Course Name: Theory of Fire Propagation (Fire Dynamics)

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Lecture – 02

Module 6 – Analysis of Fire Plumes

Fire plume – heat release, flame height:

Using the estimations of $\dot{m}_a$ and U_e:

$$s \approx \frac{\rho_\infty \times \pi \times D \times h_f \times U_e}{\dot{m}_f} \approx \frac{\rho_\infty \times \pi \times D \times h_f \times \sqrt{g h_f}}{\dot{m}_f}$$

From this, average flame height is estimated as:

$$h_f/D \approx [(\dot{m}_f \times s)/(\pi \rho_\infty D^2 [gD]^{0.5})]^{2/3}$$

Total heat release rate is $\dot{Q} = \dot{m}_f \times \Delta H_c$, using this:

$$\frac{h_f}{D} \approx \left(\frac{s c_{p\infty} T_\infty}{\pi \Delta H_c}\right)^{\frac{2}{3}} \left(\frac{\dot{Q}}{\rho_\infty c_{p\infty} T_\infty \sqrt{gD} D^2}\right)^{\frac{2}{3}} \approx C(Q^*)^{2/3}$$

Here, Q^* is a non-dimensional quantity called Zukoski number. Denominator defining Q* represents thermal energy transported by the plume, where D^2 represents area and $(gD)^{0.5}$, the velocity.

Q^*, the Zukoski number, indicates momentum dominated jet if its value is greater than 10^4, as indicated by McCaffrey. Q* also represents the ratio of combustion energy to nominal plume energy. Q* is also used to represent a ratio of a length scale as:

$$(\boldsymbol{Q}^*)^{2/5} = \left(\frac{\dot{\boldsymbol{Q}}}{\boldsymbol{\rho}_\infty \boldsymbol{c}_{\boldsymbol{p}\infty} \boldsymbol{T}_\infty \sqrt{\boldsymbol{g}\boldsymbol{D}}\boldsymbol{D}^2}\right)^{\frac{2}{5}} = \left(\frac{\dot{\boldsymbol{Q}}}{\boldsymbol{\rho}_\infty \boldsymbol{c}_{\boldsymbol{p}\infty} \boldsymbol{T}_\infty \sqrt{\boldsymbol{g}}}\right)^{\frac{2}{5}} \Big/ \boldsymbol{D}$$

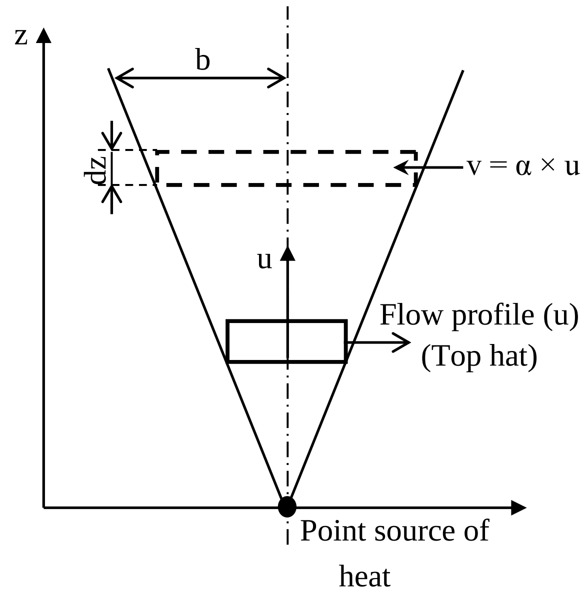
Observing the power of **2/5** for $\dot{\boldsymbol{Q}}$ in this equation, Hesteskad reported correlation using $\dot{\boldsymbol{Q}}$ in kW for flame height (in m):

$$\boldsymbol{h_f} = \mathbf{0.23}(\dot{\boldsymbol{Q}})^{\frac{2}{5}} - \mathbf{1.02}\boldsymbol{D}$$

Fire plume – theoretical analysis:

An idealized plume has a point source from which a buoyant flow generates as shown in in figure (adapted from Karlsson & Quintiere). Simplifying assumptions are invoked to obtain a theoretical framework for the problem. Change in density within the plume is negligible as compared to ambient density.

Plume density is assumed as local ambient density and buoyancy force is calculated using the difference between the ambient and the plume density. This is also known as Boussinesq approximation.



Fire plume – assumptions:

i) Radiation heat loss is neglected.

ii) Profiles of velocity, temperature are uniform and a top-hat profile is assumed.

iii) Velocity of air entraining through the boundary of the plume is proportional to plume velocity at the given height (**v = α × u**), as shown in the figure.

iv) Domain is symmetric and there is no viscous effect.

Following Morton et al. (1956), mean flow variables in an idealized plume are calculated by solving the conservation equations for mass, momentum and buoyancy.

