Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 5

Lecture - Lec29

29. Reynold's Averaged Navier-Stokes (RANS) models - II

So we will see why this happens. So the first question we are going to address is why this minus 2 third k. two-third of your turbulence kinetic energy. So, now this particular part to understand why this is introduced we will consider a flow the same problem that is turbulent plane Couette flow. So, now consider a channel flow or a plane Couette flow let us say plane Couette flow.

So, in this what are the conditions that we have seen when the flow comes to a fully developed state we have statistical stationarity and the flow will be homogeneous in two directions. So, when a plane turbulent Couette flow we get when it is in fully developed condition. So, we do get dou by dou x1 of any statistical quantity is 0 right and then we also have dou by dou x3 is also 0 that is we have statistical homogeneity statistical or statistically homogeneous turbulence in x1 and x3 directions in this flow. There are many flows like that where you can get a statistical homogeneity.

So, now we will see what happens to that if we if you are going to use this particular condition. So, now when I have shown the data that all the three normal stresses exist in a even in this kind of a flow and then redistribution rate is the cause and all these things we have discussed. But we are now that is the physics I have shown you the data from direct numerical simulation where we solved only Navier-Stokes not RANS equation. So, everything is preserved there. So we are we are not discussing the physics but our model should also represent the physics otherwise it will behave very bad correct.

So now we are going to see whether model equation are going to capture this phenomenon. So now if I take the equation model equation. So in equation set i equal to j equal to 1 in what was the equation number for this Boussinesq equation 2 in equation 2 that is your Boussinesq expression. So, that means I am taking an expression for one of the normal stress diagonal term. So, I have minus u 1 prime u 1 prime I am trying to see what the Boussinesq is giving for this particular term right.

So, now we see I get its nu t of dou ui bar by dou xj which is dou u1 bar by dou x1 because i and j are set equal to 1 here plus dou u1 bar by dou x1 and then minus 2 third k delta 1 1 right i equal i is equal to j delta 1 1 survives so now what happens to this dou by dou x1 of a statistical quantity statistically homogeneous here right so this term goes away due to statistical homogeneity so if i don't have this minus third minus two third k term what will happen to your normal stress it is saying zero your bushiness cannot set normal stress to zero that is the primary stress that is being generated that we have seen from data. So, therefore, here without this term. So, without this u 1 prime u 1 prime will be 0 that is clearly wrong from data we see that this is positive and it is the largest component in a plane turbulent Couette flow ok. So, the models that you make should not be good only for one particular problem nobody will use this you only you will use it ok. If you are going to say I have a model and go and use it, it must be working first of all for many types of flows, it must be accurate for many type of flows, it must be it must be numerically stable that means it should your code should not crash and if the model must be easy to implement there are so many considerations must happen then only your model will become famous.

This is the reason why the bussinesq is using minus 2 3rd k and therefore now this will be essentially your u1, therefore u1 prime, u1 prime is now set to 2 third k, delta 11 is 1. This is fine, so now at least 2 third k has been set. But why 2 third k? Now the other question, why 2, why 2 third k or at least we will say why k itself, why 2 and why k, two question why are we using turbulence kinetic energy? Of course, one argument is this reynolds stresses are turbulent stresses. So, obviously, I do not want to use a flow quantity right, your mean momentum is a flow quantity u bar v bar w bar and mean pressure these are all flow quantities that is not turbulent at all. Entire turbulence is lumped in reynolds stresses in the RANS equation.

So, it is always a good idea it makes sense to model the reynolds stresses based on a turbulent term. So, turbulence kinetic energy is on the kinetic energy of the fluctuating motion and therefore, that is one idea, but we will see why it is still being used. I will give you some more arguments here. So, now we will add all the three stresses in this particular equation in the Boussinesq. So, what you do now is consider equation 2 and add the three normal stresses from the Boussinesq.

So, that means we need three expressions here which is minus u 1 prime u 1 prime minus u 2 prime u 2 prime minus u 3 prime u 3 prime. I am going to add all these three coming from according to the Boussinesq. So, we already seen what this will be if I use the Boussinesq. So, I will have the first one minus u 1 prime u 1 prime is basically 2 nu t here right, it is 2 nu t dou u 1 bar by dou x 1 ok. Now if I take u2 prime u2 prime over bar this will be nu t dou u2 bar by dou x2 plus dou u2 bar by dou x2.

