Vibration Control
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Module - 4
Vibration Generation Mechanism
Lecture - 7
Numerical Problems

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. In the course of Vibration Control, we are in the module 3, in which we discussed about the main Vibration Generation Mechanism and the vibration isolation features. So now, this is the last lecture of this module 3, in which you see here, now we are going to discuss about the Numerical Problems of the vibration generation mechanism and the isolators.

We discussed about in the vibration isolation that, what exactly the insertion losses are there, how we can calculate those when they are being dissipated through even the air bone or the structural bone things. And also we discussed about the various feature of the foundation part, in which when we have a rigid foundation or the flexible foundation. And then we are putting the isolator together then how the vibration can be suppressed or we can simply reduce the vibration amplitude.

In the previous class, we were discussing about one form of the vibration generation mechanism, in which we discussed about the damping models and the measures of the damping and we found that, there are lots of factors, is directly affecting the material damping. So, when we are talking about a material damping with the constant figure that, this is the material and material is there and this is what the microstructure of the material, we can take a constant value, is not true.

And even when we are going with the different types of material then we know that, even when the transformation of the kinetic energy or the strain energy in the dissipation form you see here and which is being transforming to the heat, always the grain structure, the grain size and even the grain boundaries are directly affected by this kind of formation of heat. So, when we were discussing about the viscoelastic material, we could even find out that, because of the viscosity and with the elastic feature of that material, there is a the hysteresis loop was there, which was simply showing the loss of energy.

And this loss of energy is creating more problem in the formulation of the total amplitude of the viscosity, which is being provided by these kind of materials. Even we discussed some of the numerical aspect of this that, how the damping coefficients can be calculated in a complex form when you have n th degree of freedom system, and how the matrix multiplications are being there, just to justify the symmetric nature of our damping part.

And in that also, we discussed about the damping ratio for the under damped or for critically damped system. So, this part you see here, which we discussed in the previous class. So, today now we are going to discuss about the numerical problems in this chapter, in which we discussed about the vibration generation and the isolator feature.

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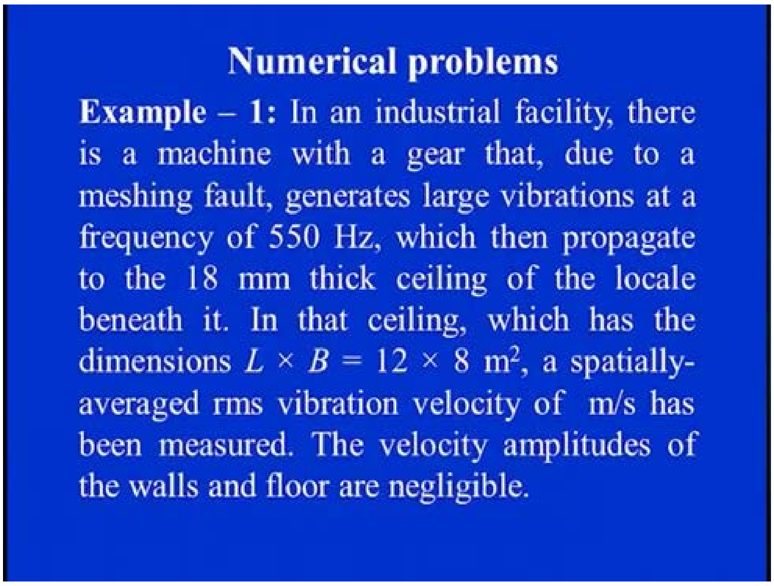

So, the first numerical is, we have in the industrial facility, there is a machine with the gear and due to the meshing fault, it generates the large amount of vibration at the frequency of 550 Hertz. So, the frequency is given to us, which is propagated to the 18 millimeter thick ceiling of the locale beneath in that. So that means, you see here at the below of this machine, we have a 18 millimeter thickness ceiling was there and in that ceiling, the dimension is L into B that means, the length into breadth is 12 into 8 meter square. And you see, the average rms velocity is also being given in the meter per second, which can be easily measured and the velocity amplitude to the walls and the floors are negligible. So, you see here, we are not going to consider the velocity component, which are being there with the walls or the floors towards the transmission feature.

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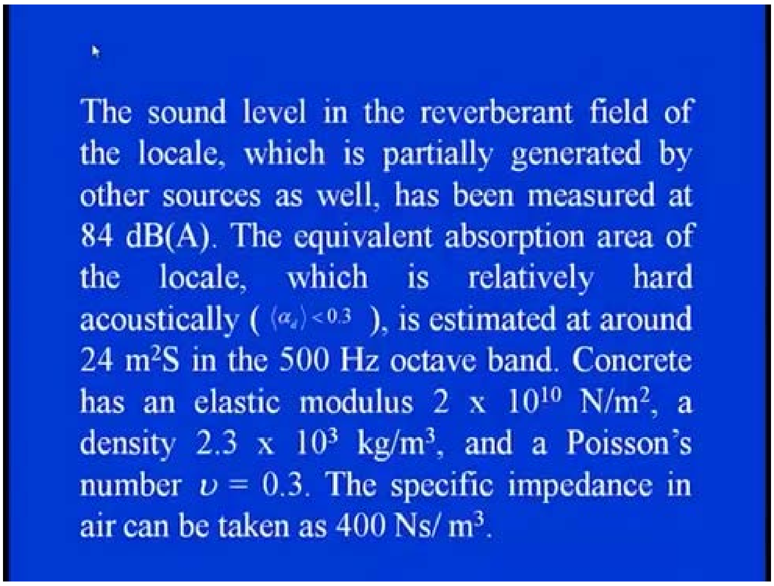

So, we are very much focused to this local part and the sound level with this is also being, we can say the partly generated as the transmission of the vibrations are there and we can say that, it is almost near about the 84 decibel unit in this. So, we can say that, the equivalent absorption area of the local, because we are just going with the local part, is almost you see here, the alpha e which is we can say relatively hard acoustically feature. Because of the material property, the alpha is less than 0.3, can be estimated at around we can say 24 meter square per second in 500 Hertz of the octave band.

And below that, we have the concrete floor, which has the elastic modulus of 2 into 10 raise to power 10 and the density is 2.3 into 10 raise to power 3 kilogram per meters cube and you see the Poisson ratio for such kind of things can be straightaway taken as 0.3. And you see here, the air which is being circulated there, we can take the specific impedance as 400 Newton second per meter cube. So, these are the input parameters which are directly interacted with the machine.