Fire plume – mass conservation:

Mass flow rate of the plume (kg/s) in a differential element of height dz and radius b at a height of z from the point source is:

$$\dot{m}_p = \rho u \pi b^2 \text{ or } \dot{m}_p(z) = \rho u(z)\pi[b(z)]^2$$

Mass conservation is written as "Increase in mass flowing up through differential element dz = Mass entrained through sides of dz per unit height".

$$\frac{d}{dz}\left(\dot{m}_p\right) = \frac{2\pi\rho_\infty b(dz)v}{dz}$$

Here, 2πb(dz) is circumferential area through which air entrains with velocity v into the differential element dz. Combining these:

$$\frac{d}{dz}(\rho_\infty u\pi b^2) = 2\pi\rho_\infty b\alpha u$$

Fire plume – momentum conservation:

Here, ambient density is taken. On simplification,

$$\frac{d}{dz}(b^2u) = 2b\alpha u \qquad (A)$$

Momentum conservation is written as "Rate of change of momentum through height dz = Buoyancy force + Viscous forces". Here, viscous force is neglected.

$$\frac{d}{dz}(\dot{m}_p u) = (\rho_\infty - \rho)g\pi b^2$$

Using the expression for plume mass flow rate, $\dot{m}_p$:

$$\frac{d}{dz}(\rho_\infty \pi b^2 u^2) = (\rho_\infty - \rho)g\pi b^2 \quad \text{or} \quad \frac{d}{dz}(b^2u^2) = \left(\frac{\rho_\infty - \rho}{\rho_\infty}\right)gb^2$$

Fire plume – energy conservation:

Energy flow (**convective heat flow, $\dot{Q}_c$**) at height z is written as:

$$\dot{Q}_c = \dot{m}_p c_p \Delta T = \pi b^2 \rho u c_p \Delta T$$

Here, **$\dot{Q}_c$ is 60% - 80% of $\dot{Q}$** & $\Delta T = T(z) - T_\infty$, T(z) is temperature at height z. Since top hat profile is assumed, T does not vary along the plume radius and only varies with z. From ideal gas law:

$$\rho T = \rho_\infty T_\infty$$

ρ & ρ_∞ are plume & ambient densities. $\Delta T = T - T_\infty$ and $\Delta\rho = \rho_\infty - \rho$:

$$\frac{\Delta\rho}{\rho_\infty} = \frac{\Delta T}{T_\infty} = \left(\frac{T - T_\infty}{T_\infty}\right)\left(\frac{T_\infty}{T}\right)$$

When T → T_∞, $\frac{\Delta\rho}{\rho_\infty} \approx \left(\frac{T-T_\infty}{T_\infty}\right)$

Fire plume – analysis:

Writing ΔT in terms of $\dot{Q}_c$ from energy equation, and using expression for Δρ/ρ, momentum conservation is written as:

$$\frac{d}{dz}(b^2u^2) = \left(\frac{\Delta T}{T_\infty}\right)gb^2 = \frac{\dot{Q}_c g}{\pi\rho_\infty u c_p T_\infty} \qquad (B)$$

Equations (A) and (B) are two ordinary differential equations having unknowns b and u for a given $\dot{Q}_c$. These can be solved simultaneously to get b and u as a function of z. Let $b = C_1 z^m$ and $u = C_2 z^n$. Using these in equation (A):

$$\frac{d}{dz}(C_1^2 z^{2m} C_2 z^n) = 2\alpha C_1 z^m C_2 z^n \Rightarrow C_1^2 C_2 (2m+n) z^{2m+n-1} = 2C_1 C_2 \alpha z^{m+n} \quad (C)$$

Using $b = C_1 z^m$ and $u = C_2 z^n$ in equation (B)

$$\frac{d}{dz}(C_1^2 z^{2m} C_2^2 z^{2n}) = \frac{\dot{Q}_c g}{\pi\rho_\infty C_2 z^n c_p T_\infty}$$

$$\Rightarrow C_1^2 C_2^2 (2m + 2n) z^{2m+2n-1} = \frac{\dot{Q}_c g}{\pi \rho_\infty C_2 z^n c_p T_\infty} \qquad (D)$$

Equating powers of z on both sides in equations (C) & (D), m = 1 and n = -1/3. Therefore, equations (C) and (D) are written as:

$$C_1^2 C_2 (5/3) z^{\left(\frac{2}{3}\right)} = 2 C_1 C_2 \alpha z^{\left(\frac{2}{3}\right)} \quad (E)$$

$$C_1^2 C_2^3 (4/3) z^{\left(\frac{1}{3}\right)} = \frac{\dot{Q}_c g z^{\frac{1}{3}}}{\pi \rho_\infty c_p T_\infty} \qquad (F)$$

Fire plume – solution:

From this, C_1 and C_2 are evaluated as,

$$C_1 = \frac{6}{5}\alpha \;\; \& \;\; \mathrm{C}_2 = \left[\frac{25}{48\alpha^2} \frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}\right]^{\frac{1}{3}}$$

Using these,

$$b = \frac{6}{5}\alpha \mathrm{z} \;\; \& \;\; \mathrm{u} = \left[\frac{25}{48\alpha^2} \frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}\right]^{\frac{1}{3}} z^{-\frac{1}{3}}$$

Conservation of buoyancy is:

$$\frac{d}{dz}\left(b^2 ug\left(\frac{\rho_\infty - \rho}{\rho_\infty}\right)\right) = 0 \Rightarrow b^2 ug\left(\frac{\rho_\infty - \rho}{\rho_\infty}\right) = B = constant$$