Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 6

Lecture – Lec33

33. Modelling of turbulent kinetic energy (k): production, destruction, and dissipation rate - II

For the epsilon term, a transport equation is solved rather than individually modelling each correlation term. correlation term in $\nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$

So, now that means we need a transport equation for it. We have not discussed that in the theory so far. For turbulence kinetic energy, there was a transport equation, fair enough, right? But for this particular term, do we have a transport equation? I did not discuss.

actually it exists. So, I have not formed that part because it is very you know it will we will deviate little bit away from the modeling chapter. But if you are interested you can go and look at there is an article which looks into this. This is Mansour, Kim and Moin, Journal of Fluid Mechanics 1988.

This is by far the only paper where I have seen a derivation of this exact equation for dissipation rate exists. So, the epsilon exact equation starting from the first principle that Navier Stokes is considered, and this is how your equation looks like. So, on the left-hand side, you have the standard material derivative for epsilon, and there are four production rate terms for this ok. And then, of course, you have the other usual suspects, which is your turbulence transport; T epsilon, you have this pressure transport or pressure diffusion rate pi epsilon, that is what they call it, and D epsilon is the viscous diffusion rate. So, they have a different name for this.

So, we have this turbulent transport or turbulent diffusion, pressure diffusion, and viscous diffusion. Those three were there any way in the turbulence kinetic energy equation also, and there is a dissipation rate term for the dissipation rate of turbulence kinetic energy. So, this capital Y e here is, of course, a sink term for dissipation rate. So, the way you should read this as a destruction rate of dissipation rate of turbulence kinetic energy. So, that term is there but there are four production rates and you can see there these are extremely complicated terms.

So, it has far too many unknowns. So, only if you have patience you can do a direct numerical simulation, and you can actually look into the budget of this fair enough to understand the physics, but to model this is actually complicated. So, we do not go and model the exact equation. So, what we do in two-equation eddy viscosity models is we construct an epsilon model equation analogous to the k model equation, that is the idea. So, we do not go and model the exact equation.

So, the takeaway is, exact it has far too many unknowns. Epsilon exact equation is not used to model epsilon. We will simply proceed in the direction of the way k model equation was constructed. So, for that let us go back to our slide here. So, we can argue here that analogous to the k-model equation.

So, what is the k-model equation? So, an epsilon model equation is constructed analogous to the k model equation. This is purely a numerical argument, the numerical argument for our own convenience. So, what is the k-model equation? k model equation let me write. So, if I have the k model right. So, if I write the k model equation, I get $\frac{\partial k}{\partial t} + \bar{u}_J \frac{\partial k}{\partial x_j}$ equal to the model component of the transport term or the diffusion rate, which was $\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}$ omitting the pressure transport term turbulent and viscous effects are considered here.

plus I have the production rate term which is $2\nu_t S_{ij} - 2/3k\delta_{ij}$ and your mean strain rate minus epsilon ok. Epsilon all this, this is of course, the entirely it is a modeled equation here. not the exact equation. So, this epsilon is required now epsilon model or simply epsilon. So, how do I construct that is as I said analogous to the k model.

So, I am going to construct a model here for the epsilon term. So, a model equation looks like dou epsilon by dou t plus I would like to have the advection rate here. So, on the left-hand side, it is straightforward to write a model equation, ok? Unsteady rate of epsilon, advection rate of epsilon. Now what do I do for the diffusion rate? Again I will fall back on the same thing. There has to be viscous effects close to the wall and then turbulence has to dominate away from the wall.

Ok? Use the same modelling idea the same gradient diffusion hypothesis also for epsilon. So, I am going to say nu plus nu t. So, ν_t is coming everywhere the eddy viscosity is present in your RANS equation, it is coming in your k equation, it is coming in your epsilon equation. So, this is probably the first thing that you have to compute when you write a code because it is having influence everywhere and it is of course coupled to k and epsilon. But here it is not the sigma k this epsilon equation.

So, we would need a model constant sigma epsilon, ok? sigma epsilon dou epsilon by dou xj plus. So, so far so good straight forward. Now, as I said, this is your diffusion rate, right? Diffusion rate, this is the production and this is the dissipation rate. all the three terms. Dissipation rate is taken care of for epsilon.

So, now production rate for epsilon what should we do? What do you think about it? So, this is your Pk production rate. We call it the Pk model. I am going to drop the model thing in writing this model everywhere. So, basically, you can say this is the Pk term production rate Pk. So what should a P epsilon should depend on? What I need now is P epsilon.

So when you construct any model, obviously as I said you need to put some arguments. One argument I can make is that I do not want excessive generation of turbulence. See the dissipation rate, what is the objective of dissipation rate? It is a sink term. So, this is going to, it is telling you the rate at which turbulence kinetic energy is taken away or dissipated or destructed in the system. So, if I have excess turbulence kinetic energy generation, the Pk term, generation rate of k is Pk.

