Vibration Control
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Module - 4
Vibration Generation Mechanism
Lecture - 5
Damping: Models and Measures – 1

Hi, this is Dr. S.P Harsha from mechanical and industrial department, IIT Roorkee. In the course of vibration and control, right now, we are discussing about the vibration generation mechanism. In which, we discussed about the source classification. We discussed about the self excited vibrations, we discussed about the rigid oblique flexible rotor, balanced and unbalanced conditions.

So, in these cases, we have seen that what exactly the physical phenomena’s are being there when we are trying to analyze these as the sources of vibration generation. And also in the last lecture we have seen that, if we have the rigid rotor or even if we have the flexible rotor, then how the things are being changing accordingly. And then if we are just applying a mass balanced condition or even the entire balancing features the machines by adding the weights or by putting you know like the different things the things are being balanced, but if we are just going on the field balancing part.

In which, you see you know like the various surrounding conditions, the operating conditions, and then you see if you are trying to balance the things, then you know like we need to adopt the different methodology, as we discussed in the previous lecture. We also discussed that how the amplitude and the phase both can be controlled effectively by putting the weight corrections in their according. So, that the principle axis where the mass moment of inertia is there and the axis of rotation can be matched equally.

So, in these cases we discussed about, the unbalanced mass, means you see the mass distribution is not uniform or even you see here some eccentricities are there or else even we discussed about the misalignment feature, we discussed about the looseness. And even we discussed about that, how to design the balancing feature? When the field balancing process is there.

So, there are various equipments in that again. It is again you know like subjective to the conditions or the equipment. Accordingly, we need to adopt equipment means the object or the machine through which the vibration generations are there. We need to adopt the process, the design and the various equipments which are being involved to correct the masses or to balance the things.

Now in this lecture, we are going to discuss about, the vibration generation mechanism. In which the various you know like the damping models and the measures are there. Now here we are trying to see that, how a damping can be adopted? Basically, from the mechanism side. So, that we can simply justify that, how the energies are being dissipated from the source to the outside. Means how the energy can be converted.

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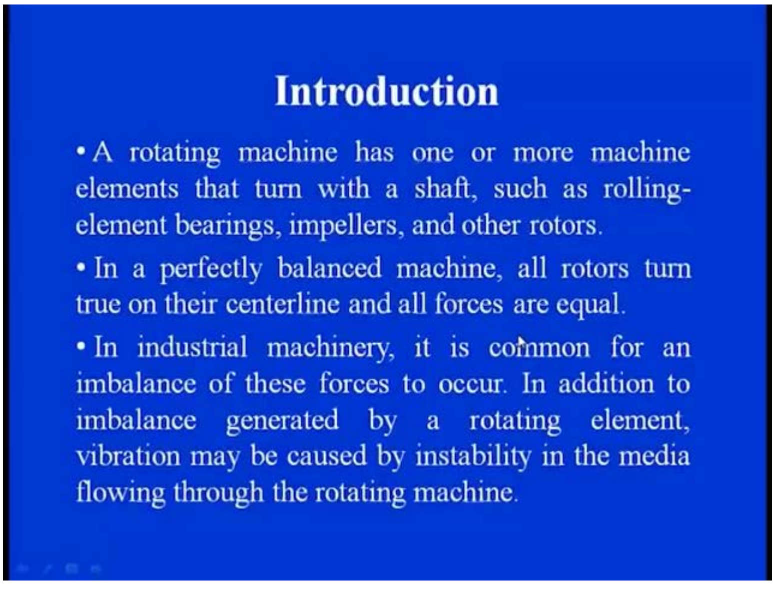

So, you see here. In this part, as we know that the rotating machine, which has you see more than one element. Under you know like the rotating feature and turned with the shaft like the bearings are there like the gears are there the impellers are there there are other you see the mounting rotors are there. So, all these you see here you know like contributing some part or even the significant, in terms of the vibrations. When we are talking about the balanced machine, all these features are absolutely in the co-plane and co-linear with the central line. What are the forces the centre line of the axis of rotation? And what are the forces which are being generated, they can be well balanced.

So, when we are talking about the industrial machinery, it is very common that the imbalance is there and these forces which are being generated due to imbalance. We need to check it out what the significance of these things are. Because, these imbalance forces are basically generated by all these rolling elements and they are causing the vibrations in the entire machine. And then even they are causing the instability features in all these components.

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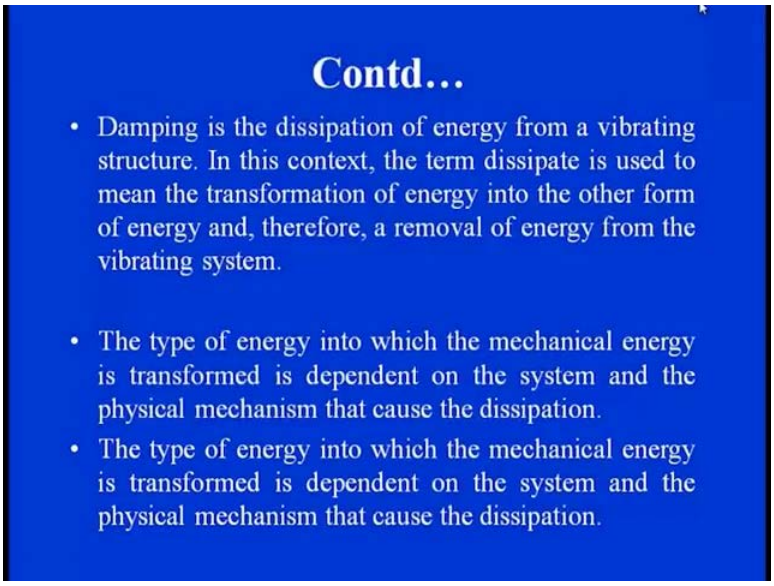

So, here when we are talking about the instability feature, the damping is one of the phenomena through which the dissipation of energy is being occurred, from the vibrating structure. In this you see here when we are talking about the dissipation of energy, that means the transformation of the energy into another form of energy. And then you see there is and you know like removal of energy from the source of excitation.

The type of energy into which the mechanical energy is transformed is absolutely depending on, what kind of systems are? What kind of the forces you know like which are being emerged out? And then how the energy is being you know like created at the time of, no not created. How the energy is being formed at the time of excitation? Sorry.

The physical mechanism, that causes this dissipation is absolutely depending on the type of damping feature, which we are adopting here. And the type of energy into which the mechanical energy is transformed, as we discussed that, is depending on the system. Then we need to see, that how these you know like the cause of these things are there.

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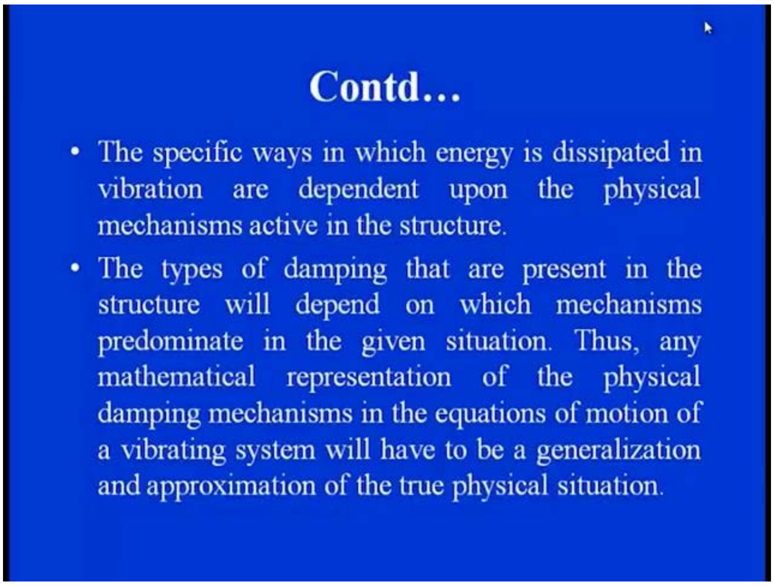

So, when we are talking about the dissipation, these are the specific way in which the energy is being taken out from the system. And these mechanism are being designed accordingly. So, the type of damping which we discussed already in the systems, are having three types of mechanism. In this present case, the structure, the structure which is depend on the entire you know like we can say the energy formation, on which the mechanism is predominant. We need to check it out that how the structure is under vibratory condition.

And any mathematical representation of the physical damping mechanism, is simply coming, in terms of the force balance or the equation of motion. We need to generalized accordingly. According to what the physical system is and we need to try to put in analogy. That how the mathematical representations are there in the physical system?

So, long back in 1970, the Scanlan just observed, that mathematical damping is really only a crutch, which does not really give the detailed explanation, about the real damping is being occurred at the time of operations. So, if you are talking about the mathematical way, there are some convenience which we are trying to relate. So, that we can justify that, yeah there is you know like, one source through which the dissipation is occurred. As we can simply put into three different classes. Just to justify that. One the damping which has the linear propoagtion. It is only working with the single degree of freedom system. In which you see we are only allowing to work this damping phenomena in one direction.

