Course Name: Turbulence Modelling

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Week - 5

Lecture - Lec25

25. Inner and Outer TBL: Order of magnitude analysis - II

Let us see this is the data that I am plotting here, both I am plotting here the mean velocity which is your essentially u bar and this is the urms. So, this is actually rms it is ok. So, U rms is here. I am plotting both to the same scale here. That means they are non-dimensionalized by the same velocity.

I am using the friction velocity or whatever inner velocity scale right now. So, I have this U bar. U bar is this red dashed line and the URMS is this black continuous line. So, away from the wall.

So, this is your not z if I am using y right y here. So, this is the wall basically the wall direction here, wall normal direction. So, wall normal direction and this becomes wall here, you are approaching the wall. So, as you go away from the wall, you see that let us say here, this is about the turbulence velocity here, urms is about let us say 2. And at that same location, if I go up and I see the mean momentum is at least 10 times larger, an order of magnitude different.

Therefore, away from the wall where delta was the lateral length scale. the velocity scale for the turbulence U rms was different compared to U bar that is in the outer layer right. So, we use this outer turbulent boundary layer in this zone the black and the dashed lines are at least an order of magnitude different 2 and this is like 12 right. let us say 10 times larger in order of magnitude. But I now come to a zone where eta w is important, the inner velocity, inner length scale.

So, let us say I come closer here, let us say at distance 0.1 or so. So, then this is about 2 and then if I go up or even I come much closer, let us say say 0.01 or 0.05 or very very small zone.

Then you can see here if I take this distance very close this and this value same order of magnitude right. So, your u rms or this u bar is of the same order of magnitude as u rms here in this zone So this is the zone of the inner turbulent boundary layer here. So the inner turbulent boundary layer zone. We actually do not know how small it is. I am just only giving a value 0.

01 or 0. We do not know. This depends on the Reynolds numbers that you are working with. But at least close to that we can see from data that they are of the same order of magnitude, mean momentum and the turbulence velocity. So, I choose the same scale convincing right same scale now we are going to use for all the velocities momentum as well as turbulence quantities. So, we go back here and therefore, I have used uw both for u bar its gradients as well as the turbulence quantities ok.

So, now with this of course, I can say I can summarize what we learnt here so close to the wall that is u bar dou u bar as well as this turbulence quantities and rms terms will have same scale that is uw Since very close the wall that is the inner layer inner turbulent boundary layer is inner turbulent boundary layer they are of the same order of magnitude they are of the same order of magnitude. So, we will start doing the analysis now, order of magnitude estimates using this. So, now if I go back and look into the equation here, the equation on the top the u momentum, I will just do order of magnitude estimates. So, I would get for u bar I am going to use order of magnitude estimates. again if I do this u bar is let us take here this is uw dou u bar is also uw over L the first term.

The second term is v bar of course, for v bar we did not do, but we can use the continuity equation again right. So, using continuity equation I can say I have the dou u bar by dou x is dou v bar by dou y continuity equation. From that I can get the scale analysis as this is u w dou u bar is u w by L is this dou v bar sorry or v bar here, v bar and changes in v bar all this I am choosing a scale, let us call it a new scale there, this will be nothing but v bar scale here by or not V bar, I would not call it, yeah, let us call it V bar because that is what we need in the U momentum equation. This is eta w. So, this is the continuity equation.

So, I would get here V bar will be essentially u w eta w by L. okay this is the uw eta w by L okay fine anything else I missed here yeah this is a v bar okay so this is the this whole term is the v bar the scale that we have taken there and then I have dou u bar which is u w by dou y is eta w. So, this is the left hand side part ok. So, the LHS term is this. Now we move to the RHS.

Again I keep the pressure term not participate here for the moment ok. I will come back to that later. So, I have the turbulent term which is this is u w square by L and the other turbulent term is uw square by eta w and then the two viscous terms which is nu uw by L square this is nu uw by eta w square. I simply introduce all the scales that we considered so far. Pressure of course, we are not letting it participate.

Now, if I look here, what did the condition that we say? We say that eta w has to be, eta w has to be much smaller than delta and delta itself is much smaller than L, right? That is the thin boundary layer. Therefore, eta w is much much smaller than L. So, the ratio of eta w by L, what will that be? Here, eta w by L will be extremely small. But anyway, so we have, if I take out this eta w eta w also, I can have this two terms removed, then the left hand side terms both are of the same order of magnitude. uw uw by L uw uw by L Now suddenly when you go very close to the wall, the LHS terms are same.

