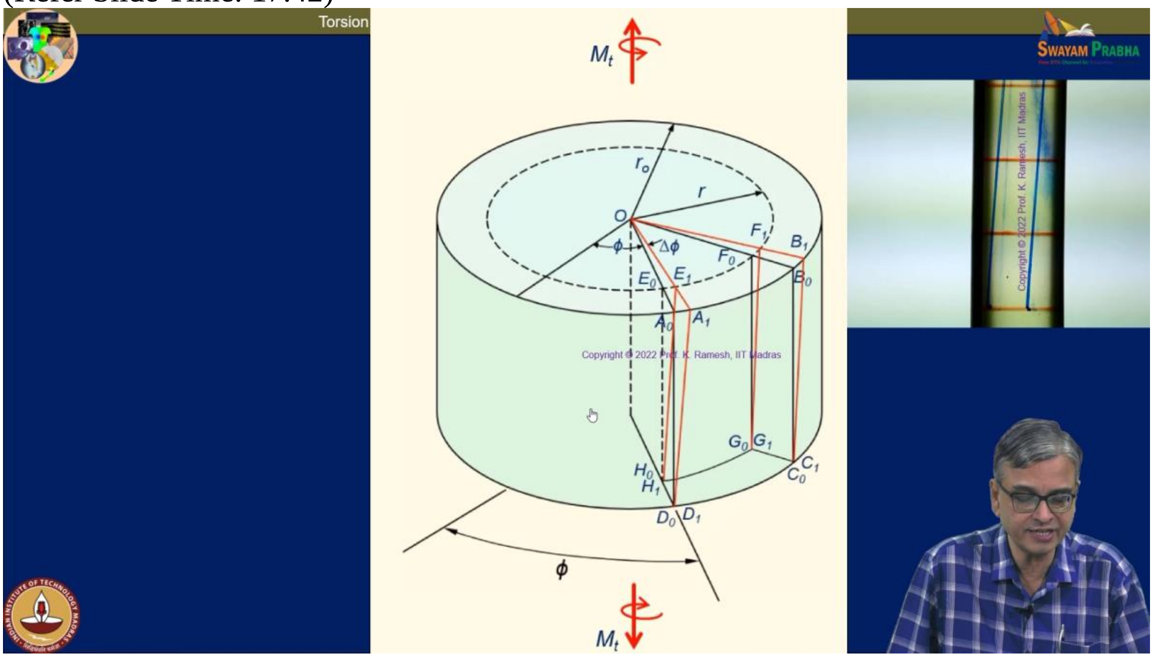

And we have also a nice animation. You look at the animation; you understand what all we have done for finding out the strain components. And this shows the experimental information and this shows the drawing in a systematic manner. So, you find out what is the strain that is existing.

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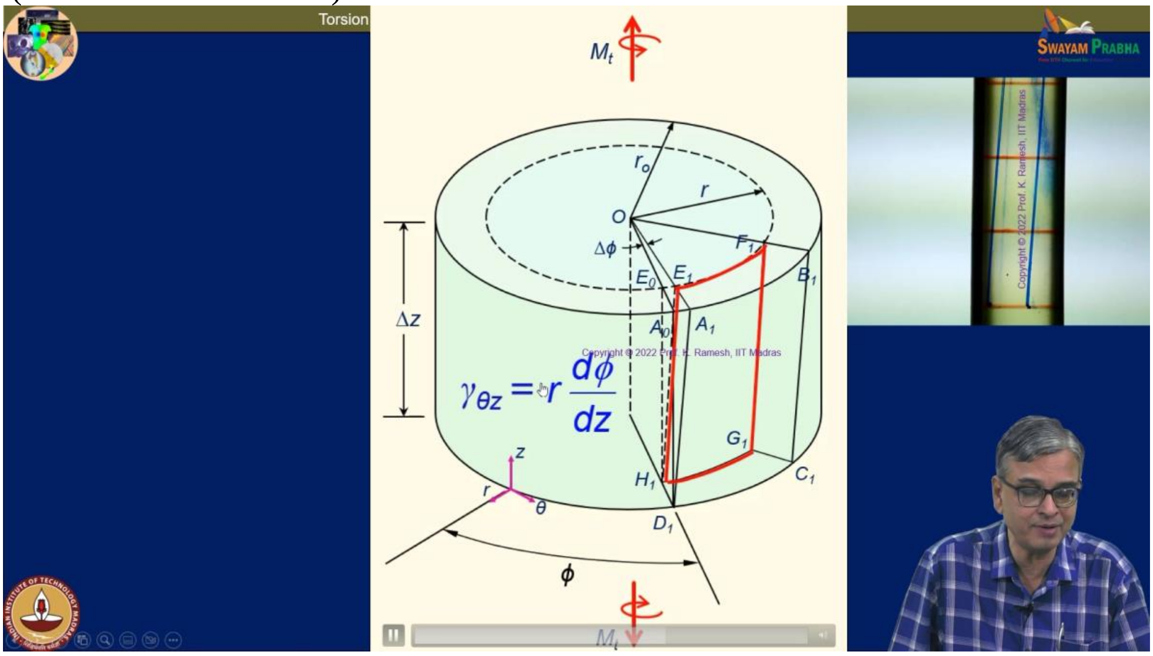

You have that as the reference axis is given as $r \;\; \theta \;\; z$. So, I have only

$$\gamma_{\theta z} = r\frac{d\phi}{dz}$$

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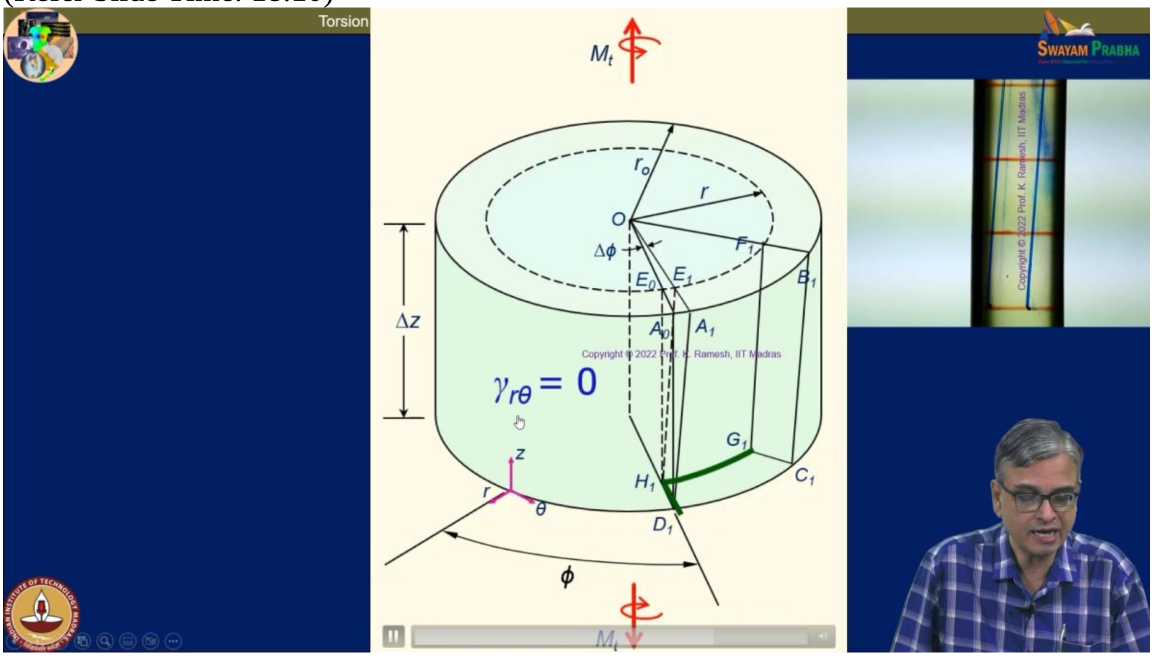

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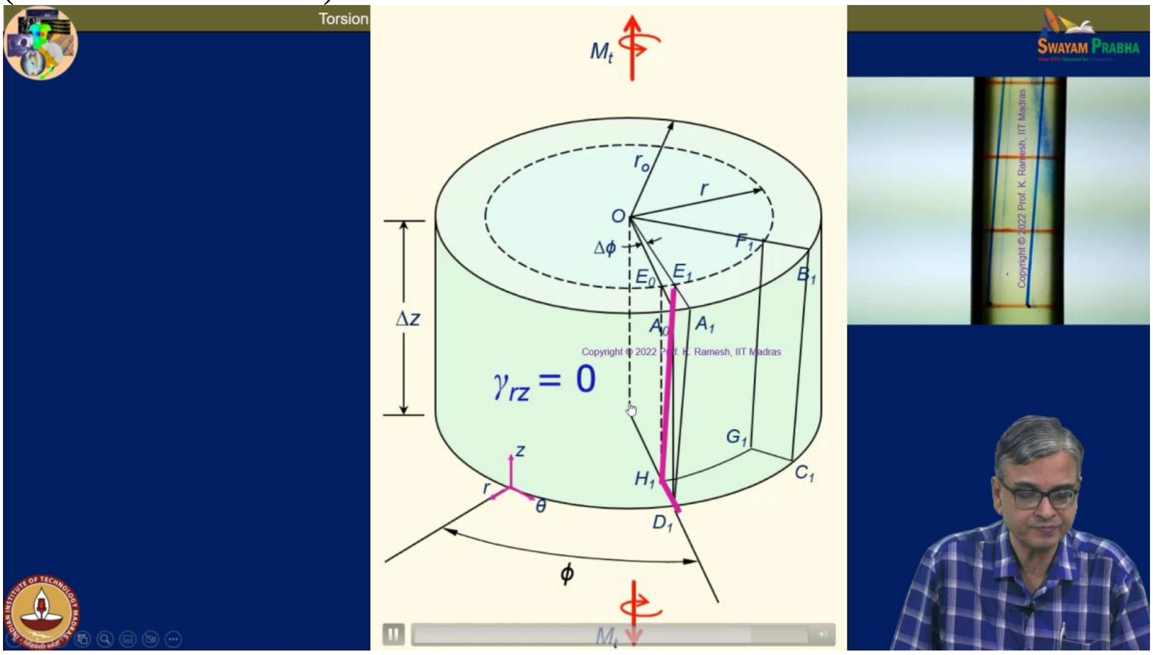

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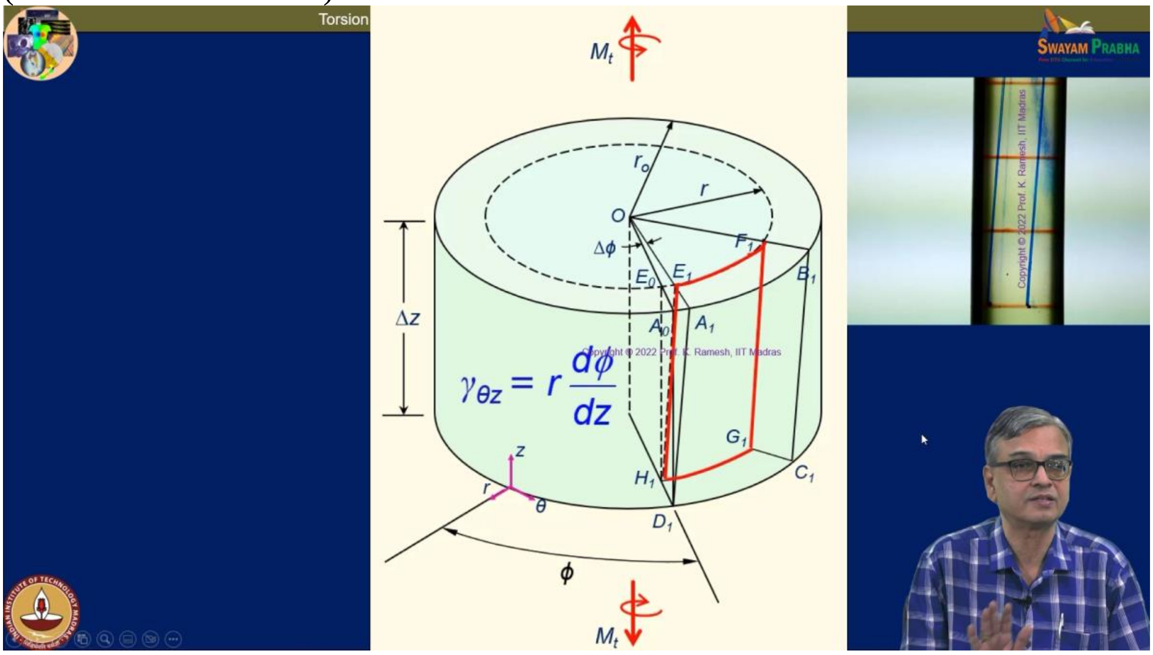

