Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 5

Lecture – Lec27

27. Inner layer equation, constant stress layer, and inner velocity scaling - II

So, η_w is a question mark, u_w is a question mark, we do not know what it is. So, we are going to introduce what is called an inner velocity scale because the moment I know u_w η_w is straightforward. So, let us introduce what is called an inner velocity scale. This u_w in literature we usually call this, this u_w is usually represented in literature as u_τ or u_*. Some of you asked many lectures before, you know the u_{rms} divided by u_τ, what is that u_τ? So, this particular u_τ or u_* has a name which is called, we call it wall friction velocity.

And u_τ by definition this u_τ, or you can say u_* square is nothing but your τ_w/ρ, which is, of course, equal to the sum of your turbulent shear stress and the viscous shear stress here. So, u_*^2 is nothing but, so, u_*^2 by definition is τ_w/ρ. So, τ_w/ρ is easy if you measure turbulent shear stress, then u_* is revealed. So, then you have a velocity scale that you can use to non-dimensionalize your mean flow or other statistical quantities, rms, mean velocity and so on, to get what is called an inner scaling, ok? So, the inner velocity scale is revealed, and that means now I have an inner length scale.

It is also straightforward to see an inner length scale. So, this inner length scale is η_w right. So, this η_w, I already said this is nothing but your ν by the u_*. Here, u_τ or u_* . And when you have this u_* is known, I can compute the inner length scale, and this inner length scale has a name which is called viscous length scale simply because it depends on the viscosity.

So, I have this is called a viscous length scale. So, it is your ν by the u_* and we also do what is called inner scaling. So, I will write down here what is called inner scaling that we already saw in the graph u_{rms} divided by u_τ, right? Many times, the data is represented in a non-dimensional fashion, all your velocities and rms. So, when we do this inner scaling, let us say your $\bar{u}$ the mean velocity. You will usually see this symbol called plus u bar plus you would have seen in literature if you look at books, you know, articles, you will see u bar.

This plus essentially indicates that it is an inner scaling that is it is not divided by your u infinity or other u reference velocity but the wall friction velocity. So, this is nothing but your u bar divided by your u_* . u bar by u_* and this particular is called inner scaling, scaling your quantities using to u_* or u_τ wall friction velocity. Similarly, I can also do let us say the inner dimensional coordinate also. So, I can also have this since this is then you would have also seen in some of the graphs what is called y plus.

Now, y is the coordinate like x, y, z and you can represent this in a viscous inner coordinate. So, this y plus is nothing but your y distance divided by your theta w non-dimensionalized y, and what will this be when I set this in? This will become $u_* \, y/\nu$. y plus is $u_* \, y/\nu$, y is known that is known from your problem kinematic viscosity is known. As long as you compute wall friction velocity you can have a y plus coordinates which is your inner coordinate ok. So, this is your inner coordinate.

dimensionless of course, inner dimensionless coordinate system $u_* \, y/\nu$. Of course, you can also have an outer coordinate system that you can use delta that you already know your y by delta will be your outer coordinates that you already are familiar with it how to use the boundary layer thickness. Now, this y plus is there a special meaning here coming in if you look at it. $u_* \, y/\nu$, what does it look like? This is non-dimensional. So, this is non-dimensional here.

Does it look like a non-dimensional number? Yes. So, it is representing a viscous Reynolds number. So, we have a Reynolds number, a local Reynolds number based on the viscous length scale η_w and wall friction velocity. using a wall friction velocity and the viscous length scale you are getting a local Reynolds number here. So, y plus simply represents.

So, in a graph you can plot instead of y how your RMS data is varying in terms of y coordinate you can plot it in terms of y plus and then you know how your RMS or turbulence kinetic energy or your velocity changing with the local Reynolds number as y plus is increasing right. So, Reynolds number is not fixed I mean you can some ask this question what is this Reynolds number in a problem. In a turbulent flow there is no fixed Reynolds number because turbulent flows are characterized by eddies right, vortices of different length scales. So, you can define the Reynolds number based on many different length scales, and we have already introduced here a few of the length scales like we have L, a very large longitudinal length scale, delta, an intermediate lateral length scale and then the viscous length scale η_w or ν/u_* which is the inner length scale or the viscous length scale right. So, 2, 3 length scales have already been introduced and you can define Reynolds number based on that ok fine.

So, now we have this and then yeah. So, I told you that this graph that there are many layers within a turbulent boundary layer. It is not just that there is an outer layer and an inner layer. This requires little bit of caution or you need to look at it carefully one by one. I will guide you to see what this actually means.

So, now we have a wall here and then I already discussed about an outer layer and an inner layer. I also discussed about what is called a TNT, which is a turbulent, non-turbulent interface, ok? So, this is nothing but a turbulent, non-turbulent interface. So, instantaneously, it looks like this and beyond that is what is called an outer flow. So, whenever in literature you see the name outer flow, it means this is outside the boundary. And inside the boundary layer, of course, that is inside your turbulent, non-turbulent interface.

This interface is extremely thin; it even has a name called the Corsin super layer. I will not go into the theory of that; that is turbulence theory, not the modelling part. Anyway, that the interface is very thin, and to know how thin we need to, we have to introduce another scale that we will introduce a little later. So, there is an interface between turbulent, non-turbulent zones. And below this interface, you have a fully turbulent boundary layer.

