Design of Mechanical Transmission Systems

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Week – 07

Lecture – 19

Lecture 19_Brake: Torque Requirement for Disc Brake Systems

So, we will continue about design and analysis of brake system. So far we discussed about the brake working methods. Here we have seen the videos for both drum brake as well as disc brake system and also we have discussed about various brake types starting from drum brake, disc brake, then within the drum brake the four variation right leading and trailing, leading, leading and S-cam and Duo type. These are four types. Within that we are extensively focusing on brake drum as well as disc brake system. So there the brake requirement aspect, brake torque requirement aspect we have done for drum brake system.

We did even couple of problem also. Today we are going to discuss about disc brake system. In drum brake system we have discussed various aspects right. Remember there is a term called right self locking that we have discussed and right self locking also we discussed and what are the terms we had in leading and trailing.

Activation force right the activation force

$$P = \frac{M_n \pm M_f}{C}$$

this is the equation. Depends on the if it is a plus for the trailing minus for the leading right this is what discussed we have done that. Now, we will go for a disc brake system right. In disc brake system I have seen this is a typical disc brake system where you have a rotor or disc right and this is generally called as a caliper. So caliper fitted with the piston so you have a brake line material either you can have a circular shape button type this is called button circular or button type.

Sometime you may have its annular type right this is called just giving just information this is annular type. So two variation you can see from disc brake aspects we will see that so this is the arrangement which a rotor is given here then the brake parts here may be it could be circular type if not annular type. So as you apply a brake through pedal right through vacuum booster, the master cylinder activated from the master cylinder the brake fluid goes and comes in this direction then this pushes your piston right this is pushes your piston. Since the caliper has in self adjusting mechanism this is this is a self adjusting mechanism okay it has self adjusting mechanism because of that the moment this one piston pushing this way to the this pad the other side also the pad also will try to move. So they move equally and make a contact against the rotating disc this how this disc brake system functions maybe I will have a couple of slides which give you a clear clear cut information the caliper squeezes patch to create a force on the surfaces of the rotor using most automotive application.

Benefit is it is a simple design as I explained self adjusting caliper and rotor venting allows faster for heat dissipation okay more importantly do you expect any self locking in disc brake system self locking yes or no, no self locking in fact. So that is the advantage no self locking because there is no leading and trailing arrangement right there is no leading and trailing arrangement if you have then that is comes into picture since there is no need of leading and trailing shoe arrangement just a button or annular pad which going to press against rotating disc so fine this is much better okay perhaps we will discuss more in detail there is one term is called braking factor BF okay a braking effort or braking factor there we will discuss in detail how this is useful particularly for a brake dynamics aspect okay. You can see the same what I have discussed just now this is given in the schematic diagram. The first step is you could see that this is the rotor and you have a pads on the both side attached with the caliper and this is the piston the fluid comes here right when the force is applied by driver to the master cylinder then the pressure from the master cylinder causes one brake pad to contact rotor that is the second stage you can see step two then the step three where the caliper then self centers causing the second pad to make a contact this is happening everything with the fraction of seconds right when what will be typical braking time you would experience is a few seconds or few minutes or what is that few seconds right so the entire operation would happen within the couple of seconds. Now we will discuss what would be the torque right similar to the drum brake system we here also we will discuss what would be the torque for disc brake system aspect and this is the geometry you could see here where this is the rotor these are the brake pads okay in fact the brake pad is the annular pad this is the in fact the annular type. So you could see there are the notations P is the force on the pads θ_1 , θ_2 and r_i, r_o dimensions of the brake pad that means I would say that r_i is the inner diameter may be in a radius more appropriately okay r_o outer radius right then θ_1 is the angle from the center where the brake learning starts you can see this is your θ_1 similar to the your drum brake system okay and θ_2 is the actual angle from the center to the end of the brake learning in fact you need,θ_2 - θ_1 that is the one will give you exact contact right for the brake aspect so that will be the contact angle okay this is the diagram taken from decision of machine elements by Bandari McGraw Hill so you could refer the text book also in fact we try to understand how the force is going to be right so the force will be here then when you apply the brake thing how the force is going to happen what we do so we will take the small strip from here and try to do the integration and from there we will find out what would be the torque right this is the dθ so I am not going to explain about derivation because the derivation is there in the already text book right to save time period I straight away give the equation from there we will solve okay and this is the pressure