I have only $\gamma_{\theta z} = r\dfrac{d\phi}{dz}$

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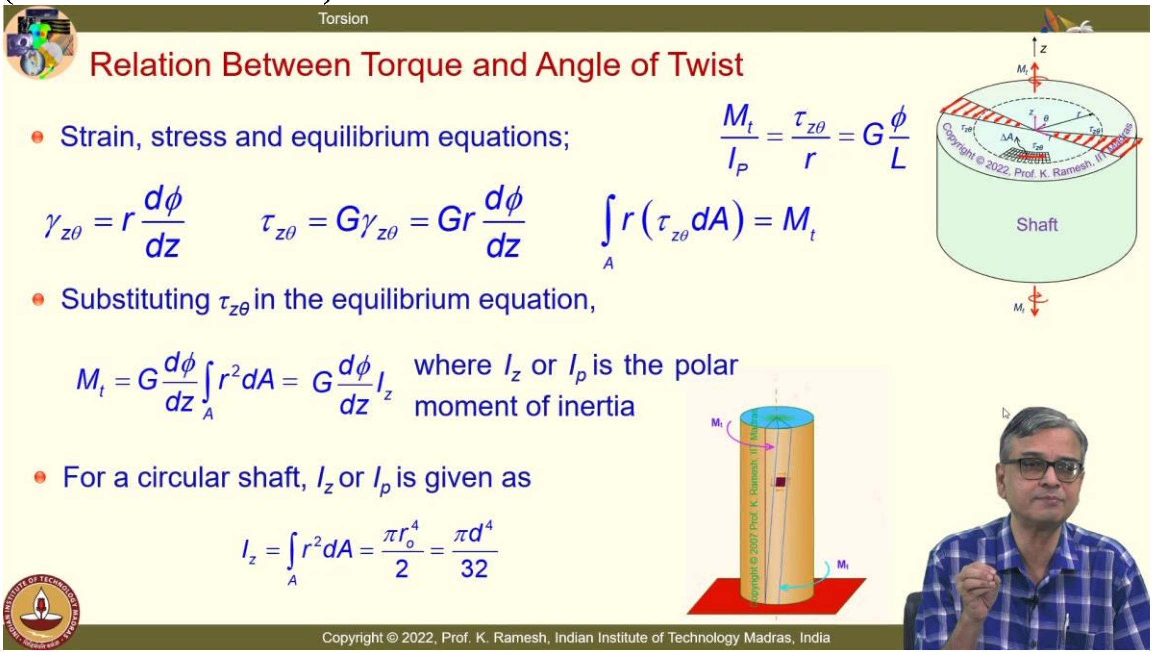

So, we have used this to develop the stresses and strain stress and equilibrium equations. And in the process, we have also looked at a quantity like this which when I substitute, I get this as $\int_A r^2 dA$, that is your polar moment of inertia. And we develop it for a circular shaft. We also extend the same ideas to a hollow shaft. The same ideas are equally applicable for a hollow shaft. And there is also interchangeably use I_z or I_p because I said that in bending of beams, I_z has a different connotation. So, it is better to look at as I_p for circular shaft when you are doing torsion and look at I_z for when you go for the bending of beams, or you can simply say that as I. If you understand how the equations are developed with respect to the coordinate system. And this is the celebrated equation that you have. This is developed for a circular shaft transmitting constant torque. And what we do is when you have a twisting moment, pick out that twisting moment at that cross-section, find out the torsional stress, ok.

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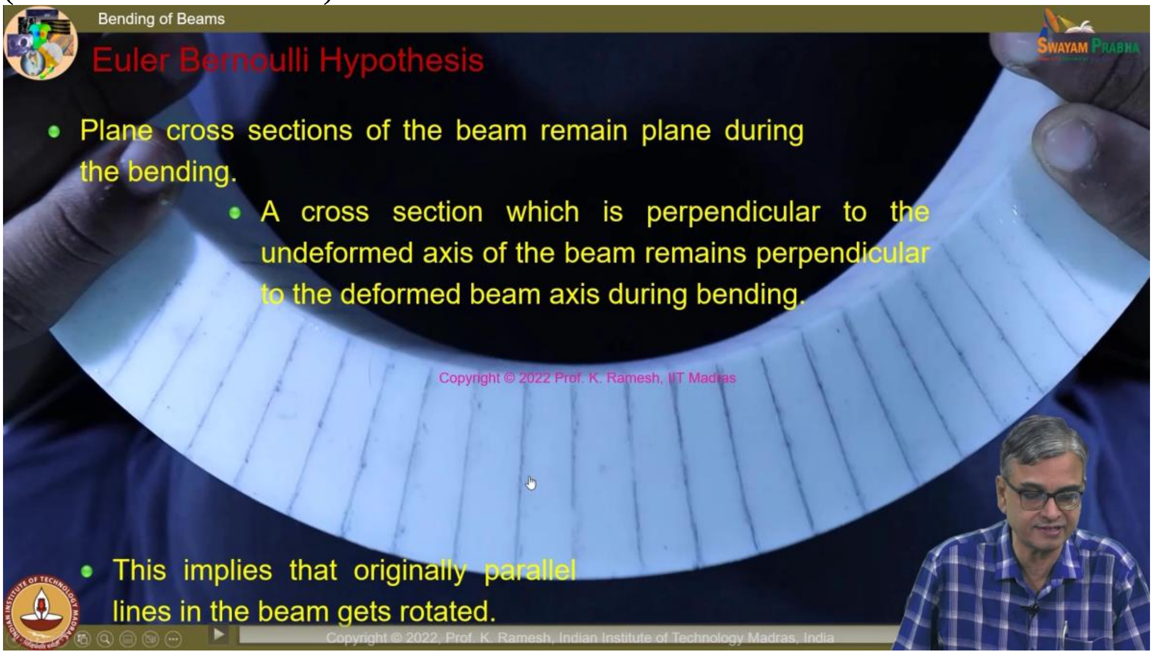

Then we moved on to bending of beams, where we have looked at a very soft beam and we could understand that straight lines get rotated. And we have understood, you know, the plane cross-sections of the beam remain plane during bending. That is what you see here; there is no warping. And you have cross-section which is perpendicular to the undeformed axis of the beam remains perpendicular to the deformed beam during bending. That is very important! Ok. This implies that originally parallel lines, the beam gets rotated, very clearly seen in this experimental demonstration.

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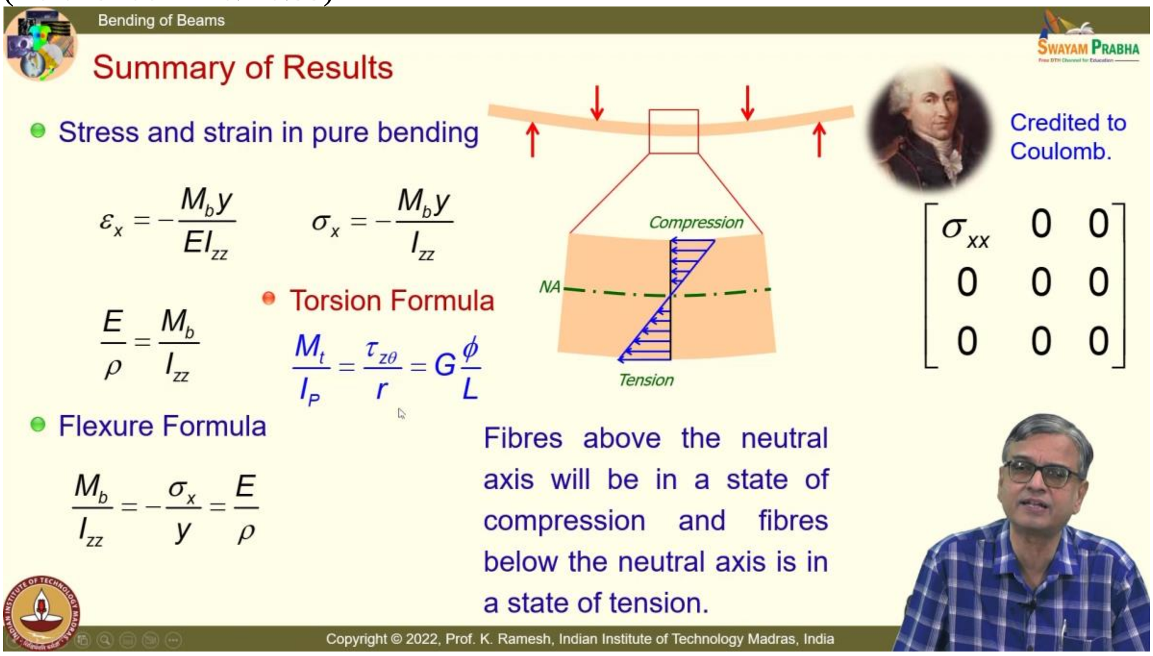

And when we go into the results, we have the stress and strain in pure bending. It is very important that theory is developed for pure bending, a beam transmitting only bending moment, nothing else. So we have the expression for ε_x, we have the expression for σ_{xx} and we have the stress tensor. And this final result is credited to Coulomb. So, you have to recognize that the stress varies linearly over the depth of the beam and the central core is not contributing to load share, which is used by nature in developing your bones. Your haemoglobin gets developed in the soft aspects of the bone. And we have the relation

$$\frac{E}{\rho}=\frac{M_b}{I_{zz}}$$

which finally, when you look at all the other expressions, you have the famous flexure formula

$$\frac{M_b}{I_{zz}}=-\frac{\sigma_x}{y}=\frac{E}{\rho}$$

Because later on, we also dropped this *zz* just to speed up your writing. Once you understand the context, what is the axis and what is the kind of moment of inertia talking in this context, it is convenient to simply write and indicate moment of inertia as *I*.
And you have the torsion formula, you compare; they are very, very similar, fine. It is easy to recollect how these are written.

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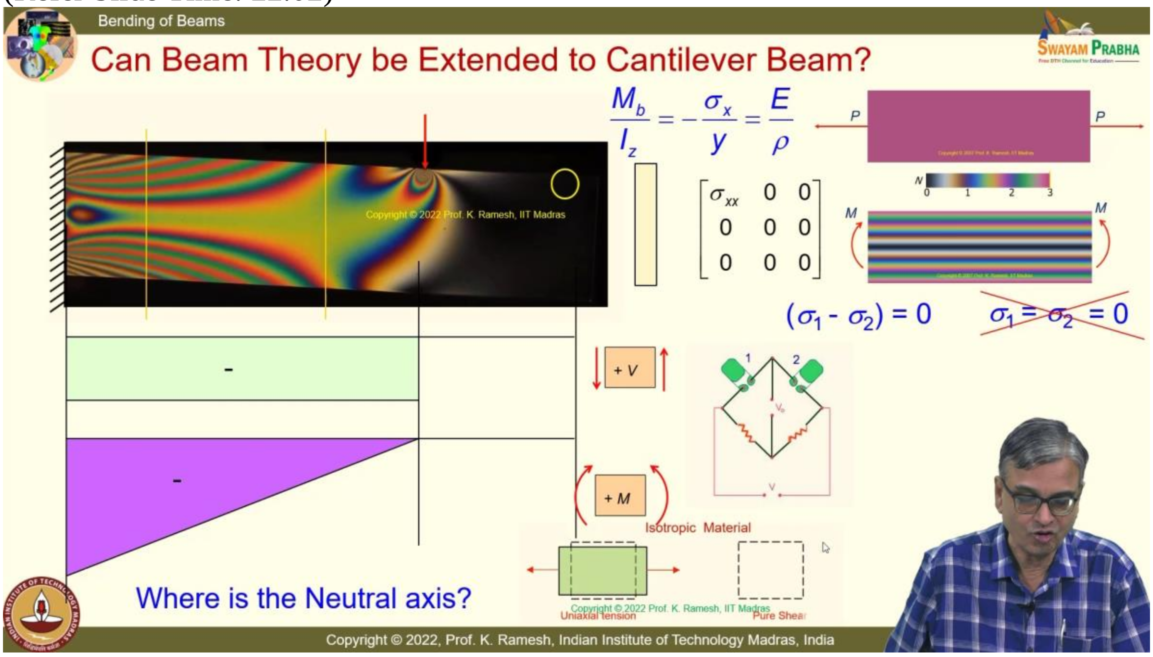

And the other question we raise, can beam theory be extended to a cantilever beam. What is the difference? In the case of a cantilever beam, in addition to bending moment, it also transmits a simple constant shear. Even a simple constant shear modifies the bending moment along the length of the beam. So, the recipe here is, you have the flexure formula and simply pick out what is the bending moment at this cross-section, then you say what is the bending stress using the flexure formula; that is permitted. And we have also seen, even though the cantilever beam warps, there is no coupling between shear and bending as long as you are having a slender member and the depth of the beam is very small compared to the length. If the depth is comparable to the length, then you can have coupling effects. That is considered as a Timoshenko beam, and you have different theory developed. Remember, such problems are solved, it is not done in this course. And we have also looked at other discussion, how do you have σ_1, σ_2 and then whether you have neutral axis and an isotropic material. You know, what is the advantage, all these related discussion we have done.

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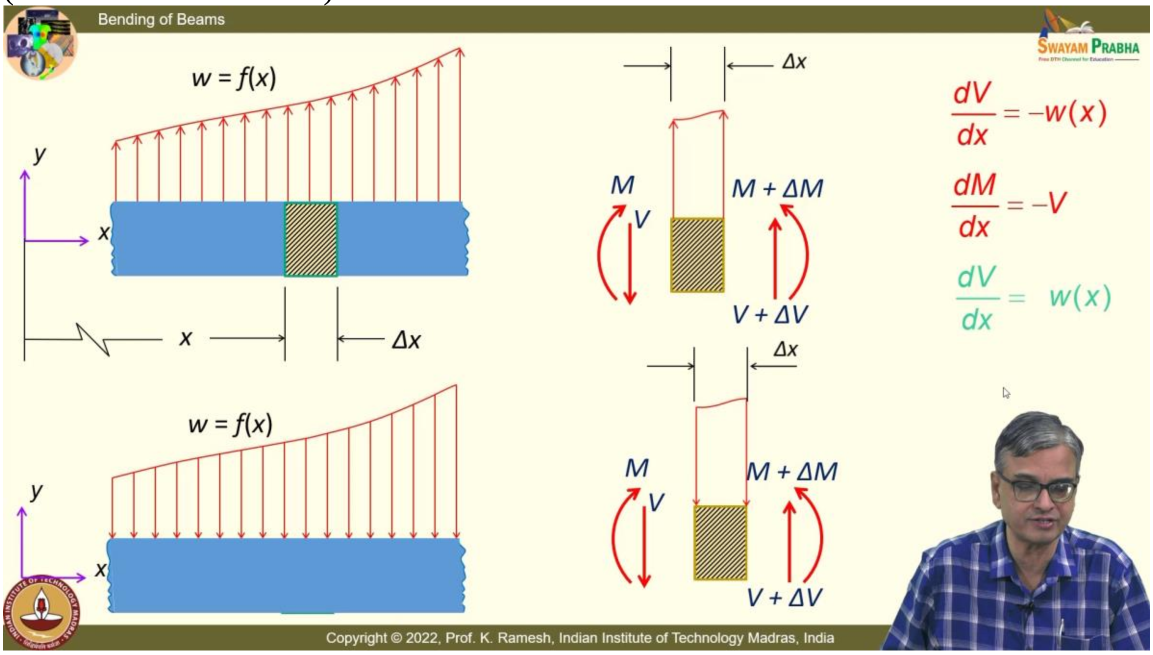

You know, whenever we have a beam transmitting a variable load or even your uniform weight, there are multiple ways books bring out the equations. You know, if I have the w acting upwards taken as positive, I will have the expression $dV/dx = -w(x)$ and $dM/dx = -V$. On the other hand, if I develop the equation with w as acting downwards, the only change is $dV/dx = w(x)$; it becomes positive. So, you will also have to understand a subtle difference. If you do the mathematics systematically, there is no problem. But if you want to interpret it based on sign convention, you have to see which way you have developed the equations.

