**Course Name: Turbulence Modelling**

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**Week - 11**

**Lecture – Lec67**

# 67. Large Eddy Simulations: Filtered Navier-Stokes Equations - II

Fine. So, now another thing important to know here is that since let us take a note here. So, we are now filtering in LES and then some residue is coming out which has to be modeled right. So, I have $\Phi$ any random $\Phi$ is now split into filtered + its residue all three random component. And the $\overline{\phi}$ quantity depends on how much you are resolving depends on how good the mesh is.

So, if you ah so, this is actually captured right. So, this is resolved called by mesh. this is modeled using SGS models the two parts. Now, depending on your mesh your $\overline{\phi}$ is changing right.

Let us say you take a mesh which is 100 cube in all the three directions together n cube is 100 cube ah and then you refine it into 200 cube. So, your your resolution is better now right you have added more mesh points resolution is better. So, $\overline{\phi}$ improves, but what happens to $\phi''$ the model contribution? Model contribution is now a function of mesh correct the mesh size itself. this is not like in the RANS. In RANS models we did not use mesh size correct we use bousinnesque which depended on eddy viscosity which depended on velocity gradient and so on.

We did not have $\Delta x$ $\Delta y$ $\Delta z$ inside the model correct, but here the model component $\phi''$ depends on your mesh size smaller the mesh size less model contribution right and more being resolved. So do you achieve a grid independent solution in LES is a question. We can just take an illustration like this. Let us say I am just taking simple mesh. Let us say this is the mesh and I have eddy.

Let us say I have one big eddy here and then one tiny eddy here. So, the bigger one is captured by this mesh ok, the mesh is smaller than the size of the eddy that is being resolved and this is being this particular eddy is smaller than that. So, this is SGS right and this is resolved resolved this is SGS modeled. Now if I just resolve the mesh I am doing mesh dependency study the same problem I will take it and then I can say. So, the same problem I can say that I am taking more mesh here I am refining the mesh.

Let us say I have refined the mesh here made it tinier. Now, suddenly this becomes resolved also right. So, this is also becoming both are resolved, this is also resolved, this is obviously resolved I did a zonal refinement there only around that. We can do global whatever it does not matter as long as I have refined it now, now this structure is being resolved I do not have to model it. Now that means, by changing the mesh size ah the the amount of contribution from these two components resolved component and model component is changing correct.

So, do you get mesh independent study? by changing the mesh amount of resolved component and modeled component changes. So the question is can you get grid independent solution in LES? What do you think? Let us say I keep on refining this and it eventually becomes a DNS mesh. do you get now theoretically speaking is your LES solution using a DNS mesh matching the actual DNS data? Does it? Why? That error is also there in a filtering part yes. In addition to that the SGS model depends on the mesh size. So, no matter how tiny it is still finite it is not infinitesimally small right even in the DNS mesh.

So, a small modeling contribution is still added on top of your resolved calculation. So, theoretically speaking LES on a DNS mesh would still not perfectly match with the DNS solution more or less let us say matching. it may look matching, but theoretically speaking. Of course, there is no point doing an LES calculation on a DNS mesh. It is more expensive actually because you have more calculations, the modeling calculations are extra here right.

In DNS that modeling calculations are not there. So, obviously it is economical to do DNS on a DNS mesh rather than LES on a DNS mesh. The whole point of doing LES is to save mesh points make computations faster ok. So, typically you should be able to do LES on workstations or your modern laptops desktops that is the idea. So, that you do not need a supercomputer.

If DNS mesh is let us say 100 million that requires a supercomputer. If you are able to do let us say the same case using 5 million mesh that you can do on your desktop or laptops today right. So, 100 million 5 million right with the 5 million mesh you can get reasonably good results why not that is the idea ok. So, that means, the answer here can you get grid independent solution in LES? but you should always then in CFD we always seek for grid independent solution we ask for it right. So, here you cannot seek for grid independence you have to check for grid convergence you change the mesh size and see is what is the trend is the trend converging ok.

So, check for you can say in LES check for grid convergence not independent. Check for grid convergence or you can say grid sensitivity sensitivity. It means as you refine see how your solution is changing. with following some trend and acceptable. So, you need to apply your mind here and then decide when to stop the refinement in LES.

It is not like monotonic like in RANS, you keep on refining till the solution is coming to independent solution and stop ok fine. So,apart from this yeah. So, there is one more thing that I would like to talk about is yesterday we discussed that filtering again a filtered quantity will not be same as the original filter quantity right. So, we did discuss that the question is is $\overline{\phi}$ equal to $\overline{\overline{\phi}}$ that was a question right and we said it is in general no right I said in general the answer is no that is $\overline{\phi}$ is not equal to $\overline{\overline{\phi}}$ except in spectral filters that I will come later. but now we can go and see a numerical argument.

