Course Name: Theory of Fire Propagation (Fire Dynamics)

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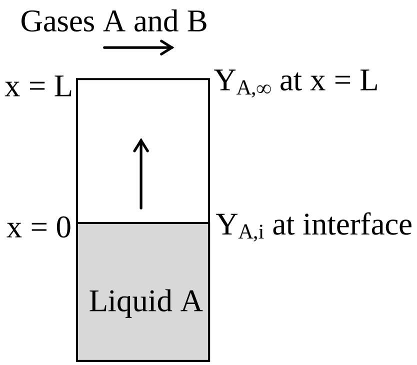
Lecture – 01

Module 4 – Burning of Liquid Fuels

Steady evaporation – Fick's law:



At the interface, the mass flux of A considering the ordinary diffusion is:

$\rho_A v_A = \rho V Y_A - \rho D_{AB} dY_A/dx$

For species B, since it is not soluble in the liquid, the net flux of B in the interface is zero:

$0 = \rho V Y_B - \rho D_{BA} dY_B/dx$

Since, the sum of diffusion fluxes of A and B is zero, it can be written as, $\rho V = \rho A v A$. Bulk velocity in x-direction is the velocity of species A. Species B is transported by diffusion in the column with a zero species velocity ($vB = 0$).

Steady evaporation – Mass fraction of A:

The final equation of mass flux from interface is

$\boldsymbol{\rho_A v_A = \rho_A v_A Y_A - \rho D_{AB} dY_A/dx}$

This can be written as:

$$\frac{\rho_A \mathrm{v}_A}{\rho D_{AB}} dx = \frac{dY_A}{1 - Y_A}$$

Product ρD_{AB} is assumed as constant.

Boundary condition at the interface is $Y_A(x = 0) = Y_{A,i}$.

Integrating the above equation and using the boundary condition to eliminate the constant of integration, mass fraction is got as:

$$\boldsymbol{Y_A(x) = 1 - (1 - Y_{A,i})\,\mathbf{exp}\left[\frac{\rho_A \mathrm{v}_A x}{\rho D_{AB}}\right]}$$

Mass flux of species A depends on the interface mass fraction of vapor A, $Y_{A,i}$, which further depends on the surface temperature. Mass flux can be calculated by using the far boundary condition, that is, at x = L, $Y_A = Y_{A,\infty}$, which is given as,

$$\boldsymbol{\rho_A V_A = \frac{\rho D_{AB}}{L}\, ln\frac{\left(1 - Y_{A,\infty}\right)}{\left(1 - Y_{A,i}\right)} = \frac{\rho D_{AB}}{L}\, ln(1 + B_Y)}$$

Here, **$(1 + B_Y) = (1 - Y_{A,\infty})/(1 - Y_{A,i})$**.

Parameter, B_Y, is called the Spalding's transfer number based on mass fraction.

$$\boldsymbol{B_Y = \frac{\left(Y_{A,i} - Y_{A,\infty}\right)}{(1 - Y_{A,i})}}$$

Steady evaporation – dependence on gas-phase:

Mass flux of the vapor from the interface is driven by the gas-phase conditions. The gradient of the fuel mass fraction at the interface dictates the mass flux of vapor A from the interface. If the vapor formed over the liquid fuel surface is transported at a faster rate into the gas-phase, then the gradient of fuel mass fraction remains higher. On the other hand, if the vapor accumulates over the pool surface, the gradient of the fuel mass fraction decreases and results in a decrease in the evaporation rate. Therefore, evaporation in a convective condition is seen to be enhanced, given the same ambient conditions.

Steady evaporation – using energy balance:

Steady evaporation can be studied using energy balance given as,

$$\dot{m}'' c_p \frac{dT}{dx} = k_g \frac{d^2T}{dx^2}$$

Here, $\dot{m}''$ is mass flux of A, $\rho_A v_A$, k_g is the thermal conductivity of the gas phase mixture adjacent to the interface and c_p is the specific heat at constant pressure of the gas at the interface. Using constant properties, integration of above equation yields,

$$\frac{dT}{dx} = (Z\dot{m}''T + c_1)$$

Here, $Z = C_p/k_g$ and c_1 is an integration constant. Separating the variables and integrating that equation yields,

$$\frac{1}{Z\dot{m}''} ln(Z\dot{m}''T + c_1) = x + c_2$$

Constants, c_1 and c_2, can be evaluated using the boundary conditions. Application of the boundary condition at x = L, $T = T_\infty$, in the above equation yields,

$$\frac{1}{Z\dot{m}''} ln(Z\dot{m}''T_\infty + c_1) = L + c_2$$

Applying the boundary condition at x = 0, $T = T_s$, in the first equation and combining above two equations yields,

$$\ln\left(Z\dot{m}''T_s + c_1\right) = -Z\dot{m}''L + \ln(Z\dot{m}''T_\infty + c_1)$$

Constants can be evaluated using the above two equations.