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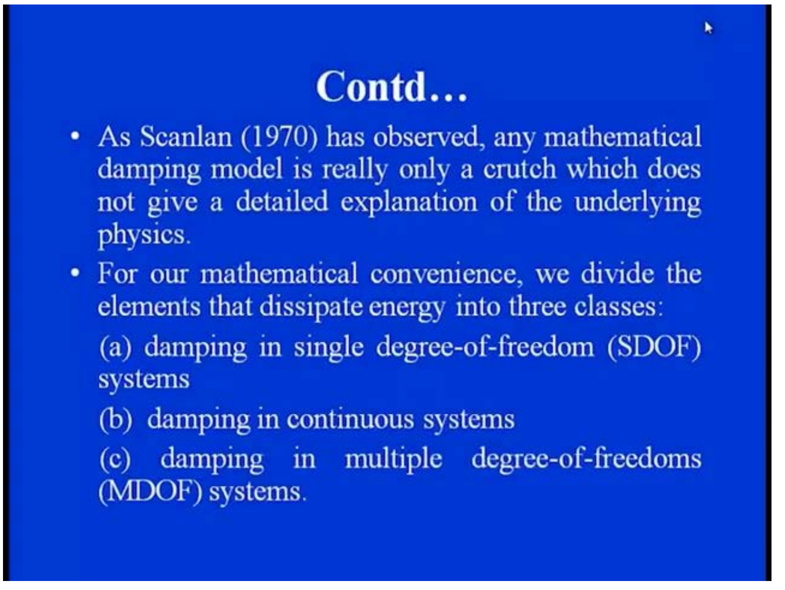

Even we can apply the damping, not to discrete system, to the continuous systems. Because sometimes we have seen that, the damping, when it is working, it is just working for the entire continuous system in the uniform way. Third, we can adopt the you know like we can say ,some kind of dissipation of energy through the elements into the multiple degrees of freeedom. According to the variable which are being available. Through which we can describe, the dissipation of energy through these coordinates. So, the damping can be designed for multi degree of freedom system.

So, again you see these three concepts are working in a proper way. We will take you see individual sections in the further part of this lecture. So, elements such as the damper of a vehicle suspension is absolutely coming into the single degree of freedom system. Because, we are restricting the motion of the damper into one direction. It has to act all these forces or to dissipate the energy or to absorb the energy in that direction only.

So, dissipation within the solid body, falls into the continuous part. Because you see here when we are talking about the solid body, it has the continuous feature. We cannot made the we cannot make the discrete points. Especially for mass damper or spring like that There is a representation which accounts for both intrinsic and the spatial distribution. So, whatever the intrinsic properties are there and it is distribution can be taken care in those kind of damping representation, when we are talking about the solid body.

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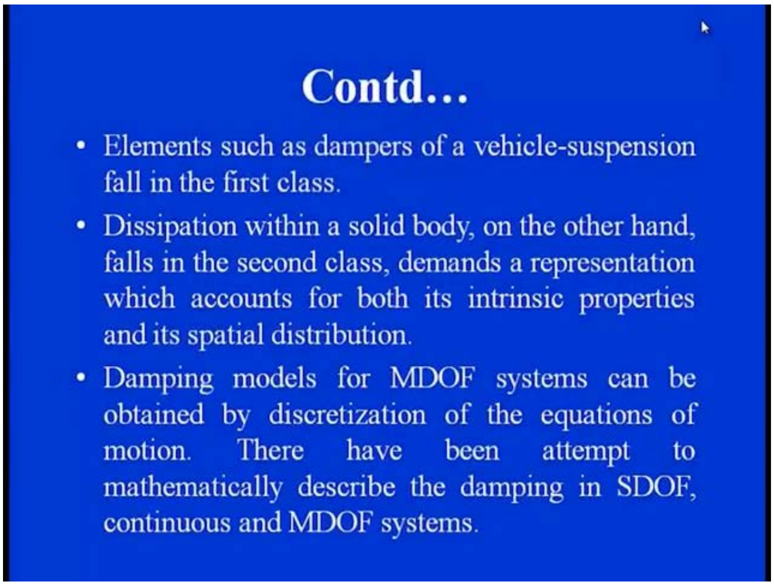

So, it is a continuous feature and the damping models for the third, like the multi degree of freedom systems. Can be obtained by the discretization of the equations of motion. We need to discretize into the various degrees of freedom, in which we are allowing the motion. So, these have, we can say the there is an attempt in the mathematical way, to describe the damping in, single degree, continuous and multi degree of freedom system. According to the operating condition and the system parameters.

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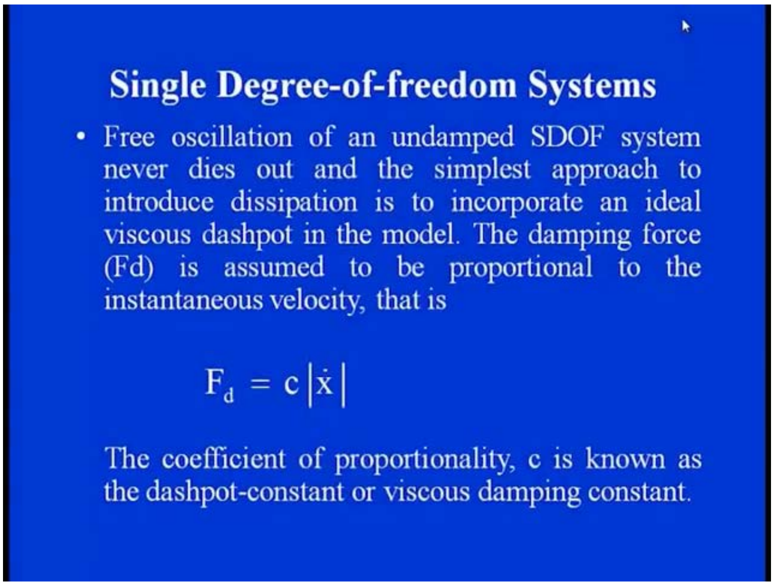

So, let us talk about first situation. That is the, single degree of freedom system. When we are talking about the damping models, we need to check it out that, how the damping is to be designed in these coordinate systems? So, say you see the free oscillation of an undamped single degree of freedom system, which we already discussed, which will never dies out. Simply we can say that, it has a clear oscillation feature which introduce the dissipation into the in incorporate an ideal viscous dashpot.

So, when we are talking about the undamped and damped system the only difference is the dashpot, which has to be there in the damping feature. The damping force, which which we assumed, to be act in one direction here, is also being assumed that it has a linear propagation with the velocity component, which is followed by a c constant and this constant is absolutely a function of the viscosity of the oil, which is being there in the damping device.

So, these assumptions are clearly showing that, we are only considering the damping in one direction and that too, the variation of velocity according to the motion of or the oscillation feature of the object is in the linear way. So, if you are talking about the viscous force. It is nothing but equals to c into velocity. And the coefficient of this proportionality, for the linear propagation is known as the dashpot-constant or viscous damping constant. So in other way we can say that, we can directly adopt those things the coefficient and the velocity. When we know that the damper is only be, valid in one coordinate system and the damper is only acting according to the desired one.

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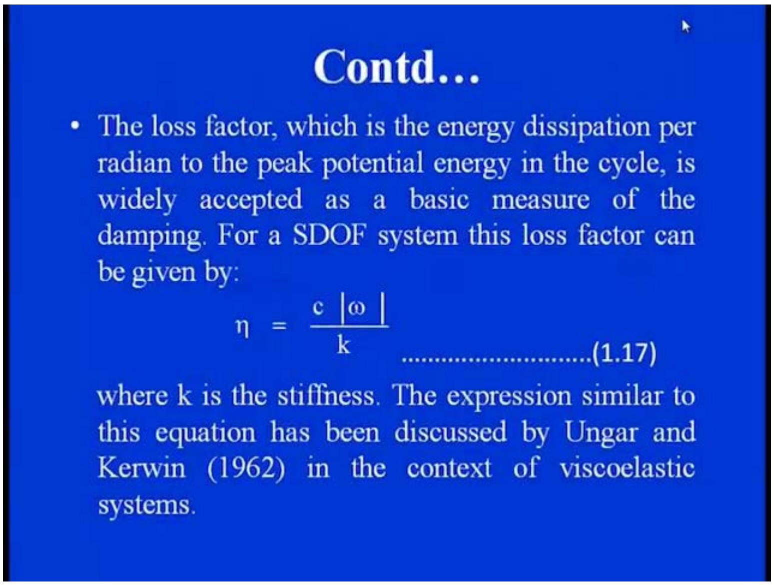

In this case there is another one performance factor, called the loss factor, which is nothing but the energy dissipation per radiation to the peak potential energy in every cycle. And you see when we are trying to measure, the damping it is absolutely you see the adoptable concept. Because, we need to check it out that, how much loss factor is being taken in these model? So, right now you see, when we are talking about the single degrees of freedom system, we know that the loss factor is nothing but equals to c into w divided by k. Where the c is the coefficient the viscous damping coefficient or we can say the proportionality coefficient for the damping forces into omega.

Omega is the natural frequency. So, when we are talking about the loss factor we need to check it out that, what is the frequency at which the excitations occurs and how much energy is to be dissipated by this, divided by the stiffness, because the stiffness is also one of the key design parameter, which needs to be there to control the entire vibrations. So, the expression this, neta, which is nothing but the loss factor is equals to c w divided by k. Is absolutely similar to the equation, which was derived by the Ungar and Kerwin in 1962, when they have taken the viscoelastic system for dissipation of the energy.

