Course Name: Turbulence Modelling

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Week - 7

Lecture – Lec39

39. New model k- ε and model constants – II

So, k model equation in k omega model, of course. because the last term in the k epsilon in the k model equation the k epsilon it has an epsilon minus epsilon that we cannot solve it here. So, I need a modified k equation. So, that is straightforward: we get essentially dou k by dou t plus uj bar dou k by dou xj equal to dou by dou xj of nu plus nu t by sigma k. dou k by dou xj and then the Pk term no change in the production rate of k because that is Reynolds stresses followed by mean strain no change occurs here Pk is appears to be the same there.

Now minus of epsilon was there instead now it has to be epsilon by beta star k that is what we used it. So, this becomes essentially beta star k omega here. So, your sink term becomes minus beta star k omega which is the only new term in the k equation, slightly modified. So, this is your equation 3.

K_{model} eqn in k-ω $\Rightarrow$ $$\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k^\omega}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \beta^* k\omega$$

So, equation 2 and 3 together will have the k omega model with the eddy viscosity the ratio of k/ω. So, this is what I have and yes. So, the model constants that we have to look at. So, the model constants we have β^* it is essentially C_μ they call it β^*here, but they give the value as 9 by 100. So, if you see in literature this is the value declared, but this is actually seen in my opinion this is nothing but 0.09, but you will see this 9 by 100 in the literature. And you will also have this σ_k^ω in the where is it here this particular one they call it σ_k. for the omega equation of course. So, σ_k^ω equal to σ^ω both is taken as 2, model constant here. And the $C_{\omega 1}$ and $C_{\omega 2}$ is what I am telling you because it is consistent with $C_{\omega 1}$, $C_{\omega 2}$.

In literature they use a different terminology for this. So, what they are using is essentially alpha or a gamma this is what they use, but I am simply using this as $C_{\omega 1}$.

This value is 5 by 9 and then I have c omega 2 this is again they use beta or $C_{\omega 2}$ the value is 3 by 40. So, these are the model constants that are required to complete the entire k omega model here. Now, we have to look at the behavior, the model is fine, but is it giving numerically stable or not.

So, let us go back and look at this equation 2 and 3 to see whether this is numerically stable or not. So, now consider this: consider k omega model behaviour that is your equation 2, 3 and then I can call this 4, then make this 3, then the 2, 3, 4 completes the whole equations here. So, that is equations 2, 3, 4 near the wall. So, if we take the eddy viscosity that is the straightforward part ν_t as y tends to 0, k tends to 0 that we know. So, this will not create any problem for you because it is k/ω, it was not creating any problem elsewhere also k square by epsilon was also ok.

So this goes to 0. So, it is stable, no problem here, similar to the k epsilon. We look at the k equation. This is equation 4 here, k model equation. So, again same arguments gradient terms on the left-hand side and the diffusion terms.

So, the gradient terms will not create any issue. Gradient terms are fine here the first three ones that is not a problem. we only look at the Pk and the beta star part. Pk is same as before, there is no change in the Pk. So, Pk is nothing but your $\left[2\nu_t\overline{S_{ij}} - 2/3k\delta_{ij}\right]\frac{\partial\overline{u_i}}{\partial x_j}$.

Only the value of ν_t is k/ω. So, that is also not a problem, ν_t goes to 0, y goes to 0, Pk also goes to 0, not a problem. So, this is ν_t is nothing but k by omega here. So, this is also ok. So, this is production rate goes to 0 stable.

So, then the last term beta star k omega is not a problem because k goes to 0 that is also fine. So, you have the sink term, which is beta star k omega. So, y goes to 0, k goes to 0, goes to 0, stable. no issue here. So, then we look at the omega equation, which is equation 2 here.

So, I can look at this particular part, let me just copy this. So, this is my epsilon or sorry the omega model equation right. This is the omega model equation. Again, gradient terms will not cause any numerical instabilities. Gradient terms are fine.

Here, $C_{\omega 1}P_k\omega/k$ this again looks like it is going to cause a problem, but we will see whether it causes a problem or not, right? So, we look at the so-called P_ω term. Let us call this P_ω, this part I will write it in the below. Production rate of omega term, and this is the dissipation rate of omega; you can call it epsilon omega or something. I am only writing this, this terminology does not exist, epsilon omega and all this, it looks little funny, but that is ok. So, let us look at the production term p omega, $C_{\omega 1}P_k\omega/k$.

So, y goes to 0, k should go to 0. It looks like it is going to blow up if you implement it like this. So, let us see, we will substitute what is Pk. So, I get $C_{\omega 1}$. What is Pk? Pk is nothing but your $2\nu_t\overline{S_{ij}} - 2/3k\delta_{ij}$.