Steady evaporation – temperature profile:

Expression for temperature profile along x-direction is given as,

$$\mathbf{T(x)} = \frac{(\mathbf{T}_\infty - \mathbf{T}_s)\mathbf{e}^{Z\dot{m}''(x-L)} - \mathbf{T}_\infty e^{-Z\dot{m}''L} + T_s}{1 - e^{-Z\dot{m}''L}}$$

When steady evaporation occurs, the heat balance at the liquid fuel surface is given as,

$$\dot{m}'' h_{fg} = k_g \left.\frac{dT}{dx}\right|_{x=0}$$

From the temperature profile, derivative of temperature at x = 0 can be calculated as,

$$\left.\frac{dT}{dx}\right|_{x=0} = \frac{\left(Z\dot{m}''(T_\infty - T_s)e^{-Z\dot{m}''L}\right)}{1 - e^{-Z\dot{m}''L}}$$

Steady evaporation – evaporation rate:

Using derivative of temperature in interface boundary condition,

$$\dot{m}'' h_{fg} = k_g \frac{\left(Z\dot{m}''(T_\infty - T_s)e^{-Z\dot{m}''L}\right)}{1 - e^{-Z\dot{m}''L}}$$

Simplification of the above equation yields the evaporation rate,

$$\dot{m}'' = \frac{k_g}{c_p L}\ln\left(1 + B_q\right)$$

B_q is the Spalding transfer number written from energy equation.

$$B_q = \frac{c_p(T_\infty - T_s)}{h_{fg}}$$

Steady evaporation – Spalding numbers:

Mass evaporation rate from mass transfer and energy transfer should be the same, That is,

$$\dot{m}'' = \frac{\rho D_{AB}}{L} ln(1 + B_Y) = \frac{k_g}{c_p L}\ln\left(1 + B_q\right)$$

Here, $\mathbf{B_q}$ and $\mathbf{B_Y}$ are written as,

$$B_q = \frac{c_p(T_\infty - T_s)}{h_{fg}}, B_Y = \frac{(Y_{A,i} - Y_{A,\infty})}{(1 - Y_{A,i})}, (1 + B_Y) = (1 + B_T)^{Le}$$

Here, $\mathbf{Le = \lambda/(\rho D_{AB} c_p)}$. Based on ambient temperature (T_∞), liquid fuel surface attains an equilibrium temperature (T_s), close to the saturation temperature; thermodynamic equilibrium prevails. Partial pressure or mole fraction of fuel vapor at the interface is calculated using equations such as Clausius-Clapeyron equation.

Equilibrium under steady evaporation:

Mass fraction of the fuel at the interface may be evaluated from:

$$\boldsymbol{Y_{A,i} = 1 - \frac{1 - Y_{A,\infty}}{\left(1 + \frac{c_p(T_\infty - T_s)}{h_{fg}}\right)^{Le}}} \qquad (\boldsymbol{C})$$

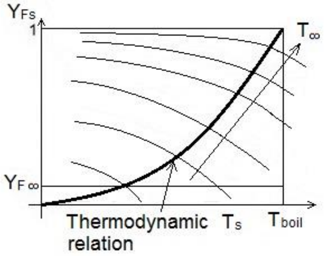
It is clear that when T_∞ tends to infinity or h_{fg} tends to zero, $Y_{A,i}$ tends to unity. Since $Y_{A,i}$ is a function of T_s, it is evaluated by solving equation (C) along with a thermodynamic relation such as Clausius - Clapeyron equation, which is written using h_{fg}, specific gas constant of the fuel and its boiling point, T_{boil}.

$$\boldsymbol{X_{A,i} = \frac{p_{sat}}{p} = \exp\left(-\frac{h_{fg}}{R}\left[\frac{1}{T} - \frac{1}{T_{boil}}\right]\right)} \qquad (\boldsymbol{D})$$

Interface quantities:

The equilibrium temperature, T_s, for various values of the ambient temperature, may be determined using the below procedure:

First a value of T_s is assumed. Using the thermodynamic relation {equation (D)}, the saturation pressure at T_s is evaluated. The mole- and mass-fractions of the fuel at the droplet surface are calculated. The values of B_q and B_Y are calculated. Substitute these values in Equation (C) and determine if the equation is satisfied. If Equation (C) is not satisfied, the value of T_s is adjusted and steps 2 to 5 are repeated until Equation (C) is satisfied.



The equilibrium temperature, T_s, for various values of the ambient temperature is shown in the figure. Thick black line is equation (D). Equilibrium surface temperatures are the intersections of curves of a thermodynamic relation and those obtained by solving Equation (C) at different ambient temperatures.