Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 10

Lecture – Lec59

59. Pressure-Strain rate modelling and wall corrections for RSM - II

So, the wall correction this is another toπc wall correction right. So, here I am going to consider let us say a coordinate system. let us call this S and N. N is the wall normal, S is a wall parallel coordinate system okay where I have a wall here. That means as I approach the wall very close to the wall I need some kind of a correction term a damπng function term here.

and I want this damπng function to be active in the range let us say from $y^+ = 1$ to about y^+ about 200 ok. I want the damπng function to be active in this part. that means it has to be automatic as it is approaching the wall the damπng function it is not like you have to explicitly model this or explicitly give a statement if y^+ is this use this and that is impossible in a generic flow problem you will not even know what is the y+ value a priori. So it should be automatic so it is approaching wall the term should become active as it goes away from the wall its contribution should be nil.

what we do is wall correction for first I am going to give for the wall normal component okay. So, for wall normal component that is $\overline{u_n'2}$ it is the wall normal right, wall normal stress that means in this particular equation for the wall normal stress equation for this component the π_{nn} slow. So, the wall corrections will be there for both rapid and slow ok. So, that means the π_{ij} itself is modeled as rapid plus slow plus it will have the wall correction for slow, wall correction for rapid term ok, sum of many terms ok. So I would call this prime here because it is a wall correction.

So π'_{nn} for the slope this is $- 2 C_1' \rho \frac{\varepsilon}{k} \overline{u_n'2} f$. Simply modeled on the Reynolds stresses because the slow terms are always depending on correlation of fluctuating quantities not on the mean strain. So it is depending on here. So essentially what it is doing is it is taking away two parts in the wall normal. So wall normal if you recall the two component limit, the wall normal you will have damping I mean it damps faster than the wall parallel components right because of the pressure damπng and viscous damπng.

So here π'_{nn} the wall correction is it is taking two parts away and this will be given one part each to the other it is as simple as that right. So this is your for the wall normal term where f is the damping function where the damping function f = $\frac{k^{(3/2)}}{2.55 x_n \varepsilon}$. So x_n is your savior. So if you see here π'_{nn} slow is a wall correction that means it has to get active as you come closer to the wall and this has f and f is inversely proportional to x_n.

So as you approach x_n, x_n is the wall normal coordinate right. So as you approach the wall f will increase so π'_{nn} slow will be active. If you are going very far away from the wall f will be so small that π_n and prime slope will be negligible. So it is a $1/x_n$ dependency just like the wall dependency we introduced for epsilon boundary conditions and other places damping functions. So this is one part here f is the damping function So, this takes care of only π_{nn} or the normal stress correction for the normal stress.

We also need it for the parallel stress. There is one more thing I have to say here where the damπng function is this and x_n is the wall normal distance. you do not have to implement this between this y+ 1 and y+ 200. All I am saying is damping function has to be active automatically. It is not like you need to go and explicitly say become active here.

This is the zone of its influence ok. So, now I need the other component for the wall parallel which is u s prime square the two wall parallel stress components right. Wall parallel for that equation the correction term is π_{ss} slow prime of course is equal to simply I am giving one part to each. taking two parts in the normal direction giving one part equal to both okay. Yeah just assume you have three kids at home one the big kid you are taking away some food from that guy and giving to two small children that is what it is happening here.

$C_1' \rho \frac{\varepsilon}{k} \overline{u'_n 2} f$. one part each to the both. So, you can write here that note that two parts are taken or you can say removed from normal direction, wall normal direction and given equally to the wall parallel wall parallel direction the s directions. So, this will be for the normal stress. wall normal removed given to the wall parallel, but there is also a shear stress term.

So, we will see that correction for the wall for the shear stress also that is for π'_{sn} that is when you have $\overline{u'_s u'_n}$. So, for the shear stress for shear stress equation that is for your

$\overline{u'_s u'_n}$. In that equation the π'$_{sn}$ slow is modelled as – 3/2 C$_1$' ρ $\frac{\varepsilon}{k}$ $\overline{u'_s u'_n}$ f. So this gives the three correction terms that are used in the or the wall correction terms for the slow, normal stress, three normal stresses and the shear stresses. But we also need it for the rapid ok.

So the wall correction for wall correction for rapid terms so here I would give out the expression similar idea taking two parts away and giving one part equally. The formula changes okay slightly because we are not going to use Reynolds stresses we are going to use the rapid term itself. So the π'$_{nn}$ rapid okay wall correction obviously is nothing but - 2 this is the wall normal component - 2 c_2'f π_{nn} rapid. So, the π_{nn} rapid is being computed from your IP model. So, minus 2 parts is taken away with the damping function of course and the π'$_{ss}$.

rapid prime indicating wall correction terms. This will be c_2'f π_{ss} rapid, okay and the π^{sn} rapid term will take - 3/2 C'$_2$the damping function and π_{sn}. So, we are going to use the corresponding the the rapid terms here for all the components. So this will complete your wall correction implementation for rapid and slow. So this is the sum total of right.