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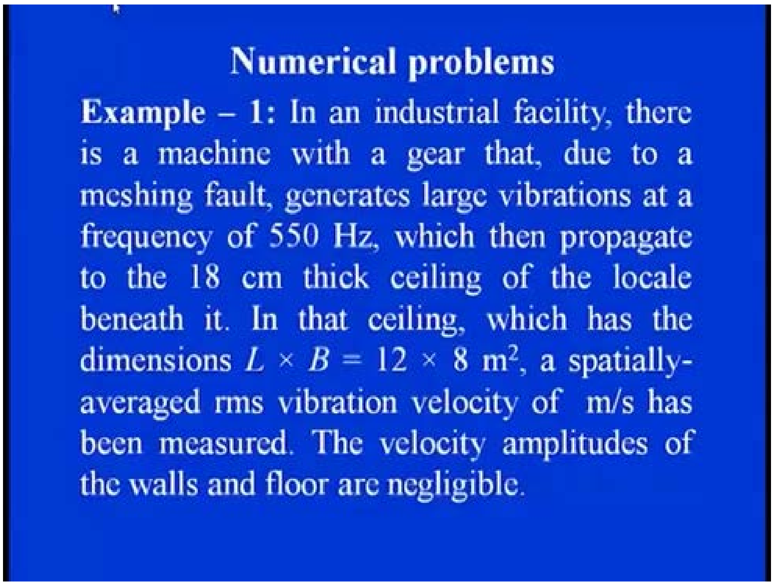

And you see, when this vibration is propagated along with, we have the ceiling which of 18 millimeter thick and then all these dimensions are there.

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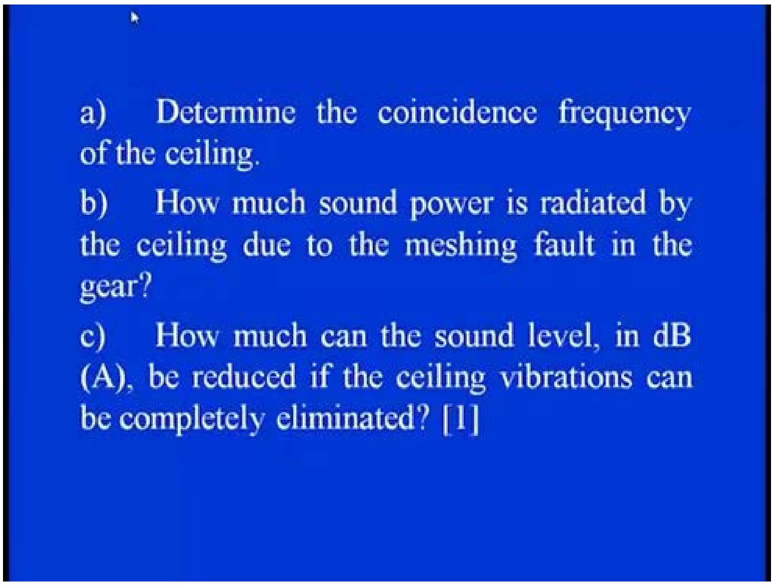

Now, the first thing we would like to find out, the coincidence frequency of the ceiling, because of the machine excitation. Second the power, the sound power which is being radiated due to the meshing fault, because we have now the exciting frequency then how much power is being radiated through this ceiling. And you see here, because and it is due to mainly the meshing fault and how much the sound level can be created or it can be reduced if the ceiling vibration can be completely eliminated.

So, when we are trying to now reduce the vibration or eliminate the vibration, will it be there any sound there itself or not. So, you see these are the three different aspects of the sound and vibration interaction when you are in the industry and it is a very common problem, practical problem is there, which was taken by the handbook of the sound and vibration of the K. T. H. Sweden. So now, we need make an strategy to solve this problem.

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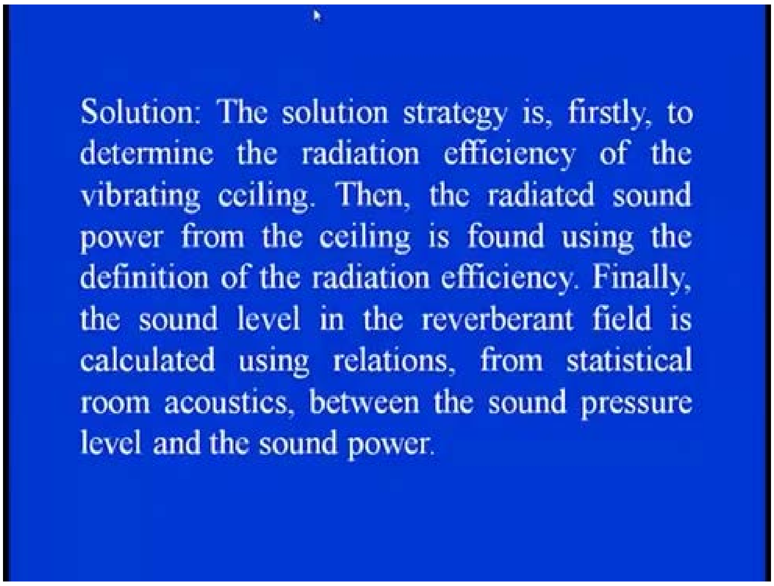

So, first of all we need to find out, the radiation frequency of the vibrating ceiling so that, we can at least find out that, how much transformation is there with the specified size of the ceiling. Then we need to go with the radiated sound, which is being transmitted through that. And then we need to find out the sound level when you see we know that, there is a clear radiation and the vibration excitation is being just passing through these ceiling towards the ground of the concrete.

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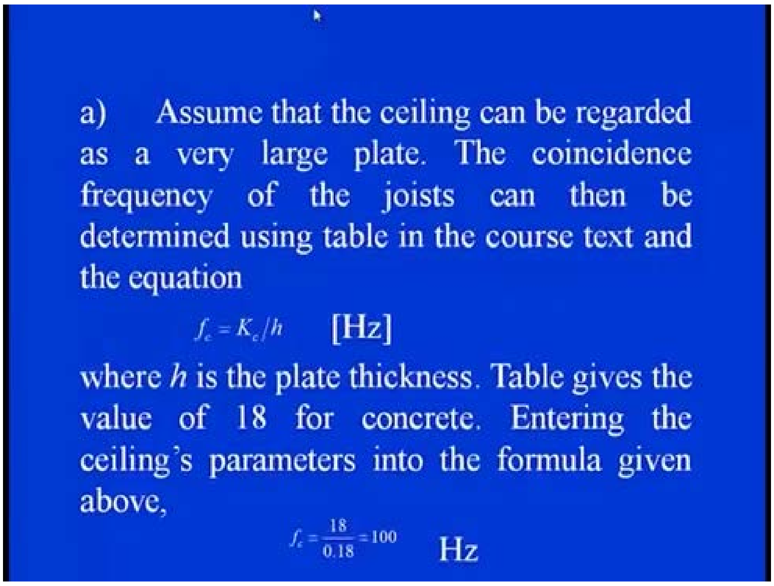

So, the first you see here, in the first case we need to see, the ceiling can be regarded as a large plate, as we can see. And the coincidence frequency of these joist, in the chapter number 15 th we discussed, can be find out by f c equals to K c divided by h, where h is the plate thickness, which we have already considered as the 18 millimeter. And when we are putting these things and we know that, this K, h values also given in this way. So, you have the plate thickness 0.18, you have the K c which is nothing but the constant value for this concrete feature. So, we can get the f which is nothing but the exciting frequency of these, we can say the ceiling f c is nothing but equals to 18 divided by 0.18, which is nothing but equals to 100 Hertz. So, this is my coincidence frequency of the joist with the ceiling and we can get that part.