I am essentially setting i equal to j equal to k to get an equation for minus u2 prime u2 prime equal to nu t of dou u2 bar by dou x2 plus dou u2 bar by dou x2. So, that is here dou u2 bar by dou x2. when I am setting i equal to j equal to 2. Now, if I set i equal to j equal to 3, I get 2 nu t dou u 3 bar by dou x 3 dou u 3 bar by dou x 3. And the last term, the sum of 3 terms, so that is minus 3 2 third k delta ij, summing all the three expressions coming from the bussinesq itself.

Now, it is ready to see one of the term goes away continuity equation right. So, this due to continuity it is going away. So, that is fine. So, I have the sum total of this three Reynolds normal stresses according to the Boussinesq is this, this is equal to now sorry this is equal to minus 2 k because this also 3 and 3 cancel out, I get minus 2 k delta ij or basically delta ij need not be there because delta 11 plus delta 22 plus delta 33. If I write it I will get the sum total is here delta 11 plus delta 22 plus delta 33 that is 1 anyway.

So, I get basically minus 2 k here. And this one what can I write this part as? This I can write it as minus ui prime ui prime over bar from your tensor rules summation of three terms. And what is this giving me? k is equal to half u i prime u i prime. This is what we learnt in the theory course turbulence kinetic energies. So, we have so it is not arbitrary here.

There is if you look closer it is revealing some nice information. And therefore, 2 third k is used here and the k being used. So, we will see here. So, sorry this is the k here, k is used here because of this particular reason. So, we will see why 2 third k is coming into picture.

Is this clear? So, basically in the Boussinesq minus 2 third k is introduced. So, now we see why turbulence kinetic energy is introduced right because we want a turbulent quantity to represent the Reynolds stresses and we actually recovered the k expression when we add the 3 normal stresses in the Boussinesq. this is what we get. So, now this is explaining this explains why k is used. So, now 2 third k now what we do is to get y 2 third k.

For that we simply look at this expression for turbulence kinetic energy which is k is nothing but your half of u1 prime, u1 prime average plus u2 prime, u2 prime average plus u3 prime, u3 prime average. Now if I am going to consider a flow where it is statistically isotropic turbulence. So, we have discussed what is a statistically homogeneous turbulence that is in a particular direction the gradient of the all the statistical quantities are 0 that we have discussed here dou by dou x1 of a mean quantity 0 statistically homogeneous in that x1 direction. But if I am going to take statistically isotropic turbulence. That means the anisotropic nature is gone.

I just need one single stress component to represent the whole flow. Whether that kind of a flow exists or not, we do not worry about it. but that is a simplification we can consider. There can be zones within a generic turbulent flow where there can be statistically isotropic turbulence can exist. Even in a generic flow where there is lot of anisotropy there may be some pockets in the system where it can be statistically isotropic.

So, we consider here consider statistically isotropic turbulence. You should never drop this statistical term ok. So, many get confused that isotropic turbulence means u prime is equal to v prime equal to w prime that is not we are talking about we are looking into the statistical nature of it. So, it is when you have this statistical isotropic turbulence what this means is the 3 rms are same. So that means it is u1 prime u1 prime over bar is equal to u2 prime u2 prime over bar equal to u3 prime u3 prime over bar.

When that occurs I get k equal to 3 half u1 prime u1 prime. When all three are same I can use only one diagonal component to represent. So this becomes three times. So, 3 half. So, what is this giving me then? What is u1 prime u1 prime now? Because that is what I want to close in my RANS equation, right, Reynolds stresses using Boussinesq.

So, now I get this as 2 third k. Now you know why 2 third k has appeared. Therefore, 2 third is used. Please note all these points in the textbooks it may not have all this information.

Please take notes. Is this clear? Any other doubts on this? No? So we will close here today and then we will continue. Any questions? Welcome. ah yeah yeah yeah okay okay okay this has to be some total will be 1 that is true it is wait if I am going to add these 3 okay sorry this is not this is incorrect because this is only minus 2 3rd k delta 1 1 which is 1 so we do not have to this is mathematically incorrect that is true correct so this is actually just so this is actually 1 in each of this in the next expression it is minus 2 third k delta 2 2. So, that is nothing but minus 2 third k delta 2 2 is anyway 1 that is correct.

So, this is that you take it out. It is used yes, Bossinesq is used for closing incompressible turbulence as well as compressible turbulence yes. So, the turbulence models that we discuss in this course is applicable also for compressible turbulent flows. it is straightforward to use the models here for let us say weakly compressible flows that is let us say in subsonic conditions Mach 0.

5, 0.6 or so on. But if you have a transonic, supersonic, hypersonic flows then the same models that we discuss will have extra terms. So, some correction terms appear some extra things are required for to accommodate those conditions. Because at the end of the day, you need to represent accurate physics as much as possible through your model. So, you will do whatever it takes to capture that. See you in the next class.