Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 12

Lecture – Lec69

69. Large Eddy Simulations: Filters and its types - II

The two types of filters has been explained and using this I can show you the one of the statements that I made that double filtering. Filtering an already filtered variable in general I told you is not equal to the original filter data in general. The exception I told you it is in the spectral filters ok. So, we can take box filter and see what happens. So, in box filter obviously $\bar{\bar{\phi}}$ should not be equal to $\bar{\phi}$.

but it should be equal in spectral filter. So, we will see that one. So, again here the question is we are just revisiting this is $\bar{\bar{\phi}} = \bar{\phi}$ that question we only looked into a numerical demonstration or illustration in the last class where I used a trapezoidal rule, linear approximation and everything to show that these two are not same in a box filter ok. So, now if I take box filter ok.

So, phi double bar $\bar{\bar{\phi}}$ is not equal to $\bar{\phi}$. How do I say this? Now we have this generic filter definition, we can use this. So, I can say now is nothing but integral minus infinity to infinity g of r phi bar dr filtering an already filtered variable. So, this is equal to integral minus infinity to infinity. What is g of r? 1 by delta ok, 1 by delta phi bar dr.

$$\bar{\bar{\phi}} = \int_{-\infty}^{\infty} G(r)\,\bar{\phi}\,dr = \int_{-\infty}^{\infty} \frac{1}{\Delta}\overbrace{\left(\int_{-\infty}^{\infty} G(r)\,\phi\,dr\right)}^{\bar{\phi}} dr = \int_{-\infty}^{\infty} \frac{1}{\Delta}\underbrace{\left(\int_{-\infty}^{\infty} \frac{1}{\Delta}\,\phi\,dr\right)}_{\bar{\phi}} dr$$

$$\therefore\ \bar{\bar{\phi}} \neq \bar{\phi}$$

What is phi bar? phi bar is nothing but again integral minus infinity to infinity g of r phi dr. So, this particular thing is Phi bar here, this is Phi bar whatever is inside the round bracket here ok. So, now this is equal to minus infinity to infinity 1 by delta minus infinity to infinity again g of r is 1 by delta. Now, obviously this is not equal to phi bar. What is phi bar? phi bar is what is inside this red which is just integral minus infinity to infinity 1 by delta phi dr.

So, this is phi bar. This is phi bar. So, I can take it here. like this. So, therefore, phi double bar is not equal to phi bar because there is a 1 by delta of phi bar coming up again.

So, even for the same filter size obviously, if you change filter size it is obvious. Let us say phi bar is at delta and phi double bar you are taking 2 delta you are taking a bigger and bigger filter size. Obviously, the averaging will smoothen out more data volume averaging. So, obviously, it will not be same that is obvious, but even for the same filter size delta you can see that it becomes 1 by delta right phi bar divided by delta will happen. Each time you filter it 1 by delta is coming and therefore, phi bar is not equal to phi double bar for a box filter for a cutoff filter or the spectral filter.

For this the phi. I will use this cap because we are working with the wave number space double bar now is equal to the same thing. I can say integral minus infinity to infinity g cap of k wave number right r is the spatial variable for the in the physical space now we are working with wave number space k. k is the wave number. So, g cap of k and then I have the phi in the wave number correct.

So, this is equal to minus infinity to infinity. What is g cap of k? It should be 1. and then I have the integral minus infinity to infinity if I take the g cap of filter phi bar. So, phi bar is now again g cap of k phi cap d k this is equal to integral minus infinity to infinity integral minus infinity to infinity this is 1 again right g cap of k is again 1. So, this becomes only p cap dk dk this will become 1 here.

$$\bar{\bar{\hat{\phi}}} = \int_{-\infty}^{\infty} \hat{G}(k)\, \bar{\hat{\phi}}\, dk = \int_{-\infty}^{\infty} 1 \int_{-\infty}^{\infty} \hat{G}(k)\, \hat{\phi}\, dk\, dk = \int_{-\infty}^{\infty} 1 \int_{-\infty}^{\infty} 1(\hat{\phi})\, dk\, dk = \bar{\hat{\phi}}$$

since it is not 1 by delta here the filter cutoff filter size is 1 you are setting it no matter how many times you are going to filter you are going to get the same here in a spectral cutoff filter. So, the only this is an exception only if you are using a spectral filter filtering again and again gives the same result otherwise it will always give a different result ok fine. Take one more note. So, you may be right now little bit confused with ok what is this filtering I have to choose a mesh first and then mesh itself is a challenge because you need to refine the mesh to see mesh convergence and everything. Now on top of it I have filter to work with I choose this filter what if I use a bigger and bigger filter right so many parameters.