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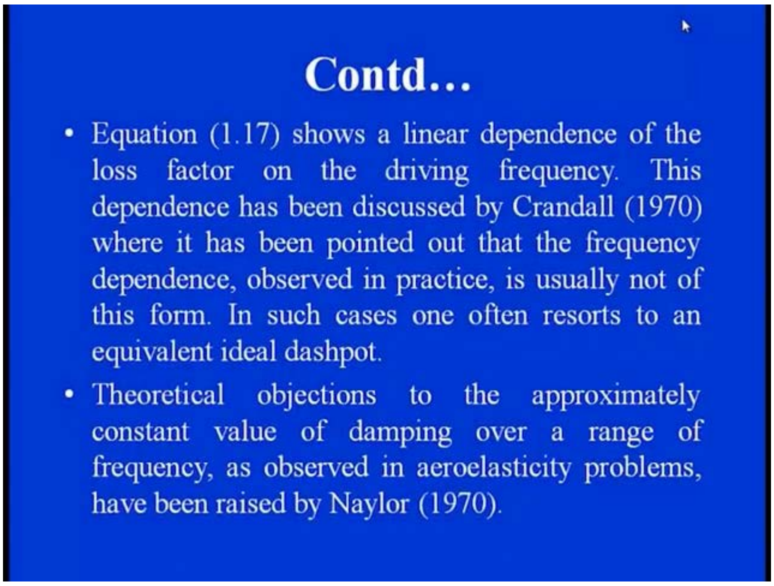

So, the equation clearly shows that, there is a linear dependence of the loss factor on the driving frequency. It has to be, because we know that, when the driving frequencies are dominating, certainly more and more loss factors are being occurred for modelling of these dampings. This dependence, which was discussed by Crandall, in which it is simply pointed out that, the frequency dependence feature is usually not in the linear way.

In some cases, we need to make an equivalent, ideal dashpot to just replace this loss factor concept. So, that is why you see here the loss factor when we are trying to formalize, we know that the system parameters and the operating conditions both are responsible for measuring this loss factor.

But there are some theoretical objections are being there, which are just trying to approximate the constant value of the c, that is the damping over the range of the frequency. So, sometimes you see, when we are talking about you know like some aeroelastic problems ,which was been discussed by the Naylor in 1970.

It was clearly shows that, how we can take the constant value of damping, which is not you see you know like the linear propagation there is a non-linear propagation is there according to the deformation or the complex form of damping. But, he clearly explained explained that how we can take, how we can make an approximation in those things. What are the clear clearcut you know like the objections in taking those things?

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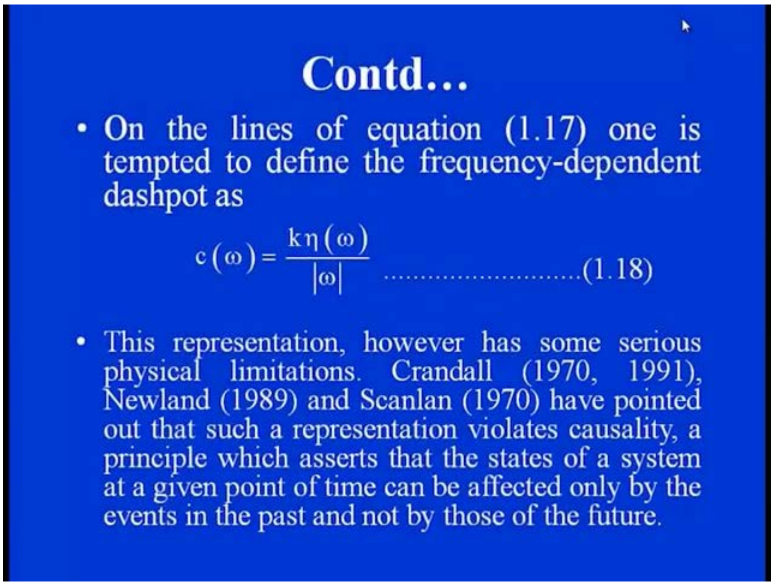

So, on that basis you see here, we can say that, the frequency dependent dashpots are nothing but c w equals to k neta w divided by modulus of w. So, it is it is it is pretty clear that when we are trying to see the damping which is the frequency dependent coefficient c w, is nothing but equals to k. That is the stiffness the neta, which is the loss factor, which is also a function of the frequency feature divided by the modulus of that frequency component.

But, again you see here, it shows the linear propagation of that and that is why there are lots of limitations are there in that. So, many of the researcher right from the Crandall to Newland to Scanlan, they simply pointed out that, you see when we are trying to show this linear propagation, whatever the casuality which is being happening there is absolutely violating this linear propagation law.

So, principle which asserts the state of the system, can be given out with respect to the time variation and it is affected only by the event which is being passed and something which is going to be happen future. So, we need to incorporate all those situations or in terms of the variation. When we are trying to formulate the the dashpot coefficient or the damping coefficient frequency dependent.

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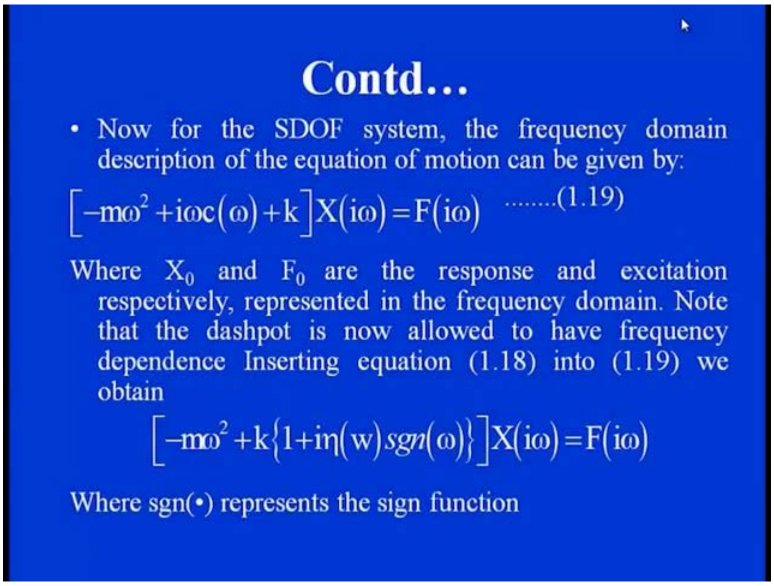

So, now you see here with this concept now, if you if you would like to formulate the problem, which is the frequency domain dissipation, there you see so we can say that, it is minus m omega square plus i omega c of omega plus k into whatever you see you know like the displacement x of i w equals to the forcing forcing feature through which the excitations are there f i w. Where we are simply see that, whatever the responses in terms of x or in terms of f, are nothing but giving us the final outcome response and the excitation feature which is being you know like there in the frequency domain.

The dashpot which is now you know like allowed to have a frequency dependence feature, we can straightaway find out that, when we are adding those, the harmonic features in that it is nothing but equals to minus m omega square plus k into one plus i neta. Again you see here we are trying to formalize what exactly the c value is into the sign omega.

So, now you see this is a clear variation with the loss factor incorporating in terms of the damping and then it is you see x i w equals to f of i w. Where this sign omega is clearly showing the variation in the harmonic way and when you see this variation is there we can starightaway incorporate the sign function accordingly. But, if you want to you know like describe this variation in the time domain feature as we discussed you see already in our previous cases.

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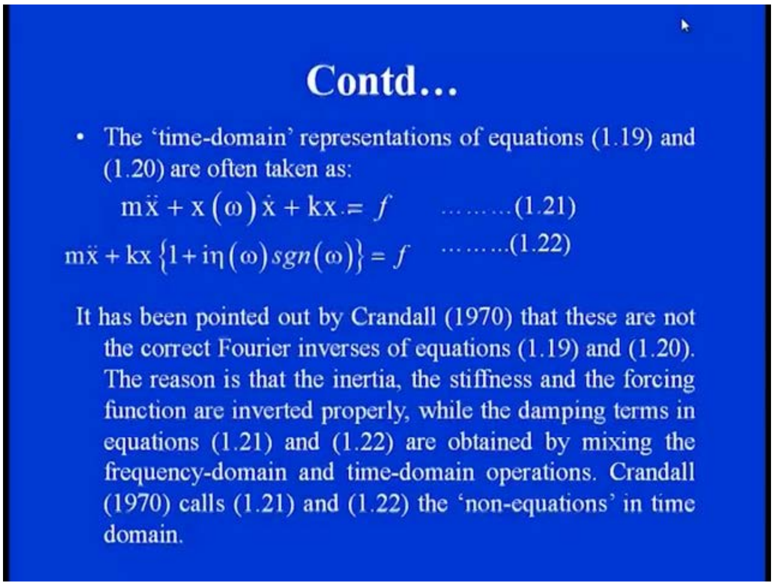

Then, it is a simple equation for all these kind of forces, m x double dot, means m d two x by d t square plus x of w x dot. So, here we can say that, this damping which is you see you know like coming as the function of velocity is a linear propagation with the dashpot forces or the damping forces plus k x equals to f. Even if you would like to add the loss factor in these equations then we have the inertia force and x double dot plus k x into 1 plus i out and eta omega sign omega equals to f.

