Course Name: Turbulence Modelling

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Week - 1

Lecture - Lec03

3. Statistical Analysis: an approach of modelling turbulence - II

So, let us believe we have access to true mean. Alright So, when I have now I have done let us assume that I have done this some experiment and somehow I have got access to the true mean ok. So, that is what that is how we are building upon and moving ahead ok alright. So, now, if you have this, then how do I some examples of your velocity vectors and so on how do I represent it? So, you can say example could be. Ensemble mean of a velocity vector because this x_n, I said, is a random realization, it could be anything, it could be temperature, pressure, whatever.

So, if I have this, then I can say $u_i(x_i, t)$ is three-dimensional velocity vector let us say u_1 in the direction of one direction of your coordinate axis is a function of x, y, z and time. So, if I use over bar, that is the ensemble mean of this.

$$\overline{u_i(x_i, t)} = \lim_{n \to \infty} \frac{\sum_{n=1}^{N} u_i^n(x_i, t)}{N}$$

So, note that this ensemble mean is a function of three-dimensional space and time.

This is not time average. Just like the way you flip dices, each one is an independent realization. So, your ensemble mean is a function of 3D space it is x, y, z and time t ok. So, here u_i can be anything. So, your u here u_i could be u_1, u_2 or u_3 velocity components ok all right.

So, now, we know how to define an ensemble mean. for a velocity vector and some may have a question then what is time average because in the laboratory we are continuously taking samples. So, a time average can also be same as an ensemble mean only in a special case. So, we will define what is a time average first. Let us call it u_t that I am averaging it in time. This is

$$u_t = \lim_{T\to\infty} \frac{\int_0^T u_i(x_i,t)dt}{T}$$

So, your time average becomes ensemble average only in a special case when your mean is not changin $_t$. For example, let us say you have two signals, let us say there is one signal which is like varying like this. ok, forever. So, this is the number of samples or we can call it time here right or let us say let me use the time since we have discussed this.

Let us say you are collecting samples in time and this is the your temperature or any other parameter. So, then the average mean will be. this this is your u_t so the mean itself is not changing in time such a flow if you have a flow where mean is not changing in time then time average can be used and time average is same as ensemble average but you can get many flows like just like the engine internal combustion engine it is the piston is compressing and expanding and therefore there will be an oscillatory variation in the temperature So, you would get a signal like this. Right? So, what is its mean then? The mean is not averaged over the cycle. So, the mean has to be, so this red line represents the mean.

So, if we have this kind of a process, so this is where you have what we call a stationary flow. The mean is no longer a function of time in this particular case then time average can be used which will be same as the ensemble average. Here the time is also oh sorry the mean is also a function of time mean is changing with time here ok. So. therefore, time average or time mean becomes your ensemble mean only in stationary flows.

So, it is perfectly works here, in this particular case. not here, so you need to do any other type of flow where is mean is changing either in a periodically, in a periodic sense or it can be in a completely random sense right then use ensemble. Ensemble mean is very generic and therefore we will use ensemble mean as a process. okay to continue to separate the fluctuation because you already see a signal like this so the question we ask is if I have to capture these turbulence or model it then I need to first know what are these oscillations around the mean, right so in these oscillations in a turbulent flow around the mean is the random component or the turbulent component I need to understand how much energy it is carrying what it does if I understand this then I can model it. For that, I am removing the mean component first.

That is the whole point of starting with this chapter: statistical analysis, ok. So, we discussed ensemble mean and then the time mean, but we will proceed with this ensemble mean further. So, now ensemble mean itself is not enough all the time. To just get it, we need some higher order statistical values. So, before that let me just give you some notations that you would see in textbooks and the notations that I will follow mostly.

If I change it of course, you please let me know. So, we look at some notations that we are using. So, the first thing is to look at fluctuations about the mean. So, if I am going to talk about mean, I mean ensemble mean. So, we will not going to use arithmetic mean anymore.

So, mean implies true mean here. So, fluctuations about the mean. So, we get some notations, notations that you see in literature. so, you can usually see, let us say there, is a random parameter could be velocity, temperature, whatever or pressure. Sometimes you will see a tilde on this $\tilde{x}$ then that is sometimes or x itself.

So, these are too popular in literature on textbooks. So, these are the instantaneous instantaneous component. Just the way you are recording the signal, the signal itself whatever we have shown that is oscillatory that is can be represented as x or $\tilde{x}$ and when they use $\tilde{x}$, they use x for the fluctuation or x', this is used for fluctuation or randomness, the random component. And then we have this X, capital X, let us say or $\overline{X}$. So, this is your mean or the statistical component.

So, in the course I am going to use this x' and then the $\overline{x}$. So, when I talk, whenever if I see u if I say u, v, w that means it is instantaneous signal, and u', v', w' means fluctuating velocity components. $\overline{u}$, $\overline{v}$, $\overline{w}$ means the true mean. So, I am going to use this notation. So, now there can be signals where two signals having same mean, but they have different randomness and therefore, we need higher order terms.