There is no big brother, small brother. Both are now twins of the same order here. When you are inside, eta w, the inner turbulent boundary layer. So, we do not know on the left hand side which is the largest term right. So, on the LHS like we can say no large term on the LHS both are of the same value.

So, and the objective of this entire inner turbulent boundary layer is that to save at least one viscous term. So, we will see, we will start from there. Let us take the largest viscous term and see which term will balance. Anyway, on the left hand side, I do not have any term which is saying this is big. So, this must be the largest viscous term here .

because eta w is much much smaller than L right and it is eta w square. So, this has to be the largest viscous term and this is the largest turbulence term simply because we have eta w is much much smaller than L. So, eta w goes to denominator means this becomes very large value and this also becomes very large value. So, these two are the biggest terms here. So, if I want to save both the viscous term and the turbulence term, I will first see let us divide the whole thing by the largest viscous term to save this.

So, I am going to say to save at least one viscous term. Let us divide throughout by the largest viscous term. If I do this, obviously the right hand side that term will be 1 here if I am dividing it throughout. So, let us do this here. So, I have uw uw by L over eta uw by eta w 1 by eta w, the first term.

So, I would these two terms goes away here. u w and u w rest survives ok. And then the next term is same which is essentially if I rewrite this I would get u w eta w by nu. So, this is u w eta w by nu and then eta w by L.

Both are same here. These two terms are same, the left hand side. So, both will be the same and question mark here. So, this is uw uw by L over nu uw by eta w 1 by eta w. So, if I do this I get one of the uw's go away and the rest of the term survive here. Yeah, actually this term yeah it is interesting here you see this particular term here is nothing but its uw square is nothing but the same as the left hand side term right.

So, this is also I do not need to do this. So, this is nothing but your uw uw by L which is nothing but the left hand side term. So, I would get the same here. So, now we are seeing that the one of the turbulence term is also same as the left hand side acceleration terms. So, I get the same uw eta w by nu and then eta w by L.

I have the other turbulent term the second largest, sorry the largest turbulence term which is uw uw by eta w over mu uw by eta w 1 by eta w. So, here this goes away, this goes away. and then the viscous term which is nu uw over nu uw by eta w eta w. So, I would get So, it is eta w square by L square which is extremely small because eta w is at least two orders of magnitude smaller. Let us imagine that delta is at least one order of magnitude less than small.

So, if L is let us say 10, delta is at least 1 and eta w is at least 0.1. So, it is 10 times smaller than l. So, eta w by L ratio will be very small and this is square of that, eta w square by l square is negligible. And then of course, the last term when I divide this, this is 1 of course, that is not that is what we are trying to see which term must balance here.

So, if I rewrite here, I would get So, this particular three terms are same here all these three are of the same value and eta w by L is very small. So, eta w by L is very small and therefore, they are negligible here all these terms are negligible, these three terms eta w by L. And this particular term is eta w square by L square. So, this is also, so I can say all these terms are second order importance or you can say negligible here. This is negligible, it is also negligible term.

this and the eta w by L whole square term also is also negligible here negligible. So, it is very clear if pressure term is not participating or at least we are saying that is also not first order importance inside the inner turbulent boundary layer which term is balancing? is only one term here. Let us see what it looks like. So, this particular term is uw eta w by nu uw eta w by nu. every term is negligible because it is a ratio of eta w by L.

So, very small except this particular term. Now, the largest viscous term is being balanced by the largest turbulence term. And what do we learn from this? What is this uw eta w by L? A Reynolds number of? Based on what? an inner velocity and an inner length scale. An inner Reynolds number is 1.

We are going to that close to the wall now. A local Reynolds number there inside the inner turbulent boundary layer uw eta w is 1. So, what did we learn here to first order? to first order the equation becomes your uw eta w by nu is 1 that is Re eta w is 1. this is to first order we have achieved inside and the equation that is balancing what is the inner equation that we have is the largest turbulence term balancing the largest viscous term ok. So, we will revisit this and then look in are there more layers inside because the pressure term we have to see whether that will also play a role. So, we will revisit in the next class.