So, life is easy if you are in finite volume method filter size becomes same as your discretization. ok. So, your mesh size becomes filter size life is easy no filtering operation required in finite volume method your discretization itself is filtering operation. So, in finite volume method that is a particular reason why finite volume method became so popular in fluid mechanics compared to finite difference or finite element ok. So, in finite volume method FVM box filtering is a default choice.

People use in finite volume method I told you spectral filters are used in pseudo spectral solvers. We are using Navier Stokes in the real space finite volume method box filtering is default. And further implicit filtering implicit filtering implicit whenever implicit means that you are not going doing an extra filtering operation here. So, what this implicit filtering means is filtering is same as discretization that is your finite volume or control volume just the mesh size. So, you do not have to do an extra filtering operation that means, whatever you are going to filter that means, whatever you are going to capture or calculate using your mesh is your resolved component.

Each time you refine the mesh you are capturing such eddies. So, it is much easy to think in physical space ok, I want to capture all the eddies from let us say meter size to down the way to let us say 1 millimeter size. then I will put a mesh size 1 millimeter that will be my filter size also anything smaller than that eddy smaller than let us say 1 millimeter will be my sub grid scale that requires modeling much easier to think now ok you right I mean you of course, you need to know all these concepts what is a filter and everything, but coming down to the finite volume method there is a big advantage filtering becomes same as discretization. Otherwise if you are using any other method finite difference, finite element you need to do an extra filtering operation on top of your meshing ok. So, then all this knowledge is useful which you know which filter to choose and all these things ok fine.

So, now we move to. So, filter types are done, filter size is what you choose and if it is I told you FVM it is a mesh size, sub grid model, sub grid scale, SGS models ok. So, we will give a little bit of one model we can try and see. So, sub grid scale, sub grid scale SGS models So what is the objective of an SGS model? You already exposed to lot of modeling, turbulence modeling now. So what is the objective of an eddy viscosity model? What is it modeling, eddy viscosity model? So, now these reynolds stresses are associated with which turbulent scale? All scales. So, eddy viscosity model is modeling all turbulent scales.

So, now as the same question here what is what should be the objective of a sub grid scale model? Modeling the sub grid scale that is the objective now here. That means, you must resolve the scales that are larger than this. Then only these SGS models will work properly. You should not expect SGS model to make your predictions better with let us say poor resolution. more scales you have to resolve in the spectrum I told you right.

So, the energetic spectrum, inertial range all this has to be captured by your mesh, then only SGS models work. So, the objective here is to dissipate energy. Unlike eddy viscosity model which is modeling all the scales, in SGS models objective is to dissipate energy. I am computing the energy in the energetic range and inertial sub range. that energy is given to SGS model and SGS model single most objective is to dissipate it.

If it is not dissipating your spectrum will not look good ok. There will be energy accumulation in the system the code can crash or you may get some funny results that is energy has to be dissipated out this is being pumped into the model the model must dissipate. So, the objective is to dissipate energy. ok unlike eddy viscosity models right. So, you can say here that is unlike eddy viscosity models whose objective is to model all scales of turbulence.

here it is only to dissipate the energy that it is being sent to it ok. So, now what is the unknown in the filtered Navier-Stokes equation? $\tau_{\text{ij sgs}}$ this looks like $\overline{u_i u_j} - \overline{u}_i \overline{u}_j$. I told you already in LES we are computing filtered velocities that is $\overline{u}_i$. right we have no access to $\overline{u_i u_j}$ this is an unknown here and therefore, this entire is resulting in 6 unknowns right. So, this has to be modeled right its entire part the $\tau_{\text{ij Sgs}}$.