So note that. So if you are implementing the let us say or I can say example here, example for π_{11} term model. will be π_{11} rapid + π_{11} slow + π'$_{11}$ rapid + π'$_{11}$ slow with the wall corrections 4, 4 terms will be there for each of this equation ok. Is this clear? Ok so, then the only thing that is left from implementing this model is the model constants. So, we look on to this have model constants. So, I have C_1 which appeared in the rapid 1.8, C_2 is 0.6 C'$_1$ is 0.5 and C'$_2$ is 0.3. boundary condition here when we have this wall correction or resolving the wall flow is straightforward right. Boundary condition is you are implementing the no slip for your velocities, dissipation rate like before you have options epsilon you know how to implement boundary condition and you have Reynolds stresses which has to go to 0 on the wall.

So it is straightforward here the boundary condition ok. So the boundary conditions you can take a note here if you want. So the boundary conditions this as well as this no slip here right, no slip condition for the stresses and epsilon you have options like evm you can have the near wall dependency introduced there that we had like 1/ y^2 dependency introduced you can use those boundary conditions we can also have wall functions if you are interested in not resolving the wall using Reynolds stresses then wall functions are also an option. So, there is a wall function derived for each of the stress here. So, suppose if you are using wall functions the grid guidelines are similar whether you do eddy viscosity model or Reynolds stress model the wall functions and the the wall correction that means resolving grid guidelines are same.

So, you need to have your first grid in the inertial sub layer for when you implement wall function. So, the linear and buffer layer are not resolved . So, when you do wall functions if you do this then your $\overline{u_1'}2$ is 3.67 u_*^2, your $\overline{u_2'}2$ is 0.83 u_*^2 that is the wall friction velocity, $\overline{u_3'}2$ is 2.17 u_*^2, and the shear stress $\overline{u_1'u_2'}$will be - u_*^2. If you have other components also you can use the same expression. In many canonical flows only one shear stress exists like the turbulent channel flow or turbulent plane squared flow that you did. The other two components I mean by theory you cannot say that only after doing the experiments or DNS you can realize that the other two are negligible or zero. only this primary shear stress exists and epsilon of course, you need a wall function for that which is same as before u_*^3 by kappa y.

So, u_* is your wall friction velocity, kappa is the wall or the von Kármán constant. y is your wall normal coordinate of course, y or x_n if x_n is the one you are using depending on what you want to call it. Is this clear? The Reynolds stresses modeling part So there is instead of solving a transport equation for Reynolds stresses some would like to go ahead with using let us say an algebraic expression like the you know we had a zero equation model right for a eddy-viscosity also like that. instead of having a transport equation for Reynolds stresses how about having algebraic expression for Reynolds stresses. This is called ASM algebraic stress models or algebraic Reynolds stress models.

I will not go much deeper into it because these are not very popular but this option exists in I think in one or two commercial codes they exist but they are not very popular. I will just give you the model philosophy how do they know why did they thought about it and what is the idea behind it. Just an introduction to that one. Algebraic stress models, the so called ASM algebraic implying algebraic expression here. So what they do here is that there is a model assumption that the transport of the Reynolds stresses is related to the transport of the turbulence kinetic energy.

Transport implying the convective transport or the advective transport and the diffusive transport. The two convection diffusion is of convection diffusion of your Reynolds stresses is related to convection diffusion of turbulence kinetic energy that is the assumption ok. So the model assumption here is that The model assumption is transport that is convection and diffusion of Reynolds stresses is related or you can express it in terms of transport of your turbulence kinetic energy. This is a model assumption here. So, that means this essentially implies that symbolically I am writing C_{ij} is your convective transport term in a Reynolds stress equation.

- D_{ij}. So, I will write it here the convection part, the diffusion part. So, C_{ij} - D_{ij} we want it to be proportional to C_k or C_{tke} - D_K transport here. So, C_k ,D_K term or I can write this as C_{ij} - D_{ij} = C_k - D_K and introduce the Reynolds stresses here. because the whole point is to get an expression for Reynolds stresses. What is the point of relating the convection diffusion of this to the convection diffusion of that? I need an algebraic equation for Reynolds stresses.

So I am introducing that here ui prime uj prime by k non-dimensional. So therefore I can get so now I can express this C_{ij} D_{ij} as what it is related to on the right hand side of those equations right. However C_{ij} - D_{ij} what is this related to on the on its right hand side? I have p_{ij} production rate term, I have π_{ij} term and then ε_{ij} and C_k - D_K is equal to P_k - ε. So, therefore, I can say that $P_{ij} + \pi_{ij} - \varepsilon_{ij}$ is equal to $\overline{u_i' u_j'} / k\,(P_k - \varepsilon)$.

So this is your algebraic expression. So each Reynolds stress you can obtain using some model for each of these terms. I am just giving the idea behind algebraic stress model not would not like to go deeper into this one. RSM is much more popular than ASMs, Reynolds stress models or transport, solid transport equation. If you are able to get convert solutions that is popular. But in the, so now we can say that I am more or less completing the statistical approach in the modeling.

right. So, statistical modeling had RANS equation solving RANS equation. That means we are interested in mean velocity and mean pressure to get that we are modeling the turbulent terms. We are lumping it as Reynolds stresses and Reynolds stresses are either modeled using a eddy-viscosity approach or solving transport equations for it. But the main objective is to get mean velocities and pressure. If that is your focus that is that is a focus for industry this is sufficient.

Of course you have to live with some error there. But beyond that if you want to capture turbulence I am not going to model these correlation terms as lumping it as some statistical equivalent. Instead of that I would like to let the mesh capture the random or stochastic component. Then we move into what is called the eddy resolved methods. In the next class I will start that part.