Course Name: Turbulence Modelling

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Lecture – Lec58

58. Pressure-Strain rate modelling and wall corrections for RSM - I

So we tried to establish a relationship between the pressure strain rate to this Reynolds stresses and the mean strain rate and that is how the link came in the IP model. Okay, let us get started again. So last class we looked at the IP model or the π_{ij} rapid model right. So the slow term is still remaining and if you recall we tried to establish relationship between the pressure strain rate with the known terms which is the mean strain rate right. So the knowns were have access to the mean strain rate okay I have access to the Reynolds stresses itself and the dissipation rate these are the knowns and we linked the pressure strain rate rapid term to these two.

The IP model is essentially linking mean strain and the Reynolds stresses and the only term where these two terms are appearing together is your production rate terms. So, the IP model has p_{ij} right. So, if you just recall the IP model was π_{ij} rapid was $-C_2\rho\left(p_{ij} - \frac{2}{3}p_k\delta_{ij}\right)$. model here.

Now the slow term has to be modeled right. So we look at π_{ij} slow term okay. So the slow term if you recall this was only a function of terms which were only had turbulent terms it had no mean strain right. So here there were no mean strain terms in the π_{ij} slope. So it is it was entirely depending on correlations of fluctuations right.

So here there may be more than one approach how to model this. So I am going to follow one particular approach here. There may be another approach in some literature you will see. So we will go ahead with what is called in the modeling part right. So we are going to model this term.

So we will take up what I call or what is commonly called as physical modeling of π_{ij} or π_{ij}, there is no comma here right, it is a tensor π_{ij}, π_{ij} slow term. this particular model has a reference which is also called Rotta's model. It is also called return to isotropy concept or model return to isotropy model. as we know pressure strain rate takes away turbulence in a direction where it is produced and tries to give it out to the directions where it is not produced. So it is trying to make the turbulence isotropic but it will not be because there are other terms active the production the production rate terms the dissipation rate terms so they will cause the unbalance between the three components of the stresses.

So here what we do is first we are going to take I am going to consider now a point here let us call it O and then I am going to say okay there are two fluid particles here with I will take two fluid particles. These two fluid particles now are bouncing against each other okay. So this is both are bouncing against each other and they are carrying a fluctuation u_1’ and I will have this coordinate like this. This is x 1, this is x 2 coordinate okay. Two fluid particles of u_1’ bouncing against each other at point O okay, they want to collide here.

So if this occurs obviously the pressure at the point O will rise right, the pressure fluctuation P’ will rise when these two will try to collide at point O. So we will use this concept to see what we can do it. So the first thing I would like to argue or model arguments is that now let two fluid particles with u_1’ bounce into each other at point O. When this occurs So now look here so you have here you have higher turbulence fluctuation the zone here I am talking about and same thing here u_1’ high when it collides you will have a lower u_1’ fluctuation right at the collision zone. So this will be u_1 low at the collision.

So what will happen to the gradient $\frac{\partial u_1'}{\partial x_1}$, $\frac{\partial u_1'}{\partial x_1}$. What will happen to this? Is it positive, negative? Negative, right? So this is negative. When this collision occurs this gradient will be negative, ok. Let us call this 1. So as a consequence of this collision as I already said the pressure P' should increase.

As a consequence P' increases at point O ok. So, therefore since $\frac{\partial u_1'}{\partial x_1}$ is already - P' being large is ok, but the entire part will be still negative. u_1' this part is already negative right. So therefore P' $\frac{\partial u_1'}{\partial x_1}$ is taking a negative value here all right. So now I will make another argument that if when locally the pressure fluctuation is higher at point O and you are trying to bring two particles bounce against it that means work is done against this pressure gradient right.

So that means the kinetic energy that is lost along the x 1 direction must be transferred to the other two directions that is what we are going in that direction the pressure strain rate is doing this. So therefore in x 1 direction in x 1 direction work is done moving fluid particle particles against the pressure gradient So this implies that the kinetic energy lost along the x1 direction must be transferred to the x2 and x3 direction. Therefore the turbulence kinetic energy lost along along x1 direction is transferred to x2 and x3 directions. So from 1 what will I get? Therefore from equation 1 what do I get? If this is negative the $\frac{\partial u_2'}{\partial x_2}$ and $\frac{\partial u_3'}{\partial x_3}$ is positive also for also by continuity right. So $\frac{\partial u_2'}{\partial x_2}$ is positive $\frac{\partial u_3'}{\partial x_3}$ is positive you can also say also from continuity.

okay. So now for this to happen the kinetic energy as I said in the x1 direction should be larger than the other two direction right. The kinetic energy lost along x1 is transferred to the other two means the kinetic energy along the x1 should be larger that the TKE has to be larger compared to x2 and x3 directions right. So for this to happen TKE along x 1 direction should be greater than or you can write symbolically should be greater than use another color here should be greater than TKE along x 2 comma x 3 directions ok and this implies this implies your $\overline{u_1'}2$ must be greater than $\overline{u_2'}2$ and $\overline{u_1'}2$ must be greater than $\overline{u_3'}2$ ok. And another argument is that now if this the amount of energy that is being transferred from one to the other two directions. should also be proportional to the difference in the TKE components.