Then, I would like to have the production rate of epsilon depend on the production rate of turbulence kinetic energy itself. So, you have excess generation of turbulence. There will be excess generation of epsilon also. So, that there is some kind of a balance. So, we want some kind of an equilibrium we are assuming this need not be the case in turbulent flows.

You can get a flow where there is far more turbulence generation, but epsilon is not dissipating enough it can exist. For the modeling sake we want Pk and epsilon to be balancing or interdependence. So, for that sake, I want P epsilon to be a function of Pk. So, I can introduce Pk here. So, what? Does it solve my problem? So, what is the unit for this? This is meter square per second to the power of 4.

Epsilon is meter square per second cube this is a time rate of change of epsilon dimensions are meter square per second to the power of 4. So, every dimension has to be the same here you can go ahead and check it will be dimensionally consistent every term here. Therefore, the production and production rate of epsilon and dissipation rate of epsilon must also have the same dimension, but Pk will give you the same dimension as, what is the dimension of Pk? Its dimensions are same as epsilon, right? Epsilon is meter square per second cube, Pk will also have meter square per second cube. So, you need a time scale that is missing. So, I need a time scale here.

What is missing for me is a turbulent time scale. Obviously, a model constant will also appear when I do this let us call it C epsilon 1. A model constant has appeared and a time scale is required here for dimensional consistency. So, then if I do this then of course, this becomes meter square per second to the power of 4. We will figure it out what that time scale is and then now I need a sink term.

Minus of this has to be dissipation rate of dissipation rate of turbulence kinetic energy. Again, what should this depend on? If epsilon is produced more, I want a term which is depending on epsilon itself to remove. So, again the same argument. So, I would like to have epsilon here. as a term for the destruction rate of epsilon.

So, this is the dissipation rate. So, note it here. This is the dissipation rate of epsilon. This is the production rate of epsilon. So, symbolically, I can call this P epsilon.

Model Eqn $$\frac{\partial \varepsilon}{\partial t} + \bar{u}_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left\{\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right\} + C_{\varepsilon_1}\frac{P_k}{\tau} - C_{\varepsilon_2}\frac{\varepsilon}{\tau}$$

2.

There is no particular symbol for this, but if you want we can just write epsilon epsilon even though I have not seen people using this we can just use it no problems ok. So, these two terms again dimensionally it has to be correct. So, now epsilon, the unit is meter square per second cube, same as Pk. Again, the time scale I need a time scale; I would need a model constant, and also, c epsilon 2 ok. So, more unknowns have come here so far in the k equation.

What is sigma k, ok? That is an unknown, and then I have in the epsilon model equation sigma epsilon? C epsilon 1? C epsilon 2? But I need a turbulent time scale. Did we use a turbulent time scale so far in the context of two-equation turbulence kinetic energy dissipation rate model? k by Epsilon, right? So, we already used the turbulent time scale. So, tau is your turbulent time scale. that we used as k/ε and we are anyway constructing k-ε model. So, why use some other time scale for modeling part? You can go ahead and do it.

The time scale can also be a flow time scale which was $\frac{\overline{du}}{dy}$ would give you according to Prandtl's mixing length model that is another time scale. But here we are using a turbulent time scale k/ε. A ratio of that actually. So, you have to be very careful here when you initialize the data, when you are writing codes, this k/ε. So, do not start with something like epsilon 0, your code will blow up.

You are looking at a ratio of this, and this ratio is very complicated in certain places see on the wall epsilon goes to maximum no problem but away from the wall, when you go to we are constructing a turbulent boundary layer, let us say and then you are going to free stream your numerical domain will go up to free stream what is epsilon there How do I give this value? Don't set 0. Set some very small. So those who have taken CFD courses and have learned this would know that never define anything like 0. Always define something like small and large values.

Small can be like 10 rise to minus 10. Small grade can be or large value can be 10 rise to 10 or 20 or something. So use instead of 0 use something like that. An extremely small value. So, anyway k/ε is our time scale. So, if I introduce that, I would get what would happen here? If I remove this, I get essentially epsilon by k, correct? $C_{\varepsilon 1}$, epsilon by k.

So, epsilon has come inside the production rate also of what dissipation rate of turbulence kinetic energy. So, the P epsilon is a production rate of epsilon which is depending on epsilon itself and also on the Pk. Similarly, the time scale here, so this becomes epsilon square over k. So, this completes the modelled equation for both k and epsilon. Of course, there are many constants that are there here and I can give out the value not a problem, but we will discuss what these values will be.

So, you will see that there are different variations of k epsilon models. There are names like standard k epsilon, RNG k epsilon, realizable k epsilon. There are many, if you search you will get many turbulence k epsilon models. And then the value of these constants slightly vary. So, in the context of the standard k epsilon model, I will go ahead and derive and tell you what these values will be.

I will not go ahead into every k epsilon model because we need to move on to other types of models. So, any questions on this I can take it up.