16. Pressure-strain-rate and redistribution of turbulence in flows - I

Welcome you all again and in the last class we looked at what is called a plane turbulent couette flow. So, we basically wanted to understand what is what is the role of the pressure strain rate term for that we consider an example which is plane turbulent quiet flow and we consider that it is fully developed. So, this is the condition that we have considered some of you ask the question why these two are 0 how do you know this a priori before calculating we do not know I will show you from the data the results yield that these two mean velocities to be 0 okay when it comes to a fully developed condition okay. So, we have considered a fully developed flow condition and of course when the flow achieves statistical stationarity and homogeneous in the two directions x 1 and x 3 using this we apply to the equations, the Reynolds stress equations for 3 different directions for x1 that is i equal to 1, k equal to 1. and i equal to k equal to 2 and i equal to 3 k equal to 3.

Three directions we applied to see what happens to the Reynolds stress equations and we found that the main takeaway was that in the flow direction that is x1 direction right that is where the plate is moving along the x1 direction. In that direction we found that the production term p11 is not 0. So, the production rate for $u_1' u_1'$ stress exists and it is non-zero. But when we applied for 2 and 3, the x2 direction and x3 direction, we find that the p22 becomes 0 and p33 becomes 0.

So, the production rates are 0 for the other two directions. So, despite that we have of course no information on what these three stresses look like from the equation. One must solve them or measure them to know what they are. But the equations itself are telling that production rates are there only along one direction and other two direction it does not exist. So, now we go back to this particular flow problem to understand what we have studied so far.

So, in this particular flow problem now, so I have taken the plane turbulent Couette flow. This is the data from my own DNS calculations. So, this is in-house DNS data, in-house will come to that what DNS means in-house DNS data is what I am presenting here. So, you are seeing here the first slide of course here the flow is going from left to right it is a three-dimensional flow right. So, the configuration what we have taken is essentially if I put it here as I mentioned or if I take it in 3D also.

it is not complicated to imagine. This is a plate, the top one and the bottom one, two infinitely long parallel plates and the top plate is of course moving along the x direction. So, we can give this coordinate system x1. this I call it x3 and the x2 is the out of plane component. So, this particular plate is now moving on the right side and the bottom one is fixed.

So, if I do this then and then I am now looking into this particular coordinate system here for this top one this is your x 1 and this is along the x 1 and this is your x 3. So, I am taking a cut plane and I am looking into at a particular time. at some time t I am showing you an instantaneous cut section how the flow looks like. So, this is the top plate is now moving in the this direction bottom one is fixed here. So, you see nice turbulent fluctuating contours.

The highest velocity is of course, on the wall because the plate is moving no slip is applied there. and the out of plane as I said so the flow is the way I have simulated is that it is infinitely long along the x1 direction infinitely long along the x2 direction x2 is the out of plane only along x3 it is 2h the height of the or the gap between the two plates is 2h here all right. So, now if I look at the data after it has come to a statistical stationary state and when it has and then I take samples and I make sure that it is statistically convergent. So, this is my u 1 velocity component at some time t and what I have here is the $\overline{u_1}$ this particular second graph here. So, you can see here that obviously the mean data is not as exciting as the instantaneous.

But the one takeaway here is that you can readily see that along the x1 this is the x1 direction and this is the x3 direction here. So, you can readily see that along the x1 direction gradients are 0 the homogeneity that we talked about. So, you have this $\frac{\partial \overline{u_1}}{\partial x_1}$ = 0, no gradients along the x 1 as I walk. So, whether I take data at this location or this location or this data will look the same as long as I am going along the same, walking along the same x 1, walking along x 1 at the same x 2 x 3 locations. and when I subtract the u1 minus so I get here this is my u_1' is u1 - $\overline{u_1}$ is this data obviously the excitement is back now you see the random components here and if I now look at the next.

view which is where the flow is coming towards you this is from flows from left to right. So, this is another view where the flow is coming towards you. So, this is basically your towards you indicates this is x3 still the wall normal direction the gap between the two walls and this is the x2 direction. which is the out of plane that we talked about. So, we are looking into the data in the out of plane right.

It is difficult to imagine in 3D that is why I have taken a cut section for you to see. So, this is an x2 x3 plane earlier you had x1 x3. So, this is flow is coming towards you same thing I have the this is u1 at some instantaneous this is my $\overline{u_1}$ and this is the u1 fluctuation component.ok so yeah again you see here this is the x2 right so i have $\frac{\partial \overline{u_1}}{\partial x_2}$ equal to 0 span wise homogeneity this is x2 is the span out of plane so homogeneity is also visible here so whatever we considered that the flow is homogeneous along x1 and x2 is valid okay only it is inhomogeneous along x3 that is the wall normal x3 is the wall normal wall normal direction ok. Now this is about the statistical part, so basically each of this data here there are like thousands of samples here I am just plotting for you to see and you can see each looks very different.

So, u1 represents instantaneous data and this red particularly indicates that it is I will just indicate what this means this is $\overline{u_1}$ is essentially this red colored one. the red color line is your $\overline{u_1}$ and this is your x2 the wall normal direction right I already told you it is 2h is the gap between the stationary and the moving wall. So, obviously this is the moving wall the highest velocity is here and this is my u1 here and at the bottom wall it is 0. it has I have given 1 at the top wall moving there and if I subtract each of this u1 minus $\overline{u_1}$ I get u_1'and they are all oscillating around 0 ok. oscillating around 0.