And then I have the strain rate part, dou y bar by dou xj followed by omega by k. So, again, eddy viscosity is k by omega. So, I can replace the eddy viscosity here by k by omega. So if I do this, you can see goes away here and also of course the two terms.

Let me just undo this. So let us expand this and see $C_{\omega 1}$, 2k by omega Sij bar. ui bar by dou xj omega by k minus c omega 1 two-third k delta ij dou ui bar by dou xj omega by k. Now, both k and omega are cancelling it out. this has a profound influence in the k omega model. So, if you look if you recall the same P epsilon term the production rate of epsilon in the k epsilon model after dividing all this the k is still remaining there in that equation.

We just go back, going two steps back here. So, you can see it here in this part here. This particular term is now c epsilon 1 2 c mu k delta k Sij bar dou i bar by dou xj. So, this k is there. So, this P epsilon is still a function of k.

So, it is coupled. You need to compute k so that that k value is used in the P_ε value. It is a coupled system. Luckily, here, the k dependency is gone. Your P_ω is decoupled from k. So, if you have if you are numerically solving equations where any term is decoupled, it is great while solving it.

This will not cause numerical issues. If, let us say, the k is actually giving bad values, your P epsilon will also give bad values because it is a function of k. But P_ω is not a function of k. So, this will also not pull it down. So, this is one great advantage of a k omega model over k epsilon.

So, no k dependency here completely decoupled. And here of course, the k k goes away only omega comes in the p omega. So, here no k dependency at all. So, this is equal to C omega 1 2Sij bar dou i bar by dou xj. So, the P omega is depending entirely on the flow terms just the strain rates here not on the turbulence kinetic energy.

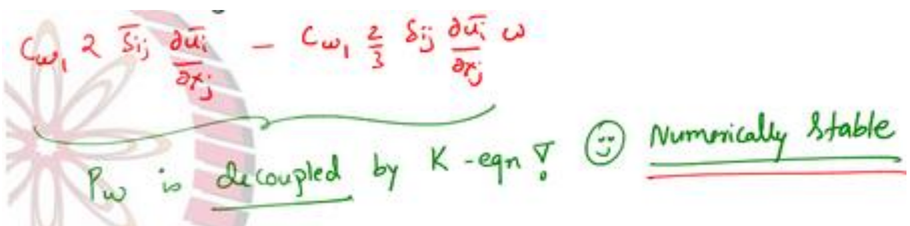
$$P_\omega = C_{\omega_1} P_k \frac{\omega}{k} = C_{\omega_1}\left[2\frac{k}{\omega}\bar{S}_{ij} - \frac{2}{3}k\delta_{ij}\right]\frac{\partial \bar{u}_i}{\partial x_j}\frac{\omega}{k} = C_{\omega_1} 2\frac{\cancel{k}}{\cancel{\omega}}\bar{S}_{ij}\frac{\partial \bar{u}_i}{\partial x_j}\frac{\cancel{\omega}}{\cancel{k}} - C_{\omega_1}\frac{2}{3}\cancel{k}\delta_{ij}\frac{\partial \bar{u}_i}{\partial x_j}\frac{\omega}{\cancel{k}}$$

$y \to 0$

$k \to 0$

So, it is decoupled whether it has a consequence because now the production rate of omega, omega is a turbulent quantity is an inverse time scale, but it is now being computed using what is this? Sij bar and dou i bar by dou xj. This is also time scales only, but a flow time scale. So, it is decoupled, numerically stable, whether it is giving accuracy that is another issue. So, you can at least say now this term is now decoupled. So, we will see the other one which is minus C omega 1, 2 third delta ij dou ui bar by dou xj omega.

So, the equation is completely decoupled here from P omega is decoupled from k equation. So, this is good for, this smiley is for numerical stability. numerically stable whenever you have a decoupled system that is fine. So, entire equation also does not have any issues of divide by 0. So, this is numerically stable decoupled that is also good two advantages here.



Now, we look into the other term, which is what we call an epsilon omega, which is C omega 2 omega square. So, there is no problem here y goes to 0, k goes to 0, k itself is not appearing here unlike in the epsilon model it is epsilon square by k, k itself is not appearing here. again, no k dependence or you can say decoupled from k. So, in a k epsilon model there is a strong coupling between the k and the epsilon equation. You need to iteratively solve this, but in the omega equation, both the production rate of omega and the dissipation or destruction rate of omega is decoupled from k, which is good for numerical stability, unlike the k epsilon.

But there are, this does not mean that standard k epsilon or any k epsilon models are unstable. So, there are ways to handle that. This is just a modeling argument of having a new model. Thank you.