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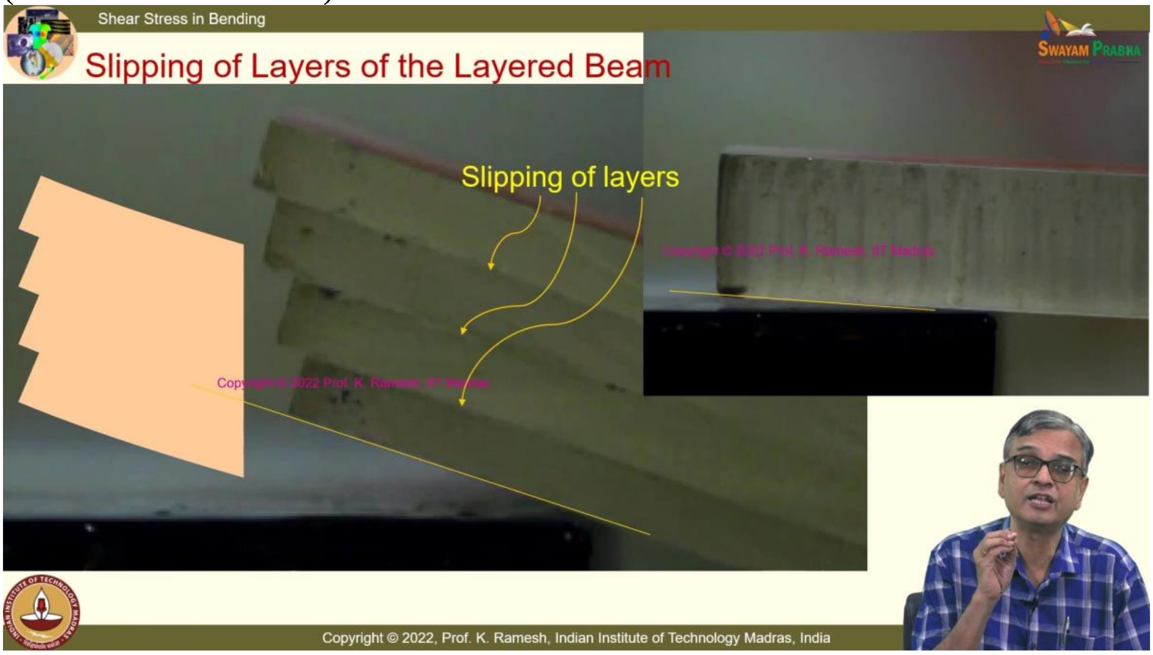

And one of the subtle points in the case of bending is how shear gets developed. To understand this, we have taken a layered beam, and we find they get, they are slipped as the loading is applied. You could very clearly see slipping of layers. And you can also imagine something is holding it; that is why the layer is not slipping in a beam which is rotated like this. Here, the rotation is very, very small and this edge remains straight, whereas the same height of the beam with four layers, you have this jagged edge. That clearly shows, that is something is holding the surface, ok. So, on that basis, we developed the mathematical expression for the shear stress, and this also shows how much is the angle or the slope when the beam is one unit but with different layers which brings out very clearly the role of shear in the beam analysis.

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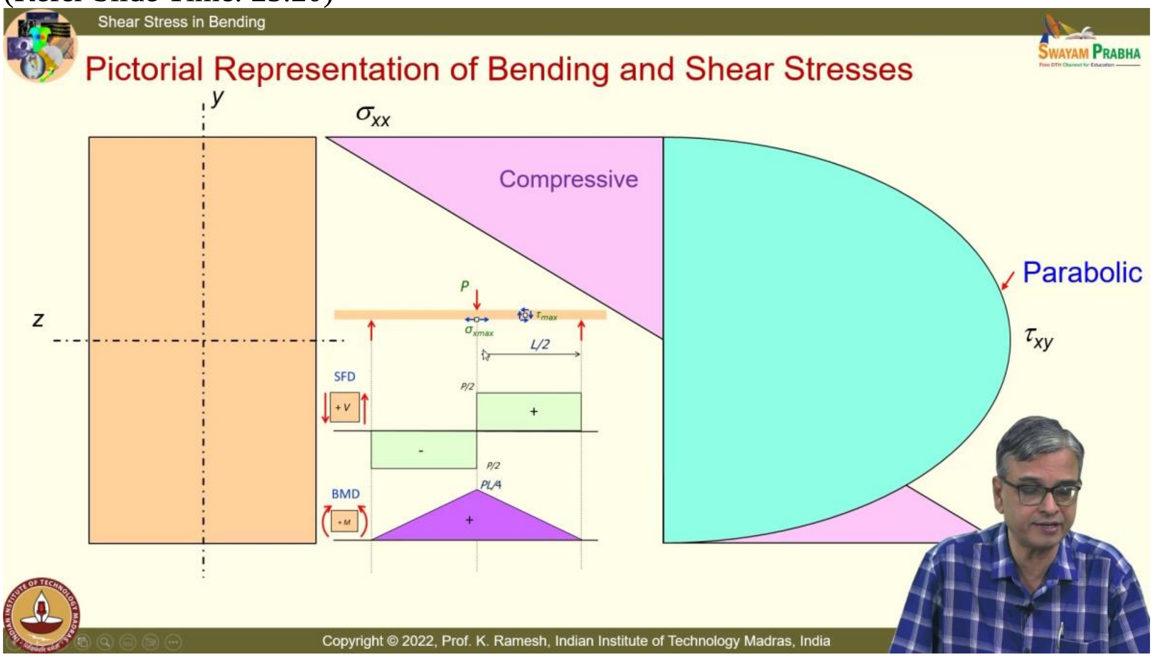

And you know, even though we have said shear is important, there are other concepts also we have learnt. A bending stress, if you plot it with some scale, is triangular and you have compressive and tensile. And in order to illustrate that shear stress varies parabolically, many books show this big, but they do not give an insight, what is the relative magnitude. That is also very important.

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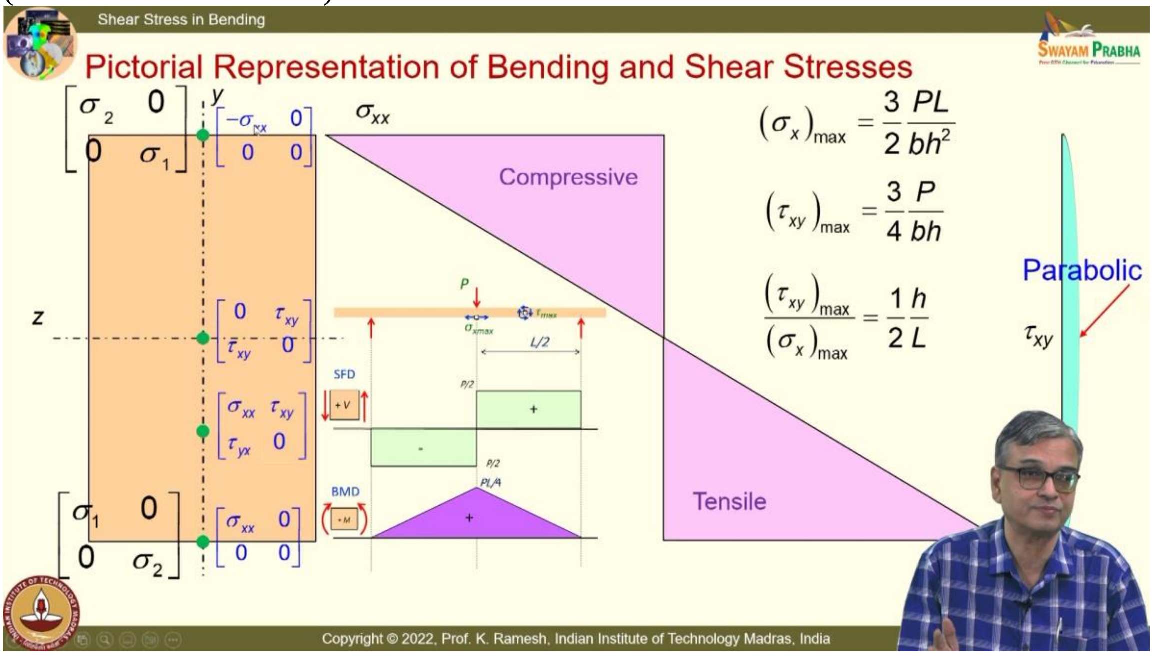

Even if you consider a simple problem of a rectangular cross-section subjected to three-point bending like this, for this problem if you calculate the shear stress, in actual magnitude if you plot, it is as small as this compared to the normal stress. That is normal stress created by bending. That is the reason, even though shear is important in certain instances, we are neglecting it because its actual magnitudes are considerably very small in many applications. And we have also looked at how to put, you know, the stress tensor. You have the comparison in terms of numbers, in terms of

$$\frac{(\tau_{xy})_{max}}{(\sigma_x)_{max}} = \frac{1}{2}\frac{h}{L}$$

So, if I have *h/L* as, if I take this as 1/10, so it is 20 times less compared to the normal stress. So, that is the idea that you should have.

And we have also looked at how to write the stress tensor at specific location of the cross-section. This is the cross-section that we are talking about. You can also write the stress tensor. This is compressive, so I have put this as $\begin{bmatrix} -\sigma_{xx} & 0 \\ 0 & 0 \end{bmatrix}$. And when I have this as the tension side for this problem, I have $\begin{bmatrix} \sigma_{xx} & 0 \\ 0 & 0 \end{bmatrix}$. And when I look at at the center, because this beam is also transmitting a shear force; so what I have here is, I have $\begin{bmatrix} 0 & \tau_{xy} \\ \tau_{xy} & 0 \end{bmatrix}$. When bending is maximum, shear is zero. When shear is maximum, bending stress is zero. That is the interesting aspect of what happens in a beam. Suppose I take a point in between, this will have $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & 0 \end{bmatrix}$. That is dictated by the problem, ok. And we have also looked at how to write σ_1 and σ_2. You know, we always say that one should be algebraically the largest. When I have this as compressive, this becomes σ_2. Very subtle point, but it is also very important. That is the convention that we have used. We will always have σ_1, σ_2, σ_3 arranged algebraically larger, ok.

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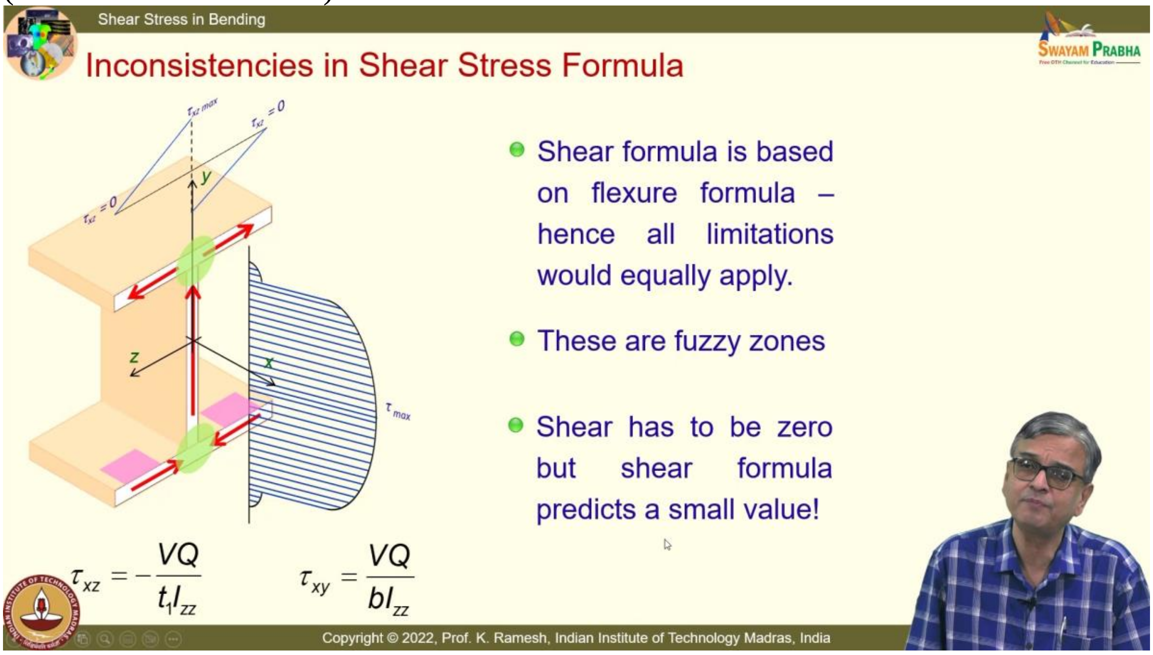

And you know, we have also looked at the inconsistencies in shear stress formula. You have an issue when I have it in the junction, ok. Nevertheless, we use these expressions, because we get the numbers reasonably for us to play with. And from your free surface concept, you know, I should have my, this flange; this should have zero stress, but your shear stress formula does not give it. And in the transition region, what you have here is, this you take the length of the flange, and you get a smaller magnitude. The moment you come to the web, suddenly these stress magnitude increases. That is correct. There is no problem. The problem comes in other zone, ok. And you have this τ_{xz} varies linearly because you take Q like this, ok.