$$P = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} pr\, dr\, d\theta = (\theta_2 - \theta_1) \int_{r_i}^{r_o} pr\, dr$$

$$M_t = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} \mu p r^2\, dr\, d\theta = (\theta_2 - \theta_1)\mu \int_{r_i}^{r_o} pr^2\, dr$$

you can see the pressure vary from internal diameter r_i to r_o and θ_1 to θ_2 pr d r dθ okay and this would be constant right θ_1 - θ_2 is constant, similarly this is the force, pressure into area would give you the force aspect, similarly the force into distance would give you a torque activated in the disc brake system so when we talk about torque we always talk about M_t is a torque is nothing but frictional torque right we are talking about everything happening due to whatever the energy raise, the kinetic energy need to be converted into heat energy or friction energy that happens due to your friction so μ is the coefficient of friction, so this is the equation pr^2 dr, you can see here then pressure is clearly given, inner, outer radius, initial, and final contact angle coefficient of friction finally you would expect this equation simplified one as this you can see that now

$$P = (\theta_2 - \theta_1) p_a \int_{r_i}^{r_o} r\, dr = \frac{1}{2}(\theta_2 - \theta_1) p_a ({r_o}^2 - {r_i}^2)$$

$$M_t = (\theta_2 - \theta_1) \mu p_a \int_{r_i}^{r_o} r^2\, dr = \frac{1}{3}(\theta_2 - \theta_1) \mu p_a ({r_o}^3 - {r_i}^3)$$

I will give you two information from before proceeding to find out actual pressure actual torque also okay in not only disc brake system and also clutch we propose two different theories one is called uniform pressure theory, the other one is called uniform wear theory okay so what is that mean, uniform pressure theory and uniform wear theory right so assuming that so when your rotor is there brand new the pad is applied what do you expect the contact the contact will be complete uniform contact okay so the entire surface is going to be the same okay so in that case of the brake pad will make a contact against rotating disc so in that case that is called uniform pressure theory that is uniform pressure theory so you assuming that you allowing the brake system to run for certain time after some time what do you expect on the brake pads do you expect the brake pad is uniform or is there any changes, yes sorry wear will be there obviously you will have a wear okay because of wear so what do you expect the contact will be uneven okay so what happen if the inner radius the pressure will be higher the outer radius pressure will be low it will fall like a parabolic curve okay, may be I will discuss those things when we go for a clutch aspect okay so that causing again not inequality which again keep changing rubbing the pad and disc at fine time the wear will be uniform across the contact area okay that is the reason so you will have a uniform pressure theory one thing and uniform wear theory another thing okay let me ask one question in which theory the torque will be more is it a uniform wear theory or uniform pressure theory yes torque will be pressure will be more in uniform wear okay we will see that anyway okay so one more thing. Let me ask question as a diesel engineer what would be your criteria you would you want to operate for a maximum allowable torque or lesser than torque yes just think about that so those things we will discuss okay as we progressing in the lectures so this is the equation you can see here so when the brakes are new the pressure allowable pressure is uniform across the contact area, then you will get the corresponding force equation and this is your friction torque equation and if you solve it finally you would expect right this is the equation you get this is the pressure equation and I think something happened here I will rewrite again that equation I will rewrite so this is the equation I will do that yeah you can see here now we will move on to the uniform wear theory I will come back that point aspect, with the older brake pads axle wear can be assumed to be constant the pressure is expressed as the maximum allowable pressure which occurs at inner radius r_i obviously smaller the area pressure will be higher right so the force will be proportional to your inner radius and outer radius with the allowable pressure by again if you solve the equation with the integration finally you would expect your force equation you can see that

$$P = (\theta_2 - \theta_1) p_a d (D - d)/2$$

$$M_t = (\theta_2 - \theta_1)\mu \int_{r_i}^{r_o} p r^2 \, dr = \frac{1}{8}(\theta_2 - \theta_1)\mu p_a d (D^2 - d^2)$$

$$M_t = \frac{\mu p}{4}(D + d)$$

$$A = \frac{\theta}{2}\left(\frac{D^2 - d^2}{4}\right) \; A = area\ of\ pad, \theta\ in\ radians$$

this is the force equation the corresponding the torque frictional torque you could see this ok where please understand the θ in radians this is the equation you have to utilize if it is uniform wear theory aspect right.