So, if I take, for example, for the fluctuations about the mean, let us say I have one signal like this, let us call it ϕ is your parameter or x or v anything. Let us say this is the signal that you have recorded, n number of samples. So, this will have a mean $\overline{\phi}$. So, now there can be another signal. Now, you get higher excursions.

The mean is same. two different signals let us say one is an experiment that you have done today and tomorrow you are repeating the same experiment, and suddenly you notice that the excursions about the mean has increased the mean is same, but somehow you see the randomness has increased okay so both have same mean here, both have same mean. And therefore, we need to define not just the fluctuations about the mean, we need more than that one. So, x' or ϕ' itself is not enough. So, we need more than that. So, both have same mean, but different randomness.

So, now the fluctuations about the mean we call it let us say x' right x'. So, if I ensemble average x' what happens? Ensemble, ensemble mean of x'. What do I get? $\overline{x'}$? 0. Yes. If

I ensemble average the fluctuating signal ok, that means I have this random signal here, the oscillator signal and then I subtract.

So, it is ϕ minus $\overline{\phi}$ or x minus $\bar{x}$ should give me x'. And if I ensemble average that x', it should go to 0. If it is not going to 0, that means you are not working with ensemble mean. You are working with arithmetic mean. The average of your fluctuating signal not going to 0 means you are always working with the arithmetic mean.

And that is the people said yeah, I mean some said right. It is small, and therefore, it is acceptable to me. But we are coming to the theory here. ensemble mean this has to be there and the proof is very simple here. So, how is x' defined as fluctuation? It is nothing but x minus $\bar{x}$ right or let me write it here.

So, it is x minus $\bar{x}$ is nothing but my x' instantaneous minus mean will give me the fluctuation. Fluctuation, this is the instantaneous and this is the mean, 3 component. So, if I am going to ensemble average, so what will this be? This will be, so the ensemble average and the subtraction operation they commute. So, I can write this as $\bar{x}$ minus $\bar{x}$ over bar. Now, $\bar{x}$ is the ensemble mean and if I am doing another operation over it, what is ensemble mean of an ensemble mean? It is the same, ensemble mean is not changing, that is why I call it true mean, right? So, this is the true mean.

Therefore, it does not change. Therefore, this is 0, right? If you are not getting 0, that means you have not worked with ensemble means, you are working with some arithmetic means ok. So, here the takeaway is. What this implies is ensemble mean of an ensemble is same, it does not have any new changes in it. And therefore, x' over bar is 0. If it is not 0, that means this has become an arithmetic mean, and take a note here.

So, if x' over bar is not 0 then you are not using true mean, but arithmetic mean. So, you are operating with an arithmetic mean, which is giving a non-zero value upon averaging ok. So, as I told you in the previous graph here, so here you have same mean, phi bar is same in the two signals, but the excursions about the mean is different. And therefore, we need another parameter to study that we call variance. Let me call it x'x' over bar or simply x' square over bar that is in cases where two signals having same mean I need to look into an higher order term.

So, is this 0? What does, what does this imply? So, the question is, is x' square equal to 0? How do you know that? What if x' is 0? Then it is 0. If all x prime is 0, then it is 0. Yes. Otherwise, it is not 0. Exactly, right? So, if x' is 0, what does that mean? What kind of flow you are working with? Laminar flow.

Laminar flow, which we are not interested in this course. So, this x’, if this is 0, that means, yes in laminar flows. So, in turbulent flows it is not. In fact, it becomes a very important parameter to know the extent of turbulence, how strong or weak my turbulence is.

I cannot know only from x’. I use the variance to know how strong or weak. For example, here both have same mean here. The phi bar is same, but one may exhibit a stronger turbulence level than the other one, ok. So, we have this root mean square right, rms or root mean square or standard deviation. This is sigma equal to x prime square average, and of course, there is a square root of it.

So, square root of mean of the square of your fluctuation rms root mean square this is also often very used in turbulence so, square root of mean of the square of your fluctuation, rms, root mean square, this is also often very used in turbulence literature. You look into this, and we will see the physical meaning for all these things right now; you know only about the mean and then the fluctuation component, which is x’ or u’, v’, p’, whatever pressure fluctuation, temperature fluctuations, but what is the physical meaning for the variance and what we discussed the third order terms like this. we would need higher-order terms. Example like x’, x’, x’ or x’ fourth order terms, third order terms ok.

So, these are called skewness and flatness. But these are general statistical terms that you would see even in any finance course or general statistical course. But what is the physical meaning of this associated with turbulence that we will see. As we go into the equations governing turbulent flows, you will see the meaning of each of these terms will pop in, ok.