And the other important limitation is, when I look at here, you know this is a free surface. I should have zero shear stress, but you have shear stress here and that you will have to take it with a pinch of salt. These are fuzzy zones. Shear has to be zero, but shear formula predicts a small value. But we have already looked at, from a mathematical point of view, we may have some number. But if you compare the magnitudes, the magnitudes are very small. And this is the reasonably a simplistic analysis of an open cross-section that we could do in the case of beams. From what we have developed for the cross-section, which has one plane of symmetry, we could exploit that flexure formula for a wide variety of problems than what we have learnt it for torsion, ok.

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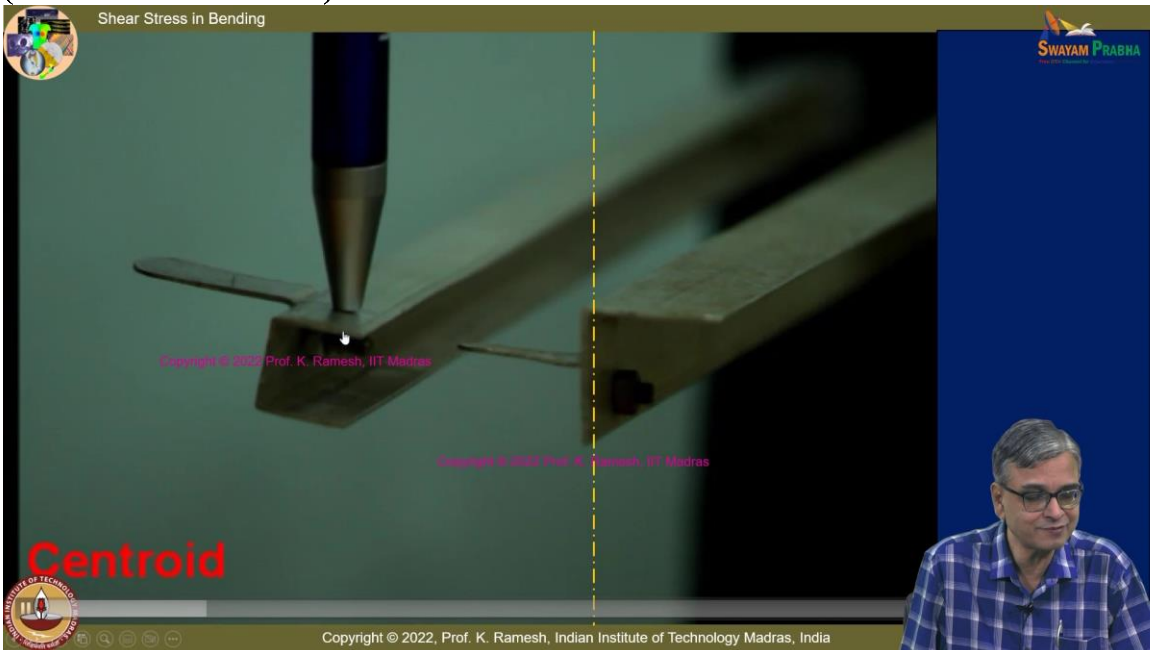

And there is also a very interesting concept, once you look at open sections. When you observe this, you find that this is bending as well as twisting.

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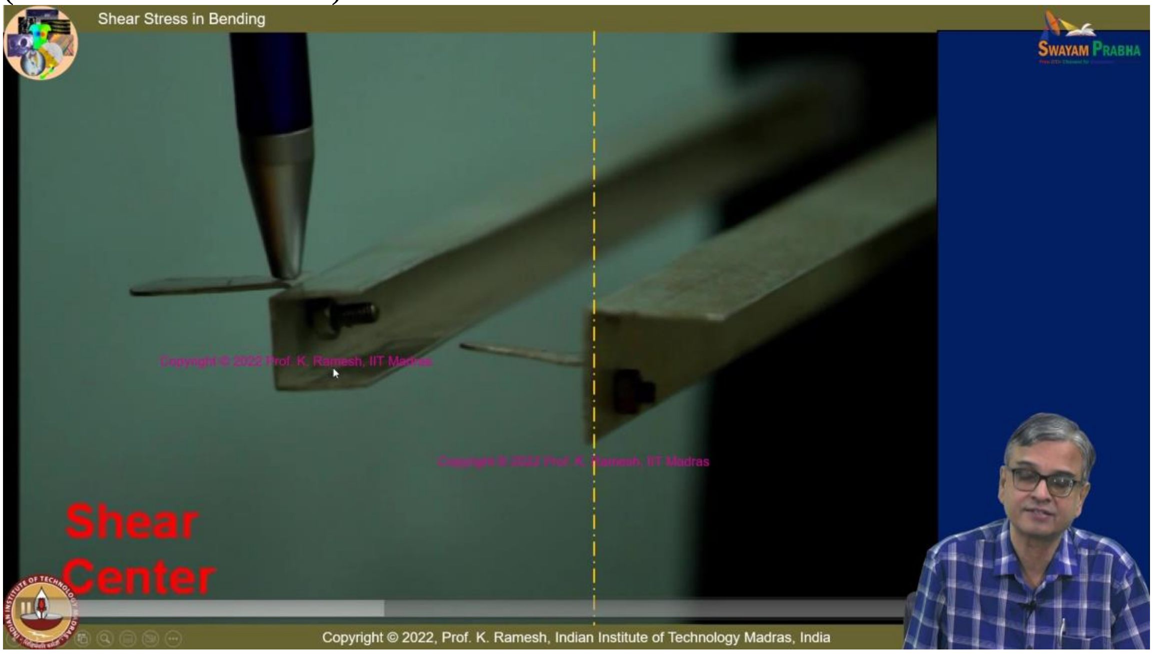

And when you apply it along the shear's center, it only bends, ok.

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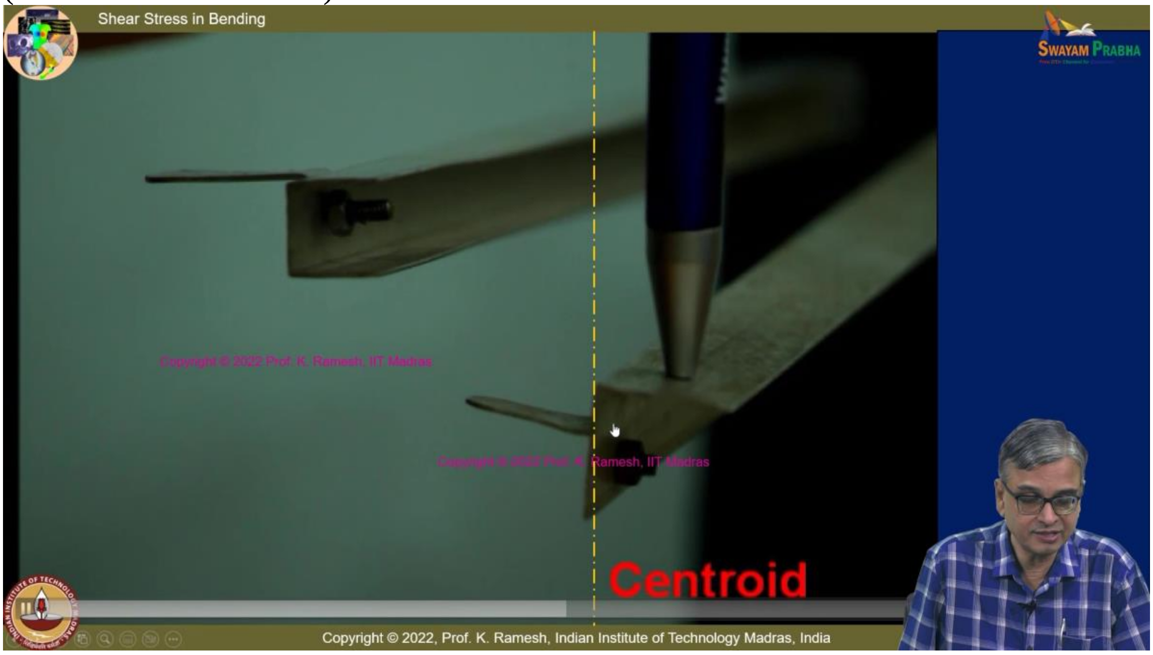

And this is another angle cross-section. And if you look at this, you know, this also bends and twists. The shear center is actually this corner; I was unable to put my pen there. So it was slipping; I just put it next to this.

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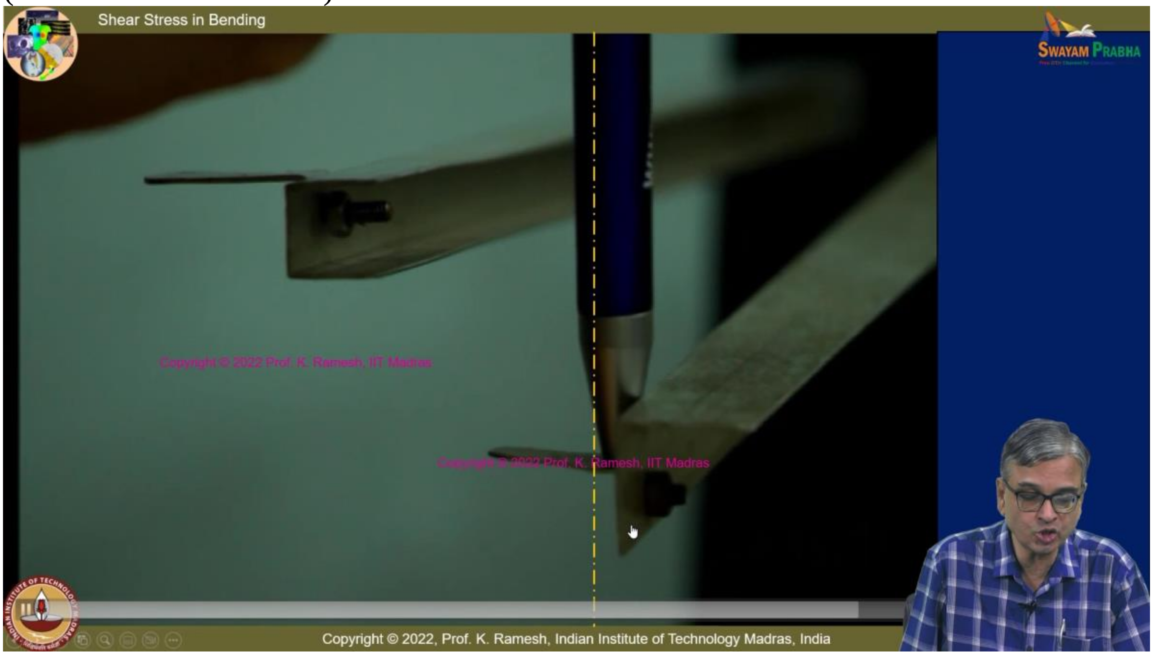

You can very clearly say that this is only bending; there is no twist. But while bending, it has bending in two planes. One plane is like this; another plane is like this. It is bending like this; it is also bending like this. And that we said it is an unsymmetrical bending. Is the idea clear? When it is bending in two perpendicular axes, you have this as unsymmetrical bending.

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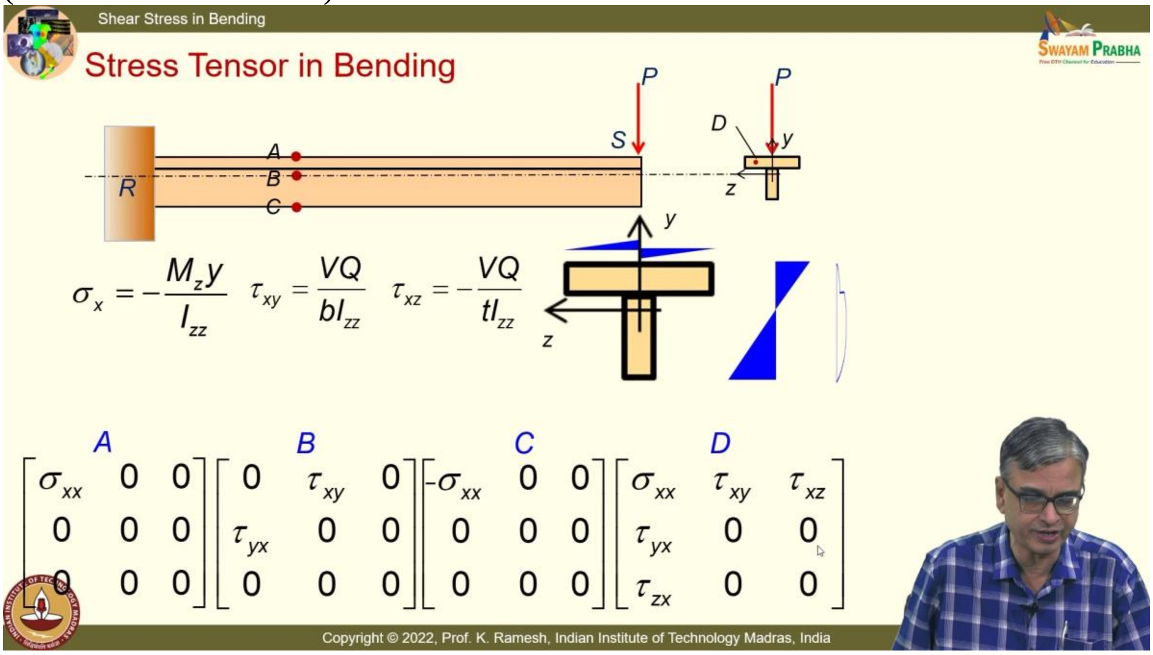

And we have also looked at another typical cross-section used in beams. You have T beams used in many, many applications. This is very interesting from the point of view of writing your stress tensor. Because you know, if you write stress tensor at *A*, it is very simple. Simply σ_x is existing, and you also have the expression from your flexure formula. And for point *B*, which is on the centroidal axis, which is also the neutral surface, this is made of one material, ok. So, I have this as only τ_{xy} and τ_{yx}. And when I, you also have the expression

$$\tau_{xy} = \frac{VQ}{bI_{zz}}$$

And your typical bending stress variation is like this. And your shear stress variation τ_{xy} is like this. And when you make this as modulated for the actual values, these values are very, very small compared to the bending stress. And when you go to the point *C*, I have put this as $-\sigma_{xx}$. And interesting aspect is what happens at a point *D*, that is what is put here. I have σ_{xx}, τ_{xy} as well as τ_{xz}, you should recognize that. The stress tensor is more populated, ok. So, we have learnt how to get the expression for stress magnitudes, whether it is axial stress, shear stress in torsion or bending stress in bending or shear stress in bending. When you get that as those quantities that appears like scalar, but you should also put it in the matrix form, so that you recognize that this is a tensor of rank 2. Not only this, when I have combined loading, if you write it as a matrix form, you can add them if you take the axis appropriately, which we have also solved a problem of a femur subjected to tension, torsion and bending. Combined loading is the order of the day, any simple problem has only combined loading. We learnt them separately.