So now let us take help from RANS to see whether we can help. RANS is a much more you know say ambitious modeling because you need to model all scales of turbulence right. If that has accomplished something then it is a much more simple job here because most of the energy you are computing most of the scales you are capturing only dissipative scales has to be modeled right. So it is much more easier here. So we can perhaps learn something from eddy viscosity models.

So, let us say analogous to eddy viscosity models. So, let us say here recall evm where you had where - $\overline{u_i u_j}$ the averaging is ensemble averaging here we are moving to the RAN's world eddy viscosity model right. This is equal to Boussinesq 2 $\nu_t \overline{s}_{ij} - \frac{2}{3} k \delta_{ij}$ correct this is a Boussinesq that we used Boussinesq. So, now this I can rewrite this as

$\overline{u_i u_j} = -2\nu_t \bar{s}_{ij} + \frac{2}{3}k$ what is k? k is nothing but ½ $\overline{u_i' u_i'}$. So, I can write this as ½ $\overline{u_i' u_i'}$ δ_{ij}.

Now, there is a problem I told you indices should not repeat more than twice. So, I cannot have $\overline{u_i' u_i'}$ δ_{ij} correct. So, I will take this as $u_k' u_k'$ it is a free index it is a sum of 3 anyway $\overline{u_k' u_k'}$ will be sum of the 3 normal stresses right. So, this will give me essentially - $2\nu_t \bar{s}_{ij} + \frac{1}{3}\overline{u_k' u_k'}\, \delta_{ij}$

Now, I can use this concept here ok. So, analogous to Boussinesq, analogous to Boussinesq. Boussinesq hypothesis, $\tau_{ij\ SGS}$ can be modeled as $\tau_{ij\ SGS}$ model. This is of course, exact what we have written above. So, this is the exact one. The model is- 2, can I write ν_t? ν_t can I write or we have to do something else here? No, ν_t accounts for all scales of turbulence, we are not doing that.

So, I need ν_{sgs}, I need is so called viscosity associated with sub grid scale motion. Again numerical argument this is not a viscosity minus 2 ν_{sgs} $\bar{s}_{ij}$. Now, where is this S_{ij} coming from? and what is the over bar meaning here $\bar{s}_{ij}$ means in LES filtered. So, this is the filtered strain rate that is available to you this is known right $\frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$ are calculating $\bar{u}_i$ filtered velocities. So, the filtered strain rates are available to you this is a known thing nu Sgs is an unknown we will see what we can do with that one plus 1/3 u_k' u_k' was there.

So, I am going to that means, here this should be τ_{kk} correct, τ_{kk} δ_{ij}. This is your closure for the $\tau_{ij\ SGS}$. - 2 ν_{sgs} $\bar{s}_{ij}$ + 1/3 τ_{kk}. So, where we can say here this $\bar{s}_{ij}$, τ_{kk} both are filtered data available to you. These two are filtered variables that is known to you.

ok filtered variables or known let us say it is ok and nu sgs is an unknown right. So, ν_{sgs} is the unknown. So, this is one option of modeling we still have not figured it out what is ν_{sgs}. So, for that we can say there is a small variation to this let us say this equation number we call it 1 here or I can say star. So, in equation star sometimes it is popular to replace this τ_{kk} like using the k like here in the eddy viscosity, but that k becomes SGS kinetic energy not the for the whole energy spectrum.

So, that is popular. So, that version is also used. So, the equation star sometimes is written as in equation 10 this τ_{kk} is sometimes replaced with 2 k_{sgs}. the reference for this in the Pope book ok C Stephen B Pope. So, τ k is replaced with 2 k_{sgs} then therefore, τ_{ij} SGS model is 2 ν_{sgs} $\overline{s}_{ij}$ + it becomes now 2 k_{sgs} δ_{ij}. This is an another option compared to the first that star equation star is one option.