So, the ceiling vibration according to this excitation can occur at the frequency of 550 Hertz, well above the coincidence frequency. So that means, the radiation energy can be easily approximated as the unit value, because the excitation frequency, which is the ceiling excitation frequency is the 100 hertz, which is the natural frequency of that and the exciting frequency due to that is 550 Hertz, is the huge differences are there. So, it can be well taken as the radiation efficiency is almost near about the 1.

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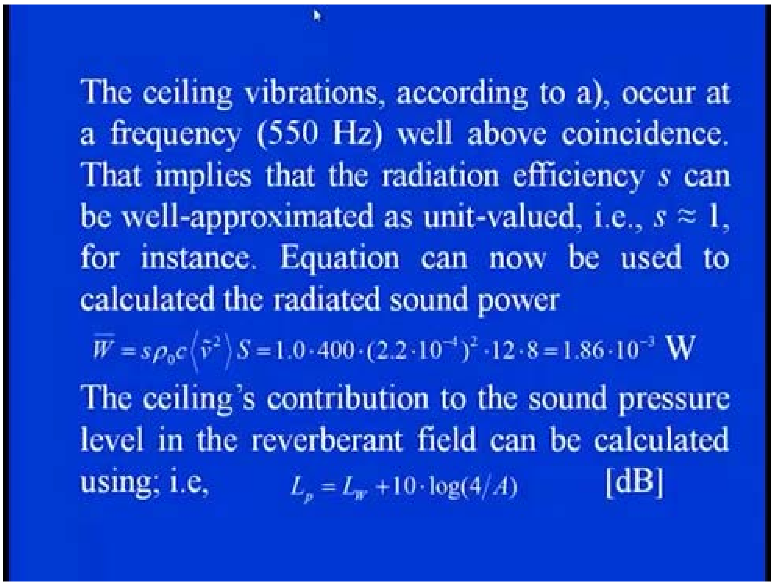

And then we can straightaway calculate that, how much the sound power in terms of the acoustical radiation. As we discussed already in the lecture 15, is as which is the efficiency into the rho 0 that is, my density into c v square here. So, we know that, the s is 1 as we know that, there is a clear difference and well above the coincidence part of that. Second, the rho 0 which was given as 2.2 into 10 raise to power this and the 400 is given as the other parts.

So, when we are formulating each, we have the sound power as 1.86 into 10 raise to power minus 3 Watt. So, this much you see, the sound power is being transmitted and generated along the machine itself by transmission of this much vibration at the 550 Hertz exciting frequency. And the ceiling's contribution to the sound pressure level in this reverberant field can be easily calculate with the sound level at that point is nothing but equals to L W plus 10 logarithmic of 4 divided by A.

So, when we are putting these values there, the sound power level is L P is nothing but equals to the 10 into log of 1.86 into 10 raise to power minus 3, which was the sound generated divided by 10 raise to power minus 12, plus that was the L W, plus 10 log 4 by A, that was the 24, so it is nothing but equals to 84.9 DB. And the contribution to the total sound level can be straightaway find out, when you are just using the weighting factor.

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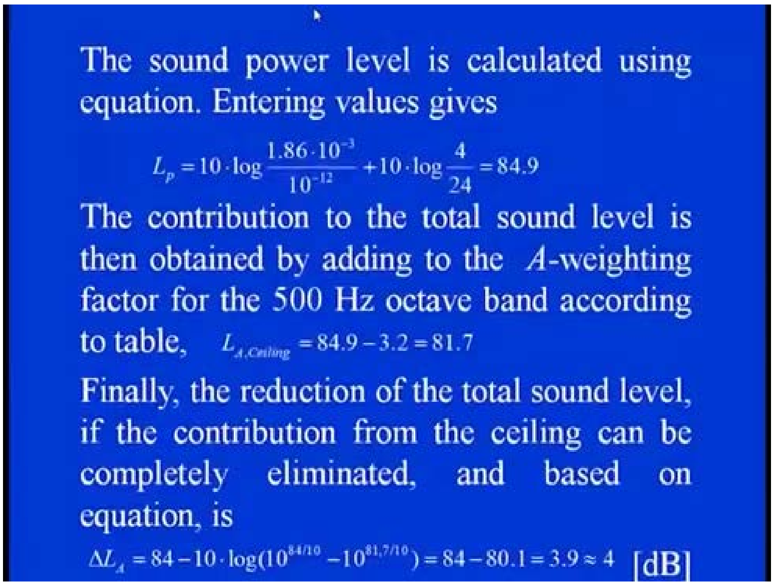

And here the weighting factor in that, which was discussed in one of the table there, the A weighting factor for 500 Hertz sound in any octave band we can say that, the L A the sound band at the ceiling is nothing but equals to the 84.9 dB, which was calculated earlier minus 3.2 that is, 81.7. So, the reduction of the total sound level if it is contributed from the ceiling, can be completely eliminated.

And based on that, we can say we can simply find that, the delta L which is L A, which we just want to eliminate that point is 84 minus 10 log 10 raise to power 84 by 10 minus 10 to the power 81.7 by 10 or else we can say that, it is 84 minus 80.1 almost near about the 4 DB. So, means that, when we are just trying to eliminate the entire vibration from this, which is being excited above 500 Hertz, we can say 81.7 when we are trying to do these things, the sound which is being there at the time is 4 decimal sound.

So that means, that much energy, the sound level, the sound power or the energy is being available when even the contribution of the ceiling is absolutely eliminated. So that means, you see here, that much amount of energy is being there due to various other sources. So, in this numerical you see here, we could easily figure out an interact, the acoustical radiation in form of sound generation and the same time you see here, how the vibration is really being contributed. And if you want to eliminate those things, how we can compute or how we can interact those features together. Now, when we are moving to the another question then we have a chimney which we discussed that, you see here a straight chimneys are there in many of the industries.

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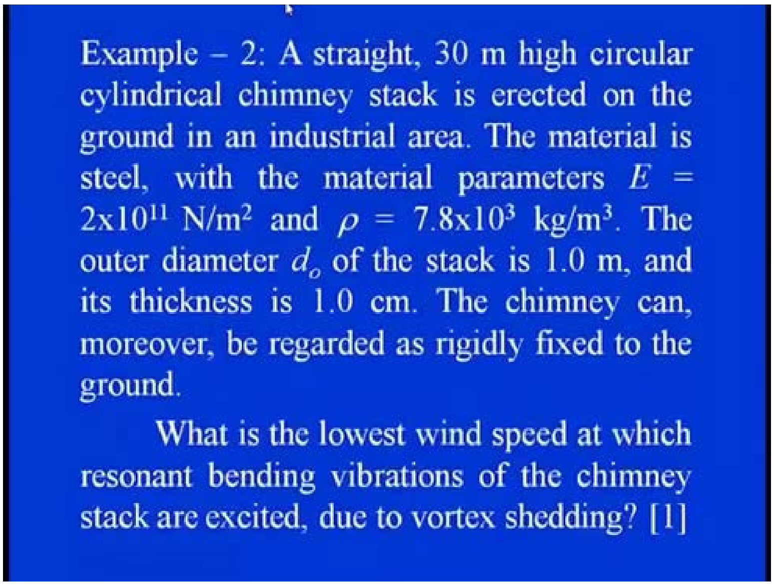

A straight 30 meter high circular cylindrical chimney stack is erected on the ground of any of the industrial part. So, we have the height is 30 meter only, the material of this one is the steel, for which you see the material property can be immediately port the elastic modulus 2 into 10 raise to power 11 Newton per meter square and also the density is 7.8 into 10 raise to power 3 kilogram per meter cube.

