Course Name: Turbulence Modelling

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Week - 7

Lecture – Lec40

40. Introduciton to wall-functions - I

Let us get started with recalling the log law region ok. I have just plotted here the data coming from a smooth and a rough boundary layer flow in a channel and this is just for you to recollect. So, the x axis is in the logarithmic scale, the y is in the linear scale. So, you see here this is what we mean by u plus is equal to y^+ the linear zone that is y^+ equal to 3 I have put a dash line here that shows the kind of a cut off. You can stretch it up to sometimes even y^+ 5, but as you see at 5 or so it starts to deviate away from the linear ok.

So, y^+ 3 is good, but in literature you will also see y^+ 5 is set as a cut off for the linear sub layer with a small error there. So, the linear zone is here u^+ equal to y^+ and then from here to here this is the buffer layer. So, there is no empirical fit there in the buffer layer part. But then we come to let us say this is like 30 right y^+ 30 is somewhere here this is the point.

Now you see this is the circles are for the smooth channel or a smooth surface and this other symbol is for a rough surface. So, obviously, in the I did mention that when you introduce roughness, the so called linear sublayer, buffer layer, they all vanish, a roughness sublayer appears. So, we will not go into that part. So, we do not have to focus on this particular line here, only look at the circles here. So, we have a linear sublayer, the red line indicating u plus equal to y^+ curve fit for the data, the circles are my own in-house DNS data.

and the red dashed line is the log law a plot plotted on top of the actual data. So, you can see here linear law that is u^+ equal to y^+ red line and the red dashed line is a log law

$u^+ = \frac{1}{K}\ln ln\left(y^+\right) + B$. And I did mention that the value of B sometimes becomes 5.5, sometimes becomes 5.

2. It is not universal It is changing and people use it what works best for their fit. And sometimes for common constant is 0.41 is also used, sometimes 0.4. So, recall this, we will use this concept now today to derive what is called wall functions.

So, let us go to wall functions. So, this wall functions also has I mean if you are going to use wall functions for your eddy viscosity model then we call them high Reynolds number formulation or high Reynolds number models ok. So, using wall functions in eddy viscosity models we get what is called high Reynolds number or the HRN formulation or HRN models. models based on wall function. So, what we mean by high Reynolds number and low Reynolds number is that this is not due to this is not with respect to your global Reynolds number Re_δ or Re_L.

This is the local Reynolds number that I am talking about. For example, if I look at the same graph that we looked at right the logarithmic part. So, you will have let us say say this is x this is y and z is the out of plane component. So, we have we have seen that the boundary layer is a turbulent boundary layer is like this and so you can define a Reynolds number based on the y^+ here a local Reynolds number So, here of course local Reynolds numbers are larger, but as you approach wall near wall zone local Reynolds numbers are lower. So, what I mean by high Reynolds number is we are looking into locally high Reynolds number right.

So, this is your locally locally high Reynolds number zone. So, this is what we aim to now use this concept to model what is called a wall function ok. So, these values that are at locally high Reynolds numbers. It is very important to remember this, it is local Reynolds number. So, you can apply this as I said for whether you if you have a Reynolds number is let us say 10,000 or 1 million of your global Reynolds number, you can still apply it.

It is nothing to do with Re_δ. This is a local Reynolds number that we are talking about. It is a local Re right. So, here of course the modeling idea is that, so why do we need to have this kind of an approach is that for example in industrial flows it becomes very complicated to have a mesh which is good enough to capture the boundary layer. So, for example, you know that for this particular condition to capture this entire boundary layer, you must have a grid like this, right.

If I am going to mention the grid on the side here. So, let us say you will have a grid like this in the wall normal direction with the first node here y^+ less than or equal to 1. So, this is the mesh that you are constructing in the wall normal direction. This puts a very stringent or a demanding mesh consideration for an industrial flow. For a canonical flow, pipe flows and all this, this is fine.

y^+ less than or equal to 1, the first node and this is the wall here. So, the first node y^+ less than or equal to 1 is fine, not a problem for canonical flows. For industrial flows, this becomes a bigger problem. So, there you would like to use what is called a wall function. For example, if I just write a schematic diagram you understand why industry would need something like that.