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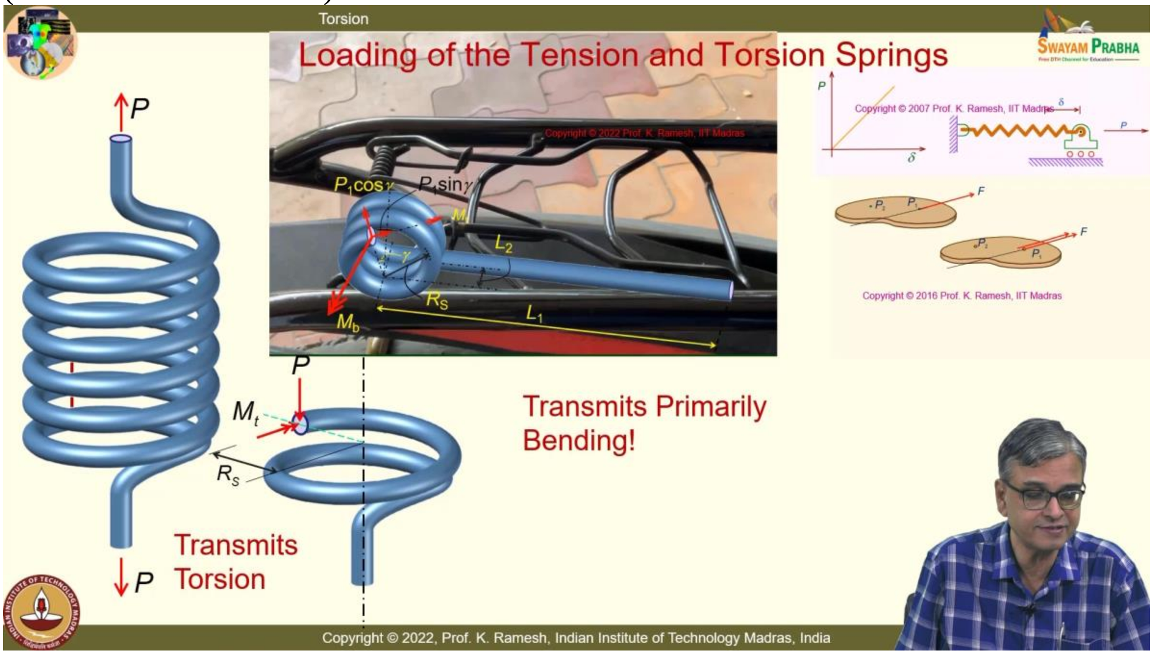

And you know, one of the concepts that I have said that you lack is how to move a force from one point P_1 to P_2. If you master this, when I move this from P_1 to P_2, I need to have a force as well as a couple. You can comfortably write the bending moment diagram and shear force diagram. You take more time because you have not mastered this, I am repeating it again and again. And even when you look at a simple tension spring, if you find what is the load which is acting, when I asked you to find out the forces, you are struggling to get it. But if you look at how to apply this concept, which I am doing it again; when I move it along the axis, nothing happens. I can move the force freely along the axis that is principle of transmissibility. But if I move from this axis to this, I have to look at, I will have a force as well as an appropriate couple, ok. Here, you are getting this couple in such a manner that this is along the axis, so it is a twisting moment. So, I will have a twisting moment as well as a shear force.

Then we moved on to how to analyze the torsional spring. You say torsional spring, but what is the major force it is transmitting? Its major force transmitting is bending. That is again, I have a force here; I move this force. So, this gives me a force as well as a couple. This couple is a bending moment and when I move from this to this, I have a force as well as a couple. This is a twisting moment. So, you have to use this aspect of moving a force from one point to another point. It is not trivial, which is not emphasized in the books. I have emphasized it repeatedly so that you get the idea of this concept better. And you can solve seemingly very complex problems in a jiffy. Now, you have the background to analyze these problems. So, do not simply say that when I have a sketch like this, a simple spring gets stretched, it is transmitting axial load. Do not say that! It is actually transmitting a twisting moment. So, this transmits torsion, and this transmits primarily bending. So, you have to know the difference.

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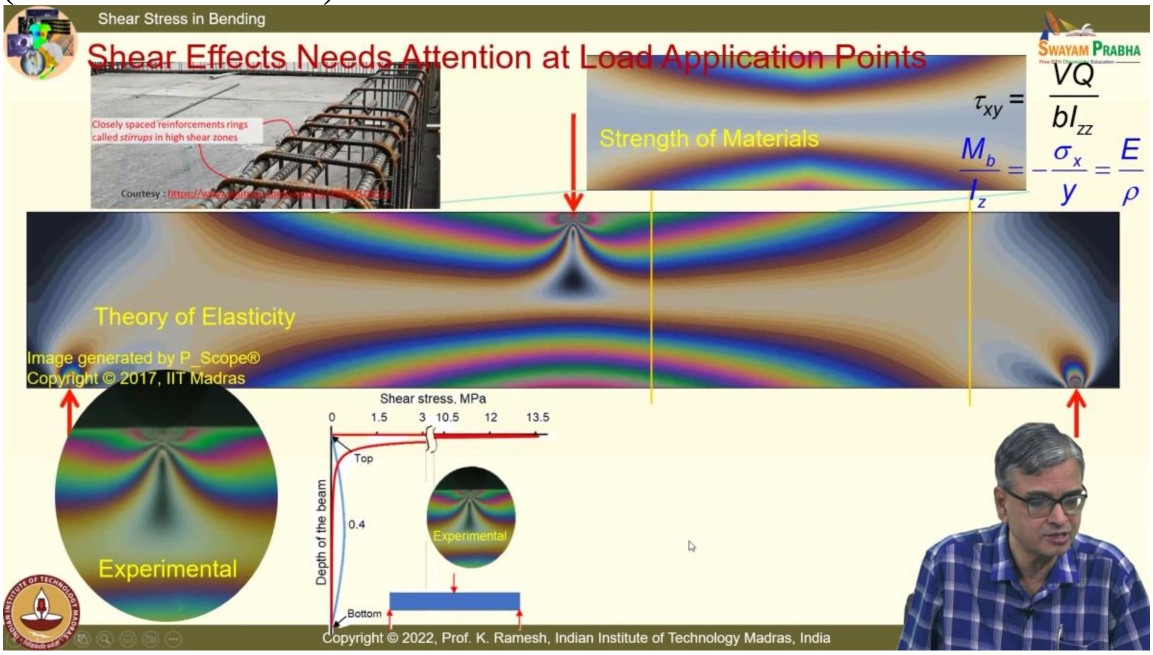

And you know, I said that even though I say shear magnitude are small, it matters when I go very close to the load application point. If at all I apply strength of materials solution, I must apply it away from the point of loading, ok. I must do it only in a zone away from the point of loading. But if I go close to the load application point, this is done from theory of elasticity solution. This is what you have in an experiment. This matches very well with theory of elasticity solution. And look at what happens in the case of strength of materials. It totally misses out this aspect, because we have never considered. We have taken shelter under Saint-Venant's principle. So, you have to consider that near the load application point, these shear magnitudes are very high. Just below the surface, it reaches a peak. At the surface, it is still zero, because it has to satisfy that free surface requirement. And these stress magnitudes are comparable to the bending stress. So, failure can definitely happen unless you reinforce these areas. And this is done in practice. People have stirrups to support this.

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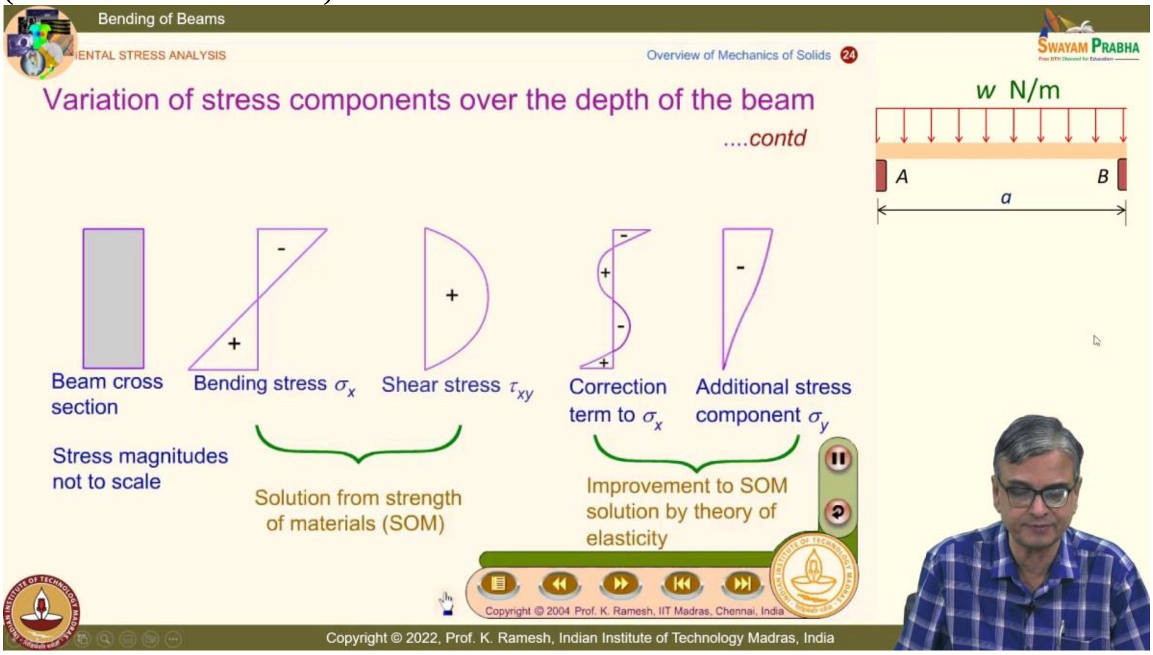

And we have also looked at another deviation. Suppose I have a uniformly distributed load. This is a common load in all civil engineering construction; you have the self-weight. And if you look at the strength of material solution, even for a rectangular cross-section, which is very well done in strength of materials, it is not in order. There is a small variation in your normal stress. It is not linear, but it is having a small non-linear component. And in addition, we have said in pure bending, there is no normal; σ_y stress component. That is correct. But when I have a situation like this, where I have distributed load, I do have normal stress. And theory of elasticity accounts for this. And here again, I want to caution you that this is not drawn to scale. None of these quantities are drawn to scale. The focus is only to show the variation, shape of variation. To accentuate the appreciation of shape of variation, it is drawn big. If you draw it in real scale, they will appear very small. All these three quantities will appear very, very small.

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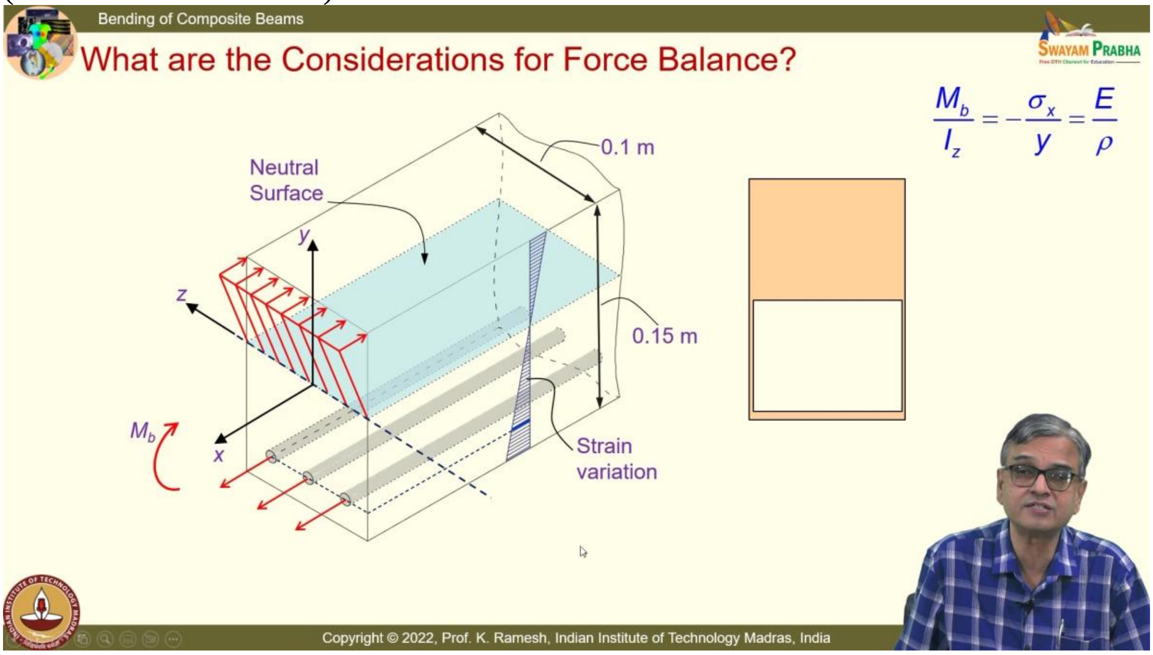

And we have also learned how to analyze a reinforced concrete beam using the same flexure formula. We imagine that the concrete beam is like a section like this, and the hollow section connected by a thin web, ok. And that is how we have analyzed it. It is all engineering analysis because only the rods; steel rods take the tension load. Concrete is very weak in tension.