The outer diameter d 0 is also given as 1 meter and the thickness is also 1 centimeter and the chimney can be, moreover we can say rigidly fixed to the ground, certainly because of the proper excitations and the rested ground part. Now, we would like to find out the lowest wind speed, at which the resonant bending vibration of the chimney stacks are excited due to the vortex shedding. This is what one of the basic cause in that, that how the vertex sheddings are basically being there to excite the entire chimney under the bending vibrations.

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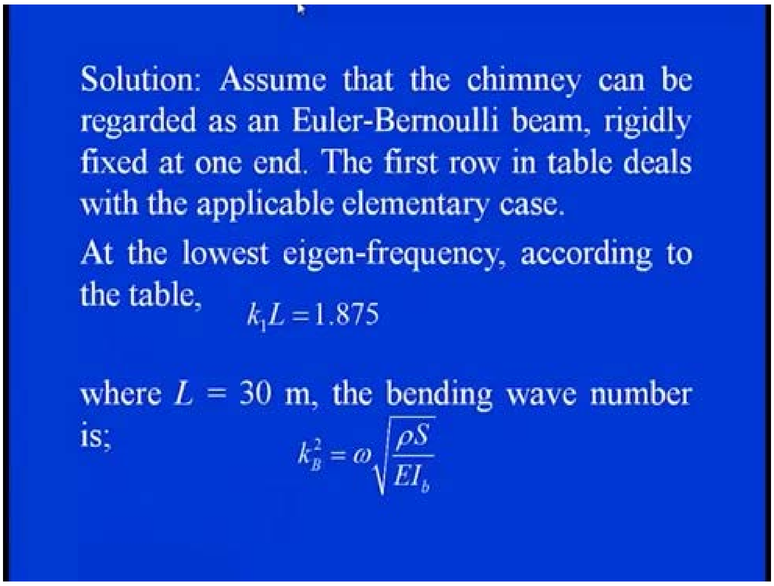

First of all, in this we know that, we need to apply the Euler-Bernoulli beam excitation theory as the 30 meter entire chimney is being there, which is at one end it is a rigidly fixed. So now, you see here, the lowest Eigen frequencies for this through these Euler-Bernoulli beam theory table is k 1 into L, as equals to 1.875. And since the L here is 30 meter, so we can say that, the bending wave number for this is the k B square equals to W into square root of rho S by E I b. So, we could easily find out that, how much wave numbers are there for this 30 meter L.

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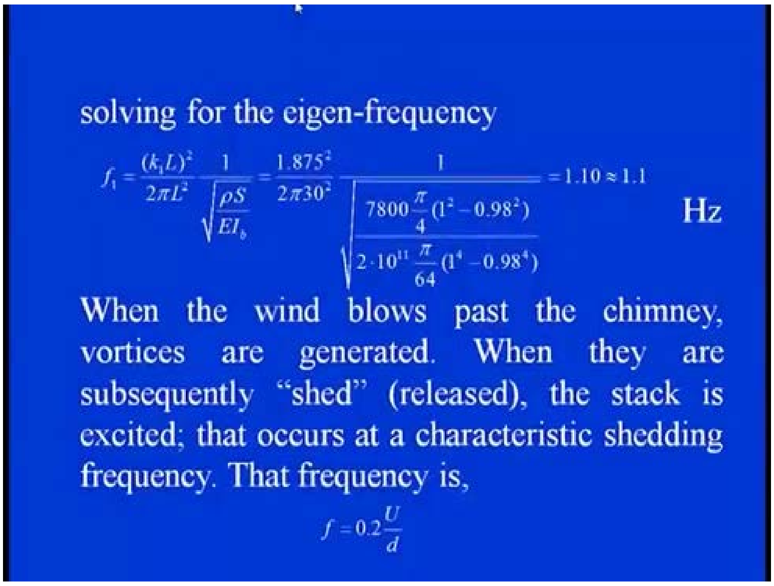

So, when we are solving this for Eigen frequency, the Eigen frequency is nothing but equals to k 1 L square divided by 2 pi L square into 1 over this, the part which we discussed, square root of rho S by E I b. E I b is nothing but equals to the modulus of rigidity for the beam and we can put entire things together you see here that, 1.875 as k L, which we calculated for the beam from the table divided by 2 pi L square is 2 pi into L is 30.

So, 30 square into 1 over the rho S, the rho is the density of that material and it is a steel one, so it is 7800. The S is nothing but equals to the total area, we can say pi by 4 1 square minus 0.98 square divided by, that is what the effective feature is, divided by the E I, young's modulus is already given to you, 2 into 10 raise to power 11 and I is the moment of inertia. So, it is pi by 64 1 to the power 4 means, because it is the cylindrical chimney which has inner and out diameter. So, one was the outer diameter 1 to the power 4 minus 0.98 to the power 4.

When we are formulating this, it is almost nearly equal to 1.10 or 1.108, so it is 1.1 Hertz. So, when the wind blows, past the chimney, the vortices are absolutely generated and when they are subsequently released then the stack is absolutely excited and the characteristic shedding frequency, at which it is excited is 1.1 Hertz. And that frequency which even we can calculate for this, it is equal to 0.2 U by d, U is nothing but the velocity and d is the path or which it is being past thus, the entire air is being passed through.

So, we can say it is the speed, where you see U, the flow or the wind velocity is coming and outer diameter of d is given to us, so we can solve for this wind speed and we can evaluate by entering these parameters. So, it is nothing but equals to we can say that, U which we would like to see, d divided by 0.2 into f, the f was calculated already the excited frequency 1.1 Hertz, the outer diameter was given as 1 meter. So, we can say that the U, this is what the velocity or we can say the blow or the flow or wind velocity, whatever we can say of the air, when this much exciting frequencies can be generated that is, 5.5 meter per second.