Then you see in these equations the Crandall clearly find found out that, when we are talking about this, the loss factor then the Fourier inverse equations like you see in which we have shown previously is not the correct representation of that. Because, the inertia forces the stiffness and whatever the forcing function which are being there, are not you see you know like clearly interpreted and inverted properly when the damping terms are being coming out in the mixing form of frequency domain and time domain operations.

So, these identifications of you see you know like the k x plus 1 1 plus iota neta w. Crandall calls these as, the non equations in the time domain. Because, they have a clear representation the frequency domain. But, as far as the time domain is concerned they are absolutely non equation form.

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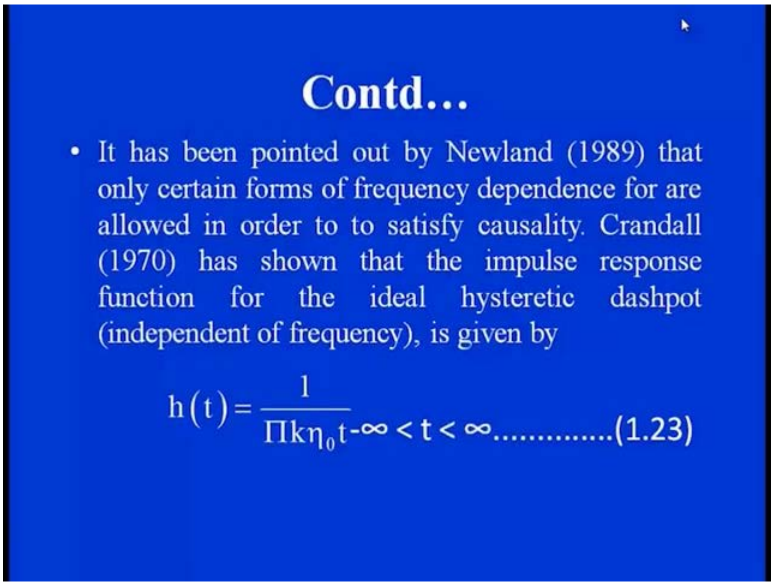

So, the Newland in 1989 says that ,only certain forms of frequency dependence is being allowed for certain kind of causality. Because, it is just satisfying those things only. So, then you see here, when we are taking those causality in form of the impulse response function. Then we can say that, for these things we can represent the ideal hysteretic dashpot, which is independent of the frequency function. We can adopt directly for these compensation. So, we can say that this is nothing but equals to the the ideal hysteretic dashpot function. It is simply giving h of t is nothing but equals to 1 divided by pi k neta 0 t. Where the t is just varying with the infinite features minus to plus.

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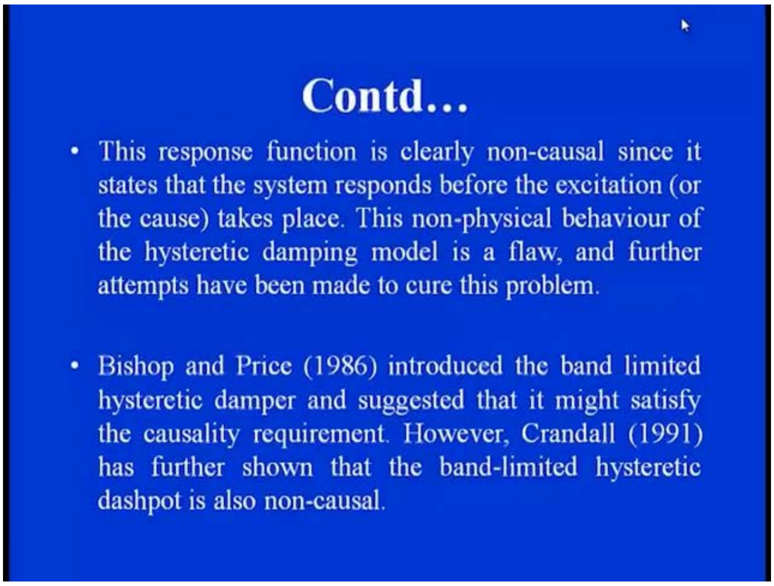

So, this response function which is clearly showing about the impulse part, is very clear about the non-causal. Since, it is states that, the system responds before excitation or any cause basically takes place. The non-physical behaviour of this hysteretic damping model is a flow, is a flaw and further attempts to be needs to rectify this problem. So, this flaw cannot be perfect we can say representation of that. Then in 1986 the Bishop and Price also introduced, some kind of you know like the band limited hysteretic damper.

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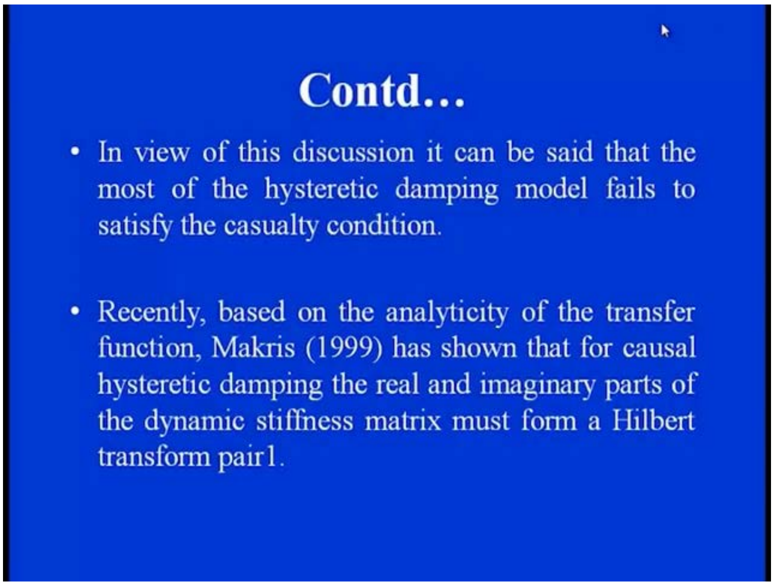

They are just trying to formulate that how you see the causality is being occurred and how we can compensate that causality requirement accordingly? Then you see here, the

Crandall in 1991 adopted those features and added that a band limited hysteretic dashpot is a non-causal.

So, what I mean to say that, in all these discussion the most of the hysteretic dampers fails to satisfy the kind of causality conditions, which is one of the requirement of the damping is. So, we can say that when we are just going according to the transfer function the Makris simply shows, in 1999 that, if we have some kind of causal features, then the hysteretic damping has the two main parts the real and imaginary. These parts are clearly showing about the dynamic stiffness matrix from the Hilbert transport Hilbert transform pairs.

So, when we are talking about the hilbert transformation, these these two features the real and imaginary feature. They can clearly show about the significance of the causality and they can be taken care. It was also shown that the continuous transformation from the linear viscoelastic model to ideally hysteretic model is really the significance part.

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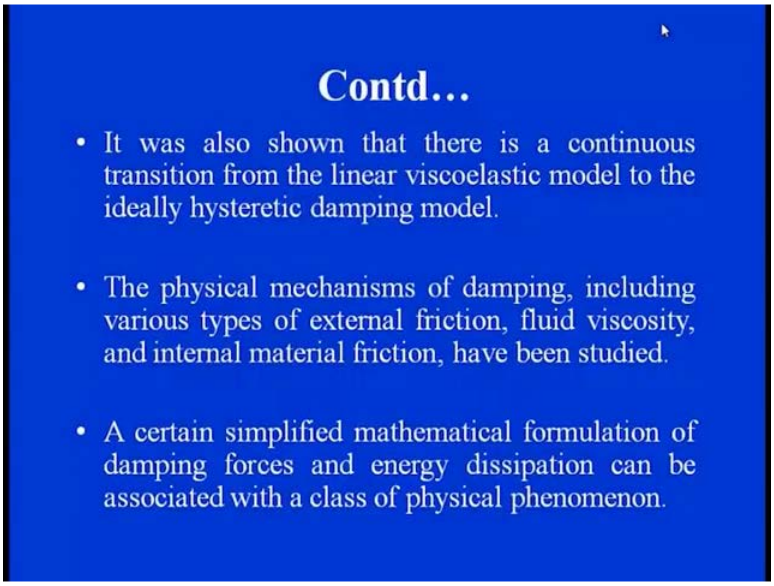

So, now you see we need to understand the basic mechanism for that and the physical mechanism of the damping, including various features like the external frictions, the fluid viscosity or any internal material frictions can also be added in those things. Then only we can understand the physical, real physical mechanism of the damping.

So, certain simplified mathematical formulation, for computation of the damping forces and their energy dissipation can be associated with a class of physical phenomena which are to be occurred either for external friction, fluid viscosity, the fluid basic property or any internal material we can say in which the molecular interactions are there. So, they can be straightaway considered in that.