So we will try to establish now relationship between this particular stresses or these three normal stresses constitute the components of turbulence kinetic energy and we already have this pressure strain rate term like $\frac{p'}{\rho}\frac{\partial u_1'}{\partial x_1}$ is one of the component. So we have to establish a relationship between this particular component to the TKE components, okay. So, we say now that the amount of kinetic energy transferred from x1 to x2, x3 directions must be proportional to the difference in the TKE components. What does this mean is I have $\overline{\frac{p'}{\rho}\frac{\partial u_1'}{\partial x_1}}$. This should be proportional to the difference that is $\frac{1}{2}\left[\left(\overline{u_1'}2 - \overline{u_2'}2\right) + \left(\overline{u_1'}2 - \overline{u_3'}2\right)\right]$

I am relating this to the difference between the components in the TKE right. So, what does this imply is that I have now, so we are achieving the model here. So, I would get essentially half of or if I sum it up I get $\left[\left(\overline{u_1'}2 - \frac{1}{2}(\overline{u_2'}2 + \overline{u_3'}2)\right)\right]$. Now I am going to add and subtract the other component that is plus, so I am adding here + $\frac{1}{2}$ $\overline{u_1'}2$ - $\frac{1}{2}$ $\overline{u_1'}2$. I am adding that because I would like to get turbulence kinetic energy recovered in the bracketed term.

So what will this give is I get $\frac{3}{2}$ $\overline{u_1'}2$ - $\frac{1}{2}(\overline{u_1'}2 + \overline{u_2'}2 + \overline{u_3'}2)$. So this gives me $\frac{3}{2}$ $\overline{u_1'}2$ - k. So this is sufficient for one of the normal stresses but we need a generic expression because the pressure strain rate itself is will have the other components right. So we have there are six stress components this is sufficient for one of the normal stress components. So we need a generic expression for shear stress as well as other stress components right.

So here if you notice here there will be a time scale missing. If I look at the the dimensions for this there will be a time scale must be missing here. That is what we will introduce later. So this will come as I mean you can easily see that the unit will be here only meter square per second square. But on the left hand side it should be meter square per second cube, correct? This should be meter square per second cube.

So at time scale that we are familiar in eddy viscosity models the $\frac{\varepsilon}{k}$ or $\frac{k}{\varepsilon}$ that part will appear in the model in a generic expression, ok. So here you can say that this is applicable, this is applicable only to a normal stress. So, we need a generic expression. Therefore, in generic form the model will look like π ij slow will be $- c_1 \rho_{\frac{\varepsilon}{k}}$ coming for the time scale part and the Reynolds stresses which is $\overline{u_i'u_j'} - \frac{2}{3}k\delta_{ij}$. So this is your π_{ij} slow term model.

This should of course be used together with π_{ij} rapid the IP model. So the pressure strain rate will have two components π_{ij} rapid π_{ij} slow. So both the models should be taken. The sum total has to be taken when you implement it ok. The IP model plus Rottas model.

there may be other options I am just giving you an illustration of how the models are coming out that is relating the unknowns to the known. There are other possibilities it is not just that you have to use IP and rotta model but these are popular models there are other models as well. But somehow we have to establish only links to what is known and known is only Reynolds stresses, mean strain rate and dissipation rate of k this is what is known. So it will be some form of only this nothing more ok. So this will complete the only the the pressure strain rate term, the rapid and the slow part.

But now we have to also have an option like this eddy viscosity model we had option to resolve the flow all the way to y plus equal to 1. We also had options for wall functions. Similarly you can do it here. If you are going to resolve all the way to the flow, all the way to the wall then you need some wall correction terms here. So this that will be addition of sum of two more terms.

So π_{ij} rapid + π_{ij} rapid wall correction term plus π_{ij} slow plus π_{ij} slow wall correction term ok. It will go like that. So we will see what this wall correction terms are. This is clear so far the modeling the slow term and the rapid term ok. There is as I said different models will have a slightly different approach.

These two are what I am giving a flavor of how it is modeled this particular term. okay. So we look at sorry I will give you all the model constants at the end okay because more model constants will come now in the wall correction and everything. I will give you all model constants at the end okay.