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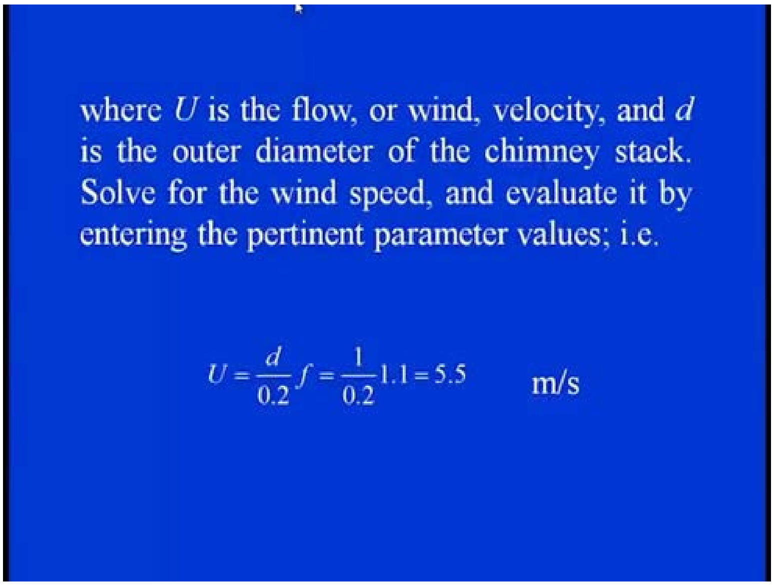

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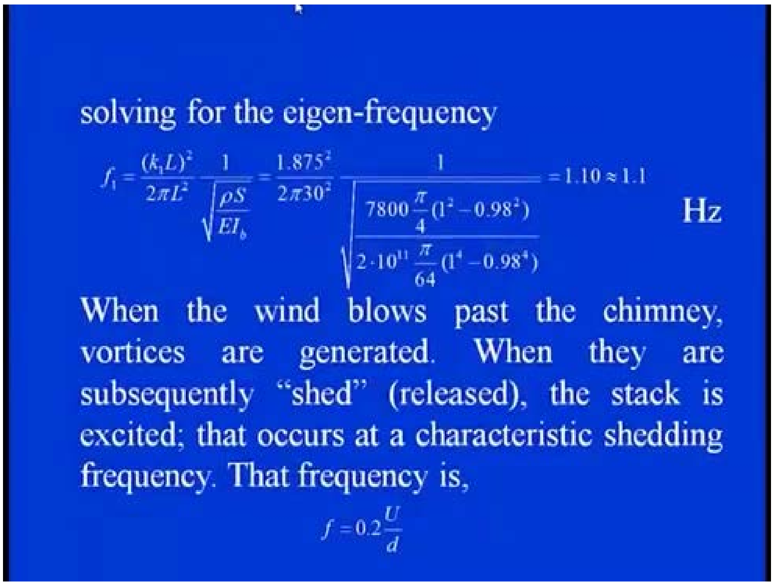

So, we can say that, this shedding frequency of 1.1 Hertz can be straightaway find out when you have an very specific flow velocity together. And this wind blows are the clear cause of this exciting frequency, when these winds are being released throughout the chimney and it is being excited there. So, this was the problem, in which you see here, there is a clear excitation of the chimney when the air flow is there. And that part, which we discussed in the flow induced vibration, in which the air was flowing or the fluid is flowing through the chimney or the pipe or any kind of the closed conduit.

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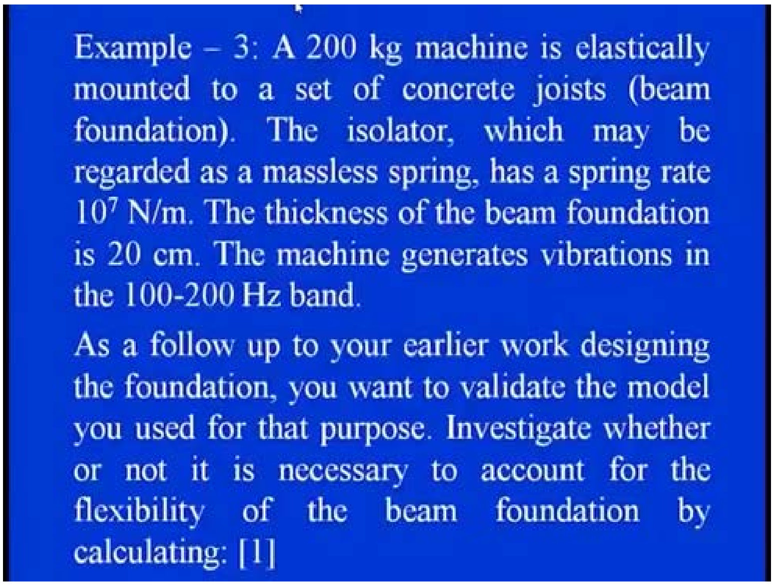

Now, in the third problem, a 2 kilogram machine is being elastically mounted to the set of concrete joists or the beam foundation, which is very a common device you see in the any of the industry. The isolator which may be regarded as the massless spring, has the spring rate as 10 raise to power 7 Newton per meter, that is what my stiffness of springs are, the material property. The thickness of the beam foundation, where the machine is being installed is 20 centimeter. Now, the machine is generating the vibration in the band of 100 to 200 Hertz only. Now, you see here, we would like to see that, how we can modeled these things, whether it is perfectly ok or not.

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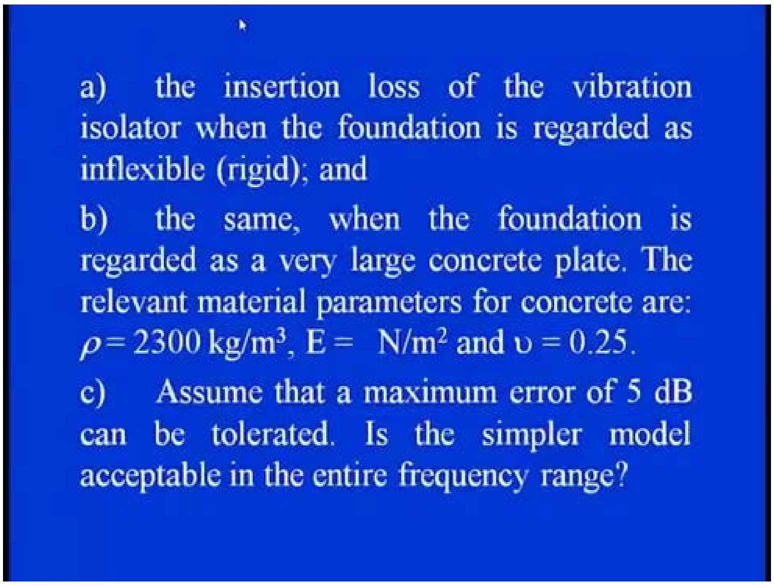

And we just want to see that, how we can take the necessary step to account the flexibility of these beam foundation by calculating what the insertion losses of the vibration isolator when the foundation is regarded as a rigid first. Second, the same you see here, when the foundation is regarded as the very large concrete plate and the relevant material property of the concrete is density as we already taken 2300 kilogram per meter cube, the Young's modulus as it is given you see here, 2 into 10 raise to power 11 Newton per meter square and the Poisson ratio is 0.25. Assume that, now in the third case, that the error of 5 decibel can be tolerated, so with these assumptions, is the simpler model can be acceptable for the entire frequency range of 100 to 200 Hertz or do we need to change in the model to formulate this thing.