And inside the turbulent boundary layer, which is of thickness delta as a function of x, I have two layers which is outer layer and inner layer. And as you imagined these, there is no sharp cutoff between these two layers, inner and outer. You are thinking there is a zone where the outer layer is there, and there is a zone where there is an inner layer. No, there is actually a gray zone. or what is called an overlap zone.

That is because, in the outer layer, the equation what was the equation we had this particular equation here was u bar dou u bar by dou x. This was balancing the turbulence term, right? So, this term was dominant in the outer layer equation. And in the inner layer, the turbulent term is dominant, but it is balanced by the viscous term. So, I have the viscous term, which is nu dou square u bar by dou y square v prime that is in the inner layer. But there is a zone where both the viscous and or the turbulent effects are also there.

The viscous effects are of course, diminishes as you go away from the wall. So, there is a zone above which viscous effects are minimal, only inertial effects prevent or prevail. So, there is a zone and this particular coordinate I am writing this whole part as y plus. So, we can take this coordinate system here. Let us say I write this as can take this.

This is your x direction and this I am plotting in terms of y plus. I will not use x plus because that is not a wall normal coordinate. Viscous length scale is defined for the wall normal component. So, I have this y plus. So, in y plus if I go there is a zone at which the viscous effects become negligible and beyond that this outer layer becomes dominant.

So, I can now give values for this. So, there is this particular zone where this inner layer will cease to be dominant when I call this y bar; y bar is nothing but y by delta. which is equal to 0.1, about 10 percent of your boundary layer thickness. So, you can see it is already boundary layer itself is very thin.

In a turbulent boundary layer, it is even more thin, extremely thin, sharp gradients you will see in a turbulent boundary layer. So, boundary layer is thin and this is 10 percent of that turbulent boundary layer is your inner layer. So, very difficult to measure data in this for experimentalists. Extremely close micro few. I know 1500 micrometres or so depends on Reynolds number, of course.

So, you are going very very close to the wall. So, this is the cutoff for this one and of course, the outer layer also ceases to be dominant when it come approaches this particular zone where that value we define as y plus equal to 30. Now I am using the inner coordinate y plus not y bar because I am inside the inner layer. So, y plus 30 is when outer layer is not so dominant. So, in between y plus equal to this is y plus equal to 0 and y plus equal to 30, you have two layers.

One is called a buffer layer; the other one is called a linear sub-layer. The linear sub-layer. You will have what is called u plus equal to y plus. This is amazing here. u plus is y plus that is the name linear.

So, your velocity there is nothing but your wall normal coordinate y plus, y plus is your coordinate wall normal direction. So, the behavior of the velocity is linear there, the linear and the cutoff for that is about y plus equal to say 3, sometimes 5 in the literature 3 to 5. So, within this zone y plus equal to 3 you have a linear sub layer y plus is equal to sorry u plus is equal to y plus. So, now we have already seen the data from how it looks like from wall to away of both the turbulent shear stress and the viscous shear stress, and then you saw that there is actually a zone where there is a cut-off.

So, if I go back to that graph here. So, if I go back now, the values are you already know what this particular thing is. Now let us say it is in y or y plus it is actually y plus it is not plotted like this. So, now you know that it is y plus 30 this is 30 here. So, above 30 we said it is the turbulent shear stress is dominant here. So, there is a zone we said within y plus 30, and there are two layers.

One is a linear sub-layer, which is y plus 3. This is probably the first position, or within less than that, turbulence is negligible. Only viscous shear stress is dominant. So, u plus is y plus here, but there is a zone that is called buffer layer where both turbulent shear stress and the viscous shear stress are equally dominant.

This zone is called buffer layer ok. This is a very important layer for turbulence physicists as well as the turbulence modelling community because this is the zone where maximum turbulence generation occurs. So, the peak of your turbulence generation rates occurs in this zone. You would have not recognized this when I showed the graphs. I showed the budgets of the RMS quantities. You would have not recognized where is that peak located.

The peak was not on the wall for the production rate. The production rate was going up and then coming down like a hill and the peak was located actually in the buffer layer. And the dissipation rates were maximum on you would have seen that it is negative all the way but going maximum on the wall because it is a viscous dissipation rate. So, now you understand from the y plus coordinate point of view that this is the buffer layer where these two terms are dominant and linear sub-layer only viscous term is dominant and far away only the turbulent quantity is dominant. So, now we quickly take note of these three layers: that is linear sub-layer, a buffer layer and an inertial sublayer.

This inertial sub-layer also has or this overlap zone has a name, which is some people call it power law zone or a log law zone. So, there is a logarithmic law. What is this logarithmic law or a power law is the we want to relate this u bar? in a logarithmic of y plus or a power law of y plus that is what they want to do. So, an inertial sub-layer or this overlap zone is called an inertial sub-layer because inertial terms are dominant here compared to the viscous terms.

So, this logarithmic or overlap zone, this particular zone where you have this tau w tau w by rho, this is nothing, but your u_* square is minus u prime v prime over the bar that we have seen away from it; this is the dominant term here. And in the buffer layer, we had this tau w by rho equal to u_*^2 is a combination of both. this plus your nu du dy. Both terms are equally dominant in the buffer layer, and here, of course, the tau w by rho equal to u_*^2 is nothing but your; only the viscous term is dominant in the linear sub-layer, ok?