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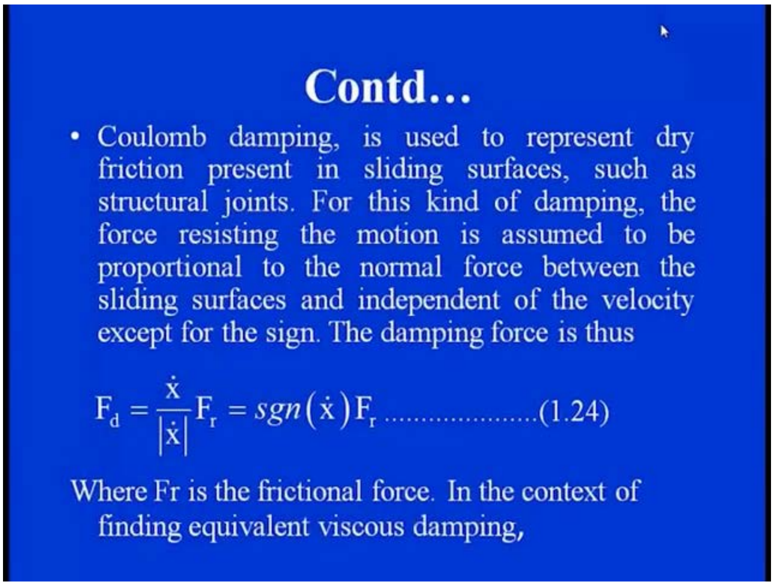

So, when we are going towards that, the Coulomb damping, which is a clear representation of the dry friction, available at the sliding surfaces or any structural joints we can straightaway incorporate those things. For this kind of damping, the force which is coming out due to the resistance of motion is assumed to be proportional to the normal force, which are being acting in between the sliding surfaces. But, it is absolutely independent of the velocity, except for you know like we can say we are just using for the sign and all.

So, for these cases, the damping force is nothing but equals to the x dot, the velocity, divided by the modulus of that x dot into F r, the frictional resistance force or else we can say it is nothing but equals to the sign omega. Only because we are just going with the direction, when we are dividing the x dot by the modulus. Sign omega or sign into whatever the velocity function into F r, which is the frictional force.

And in this we can say that the frictional force is nothing but equal to the equivalent viscous damping force. So, when we are talking about this, we need to check it out that, what are the mathematical models which are basically, put the analogy for such kind of physical damping mechanism in single degrees of freedom.

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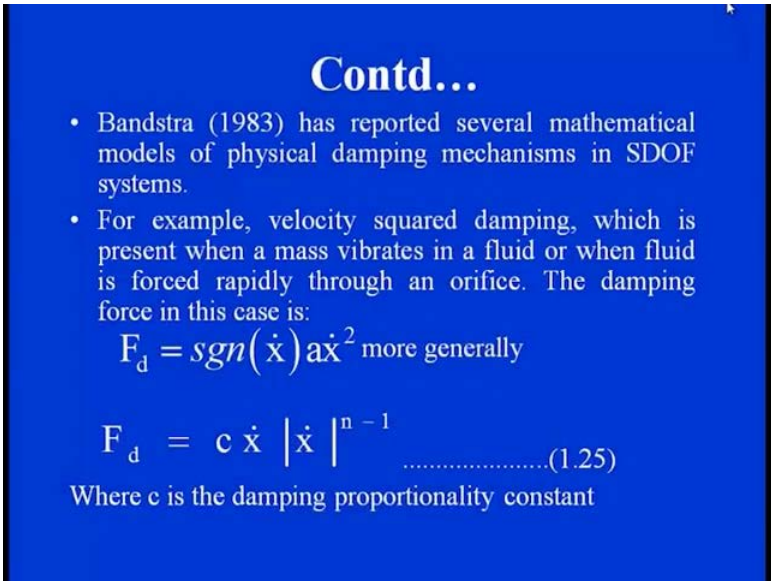

So, if you are taking you see from the Bandstra in 19983, he said that, velocity squared damping, which is simply a representation of the mass vibrate in the fluid or when the fluid is forced rapidly through an orifice. Then we can represent such kind of damping forces F d equals to sign omega into a velocity square a x dot square or else in general we can say that the F d is nothing but equals to c x dot x x dot to the power n plus n minus 1. Because, you see it is just you know like we need to multiply with this. Where the c is the damping a simple damping coefficients or the proportionality coefficients.

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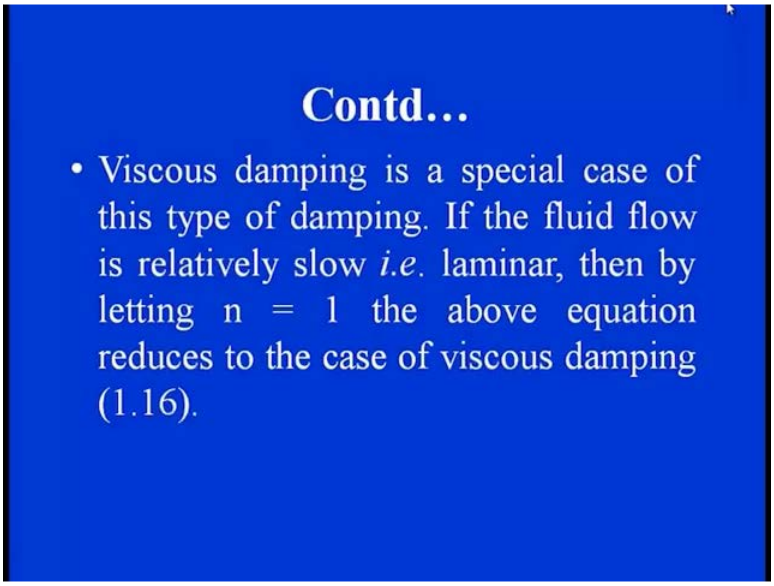

The viscous damping in these cases, are one of the significant damping, when the fluid flows are there. So, if the fluid flow is relatively low in terms of the Reynold number also that means the laminar feature. Then we can say the n equals to 1. Then the whatever the viscous features are there they are absolutely due to the viscous dampings.

But, if the turbulent features are there then the n becomes more than 1 and we can we need to consider the other factors of the damping along with the viscous dampings. So, this is what you see the single degree of freedom system, in which either way whether the system is for the fluid flow or a normal part, we need to consider the loss factor the damping and even the fluid flows are there then the different way of damping consideration.

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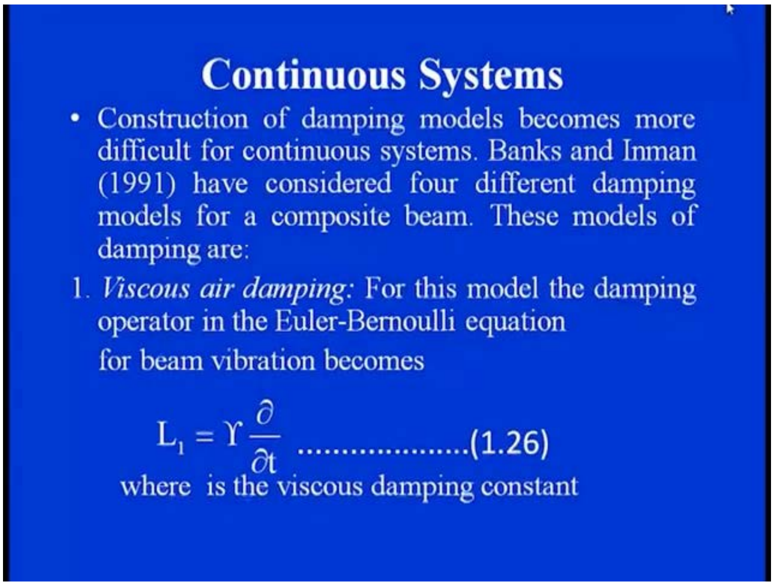

Now, we we are moving to the second feature in this is a continuous system. So, for these systems, when you cannot make the discrete value of the damping stiffness and the mass the damping model construction is very difficult of the complex phenomena. Because, in the continuous system the properties are so linked, that the interdependence are one of the important feature in this. So, professor Inman from Virginia Tech along with the banks in 1991, they considered the four different damping models for a composite beam which is simply a representation of a continuous system.

So, in that they are going to discuss briefly about those here. The first model which they considered here is, viscous air damping. For this damping, they adopted the Euler-

Bernoulli equations for this. So, let us say we have a composite beam, which is vibrating. Then we can say that the beam vibration becomes gamma into del by del t. Where we can say, the gamma is the viscous damping coefficient.

So, when we are taking those things, we know that it is a clear variation of the damping operator, along with how the damping coefficient is being concerned. The property we can say that is you know like the propagating property what exactly the state of state is. Means the variation of that. So, this damping operator is one of the important feature for the air damping part.