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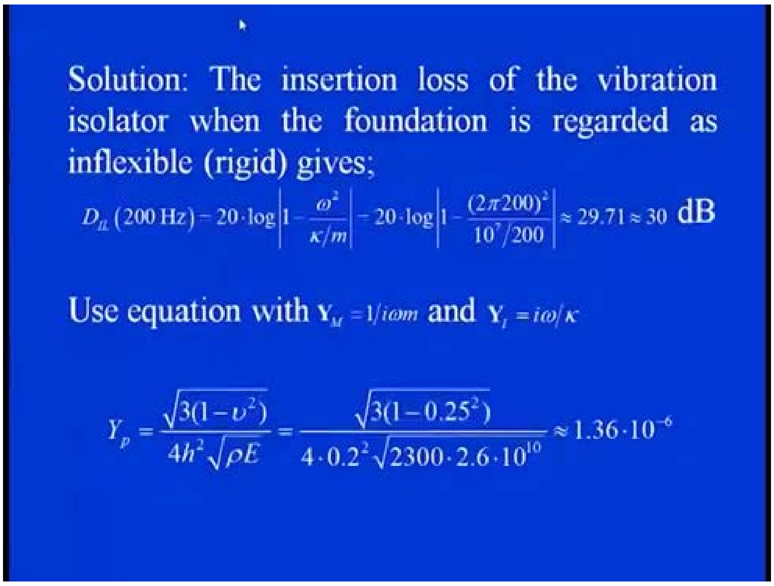

So, as we know that, the insertion losses of the vibration isolator can be straightaway calculate when the foundation is solid, is nothing but equals to 20 log 1 minus omega square divided by k by m, where we can calculate the omega as the exciting frequencies when we know that, 2 pi f square. And the stiffness is given to us, the mass is given to us, so for 200 Hertz say, now we are going towards the higher mode of this excitation.

We can say the insertion loss for 200 hertz is 20 log 1 minus, since it is 200, so 2 pi 200 square divided by, the k is given as 10 to the power 7 divided by, the m is given as 200, so it is almost the insertion losses are 30 decibels. Now, we know that, we need to act in such a way that, what exactly the transmissions are. So, Y of M is nothing but equals to 1 divided by i omega m and when Y of I through that the transmission part is nothing but equals to i omega of divided by k.

So, when we are talking about the machine feature, it is nothing but we can straightaway calculate that, what exactly the exciting frequencies are related to the mass. So, that is why you see, it is 1 by i omega m and when we are talking about the restoring features then it is i omega by k. So, Y of P of this that means, you see here, the particular feature of this Y of P is nothing but equals to the square root of 3 1 minus Poisson ratio square divided by 4 h square square root of rho into E.

So, we can straightaway calculate this by keeping the 0.25 S, the Poisson ratio. We know the h, 0.2 which was given earlier and rho is 2300 and E is given as 2.6 into 10 raise to power 10 as elastic property of this. So, Y P is nothing but equals to 1.36 into 10 raise to power 6 minus 6 meter part was there. So, when we are now just adopting this insertion loss at the 200 hertz, so now, we can put these entire features in, means the Y M, Y I and Y P together you see here, when we are keeping these things.

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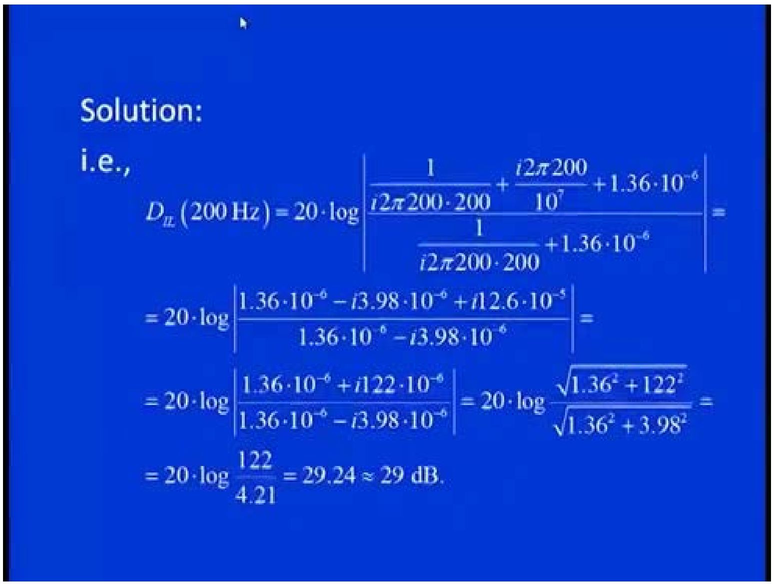

Now, our insertion loss at 200 hertz is nothing but equals 20 into log of 1 divided by 2 pi, since it is 200, so omega square, so it is 200 into 200 f square plus i omega times 2 pi f divided by 10 to the power 7. That means, you see here, we have a clear stiffness variation, so 2 this IOTA of times 2 pi as we know that, you see here which we discussed, IOTA 2 pi omega divided by k. So, that is the Y I and Y M is 1 by IOTA 2 pi into m, so we have this one.

So, this is what it is, plus the Y P that is, 1.36 into 10 raise to power minus 6 divided by the entire feature means, we have all three Y I plus Y M plus Y P divided by Y M plus Y P, so it is 1 by IOTA 2 pi 200 and 200 plus 1.36 into 10 raise power 6 minus 6 as Y P. When we are evaluating this in formation of IOTA terms and making equation of the features with this all numerical manipulations, we can say it is equals to 20 into log 122 divided by 4.21 or else it is nearly equals to 29 decibel.

So that means, you see here, now we know that, the insertion loss of the vibration isolator at the 200 Hertz frequency when we are computing all means, the D IL 200. When we are computing D IL 200 Hertz is Y M, Y I and Y P, they are giving clear feature of 29 decibel. So, even when you are computing with the individual part of this or when we are comparing these things, they are clearly giving a well features, that you see how much insertion losses are there when you just combine together. So, this was the second part was there when the foundation is just a large concrete plate and everything was there. So, 29 decibel was there and when you have simply taken the rigid foundation then 30 decibel was there. And now, you see here, when we are saying that, this is what the model, through which we can simply control the entire part.

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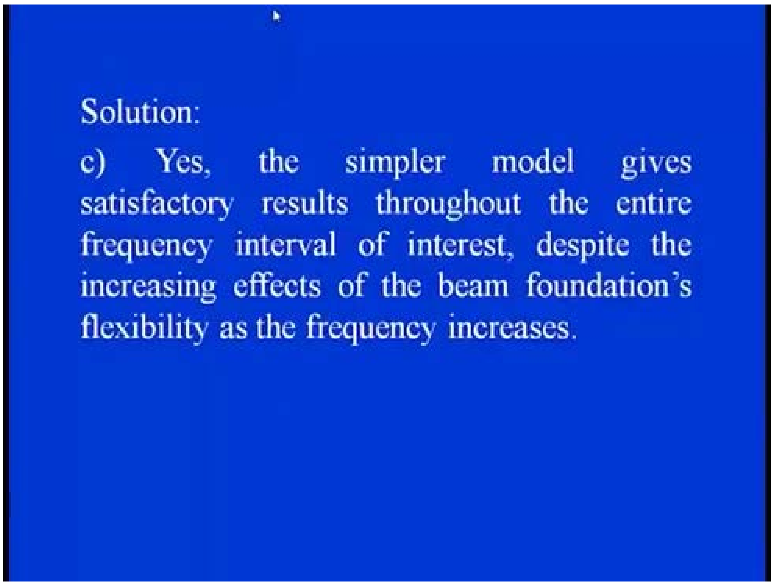

Or it is acceptable feature, we can say the simpler model can give you an accurate, not exactly so but almost accurate solution to the entire frequency interval of 100 to 200 Hertz. And but when there is an increasing effect of the beam foundations are there in terms of flexibility, we know that the frequencies will be increases. And at that time, we need to put a clear frequency check and the same time, insertion losses with these models. And we need to add those features along with Y P, Y I and Y M in computation of these insertion losses. So, that is what you see here, the transformation of these two, we can say sound when it is propagating at these exciting frequencies.