So, therefore $\overline{u_1}$' if its ensemble average should be 0 here it will be very small value since it is a time average here it will be non-zero but small decimal maybe 10 raise to minus 3 or something like that depending on how many more samples you can take but it will never be 0 since it is not a ensemble mean. and this is the other velocity component u 2. So, u 2 is the out of plane velocity and of course the mean is the $\overline{u_2}$ is this red color again which is 0 here. So, u 2 and u_2'are essentially the same here right because the mean is going to be 0. we are looking into the wall normal direction.

So, this is your x 2, sorry x 3 here, x 3 is the wall normal direction. The values are 0, the mean and this is the wall normal velocity x 3, we are looking into u 3 velocity which is the wall normal velocity. Again its instantaneous and fluctuation look the same because the mean $\overline{u_3}$ is this one the red color one $\overline{u_3}$ is 0 $\overline{u_3}$ is also 0 like previous slide here $\overline{u_3}$ that is what I mentioned when the flow comes to a fully developed state the $\overline{u_2}$ and $\overline{u_3}$ the mean velocities these two velocities are 0 only $\overline{u_1}$ velocity exists And that is why the production rates P22 became 0, P33 became 0 because of these two velocities do not exist and therefore there is no strain you need strain of that velocity right that gradient did not exist because it is 0, 0 here. Now I am just looking into a contour. So, this is if I take the 3D view the way I drew before right.

So, if you have the So, this was your top top wall that was moving and the bottom wall was the fixed wall right. So, I am now cutting a section at the mid plane. So, I am looking into a plane here and then I am showing you the data how it looks like a mid plane or a wall parallel plane. a plane that is parallel to the walls and I have taken somewhere in the middle of the channel height here and this is your u 1 the left hand side, right hand side is u 3. So, this will become essentially this coordinate is basically your x 1 and this is your x 2 the span.

So, I am looking into all the wall parallel planes here x 1 then x2 and we are looking into u1 and u3 velocity, u1 obviously exists. So, u1 instantaneous and this is $\overline{u_1}$ at that particular plane and if you see here it looks green, but if you see the value of green is 0.5. So, it is exactly at the mid plane middle of the channel and this is the fluctuation this is u_1'here.

plotted. On the right hand side I have the u 3 velocity component and its mean at the mid plane is 0. It is 0 everywhere basically it is not at the mid plane. So, green indicates here it is 0 and it is this is the $\overline{u_3}$ this is the u 3 fluctuation. $\overline{u_3}$ is 0 throughout just to illustrate how it looks like. And you can already see from the fluctuations from the left and the right.

So, this is u1 prime fluctuation and this is u_3' fluctuation. You can see the nature of these two are different. The u_1' is showing some elongated structures that we call turbulent streaks and those are absent in u_3'. they have more or less tiny turbulent eddies. But u_1' has this red and blue elongated red of course indicates high speed blue indicates low speed.

So, it has high speed low speed long structures this is characteristics of this particular flow. So, do not assume that this kind of structure exists in if you go to a turbulent jet or a plume or something it completely it is a different flow. This particular flow is showing that the fluctuations u n' u n' are different. The anisotropy is visible even instantaneously even doing any statistics. So, this graph finally concludes I am looking into the three RMS fluctuations.

right So, essentially this is my if I square this I would get u_1' u_1'. So, what I am looking into is this. So, this is non-zero. So, we did not have data when we looked at the three equations, we only found that p 11 is non-zero there when we looked in the Reynolds stress equation and the stress is also non-zero. Right And then we have the v and the wrms in that in the context of what we are doing I can just change this, this is the x3 in our notation.

So, this is the u_1' u_1' and we have this, this is the 2, u_2' $\bar{u}_2'$ is also non-zero. is the out of plane component along the x 2 direction and x 3 was our wall normal which is u_3' u_3' is also non-zero. All three stress exists, but p 22 was 0 for this problem and p 33 is 0 for this problem. So, the question is who is producing this particular stress. So, the question is who is producing this u_2', $\bar{u}_2'$ and u_3', $\bar{u}_3'$ what terms are responsible for this, how did this come into existence and they are not tiny they are almost like 50 percent of this and the other one is like let us say one third of that.

So, they are not very you cannot say that they are negligible or something they are dominant. And we also see the anisotropy that is we see U rms or the u 1 prime u 1 prime average is not equal to the u_2' , $\bar{u}_2'$. Similarly, so anisotropy you can see. anisotropy is visible even in a flow where in the one of the simplest definition of a flow it is not a engineer engineering problem even in a canonical flow you can see where the flow is going unidirectional you can see anisotropy the stresses are not equal. So, now what is causing this? So, we will come back to see what term is responsible for this.

So, the question we are asking is what terms are responsible or the which term is responsible for the presence of u_2', $\bar{u}_2'$. and u3 prime, $\bar{u}_3'$. For that what we do is we look at here of course we learnt already that p22 is 0, p33 is 0. This is the scenario we have. So, let us look at the pressure strain rate term itself.

Pressure strain rate term. What does this term look like? So, we had this $\emptyset_{ik} = \frac{P'}{\rho}\frac{\partial u_i'}{\partial x_k} + \frac{\partial u_k'}{\partial x_i}$ average . And we of course wrote the three different terms Ø11, Ø22 and Ø 33 correct. So, for when i equal to 1, k equal to 1 that is for the one of the stresses Ø 11 became 2 P' by rho dou u1 prime by dou x1 average and phi 2 2 was 2 $\frac{P'}{\rho}\frac{\partial \bar{u}_2'}{\partial x_2}$ and Ø 33 was 2 $\frac{P'}{\rho}\frac{\partial \bar{u}_3'}{\partial x_3}$.