This is an another option where both ν_{sgs} and k_{sgs} are unknowns. Here now, so here ν_{sgs}, k_{sgs}. are unknowns in this version, the second option you can say. And we can model this new SGS analogous to or the like dimensional grounds that we do right. Since it is kinematic viscosity it has to be meter square per second ok.

So, we can say new SGS is equal to using let us say mixing length hypothesis ok. So, let us say analogous to mixing length, mixing length model that is where ν_t was set equal to a mixing length followed by a time scale which was your du dy ok. So, therefore, it becomes m^2 (1 / s). So, analogous to this I can say now ν_{sgs} let it be equal to I need a length this length is should be sub grid scale length. What is a sub grid scale length? I know the grid size.

So, I am going to use the grid here itself. This is the one factor not one factor this is the factor which makes LES solutions not grid dependent sorry grid independent solutions are not possible in LES because the length scale that is actually the mesh that is going in here ok. So, the length scale has to be a sub grid length correct. So, the L has to be a sub grid length and I am going to use Cs Δ. So, this is the sub grid scale length or mixing length or whatever you want to call it mixing length or SGS length scale. So, obviously, it has to be smaller than delta right some factor Cs has to be.

some decimal. So, Cs Δ^2 because it is L^2 m^2right and the strain rate I can use that is the resolved component I have access to. So, I take help from the resolved strain rate. So, that is equal to Cs Δ^2 this is nothing but $\sqrt{2\overline{s}_{ij}\overline{s}_{ij}}$ So, this gives me m^2 (1 / s). dimensionally sound right. So, here note that this particular Δ, Δ is filter size that is a numerical length scale.

So, your ν_{sgs} is depending on the mesh. So, it is mesh dependent right mesh dependent. Grid independent solution is not possible now because you are always computing ν_{sgs} no matter how small delta is some contribution is going into this one a numerical length scale has been put Δ and S_{ij} is the as I already told you this is the filtered strain rate ok. That we already know what it is it has to be $\frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$ and what is the filter size delta. So, this filter size delta is nothing but your $\left(\Delta v_{ijk}\right)^{\frac{1}{3}}$ that is nothing but your $(\Delta\, x\, \Delta\, y\, \Delta\, z)^{\frac{1}{3}}$ $\left(\Delta v_{ijk}\right)^{\frac{1}{3}}$ is the your index that you are using while coding ijk index and a control volume size delta v.

So, local grid size is being used here local grid size. So, this is implicit box filter infinite volume method right. So, this is what you are using one disadvantage in this model ok. I did not name this model that is strange this is actually called Smogoransky model I left out the main thing here ok. So, this particular model is the is called Smogoransky model.

whatever we had discussed so far the SGS model part right. So, this is the Smagorinsky modeling the reference for this is Smagorinsky 1963. So, this precedes almost one decade for the your standard k epsilon model. So, an SGS model was already available before and that is true that the first CFD calculations if I am not wrong were done for were done by meteorologists. They were calculating the weather using the so called very large eddy simulations VLES or even VVLES very very large eddy simulation because the mesh was so large you know probably a 10 kilometer is your mesh size or something like that. So, in those times so they were calculating much ahead before RANS models came.

So, it is much earlier 1963 and one disadvantage of this smagoransky model is that disadvantage is the model constant Cs right. So, the Cs value is user dependent Cs is user dependent Obviously it has to be a fraction because the length scale has to be smaller than the delta at all. So, Cs has to be some value and there are some grid guidelines you can take or you can say it is like best practice guidelines. So, for some flows it is available what this value has to be. So, the Cs can take for example, example the value can take Cs is equal to 0.

065. this is in channel flows ok. Similar to like pipe flows, plane squared flows, Poiseuille flows all these things and when you are going to open share flows, free share flows like jet it becomes 0.25 in a jet turbulent jet flow. So this is the disadvantage of the Smagorinsky model C_S has to be known if C_S is known it is straightforward the model is very robust and stable right and not much calculation is required you just have access to the filtered strain rate you know the delta the mesh size at any given point C_S if you happen to know what is good for a particular flow it is straightforward to use Smagorinsky model. for the SGS contributions ok. Any questions on this I will take before we move to slight variations of this model as well as other types of models, other class of models.