So in general we will move on to the irrespective of whether use a uniform pressure theory or uniform wear theory the frictional torque will be

$$M_t = \mu P R_f \quad R_f \; is \; friction \; radius$$

$$R_f = \frac{(D + d)}{4} \; for \; uniform \; wear \; theory$$

$$R_f = \frac{(D^3 - d^3)}{3(D^2 - d^2)} \; for \; uniform \; pressure \; theory$$

$$P = p_a * A$$

where of R_f is the frictional radius or friction radius that radius will change for both uniform wear theory as well as uniform pressure theory look at this R_f equal to d by d by 4 which is for the uniform wear theory of course D is your outer dia similar to the dr_o and small d is the inner dia ok. And the moment you go for a uniform pressure theory the R_f would be this is what happen so same thing you can represent in terms of right r_o

$$R_f = \frac{2({r_o}^3 - {r_i}^3)}{3({r_o}^2 - {r_i}^2)} \; for \; uniform \; pressure \; theory$$

square r_i square with radius also sorry this should be cube and this is square minus r_i square this is the way also you can do that and pressure p equal to sorry this is allow pressure ok and the force acting on the break pad *P* equal to you will allow pressure into the area already we know what is the area if it is a button pad straight away $\Pi d^2/4$ that is we can do that if it is an another pad θ_2 minus θ_1 θ_2 - θ_1 into that area things that is what is coming ok. So this is the torque requirement for disc brake system when you use it uniform pressure theory and uniform wear theory ok and probably we can do some problems also in this regard before going to the problem there is one more information I would like to give you can do that you can see that there is a interesting information ok this is your rotor or disc vice versa same ok do not confuse when I say disc or rotor both are same right and this is your button pad right so the eccentricity also will play a vital role so depends on where you want to keep it the button pad from the centre of the rotor that will alter the pressure variation ok that will alter the pressure variation where r is the radius of the pad e is the distance of the pad center from the axis of the disc so this is the relationship in fact this give you the information about the ratio of the pressure things you can see that the friction radius R_f equal to δ*e delta by e ok and R_f / e that is given the ratio how much variation you can do that right so as you increasing the R_f / e ratio the friction radius again decreases as a friction radius decreases the ratio between the maximum pressure to the average pressure also inversely proportional increases that is what you would see that this is taken from the reference of all circular spot breaks ok from G. A. Fazekas transactions of ASME Journal of engineering for engineering for industry so this is the reference I have chosen what do you understand from here you have a rotor here right then put the pad as move the pad from centre to away from what do you think, what is happening let me ask question if the pad is matching with the disc axis what what happen to the pressure, will be maximum right as we move on to from the away from the centre of the disc right if you move on this is the first position assuming that this is my rotor this is axis so this is my first position this is the pad right, the another scenario I will give you this is the second scenario tell me where do you expect the pressure variation right, maybe you can refer this paper that will give you more information ok yeah right, can we do a problem please following data is given for a caliper disc brake with the annular pad for the front wheel of the motorcycle, the torque capacity is given as 1500 Nm, outer radius of pad 150 mm, radius of pad 100 mm, the coefficient of friction μ is given as a 0.35. The average pressure on pad is 2 MPa, the number of pads are given by two numbers, calculate the angular dimension of the pad is a small is a small problem ok. So we have a caliper disc brake with the annular pad ok the torque is given 1500 Nm right, the dimension is given both inner and outer radius of the pad is clearly given the coefficient of friction of friction also given and pressure is given average pressure is 2MPa, the number of pads are 2 ok. So you have to find out the only annular area of the thing is a simple thing ok what pressure theory, what theory we can use is a pressure theory or a uniform wear theory which one can we use it yeah yes pressure theory ok it did not say anything it is a brand new or uniform theory did not say anything. So we can go for straight away with the uniform pressure theory, we will use the uniform pressure theory.