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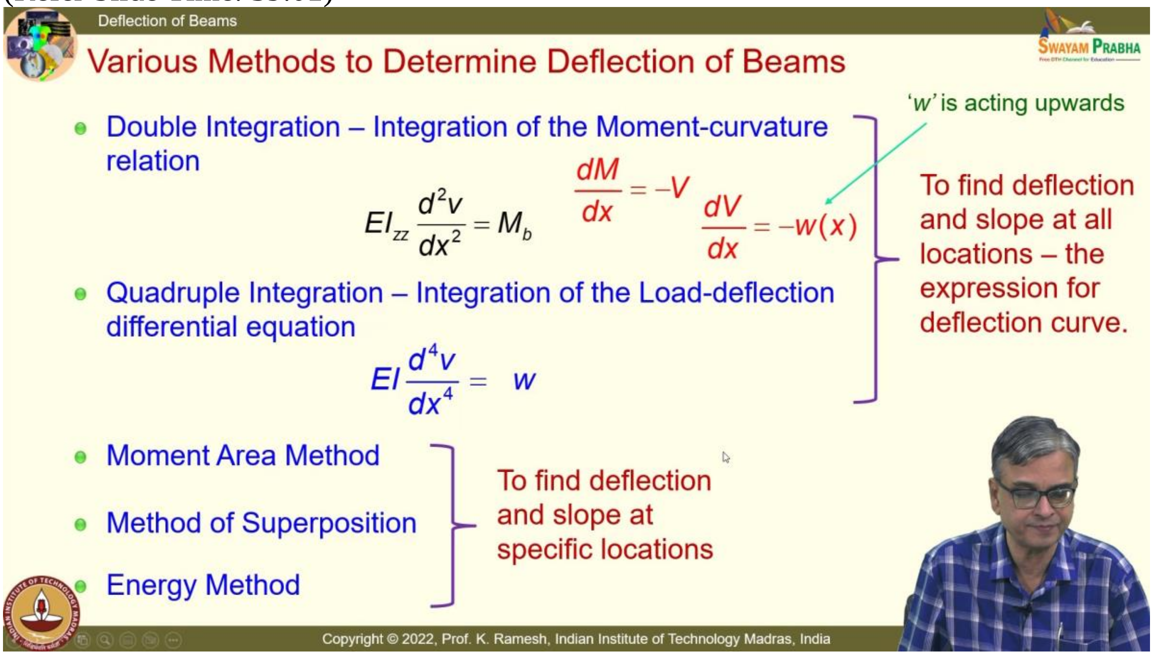

And we have also looked at various methods to determine the deflection of beams. So, we have double integration

$$EI_{zz}\frac{d^2v}{dx^2} = M_b$$

And if you use the interrelationship $dM/dx = -V$, $dV/dx = -w(x)$. Then I get a quadruple integration that is the load-deflection

$$EI\frac{d^4v}{dx^4} = w$$

We have been able to take out *EI* out because we are discussing a beam of constant material and constant cross-section. Otherwise, *EI* would be inside; d^2v/dx^2 will be there. That way also you can analyze. And you know, when you do this double integration or quadruple integration, you get the variation of slope and deflection along the length of the beam conveniently; no problem. And we have also looked at that this is useful even for statically indeterminate problems. Then we also looked at moment area method, method of superposition and energy method. These are applicable for at specific locations if you want to find out the deflection and slope, you are in a position to do it quickly, ok. And moment area method people also say even if I have *EI* changes, I have to scale up my bending moment diagram in that region with appropriate *EI*. So, beams of variable cross-section or if you change the material moment area method is also a good choice for you to find out the deflection at specific locations.

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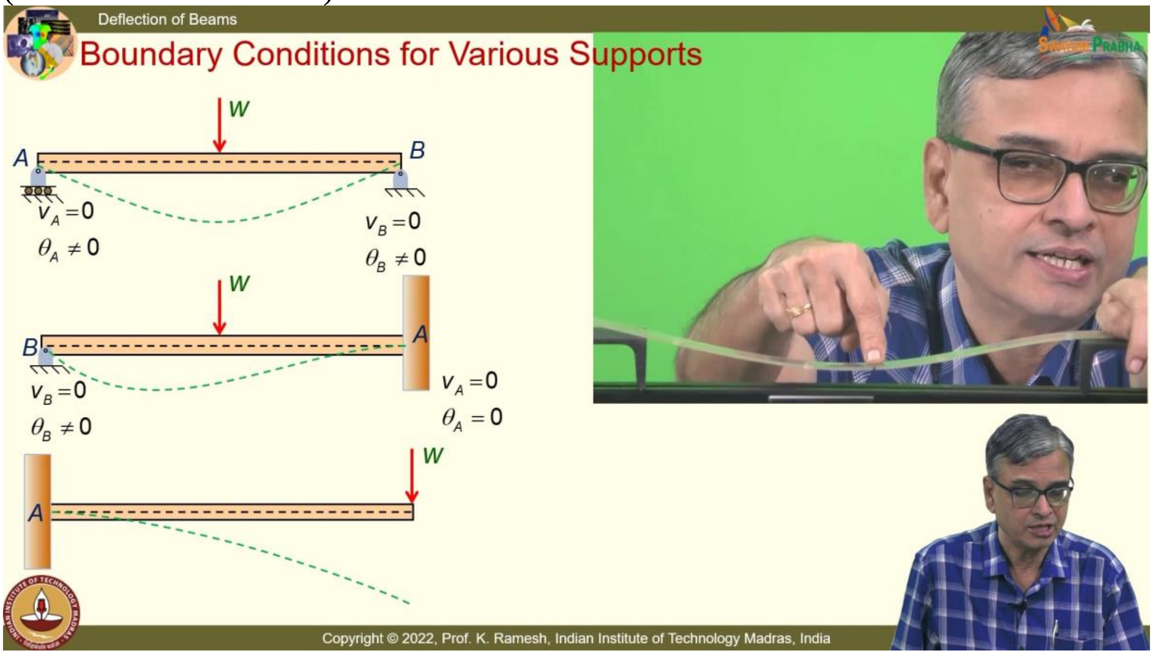

And one of the aspects in bending is deflection, is you have to find out what way the boundary condition to be written for a simply supported end. It will have a slope; it will have deflection as zero. On the other hand, if I go and do that as a fixed end, you find immediately the slope goes to zero. Your deflection is zero, but slope is also zero. And you know, you will also look at what happens in a free end. So, if I have problems of this nature, you should know how to write the boundary conditions. It is needed even for your appreciation of how the deflection can be because one of the training in this course is to learn how to use method of superposition. There the training is how to draw the deflected shape, ok. So, you should practice this and then visualize how the beam can have a deflection. That is a very important idea. It helps you to even verify your mathematical development, ok.

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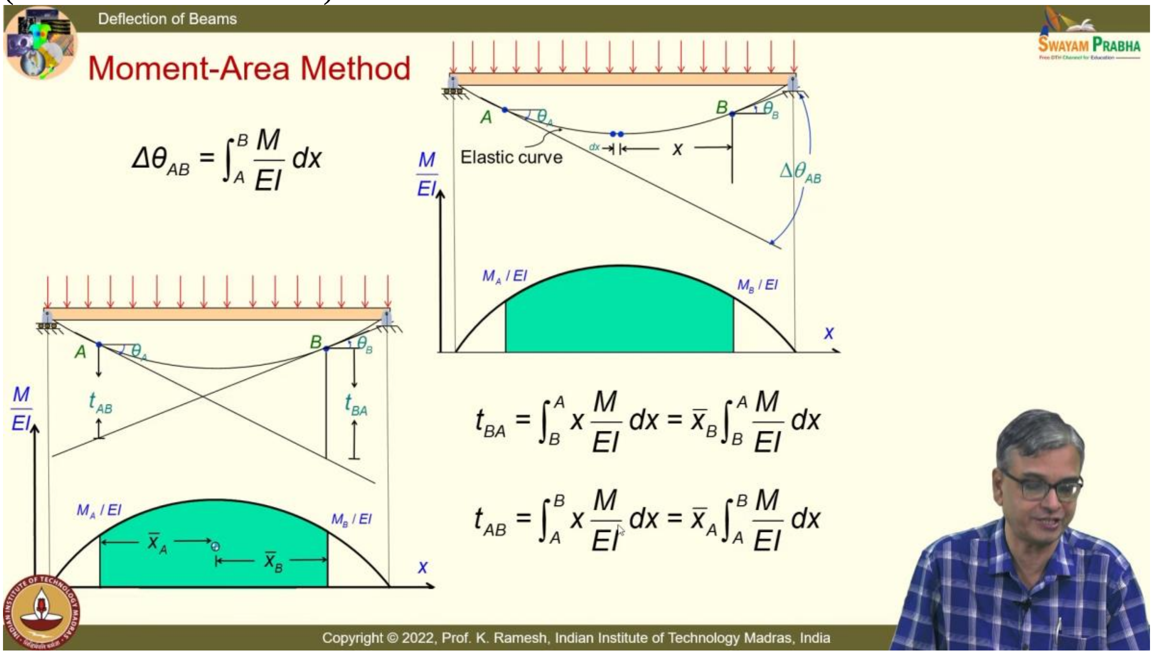

And you know, we have also looked at in the moment area method, you get only the; if you calculate the area, you get only the difference in angle of the slopes. You do not get the absolute slope. Similarly, you get only tangential deviation when I take the moment. If I take t_{BA}, I have to multiply by $\bar{x}_B$. If I have t_{AB}, I should multiply by $\bar{x}_A$. I get only tangential deviation. You should know how to use this effectively to get the actual deflection or actual slope.

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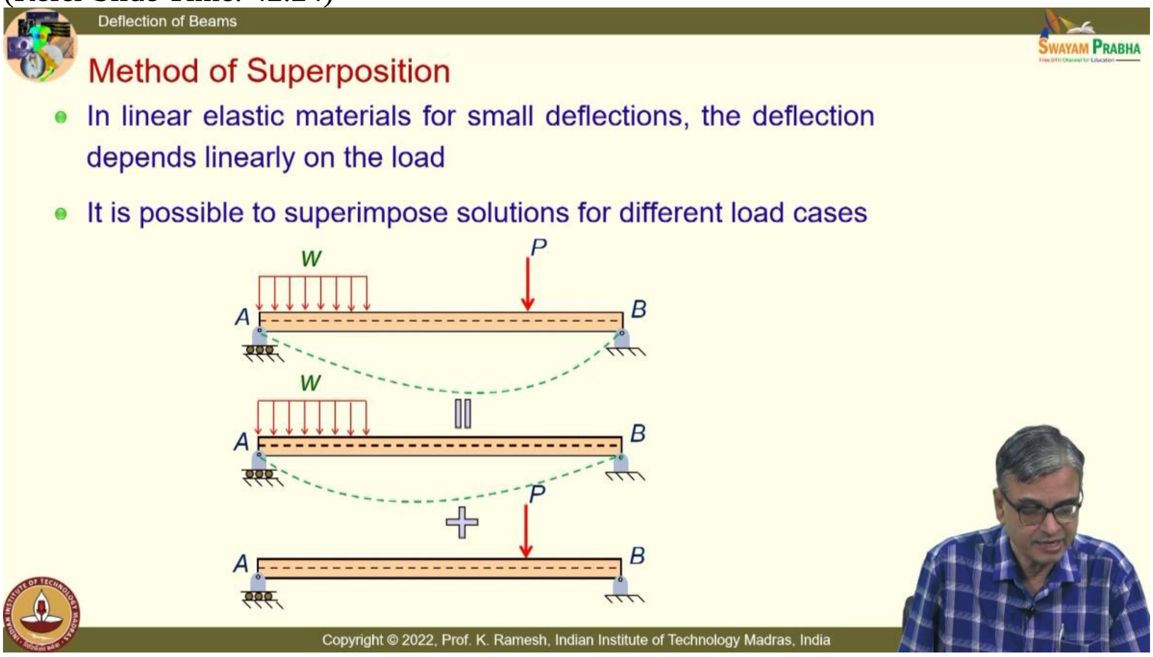

And in the method of superposition, as I said, you know, because we are living in a linear elastic regime, I can superimpose independently and add all of them; analyze the problem independently so that the problem is easy to handle and add them together in a systematic fashion. So, you should know how to draw this deflected shape. You should visualize that. That is one of the important training that you learn in this course.

