Course Name: Theory of Fire Propagation (Fire Dynamics)

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Week – 09

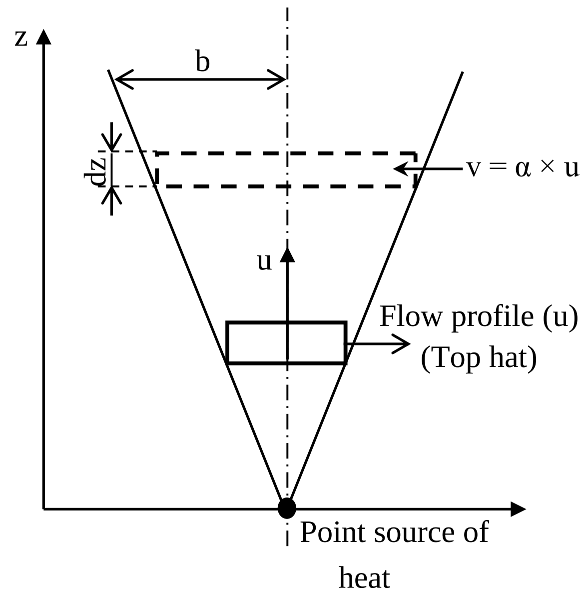
Lecture – 03

Module 6 – Analysis of Fire Plumes

Fire plume – theoretical analysis:

An idealized plume has a point source from which a buoyant flow generates as shown in in figure (adapted from Karlsson & Quintiere). Simplifying assumptions are invoked to obtain a theoretical framework for the problem. Change in density within the plume is negligible as compared to ambient density.

Plume density is assumed as local ambient density and buoyancy force is calculated using the difference between the ambient and the plume density. This is also known as Boussinesq approximation.



Fire plume – assumptions:

i) Radiation heat loss is neglected.

ii) Profiles of velocity, temperature are uniform and a top-hat profile is assumed.

iii) Velocity of air entraining through the boundary of the plume is proportional to plume velocity at the given height (**v = α × u**), as shown in the figure.

iv) Domain is symmetric and there is no viscous effect.

Following Morton et al. (1956), mean flow variables in an idealized plume are calculated by solving the conservation equations for mass, momentum and buoyancy.

Fire plume – mass conservation:

Mass flow rate of the plume (kg/s) in a differential element of height dz and radius b at a height of z from the point source is:

$$\dot{m}_p = \rho u \pi b^2 \text{ or } \dot{m}_p(z) = \rho u(z)\pi[b(z)]^2$$

Mass conservation is written as "Increase in mass flowing up through differential element dz = Mass entrained through sides of dz per unit height".

$$\frac{d}{dz}\left(\dot{m}_p\right) = \frac{2\pi\rho_\infty b(dz)v}{dz}$$

Here, $2\pi b(dz)$ is circumferential area through which air entrains with velocity v into the differential element dz. Combining these:

$$\frac{d}{dz}(\rho_\infty u\pi b^2) = 2\pi\rho_\infty b\alpha u$$

Fire plume – momentum conservation:

Here, ambient density is taken. On simplification,

$$\frac{d}{dz}(b^2 u) = 2b\alpha u \qquad (A)$$

Momentum conservation is written as “Rate of change of momentum through height dz = Buoyancy force + Viscous forces”. Here, viscous force is neglected.

$$\frac{d}{dz}(\dot{m}_p u) = (\rho_\infty - \rho) g \pi b^2$$

Using the expression for plume mass flow rate, $\dot{m}_p$:

$$\frac{d}{dz}(\rho_\infty \pi b^2 u^2) = (\rho_\infty - \rho) g \pi b^2 \quad \text{or} \quad \frac{d}{dz}(b^2 u^2) = \left(\frac{\rho_\infty - \rho}{\rho_\infty}\right) g b^2$$

Fire plume – energy conservation:

Energy flow (**convective heat flow, $\dot{Q}_c$**) at height z is written as:

$$\dot{Q}_c = \dot{m}_p c_p \Delta T = \pi b^2 \rho u c_p \Delta T$$

Here, **$\dot{Q}_c$ is 60% - 80% of $\dot{Q}$** & $\Delta T = T(z) - T_\infty$, T(z) is temperature at height z. Since top hat profile is assumed, T does not vary along the plume radius and only varies with z. From ideal gas law:

$$\rho \mathrm{T} = \rho_\infty T_\infty$$

ρ & ρ_∞ are plume & ambient densities. $\Delta T = T - T_\infty$ and $\Delta\rho = \rho_\infty - \rho$:

$$\frac{\Delta\rho}{\rho_\infty} = \frac{\Delta \mathrm{T}}{T_\infty} = \left(\frac{T - T_\infty}{T_\infty}\right)\left(\frac{T_\infty}{T}\right)$$

When T → T_∞, $\frac{\Delta\rho}{\rho_\infty} \approx \left(\frac{T - T_\infty}{T_\infty}\right)$

Fire plume – analysis:

Writing ΔT in terms of $\dot{Q}_c$ from energy equation, and using expression for Δρ/ρ, momentum conservation is written as:

$$\frac{d}{dz}(b^2u^2) = \left(\frac{\Delta T}{T_\infty}\right) g b^2 = \frac{\dot{Q}_c g}{\pi \rho_\infty u c_p T_\infty} \qquad (B)$$

Equations (A) and (B) are two ordinary differential equations having unknowns b and u for a given $\dot{Q}_c$. These can be solved simultaneously to get b and u as a function of z. Let $b = C_1 z^m$ and $u = C_2 z^n$. Using these in equation (A):

$$\frac{d}{dz}(C_1^2 z^{2m} C_2 z^n) = 2\alpha C_1 z^m C_2 z^n \Rightarrow C_1^2 C_2 (2m + n) z^{2m+n-1} = 2 C_1 C_2 \alpha z^{m+n} \quad (C)$$

Using $b = C_1 z^m$ and $u = C_2 z^n$ in equation (B)

$$\frac{d}{dz}(C_1^2 z^{2m} C_2^2 z^{2n}) = \frac{\dot{Q}_c g}{\pi \rho_\infty C_2 z^n c_p T_\infty}$$

$$\Rightarrow C_1^2 C_2^2 (2m + 2n) z^{2m+2n-1} = \frac{\dot{Q}_c g}{\pi \rho_\infty C_2 z^n c_p T_\infty} \qquad (D)$$

Equating powers of z on both sides in equations (C) & (D), m = 1 and n = -1/3. Therefore, equations (C) and (D) are written as:

$$C_1^2 C_2 (5/3) z^{\left(\frac{2}{3}\right)} = 2 C_1 C_2 \alpha z^{\left(\frac{2}{3}\right)} \quad (E)$$

$$C_1^2 C_2^3 (4/3) z^{\left(\frac{1}{3}\right)} = \frac{\dot{Q}_c g z^{\frac{1}{3}}}{\pi \rho_\infty c_p T_\infty} \quad (F)$$

Fire plume – solution:

From this, C_1 and C_2 are evaluated as,

$$C_1 = \frac{6}{5}\alpha \ \ \& \ \ \mathrm{C}_2 = \left[\frac{25}{48\alpha^2}\frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}\right]^{\frac{1}{3}}$$

Using these,

$$b = \frac{6}{5}\alpha \mathrm{z} \ \ \& \ \ \mathrm{u} = \left[\frac{25}{48\alpha^2}\frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}\right]^{\frac{1}{3}} z^{-\frac{1}{3}}$$

Conservation of buoyancy is:

$$\frac{d}{dz}\left(b^2 u g \left(\frac{\rho_\infty - \rho}{\rho_\infty}\right)\right) = 0 \Rightarrow b^2 u g \left(\frac{\rho_\infty - \rho}{\rho_\infty}\right) = B = constant$$