So M_t is given 1500 Nm this is for how many pads, for 2 pads right. So how but for one pad per pad the M_t would be tell me how much would be 750 right, 750 Nm that will be the things ok that is one thing is there, then you have r_o is given 150 mm r_i is given 100 mm ok then μ is given as a 0.35 then the pressure p ok. So this is the pressure p is given as 2×10^6 N/m^2 yeah ok. So clearly given now we need to find out what is the equation we have general equation

$$M_t = \mu R_f P$$

M_t equal to mu into R_f into P right, P is the force R_f is the frictional radius mu is the coefficient of friction ok. So which is known to you M_t is known to you right torque is known to you μ is known ok and which is unknown for you P is unknown for you right that we need to find out ok that is we need to find out. So for uniform pressure theory what would be the

$$R_f = \frac{2({r_o}^3 - {r_i}^3)}{3({r_o}^2 - {r_i}^2)} \; for\ uniform\ pressure\ theory$$

this is your R_f can you tell me what will be the R_f for this you substitute already you know that what is the r_o what is the r_i right and if you substitute can you tell me what will what would be the value of R_f frictional radius. So are you getting R_f equal to 126.67 mm are you getting this value right that is fine ok excellent.

$$M_t = \mu R_f P$$

$$750 = 0.35 \times 126.67 \times 10^{-3} \times P$$

$$P = 16916.85\, N$$

So what is the force is acting here so we are doing for the one pad please understand ok the entire torque 1500 Newton was given for the two pads so per pad how much is going to be half of that right. So what is the force are you getting are you getting 16916.85 N. 16916.85 N ok you would expect 16916.85 Newton that should be there ok so that is a pressure sorry I am sorry that is a force so when you say force ok we want to know the finally we need to find out the area also. So allowable pressure we know that p_a into area right what is the p_a is given for us 2MPa right that was given in the problem. So 2×10^6 N/m^2 I think I made a mistake earlier I said N/mm^2 square supposed to be N/m^2. I guess yeah is it my dimensions are correct no 10^6 means N/mm^2 or N/m^2. Meter square ok for 2MPa right so can you give me the what is the area now we are getting the area ok so 8458.42 mm^2, you are getting millimeter square. Yeah that is a your area ok that is clearly given. So now we want to know, this is the your annular pad right, this is the θ yeah this is the θ thing, so what is this actually this value r_i or r_1 does not either way you can say that and this would be your r_o. So what do you expect the formula the area would be

$$A = \frac{1}{2}\theta({r_o}^2 - {r_1}^2)$$

So that should give me in terms of radiance, substitute can you tell me what is the θ are you getting are you getting 1.3533 radians yeah are you getting 1.3533 radians ok in terms of degree it should be 77.54°. So that is some you know fraction. So in manufacturing will be difficult you do not want to have it. So what you could do you can make it is a round off to 80°. So the annular pad size would be 80° this should be your answer this is your answer ok. Any doubt in this problem? So what is the problem right shall we move on to the second one yeah yes.

So you have another problem a button pad disc brake uses dry sintered metal pads, the pad radius is 12 mm and it is center is 45 mm from the axis of rotation of the 88mm diameter disc. So that is also given using half of the largest allowable pressure half of the largest allowable pressure the p_{max} would be 2.4 MPa, find the activating force and the brake torque, the coefficient of friction is given as 0.31 ok. So this is very different from the previous one, the previous one was annular pad where as this one is the button pad that is the difference ok. What information we have now? The pad radius given R is the pad radius R equal to 12 mm is given ok that is clearly given fine. What is this 48 mm? 48 mm from the axis of rotation of 88 mm disc is clearly given e value exactly. So the e value is given 48 mm ok that is clear that is fine. Then what is 88 mm? That is a disc diameter ok yeah disc diameter. D you can say that ok D is clearly given 88 mm is given ok and p_{max} clearly given as a 2.4 mega Pascal ok and μ is given as 0.31 yeah that is a thing ok. So we need to find out the R_f that is very important thing right. So if we want to find out what is the equation