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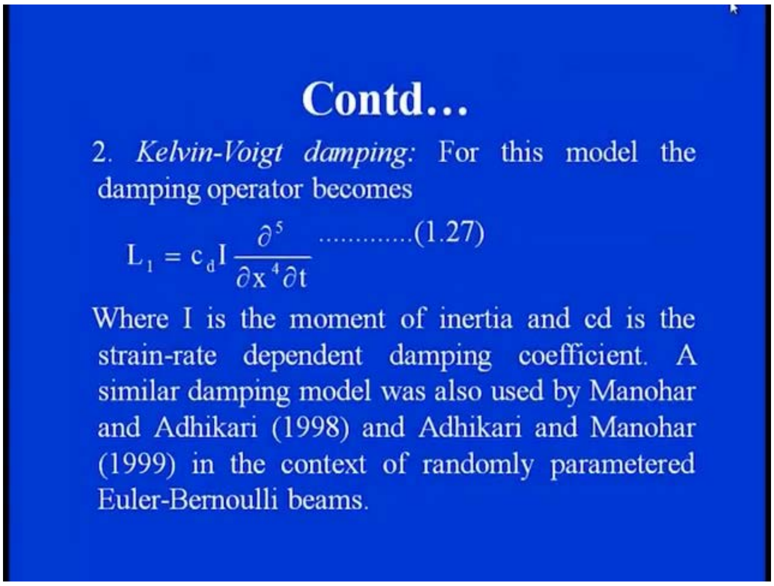

The second damping model is considered as the Kelvin-Voigt damping. In this you see here the operator becomes now the, c d into I del power 5 divided by del x power 4 del t. Means in this damping operator not only we have the coefficients, but also the propagating property has a clear dependence on the space.

Means we are considering the gradient of that and with the time as well. Where I here is the mass moment of inertia and c d is nothing but the strain rate, depending on the damping coefficient. So, this is something you see you know like the model which was further you see modified by the Adhikari and manohar in various ways for you know like the a random or any complex situation for the same Euler-Bernoulli beam.

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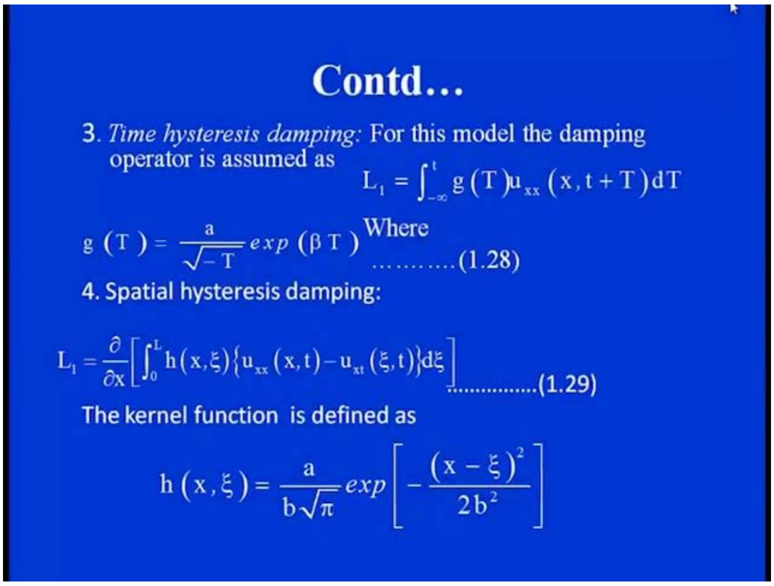

In third way we can say that this is the time hysteresis damping. So, in this third model the hysteresis values are considering which has the variation with the time. We can consider this coefficient L for entire summation of minus infinite to the t variation g of T u x x with x of t plus total time into dT. Where g of T is a function or operator we can say which is nothing but equals to a e to the power B T divided by minus t.

So, we can say that when we are considering these things, the functions they have a clear expansion. In the exponential feature along with what the operating functions are. Finally, you see the model which was discussed with the spatial hysteresis damping, in which, if you more you know like we can say the variations are being considered. Like the operator L 1 is nothing but equals to del by del x. Means now the entire property has the variation with the space, and that property is integral of 0 to L h of x comma zeta. Means now the height the thickness of the you know like the damping feature, is the variation of x and one normalized coordinate.

Then you see the velocity component that u x x is depending on the space and the time u x comma t and also you see here the u x t is there which has in the normalized, we can say direction zeta comma t d zeta. So, in that they considered the kernel function and this kernel function h which is which is depending on x comma zeta, is nothing but equals to a by b square root of pi exponential e to the power minus x minus zeta whole square divided by two b square. Where a and b are the coefficient depending on the system parameters and the operating conditions.

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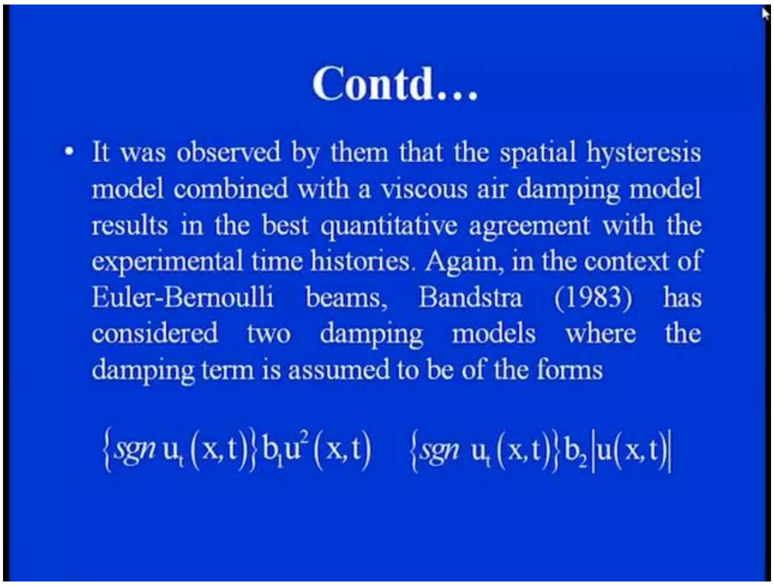

So, in these you see the spatial hysteresis model final one, which is simply combination of viscous air damping model and the other models. Just gives the best quantitative agreement with the experimental time histories. Again in the context context of the Euler-Bernoulli beams, which was discussed by the Bandstra in 1983, the two bending models have been assumed here.

One that is the sign u 1 x 1 comma t b 1 u 1 square x comma t. That is one you see in which the sinusoidal feature of the velocity along with you see the parabolic variation is being considered. Second you see the form, in which the damping term is assumed there in the damping model. Is sin of u x 1 comma t into b u b 2 u of x t. That means you see here here they are considered only the modulus feature the linear propagation of the velocity along with the sinusoidal part. So, these spatial hysteresis model in such cases, are the perfect one you see here in which there is a clear consideration. As you can see that there is a clear consideration of the kernel functions and how the velocity variations are there in this operator.

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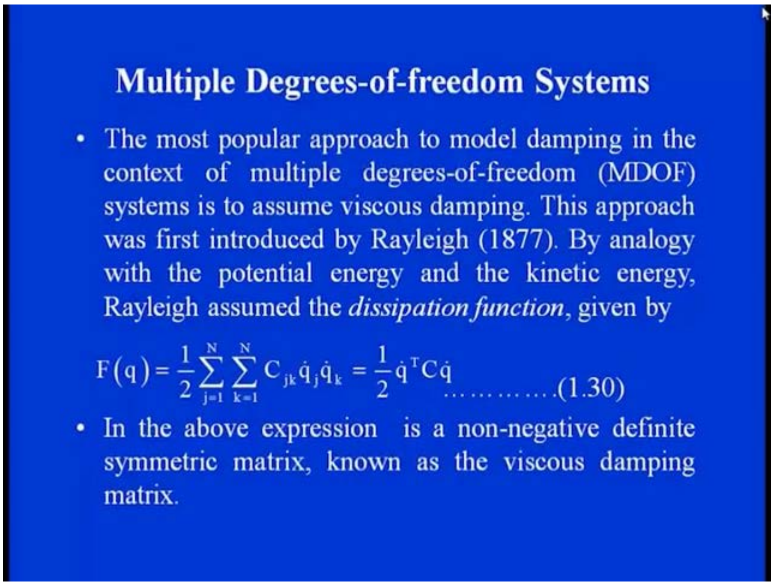

When we are going to the third model there that is the multi degree of freedom system. Then we know that, in the multi degree of freedom system most of the popular model or approaches are simply going with the viscous damping. So, even when in 1877 the Rayleigh just gave the theory, they also considered the potential energy and the kinetic energy.

Then you see they assumed, the dissipation function in those form so if you are talking about the dissipation function in this ,which is nothing but equals to half this summation the double integration of the C j k and the 2 state space coordinates the q of j dot and q dot k . Or else you see if you are just going with the matrix phenomena then it is half q dot transpose into c q dot. In this it is a non negative we can say the definite stiffness symmetric matrices, which is simply giving the damping viscous damping matrices and it it has to be you see symmetric as we are discussing for common. So, that is why you see here in these cases, the viscous damping is always being dominated feature.

It should be noted that for all forms of, viscous damping matrix, here there are various non there are various classical models which are not being considered. If you want to consider that, then the solution method which is related to the viscous damping matrices can be again divided into the classical and non classical damping. So, that is why it is important to avoid the widespread misconception that viscous damping is the only linear model of vibration damping in the context of multi degree of freedom system. Any causal model which makes the energy dissipation function non negative is the possible feature of our damping model is.