I will show you both the theoretical proof a mathematical proof for it. We can also look at a numerical proof for this an approximation to see. So, illustration for this is like this. Let us take a numerical illustration ok. So, let us take data points like this.

Let us say I have a mesh looking like this, this is i, i + 1, i - 1 ok, 3 points here can make it like this data points and now I am going to put a filtering operation about i and look at $\overline{\phi}_i$ r So, I will take a filter size let us say something like this I am taking a filter size here called $\Delta$ x let us say filter size is $\Delta$ x this is your filter size. what I have used is called a box filter that I will tell when we go to filter types. Right now consider that there is a filter around this and I am computing the filtered quantity here ok. So, let us call this $\Phi$ $\overline{\phi}$ is being evaluated at i ok. So, let $\overline{\phi}$ be evaluated at i node i here and I am considering a 1D filtering operation here.

So, consider 1D filter and also homogeneous filter. that means, the filter size is constant that is constant size filter size is uniform. So, if I do this then the filter $\overline{\phi}$ filter bar the $\overline{\phi}$ is already evaluated I am going to double filter. So, the $\overline{\overline{\phi}}$ will be 1D filtering operation that I will do which is $\frac{1}{\Delta x}\int$- $\Delta$ x by 2 to $\Delta$ x by 2. So, $\overline{\phi}$ is already evaluated I am doing double filtering.

So, I have $\overline{\phi}(r)$ , r is a special variable $\overline{\phi}(r)dr$ here ok. So, r is your special variable r is a special So, this is nothing but I can now split this into two different integrals

$\frac{1}{\Delta x}\int \overline{\phi}(r)dr$ (- Δ x / 2) to 0 $+ \int \overline{\phi}(r)dr$ 0 to (Δ x /2). Now I do some numerical approximation I use a trapezoidal rule to get this. So, I can say I have a point here I call it a b. So, I can approximate this using trapezoidal rule as these two parts are sum of $\overline{\phi}_a + \overline{\phi}_b$ ok.

So, using trapezoidal rule I can say this is equal to 1/ Δ x This is Δ x / 2 $\overline{\phi}_a$ + Δ x/ 2 $\overline{\phi}_b$. Numerical approximation here. Numerical approximation using trapezoidal rule. second order accurate ok. So, the $\overline{\phi}_a$ also I can produce now using linear interpolation between i and i - 1 and i and i + 1.

So, I get this is equal to nowt his gets cancelled Δ x Δ x. So, essentially this becomes ½( $\overline{\phi}_a + \overline{\phi}_b$) which is equal to ½ the $\overline{\phi}_a$ the filtered quantity at a I can use linear interpolation and say it is ¼ ( $\overline{\phi}_{i-1}$) + 3/4 ( $\overline{\phi}_i$) ok. So, I say that it is ¼ ( $\overline{\phi}_{i-1}$) + 3/4 ( $\overline{\phi}_i$) linear interpolation of what is a $\overline{\phi}_a$ . ok + I have the other component which is 3/4 ( $\overline{\phi}_i$) + ¼ ( $\overline{\phi}_{i+1}$) I have used linear interpolation here. the two parts are split this two linear interpolation.

So, now, if I rewrite this I get the $\overline{\overline{\phi}}$ is equal to I have 3/4 of these two. So, which is I get ½(2* (3/4) ( $\overline{\phi}_i$) . So, this is (3/2) ( $\overline{\phi}_i$)correct + ¼ ($\overline{\phi}_{i-1}$ + $\overline{\phi}_{i+1}$) ok. So, therefore, $\overline{\overline{\phi}}$ is now coming out to be you will have 3/4th $\overline{\overline{\phi}}$ + 1/8th of $\overline{\phi}_{i-1}$ + $\overline{\phi}_{i+1}$ here. Now, the question is this what $\overline{\phi}$ is $\overline{\overline{\phi}}$ is now you can see it is not equal to $\overline{\phi}$ alone.

We started with the $\overline{\phi}$ . Of course, this is a numerical approximation. I will give you a mathematical proof also that these two will not be same whenever you use any filter type other than spectral filter ok. So, now you can see the $\overline{\overline{\phi}}$ is not $\overline{\phi}$ because $\overline{\phi}$ was essentially what was your $\overline{\phi}$ ? This is essentially nothing but 1/ Δ x of this Φ( r) dr here. So, you should have essentially got here just $\overline{\phi}$ . So, I can you can also write it as i here right.

So, the $\overline{\overline{\phi}}$ is now (3/4) ( $\overline{\phi}_i$)+ 1/8 th of its neighbors this is not same as $\overline{\phi}$ ok. This is a numerical illustration or a proof you can think of. We will also have what to say mathematical illustration for this. For that we have to understand the different filter types.