For example, let us say you have you are looking into flow over an atmospheric flow over a terrain or an industrial or region or even a city. So, you will have let us say some hill right we will have some buildings maybe a car we will have a tree right so complicated geometry if you are looking into flow over an urban canopy or a rural canopy to compute your whatever whatever is your need it could be even industrial flow like flow in a very complicated looking into heating or cooling component inside your engine vehicle part. So, all these are complicated geometries, it is a complex geometry in industrial flows. So there having this y^+ less than or equal to 1 is a very demanding task. It takes lot of time to actually have a mesh with this need to sit and make a mesh and it becomes very expensive also right.

So this demand for a mesh like this, this particular mesh is very demanding right. So So a mesh of let us say Δy^+ less than or equal to 1 in an industrial flow is very demanding and computationally expensive. So for that reason they come to what is called this wall functions. So what we do in wall function is that we are going to first introduce the mesh guidelines because the mesh is the criteria here. I do not want to mesh where my first grid

Δy is y^+ less than 1.

I would like to use a bigger mesh. That is the idea. So the mesh guidelines is I can say here first the mesh guidelines. So you need to construct a mesh like this in a wall function flow where the first mesh node, if you follow finite volume method you are familiar with what is called a P node, you know node n and so on, north node, south node and so on. So a P node is what is you would follow right.

So the P node here must be in the above the buffer layer. This should come in the inertial sub layer or the logarithmic layer zone. So the first node is far far away. So you are not resolving the viscous sub layer. You are not capturing viscous sub layer when you use wall functions.

So the P node here should be 30 less than or equal to y^+ less than or equal to 100. depends on the Reynolds number. So, the upper limit depends on your local Reynolds sorry your global Reynolds number Re, Re global. It could be Re_δ, Re_L whatever you are using ok. So, you see that this is now p is now coming inside the inertial sublayer right to log law region or you can say inertial sublayer right.

So, that means you are not resolving the, when you use wall functions you are not resolving viscous sublayer. So, viscous sublayer is not resolved, resolved meaning you are computing right resolved or captured in the CFD calculation . So what do you do then? If I am making a mesh where the first point is sitting far far above, so then the idea here is that you need to account for the wall effects. You cannot simply ignore the effect of the wall. For example, in combustion the effect of wall is to quench a flame.

So, you cannot completely ignore that. So, whatever the physical phenomena a wall offers you must account for it far away from the wall that is your wall function. So, the modeling idea is to account for wall effects whatever the wall is doing. in in our case it is providing viscous effects right, wall effects away from the wall. That means the node P here must consider the wall effects as long as you are able to do that.

That means you are going to give a boundary condition at node P not on a node that is lying on the wall ok, node on the wall is not important here. The node P becomes your you know default boundary condition and that is what you are going to apply using a wall function ok. So, idea of course relies here, idea relies on log law. So the wall function that we are going to derive works for smooth turbulent boundary layers a flat plate turbulent boundary layer ok. That is works for smooth turbulent boundary layer and the focus is on high Re inertial zone not on the viscous sublayer.

But the buffer layer is when is where you have peak turbulence kinetic energy production and other phenomena. We have seen that the two stresses become dominant viscous stress and the turbulent stress becomes dominant in the buffer layer and that is where the peak of your turbulence kinetic energy the production rates appear. So, we are ignoring that ignoring the sense we are not calculating it, it has to be modeled. Somehow that effect has to be accounted on node P ok. So, this is a pragmatic approach simply because the geometry is far too complicated.

If you can afford to have a mesh you do not have to use this, it is simply a industrial or a pragmatic consideration. And the first node here, first node in log law region that is it must be y^+ greater than 30 for sure. how high you want to do this depends on your Reynolds number. Higher the Reynolds number you have, global Reynolds number, you can push the y^+ value above and above.

At least it should be above 30. So it is computationally cheaper. So we will see how we are going to do this how we displace the effect of wall away from the wall.