$$R_f = \delta * e$$

So which one what is the R you are getting now here? $\frac{R}{e}$ can you tell me what is the $\frac{R}{e}$ value? R equal to 12, this is 48 mm, that should give me 2.5 right that should give me 2.5 sorry 0.25 yeah sorry I mean to say 0.25 right you have the value 0.2 right you have the value for 0.3. So you have to do the interpolation right if you do the interpolation ok. So for $\frac{R}{e}$ equal to 25 right from the table using this table right.

So you would expect the delta would be the δ would be 0.963 after the interpolation please this is 0.963 right it is a 0.963, the corresponding the $\frac{p_{max}}{p_{av}}$ ok equal to 1.290. So 1.290 this is between these two areas. So now we have taken the $\frac{R}{e}$ interpolation 0.2 to 0.3 we got this value 0.963 the corresponding $\frac{p_{max}}{p_{av}}$ is given point sorry 1.290 ok 1.290 is clearly given. The question is what is the question half of the maximum pressure right that is what clearly given you given half of the maximum pressure that is what we need to find out yeah ok. So now we know the values here so our intention is we need to find out right so we need to find out the M_t,

$$M_t = \mu R_f P$$

this is what we need to know that ultimately we need to find out what will be the activation torque that is it that is the thing ok that is clearly given. So R_f would be what happen R_f for this disc brake aspect δ*e right set away you can do δ*e, we got what is the value δ 0.963, 0.963 into 48 the R_f is gives me 46.2 mm, then we need to find out the average pressure right the average pressure we need to find out, what is the average pressure

$$p_{av} = \frac{\frac{1}{2}p_{max}}{1.29}$$

that is what we need to know that half of the P max right divided by what how you found out the value 1.29. So with that you would expect this should be 0.155 MPa ok,

$$P = \pi R^2 \times p_{av}$$

which is what we are getting here P average we know that. So pi into what is the R value your R will be 12 right button it is a button type 12 square right into straight away I am multiplying in terms of millimeter 1.55, if I do that that is give me 70 N ok that is give me 70 N this is for the one side alone we are talking about one side pad one side pad. Then the brake torque

$$M_t = \mu R_f P = 0.31 \times 46.2 \times 70$$

$$M_t = 1000.2\ Nmm$$

μ 0.31, R_f already we know that right 46.2 then 70 Newton. So that is give me m t equal to how much you are getting 1000.2 Nmm. So this much torque is produced if you use the button brake system right in that within the disk brake system I think I will stop now ok tomorrow onwards we are going to important topic dynamics of brake system that is very critical one ok.

This one is gives you understanding about what you have learnt during your undergraduate ok. So undergraduate would have done the basics of disk brake drum break right the torque aspect ok. So now you know that how torque is supplied through activation force in the drum brake and also through piston right through caliper system to the rotor disk in the disk brake system that you understand. How this torque is going to affect the dynamic analysis then what are the dimension of the vehicle system play a crucial role ok that also we are going to discuss and also we are going to discuss when the moment you apply a brake how much mass is going to transfer from rear axle to the front axle that is also we are going to discuss. Then the when mass transfer is transferred then how to optimize the braking effort remember we need to have more breaking effort at the rear front axle then the rear axle.

So how to optimize this in the dynamic analysis breaking proportion aspect ok that is one thing and also how to design such way that we want to have a one particular axle either front axle or rear axle to control the motion of the vehicle right that is also we are going to discuss. So for everything right what are we discussing right it start from the basics of the dynamic analysis so but tomorrow so that is why do not skip it try to attend it will be very interesting ok. We will discuss so many aspects so far you know μ equal to coefficient of friction of your friction of your brake lining right then you will have a μ equal to the rolling radius that is also come in the picture then you have a μ_s equal to your road friction those things will come so we will try to find the relationship between these three right so it will be very interesting ok yeah thank you.