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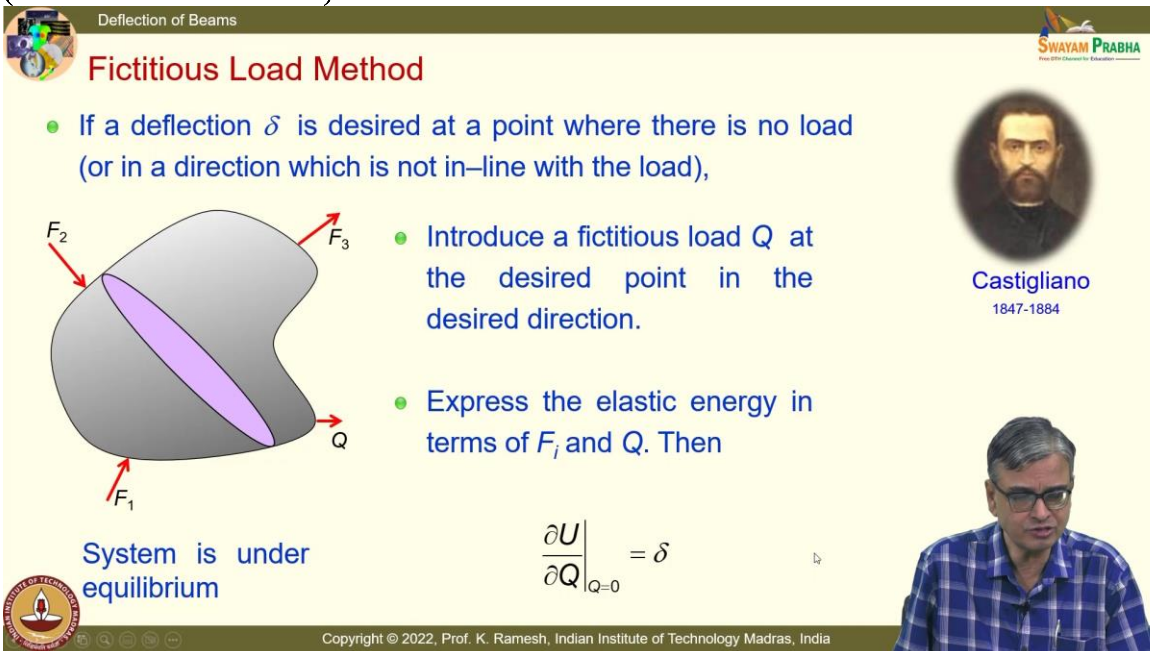

And we have also learned fictitious load method. This is due to Castigliano. He is a very young scientist. You know, he passed away very early in his life and this theorem was available in his PhD dissertation. It has become very popular and many developments using energy methods later on owe allegiance to his contributions. So, even if I do not have a particular point; load is acting, if I want to know what is the deflection in a direction or in any other point. I can add a load, introduce a load Q, determine the energy and then differentiate with respect to find out the energy, differentiate with respect to Q, substitute Q equal to zero at that stage, then what you get is a deflection along that direction for that point of interest. So, very useful method.

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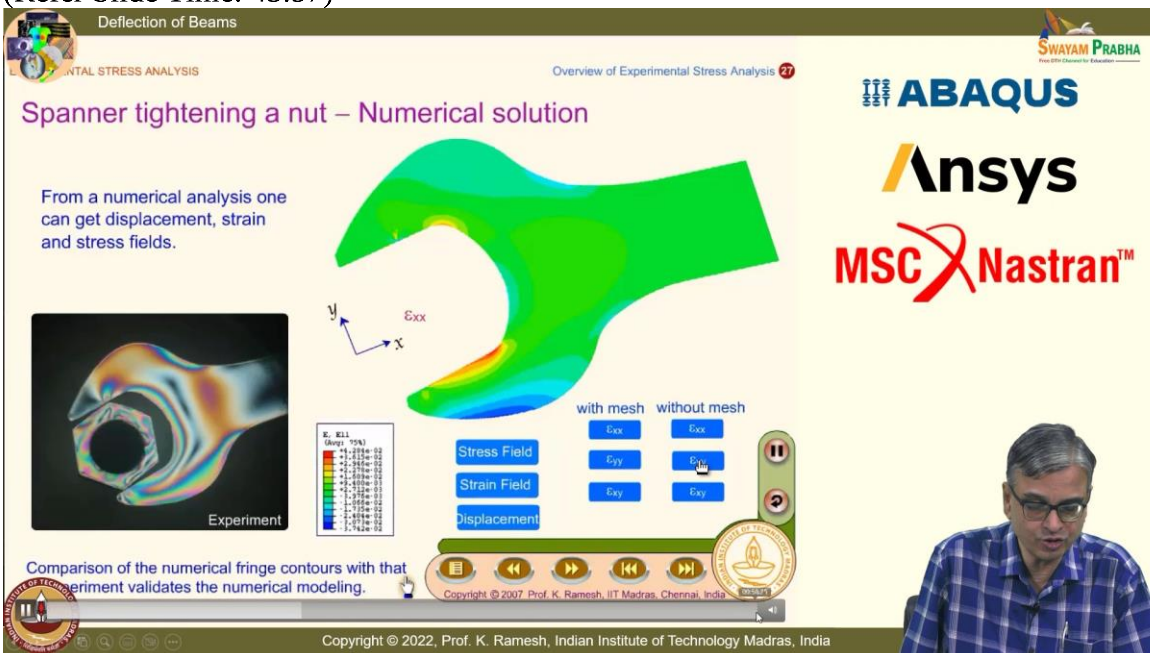

And I also said an extension of this is the development of the finite element method and experiment is truth, ok. But when I have the finite element method, I can get the displacement contours, I can get the strain contours, I can get the stress contours - all of that I can do, ok.

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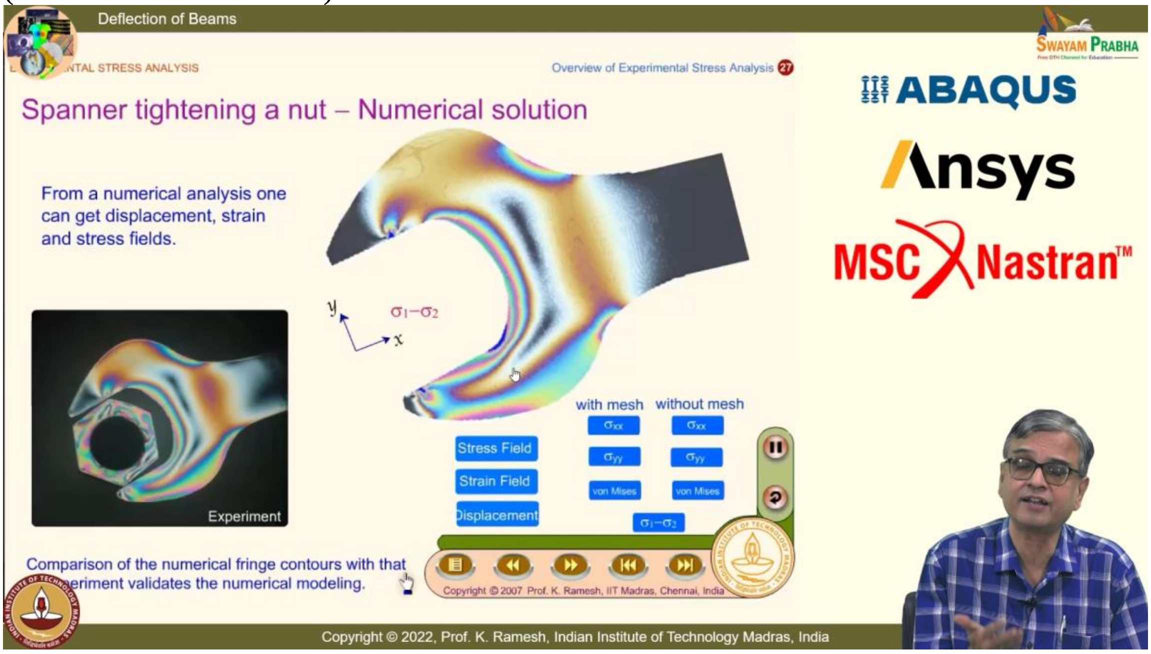

And finally, you know, if I plot the stress contours in terms of $\sigma_1 - \sigma_2$, you see this is from an experiment, this is from a finite element analysis. The match is very impressive. You can clearly say that my numerical analysis has brought out the results correctly. What is the difficulty here? You know, you have a boundary of the spanner which is very arbitrary. Even though theory of elasticity we have used as a basis to verify our strength of material solution, theory of elasticity demanded how to write the boundary condition in an arbitrary geometry. That was very difficult. It can handle simple geometries like circles, ellipses and so on. So, one of the greatest advantage of finite element is, I can discretize it using an element and handle any arbitrary geometry comfortably, but the solution is approximate. It is not an exact solution, but you go asymptotically to the actual solution, and you have many packages available, and you can learn some of them as part of your future courses.

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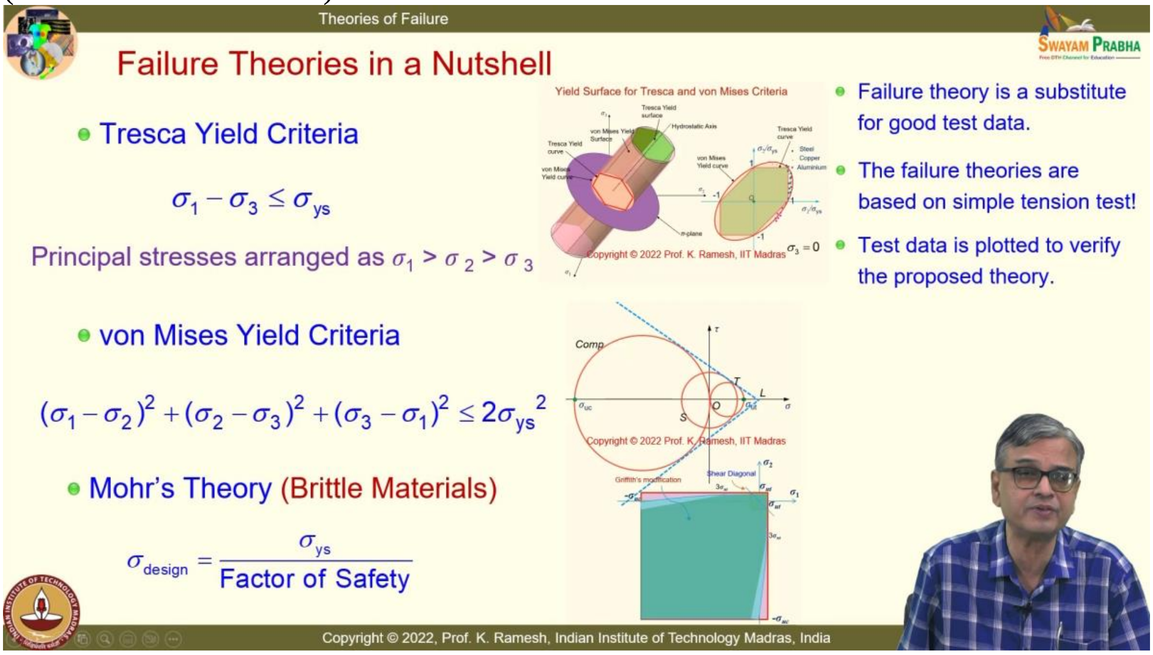

And ultimately why we have developed all these concepts of stress, strain and principal stresses? We wanted to investigate whether the component that I make, whether it will withstand the loads or not, whether it will be strong enough. So, you have failure theories, and you have Tresca yield criteria. I said, if I have σ_1, σ_2, σ_3 arranged in this fashion, you will never make a mistake if I have

$$\sigma_1 - \sigma_3 \le \sigma_{ys}$$

I caution in the case of your simple pressure vessel, both the principal stresses are positive. There is every room that you can apply Tresca yield criteria wrongly to that. You may simply use σ_1 and σ_2. You should not forget σ_3 is zero. And you should also appreciate failure theory is a substitute for good test data. I said for complex structures, people develop very expensive loading rigs and only then the design is released for human use because human lives are very precious. And failure theories are based on simple tension test. How did they approve the theories? By conducting complicated experiments and verifying that the experimental results for unknown load situations which have not accommodated in simple tension test, also falls within that yield locus or very close to the yield locus, ok.

And for brittle materials, because you know for ductile materials, you have Tresca; you have von Mises which are shown here. One is an elongated hexagon, another is an ellipse and this appears like a circle under the hexagon in the three-dimensional σ_1, σ_2, σ_3 space. And I also said that von Mises yield criteria is same as a distortion energy theory as well as limiting value of the octahedral shear stress, ok. So, people have verified it from multiple viewpoints. And for brittle materials because they have various different values of tensile yield strength and compressive yield strength, they are very strong in compression. And you have a failure locus which is modified by several people. Initially, you had one developed by, long time back Coulomb, but modified by Mohr. Then finally, it was modified by Griffith, that is also shown here. And so, you have to use principal stresses very effectively to verify whether a material, whether it is brittle or ductile, you have failure theories. And you also have in design what is known as factor of safety. I said factor of safety is a very, very important design parameter.

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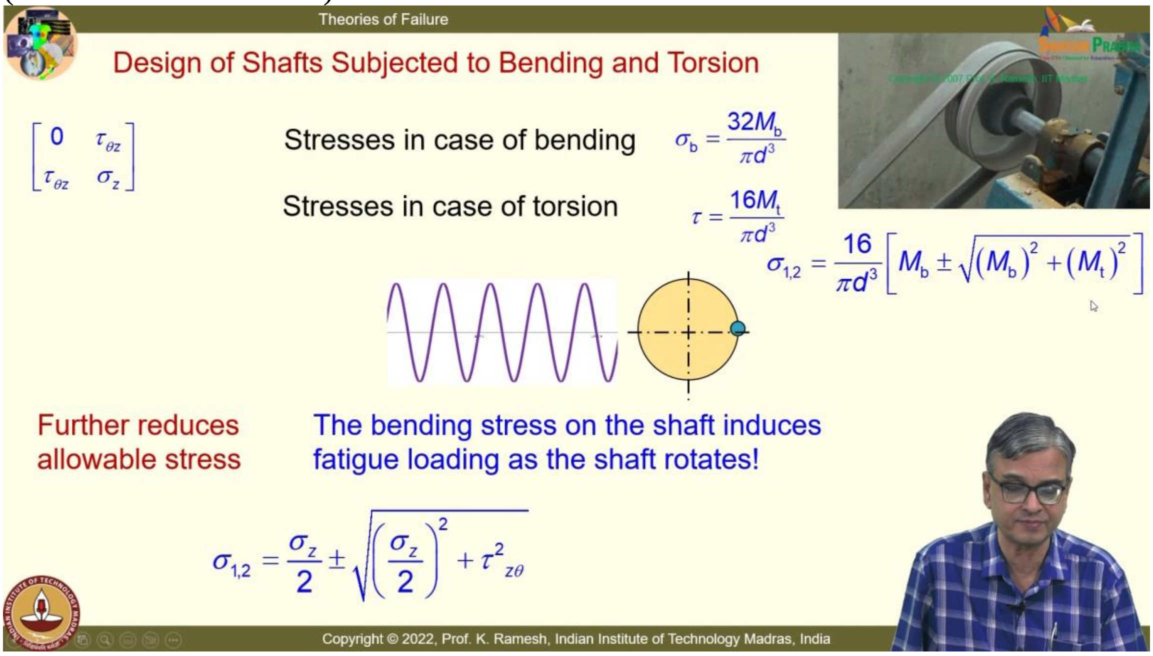

Even a simple atta-chakki can throw you surprises. So, when I have this, I have the stress tensor in this form. And you can write; in the design courses, they simply express it in terms of the bending moment and the twisting moment. And because the shaft is rotating, you have to recognize the bending introduces stress nature that varies cyclically. Because this shows you have the fiber, how the fiber when due to rotation experiences load, which is a very subtle point, ok. You may miss it. And what is the consequence of this? The bending stress on the shaft introduces fatigue loading as the shaft rotates. In view of this, it reduces further the allowable stress. And all your design courses basically use what is the principal stresses from this stress tensor and expresses this in terms of your bending moment and twisting moment.