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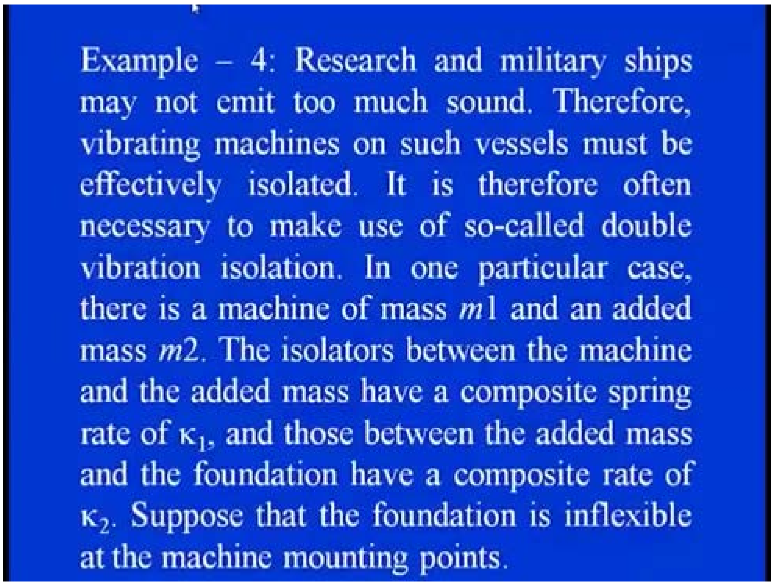

In the next model, now we can say that, we have a military ship, which is being just moving at an certain speed and the vibration machines of such vessels must be isolated. And for that, we need to take an a necessary action, we can say some kind of double isolation or even you see in the railway feature also, we know that when the excitations are so high and we just want to comfort the situation inside the cabin, there are double isolations are there.

So, this is what you see in some kind of even, not only in the military ships, but also you see in the various vehicles, where such kind of requirements are required or essentially, the steps should be required to eliminate such kind of vibration. We are taking now such kind of example that, we have a machine and you see here, some added mass is there. So, machine mass is m 1, the added mass is m 2 and now you see the isolator in between the machine and added mass, is just a composite spring say at the rate of k 1.

And those in between the added masses, and the foundation is also having the composite spring at the rate of k 2. So, we have the four parameters here, one m 1 and m 2, m 1 is the main machine mass and m 2 is the mass which is simply added mass. The spring k 1 is in between the machine and mass, and k 2 in between the added mass and the foundation. Now, if we are assuming that, this is whatever the mounting points are there, they are not providing the some kind of flexibility, so they are the rigid one.

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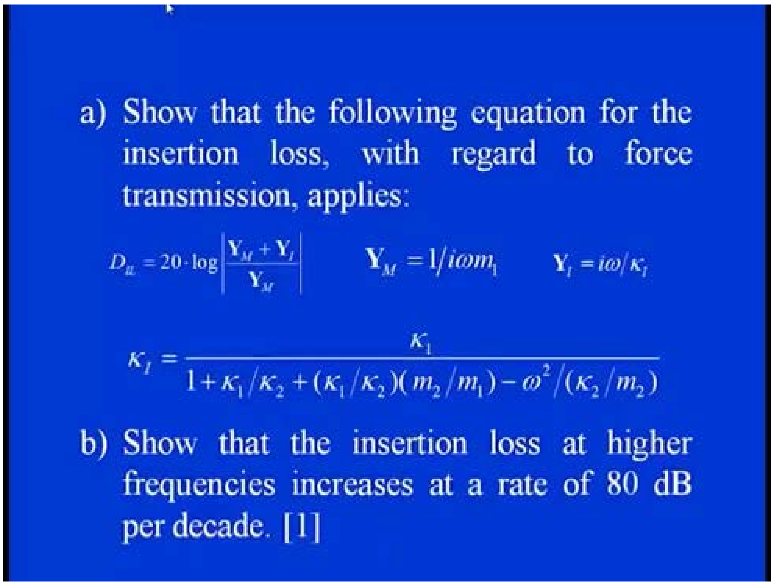

Now, we just want to prove in this case that, the insertion losses for the force transmission is D IL equals to 20 log Y M plus Y I divided by Y M. There is no Y P here, where the stiffness, we can say the equivalent stiffness spring rate k 1 is equals to, the k I rather we can say, the k I integrated one for this Y I equals to k 1 divided by 1 plus k 1 by k 2 plus k 1 by k 2 into m 2 by m 1 minus omega square k 2 by m 2, so this part which we need to show. Also we need to show the insertion loss at the higher frequency, when it is being increases at the rate of 80 decibel per decade.

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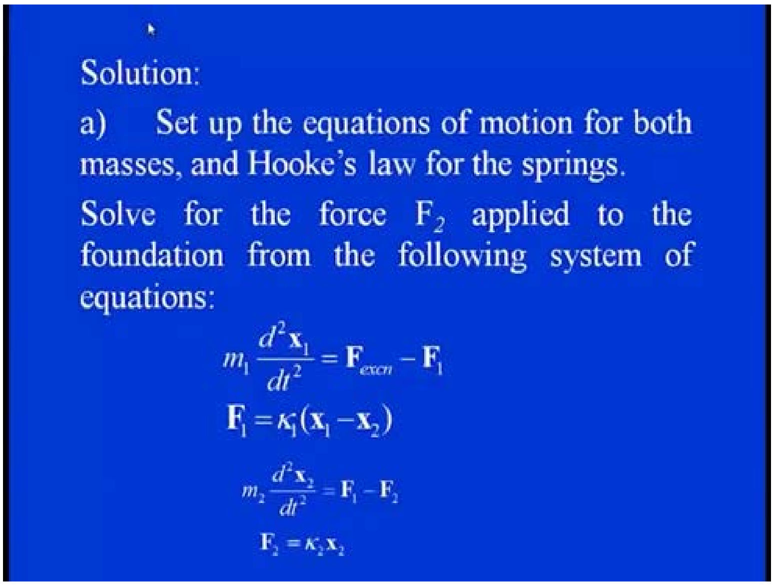

So, in the solution now, first of all we know that, the two springs are being acted in between machine to foundation and these springs are the elastic springs. So, we can straightaway apply the forces in between this, so when we are saying that, in between you see, the machine to the added mass. Since machine mass is m 1, so m 1 into d 2 X 1 by d t square inertia force equals to the excitation force minus the F 1 force. And the F 2 force, which is being applied to the foundation from the equation from the entire excitation of the machine then we can straightaway put the F 2 at the lower one.