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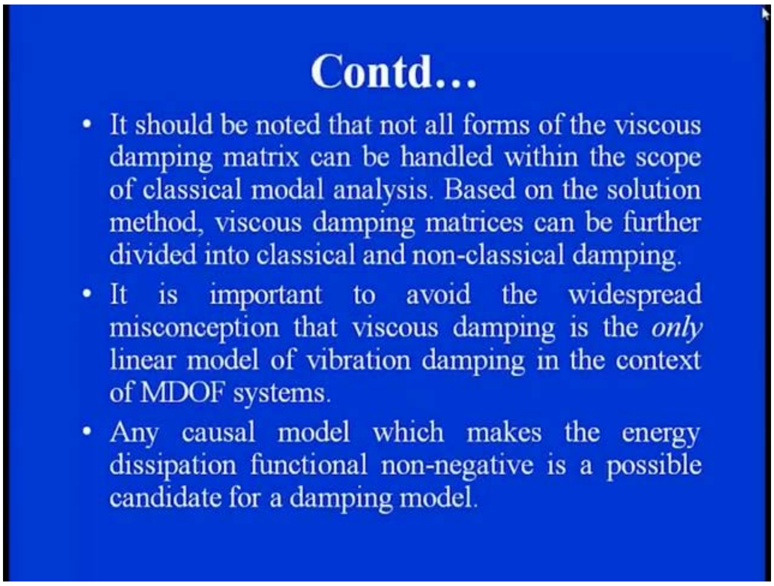

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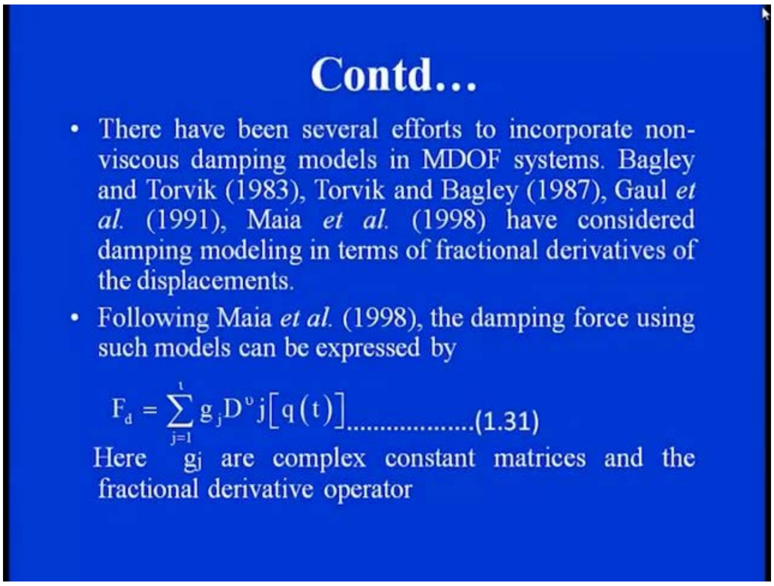

So, again you see we need to redefine those things for that and that is why you see here, the various efforts have been made to formulate the non-viscous damping models in the multi degree of freedom systems. Right from the Bagley and the Torvik in 1983 or the Torvik in even in the further paper or even you see the Gaul et al or even the Maia et al. These guys they considered the damping modelling especially for the non-viscous formation. Just in terms of the fractional derivatives of the displacements. They expressed the damping force for such systems is nothing but equals to the integral of or the summation of g of j, which is nothing but the complex constant for the matrices in which you see you know like the fractional derivatives are there into D of v comma v of j into you see q of t.

So, where we have a clear fractional derivative operator D, which is you see you know like the velocity operator. So, it is D of v and we have the g which is the constant complex constant matrices. In this you see here, they were just trying to adjust, the non viscous model of the viscosity into the real damping feature with related to the you know like we can say the displacements.

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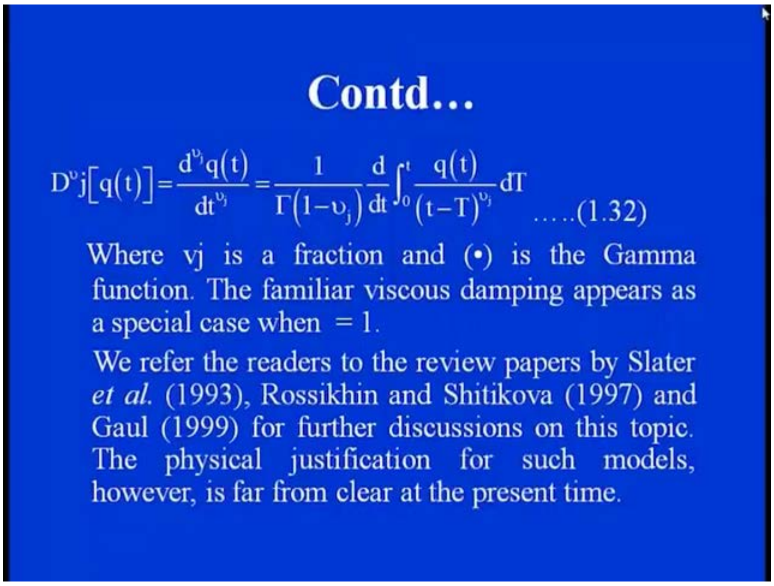

So, in this you see here we can say that the fractional derivative operator which is being there, del of j d of g into q of t is nothing but equals to the q the D of v divided by d t. Else even we are when we are trying to describe these things and we have 1 by pi 1 minus v j d by d t of the all summation of variation of q t divided by t minus t. That what exactly the times time stamp is into d t. where v j is the fractional feature. The dot is basically a gamma function, which is supposed to be you know like evaluated the entire specific geometry. The familiar viscous damping appears, to be the case, when we have the zeta equals to 1.

So, you see here in various papers it was clearly discussed, that the physical justification for these models are to be required when you are considering the non-viscous damping model in the entire damping phenomena for such cases. Most general way to model the damping within the linear range, is to consider the non-viscous damping models.

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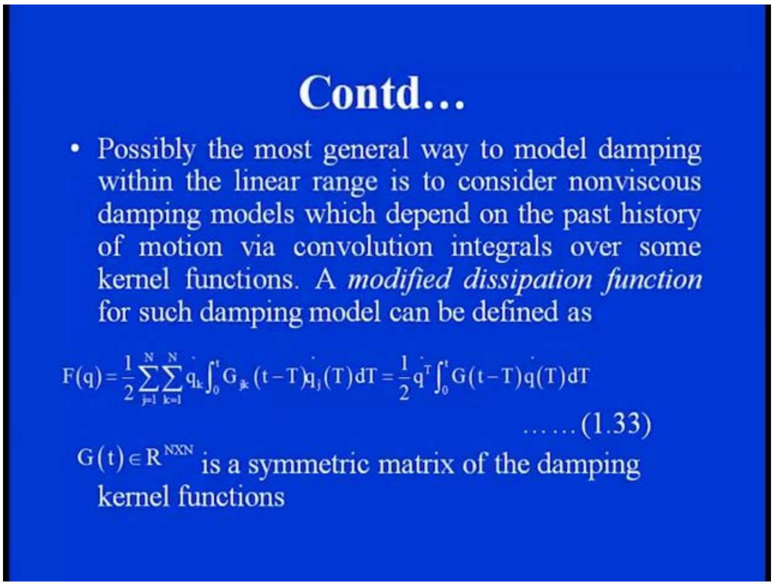

Which absolutely depending on how the system is reacted according to the force or what is the past history of these motions via convolution integrals over some kernel functions. So, then we need to adopt a different methodology for such cases. So, here we have a modified dissipation function, for such damping cases in which the forces are to be computed based on the multiple integral of the q dot k.

In which the clear integration of the continuous function like G of integration of 0 to t G of k t minus t q of j, which is simply a function of time into d t. Or else even we can say that when you have, a symmetric matrix ,which is simply calculate based on the some kernel function. So 1 by 2 q of T transpose integration of this matrix the based on the kernel function g of t minus T q dot T into d T. So, these are you see, the representation of a dissipation function for these kind of damping model. In which the damping is considered with the non-viscous feature and also you see here, they have clear the kernel function variation along with the time.

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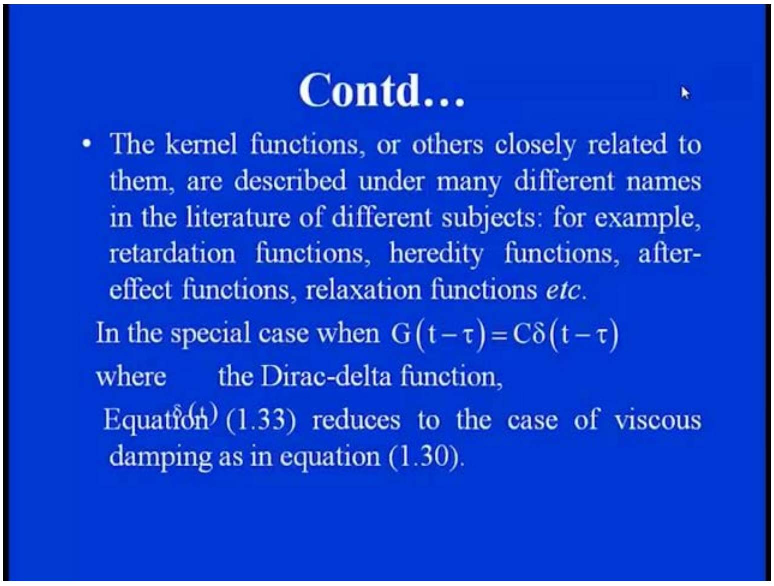

The kernel function or any other closely related functions can be described with the various we can say the name like you see here. The retardation functions, the heredity functions or anything which is close to the real feasibility of the system. Relaxation function, the effect function, after effect function, various things are there and in the literature we will find that various function which are compatible to the things which are being happening during the damping occurs.