Using energy equation as below, constant B can be determined.

$$\dot{Q}_c = \dot{m}_p c_p \Delta T = \pi b^2 \rho u c_p (T - T_\infty)$$

From this, B can be determined in terms of $\dot{Q}_c$ as:

$$b^2 u g \left(\frac{\rho_\infty - \rho}{\rho_\infty}\right) = b^2 u g \left(\frac{T - T_\infty}{T_\infty}\right) = B = \frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}$$

From the above equation, $\Delta\rho/\rho_\infty = \Delta T/T_\infty$ can be evaluated.

$$\left(\frac{\rho_\infty - \rho}{\rho_\infty}\right) = \frac{B^{\frac{2}{3}}}{\left(\frac{6}{5}\right) \alpha^{\frac{4}{3}} z^{\frac{5}{3}} \left(\frac{9}{10}\right)^{\frac{1}{3}}} = \left(\frac{5}{6}\right) \frac{\left(\frac{\dot{Q}_c g}{\pi \rho_\infty c_p T_\infty}\right)^{\frac{2}{3}}}{\alpha^{\frac{4}{3}} \left(\frac{9}{10}\right)^{\frac{1}{3}}} z^{-\frac{5}{3}} = 0.863\, B^{\frac{2}{3}}\, \alpha^{\frac{4}{3}}\, z^{-\frac{5}{3}}$$

Fire plume – mass flow rate:

Mass flow rate of the plume as a function of z is calculated as,

$$\dot{m}_p = \pi b^2 \rho u = \frac{6}{5} \left[\frac{9}{10} \pi^2 \rho_\infty^2 \frac{g}{c_p T_\infty}\right]^{\frac{1}{3}} \dot{Q}_c^{\frac{1}{3}} \propto^{\frac{4}{3}} z^{\frac{5}{3}}$$

These are approximate plume solutions considering point source.

Improvements: Here, buoyant flow originates from a finite area (fuel bed) that is located at a distance z_0 from virtual origin.

Instead of top hat temperature and velocity profiles, more realistic Gaussian profile is used. Flame radius is defined as the radial location where temperature has reduced to half of the centreline value at a given height. Plume radius, defined on the basis of velocity profile, is the radial location where the axial velocity has reduced to half of its centreline value.

Fire plume – improved solution:

Expressions for plume radius and temperature difference are obtained with the support of experimental measurements. The plume radius (b_T) based on temperature profile is expressed as,

$$b_T = 0.12\left(\frac{T_0}{T_\infty}\right)^{0.5}(z - z_0)$$

Here, T_0 is the centerline temperature at the axial distance of z.

Difference between the centerline temperature and the ambient temperature [$\Delta T = T(z) - T_\infty$] is expressed as,

$$\Delta T_0 = 9.1\left(\frac{T_\infty \dot{Q}_c^2}{g c_p^2 \rho_\infty^2}\right)^{\frac{1}{3}}(z - z_0)^{-\frac{5}{3}}$$

Centreline velocity as a function of plume height is expressed as,

$$u_0 = 3.4\left(\frac{g\dot{Q}_c}{c_p \rho_\infty T_\infty}\right)^{\frac{1}{3}}(z - z_0)^{-\frac{1}{3}}$$

These modified equations are called strong plume relations. These expressions are valid only in the plume region, when $\mathbf{z} > \mathbf{h_f}$. Virtual origin ($z_0$) is estimated when the centerline temperature as a function of z is known accurately. As per the relation for ΔT_0, a plot of $\mathbf{(\Delta T_0)^{-3/5}}$ as a function of z is a straight line, which intersects the z-axis at z_0. A correlation for $\mathbf{z_0}$ (in m) following Hesteskad, is written in terms of total heat release rate, $\dot{\boldsymbol{Q}}$ (in kW) and D (in m), as:

$$z_0 = 0.083\,\dot{Q}^{2/5} - 1.02D$$