So, when I have a shaft transmitting torsion and bending, you have a simple expression. They will do and estimate the diameter of the shaft. Just by using this, there is no torsional stress or bending stress in the expression. They are hidden, ok. And you do not have to memorize this in your later course. You can derive and people use this in their design books like this, ok.

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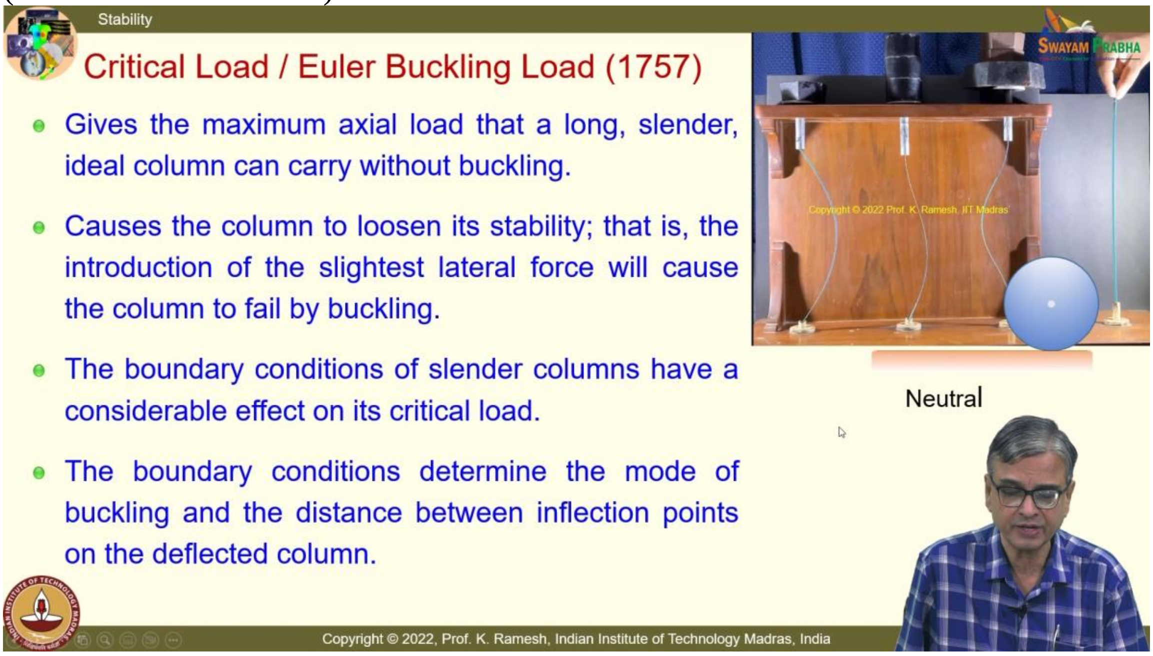

Then you know, we said that when you do not have a warning, any failure is difficult to handle. So, one of the functional failure loss is, you have a column that suddenly buckles. There is no material separation, but suddenly buckles. And we have seen that this is developed by Euler in 1757. And buckling is one aspect where we have to analyze on deformed coordinate system. Second aspect is the critical load at which buckling tapes is very sensitive to the boundary conditions. And we have looked at how the, when the boundary condition changes, how the shape changes. And we also said that these are all neutral equilibrium positions, ok. And the experiment gives you truth and this is one of the experiments which is very, very expensive.

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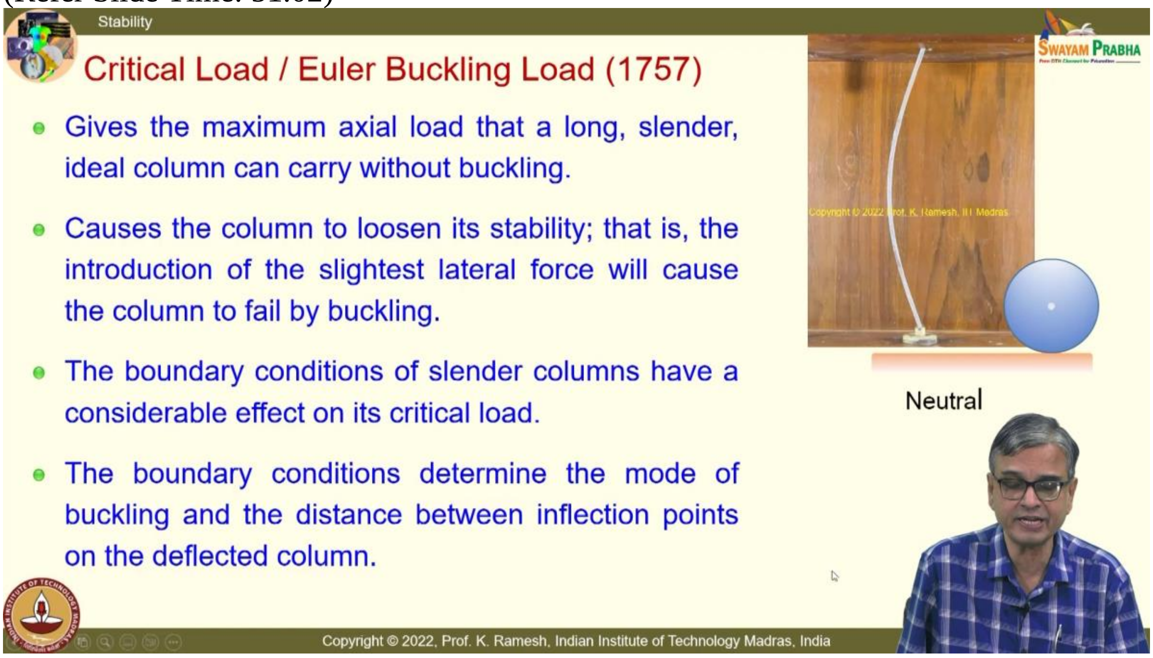

And people also found application for buckling. You know, engineers are very clever. You know, they do not believe any physical phenomena, go unutilized.

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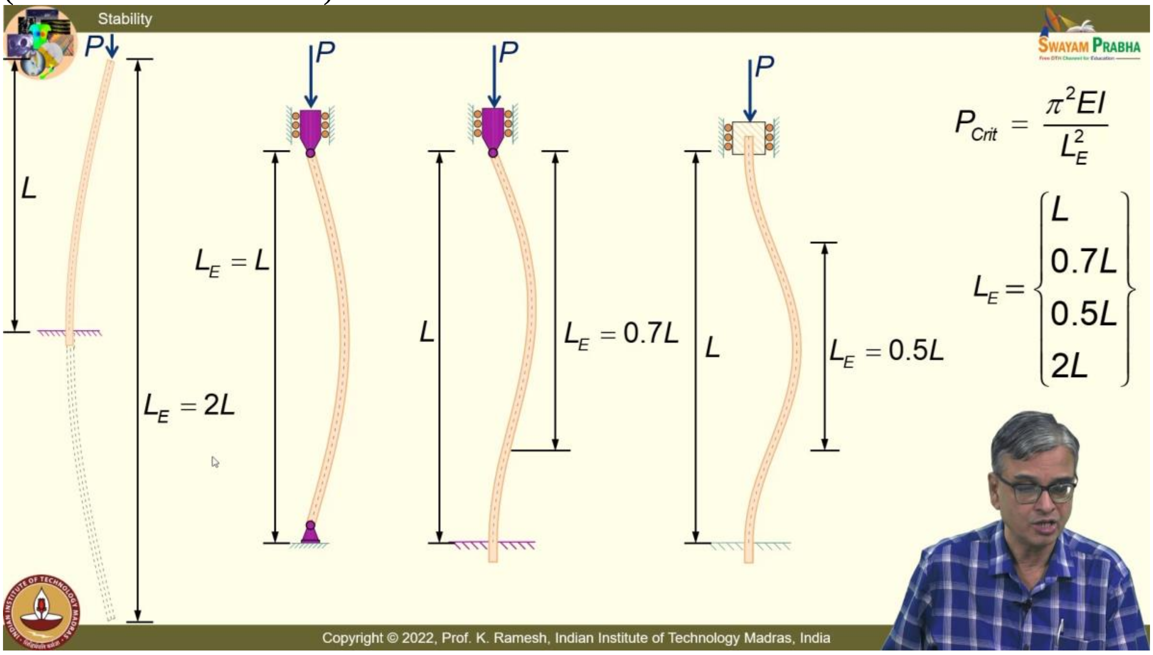

And you view the critical load from a different perspective you take the hinged-hinged column as a basis. And you have

$$P_{Crit} = \frac{\pi^2 EI}{L_E^2}$$

And you visualize other boundary conditions with an equivalent column length. If I have a column of fixed pin; I view this as equivalent length of $0.7L$. If I have a column which is clamped-clamped; I view this as a column of length $0.5L$. And if I have a cantilever; I view this as a column of $2L$. Longer the column, more it is prone to buckling, ok.

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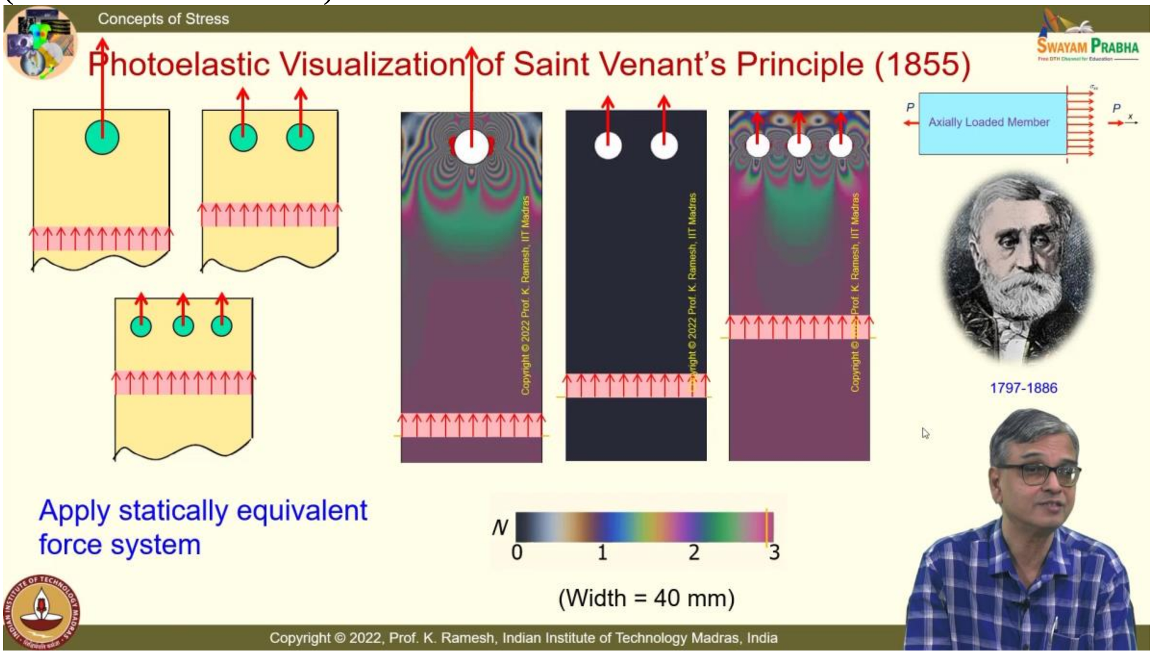

And you know, last but not the least, we have one of the best explanation for Saint-Venant's principle in this course, because you have been trained to look at photoelastic fringe patterns. So, in these three cases, I have a statically equivalent system applied. And what you find is, the disturbances die down after a distance and this distance becomes smaller, when this distribution is close to the assumed distribution of uniformly distributed; is the one which we require. We say that axial load resistance is developed like this. And this happens at different distances.

See, what is the distance that you have to take? One thumb rule is it is equivalent to the longest cross-sectional dimension. It is only an empirical approach, ok. There have been many discussions. There are also certain specific instances where Saint-Venant's principle does not work. That also you should know. But for a large variety of problems, Saint-Venant's principle is a very useful principle. And you are able to relate it easily by looking at the fringe pattern, because in an axial load, I should have a constant color. And how the constant color is obtained, you have to look at. And this is by Saint-Venant. You know, unlike Castigliano, he had the longest life, about 90 years, very active till the last minute. Very, very few people in life are gifted to pursue their passion till the last minute, ok. So, with these observations, your overview on strength of materials is brought out. So, if you look at these two lectures, one after the another, in a nutshell, you know what are the important concepts that we have looked at in this course. Thank you very much.