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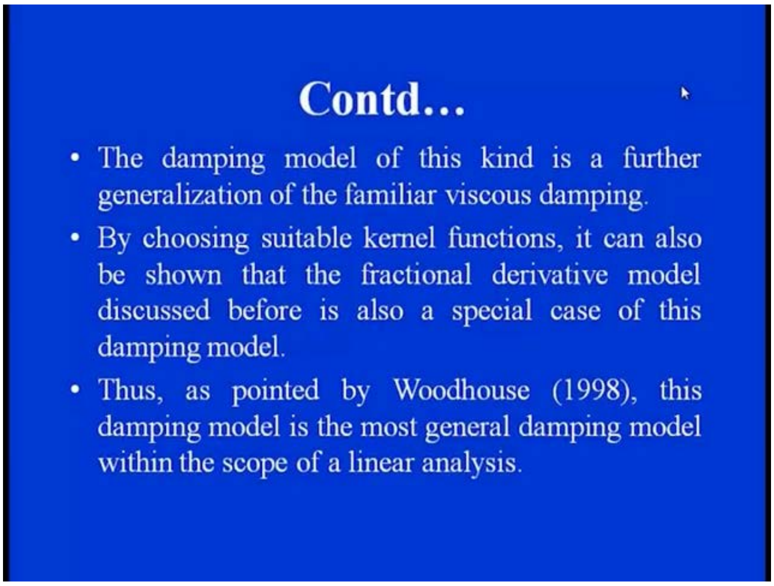

So, we can even go for some of the cases. A special case says that, when this kernel function operator G of t minus tau is equals to C delta t minus tau, then we have a direct delta function for these things. In that you see here, the damping equation is you know like critically play its role for formulation of that.

Because, you see here this is a generalized way of expressing the damping feature with viscous and non-viscous part. By choosing the suitable kernel function or anything you see here, we can also show the fractional derivative model as a special case of damping model. This damping model is the most general model, when you are just describing the damping variation in the linear part, even for the multi degree of freedom system. So, the various you see you know like the damping models are being available in the context of viscoelastic structures also.

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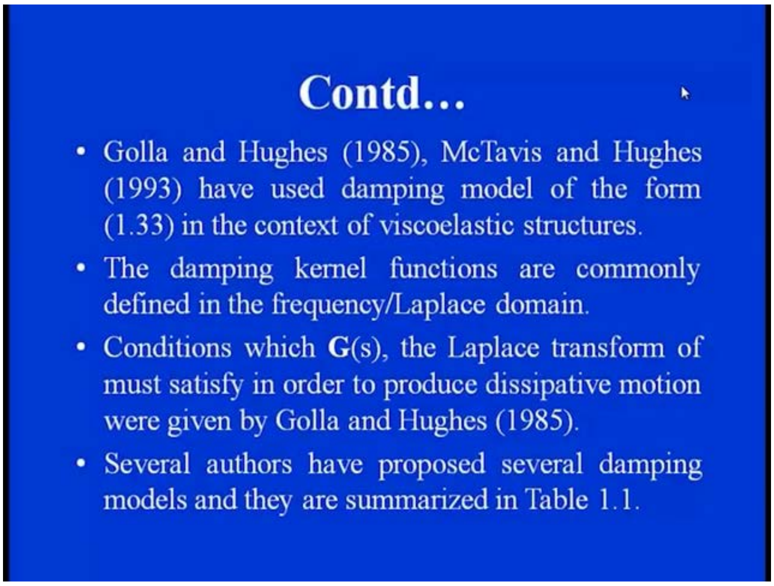

And the damping which are being considering on the kernel functions are commonly defined, in the frequency oblique the Laplacian domain. Because, in these cases we can clearly interact about the energy formulation or on those things. So, the conditions in which the kernel function G of s is considered in the laplace transformation, it has to satisfy the whatever the dissipation motion is being there along with the entire part. So, you know like the several authors just you know like gave, the various models in this.

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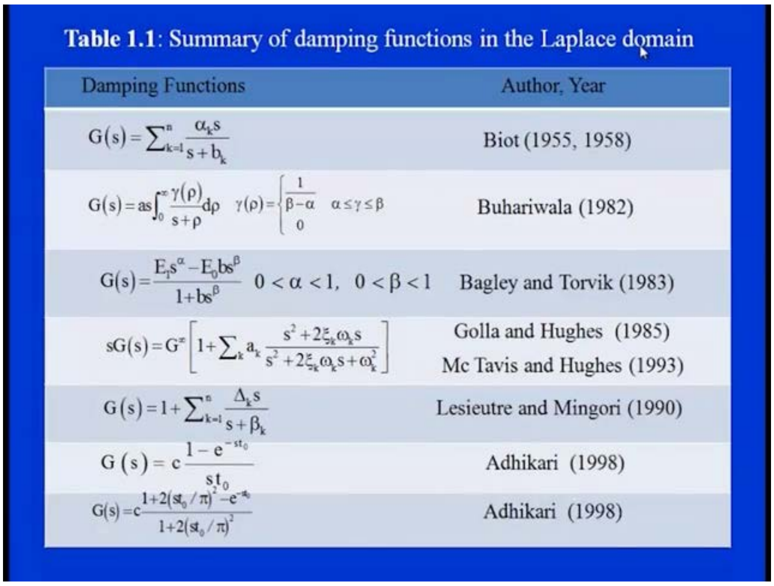

Table 1.1: Summary of damping functions in the Laplace domain

Damping Functions	Author, Year
$G(s)=\sum_{k=1}^{n}\frac{\alpha_k s}{s+b_k}$	Biot (1955, 1958)
$G(s)=as\int_0^{\infty}\frac{\gamma(\rho)}{s+\rho}d\rho \quad \gamma(\rho)=\begin{cases}\frac{1}{\beta-\alpha} & \alpha\le\gamma\le\beta\\ 0\end{cases}$	Buhariwala (1982)
$G(s)=\frac{E_1 s^{\alpha}-E_0 b s^{\beta}}{1+bs^{\beta}} \quad 0<\alpha<1,\ 0<\beta<1$	Bagley and Torvik (1983)
$sG(s)=G^{\infty}\left[1+\sum_k a_k \frac{s^2+2\xi_k\omega_k s}{s^2+2\xi_k\omega_k s+\omega_k^2}\right]$	Golla and Hughes (1985) Mc Tavis and Hughes (1993)
$G(s)=1+\sum_{k=1}^{n}\frac{\Delta_k s}{s+\beta_k}$	Lesieutre and Mingori (1990)
$G(s)=c\frac{1-e^{-st_0}}{st_0}$	Adhikari (1998)
$G(s)=c\frac{1+2(st_0/\pi)^2-e^{-st_0}}{1+2(st_0/\pi)^2}$	Adhikari (1998)

As you can simply see in this table the summary of the damping function. Especially in the laplacian domain. So, it started from somewhere 1958 by Biot. In which it is clearly showing that the Laplacian function the transformation function G of s is nothing but equals to summation of a s by s plus b of k.

You see here 1982 the blow the Buhariwala is also showing, the different variation of gamma which is nothing but you see you know like the linear dissipation model into rho divided by s plus rho d rho. Where he defined about the gamma, is again if it is, you see you know like less than if alpha is less than gamma then you see it is beta minus alpha. Or if it is you know like working in between that it, can even gives the 0 value. Then you see in 1983 to you know like all upto 1998 by Adhikari, the lots of functions are being added in terms of the non-viscous model for the entire damping formulation in the Laplacian domain.

So, the Laplacian domain is always being considered, when you know the system dynamics and the domain is really feasible. In which you see we can simply considered the complex form of the damping. So, even in the Adhikari you can see that the last one the G of s is nothing but equals to the c into now he considered the exponential feature with the you know like other part of the damping available.

The coefficients particular. So, in this lecture you see here we discussed about the various damping models. That how we can compute the damping, because as we know that if you want to control the vibration, we need to extract we need to dissipate the energy from the source. The damping is one of the critical phenomena when the resonances are occurred the by this putting the damping it can be controlled effectively.

But, how do we get the damping ? It is not a constant value which we are always keeping and saying that, yeah this is sufficient. No! We need to check it out that, whether we are considering the damping for single degree of freedom, whether we are considering the damping for continuous system or whether the damping is considered for a multi degree of freedom system. Then how to model these damping, not only for the linear propagation, but also whatever the casualities are there how it can be incorporated in that.

Then, we can represent this damping in the frequency domain function, the time domain or even the Laplacian domain. We just want to make the damping compatible to the real feasible system and what the physical system and what the feasibilities are there to vary the operating parameters.

So, this is all about this lecture. In the next lecture now again, we would like to put few more damping model or to measure the damping. And then we would like to consider some of the factors, which are being straightaway affecting the damping which is available for vibrations suppression. Also along with these conceptual features we would like to solve some of the numerical problems. So, that we can simply find out that what is the effective measure of the damping is to suppress the vibration excitation from the source.

Thank you.