So, we have the two masses and the two spring in between the entire feature, m 1 is having the X 1 displacement, m 2 the added mass having the displacement of X 2. So, F 1 which is the force with the mass m 1 is nothing but equals to the restoring part k 1 into X 1 minus X 2, the difference of these two displacement, the effective formulation of restoring forces. And when we are talking about the mass m 2 the added mass, m 2 into d 2 X 2 divided by d t square equals to F 1 minus F 2.

So, either when we are making force balance for mass m 1, which is nothing but in between the restoring force and excitation force, and when we are talking about the mass balance in between means, for this added mass in between the k 1 and k 2. So, we have F 1 and F 2, the resultant of these two restoring forces, where F 2 is k 2 X 2, F 1 is k 1 X 1 minus X 2, so these are the basic force balance equations for such systems.

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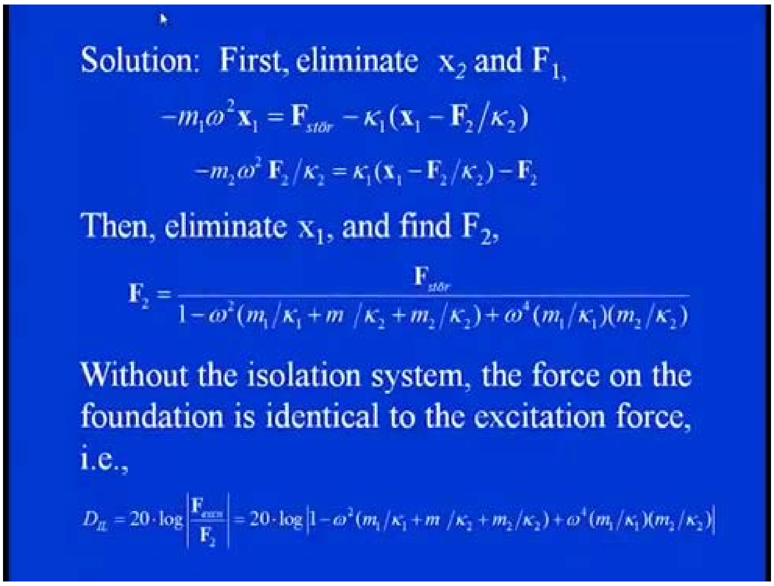

And when we are trying to eliminate the X 2 and F 1 then we have minus m 1 omega square X 1 equals to the F, whatever the storage feature minus k 1 into X 1 minus F 2 by k 2. And when we are just making balance of these things, we have minus m 2 omega square F 2 by k 2 equals to k 1 into X 1 minus F 2, which is the excitation force at the foundation divided by k 2 minus F 2, this is what it is you see here, we are trying to modify the equation based on the elimination of X 2 and F 1.

Similarly, we can go with the elimination of X 1 and F 2, so when we are trying these things, so F 2 which is nothing but the force at the foundation from the excitation of the machine is nothing but equals to the storage F. F storage divided by 1 minus omega square m 1 divided by k 1 plus m divided by k 2, whatever you see the masses, which are being coming out to do that, plus m 2 divided by k 2 plus omega square of this part. So, you see here, when we are trying to see these things we know that, the exciting frequencies in between k 1 and k 2 part spring can be straightaway get.

And when we are trying to see the force on the foundation when they are just coming, equal to the excitation force, the sound propagation is used. And if you want to find the insertion loss D IL, it is nothing but equals to the 20 log F excitation divided by F 2. So, how much excitation force is there and how much force is being transmitted to the foundation if there is no, we can say isolation system, immediately it will transfer out and we can get the 20 log 1 minus omega square, no k by m is there in that case.

So, now you see, but if you want put the entire system then we have 20 log 1 minus omega square m 1 k 1 m 1 divided by k 1 plus m 2 into k 2. There is m 1 into k 2 plus m 2 by k 2, plus whatever the omega 4, the down feature omega 4, m 1 divided by k 1 into m 2 divided by k 2. So, this is what you see, a clear transmission of the forces from excitation to foundation through these springs upto the ground part. And when we are trying to put these values, we know that, it is nothing but equals to 20 log.

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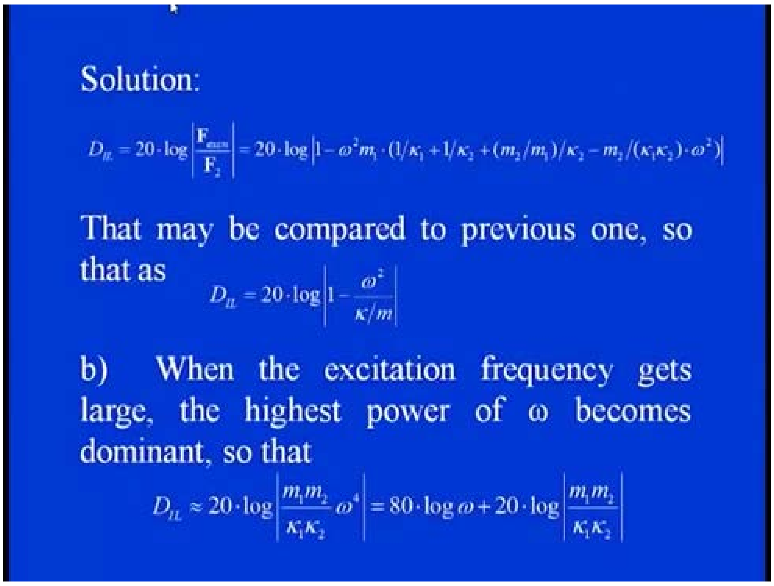

Again you see, when we are trying to modify these things, 20 log 1 minus omega square m 2 plus 1 minus 1 divided by k 1 plus 1 divided by k 2 plus m 2 by m 1 into k 2, we can say and minus m 2 k 1 k 2 omega square. So, when we are trying to compare those things we know that, it is a clear transformation of the entire forces from the excitation to the sink. And when we are comparing these things we know that, the insertion losses in such cases is 20 log 1 minus omega square by omega square by k by m.

So, this is what you see in these connection that, how the insertion losses can be computed. And when these excitation frequencies is just gets large then certainly we know that, the highest power of my exciting frequency omega becomes certainly dominant at the higher harmonics load. So, in that case, the insertion losses will be almost equal to all other feature like omega square m 1 and then this m 2 by m 1 k 2, they are all you see, though they are present, but the dominancy will be coming from the omega 4.

So, when we are computing this, the insertion losses, this is almost nearly equal to 20 log m 1 m 2 by k 1 k 2 omega to the power 4 or else we can say that, it is nothing but equals to 80 log exciting frequency plus 20 log m 1 m 2 by k 1 k 2. So, from this we can immediately compute that, you see what is the insertion losses when we are featuring out at the higher harmonics node.

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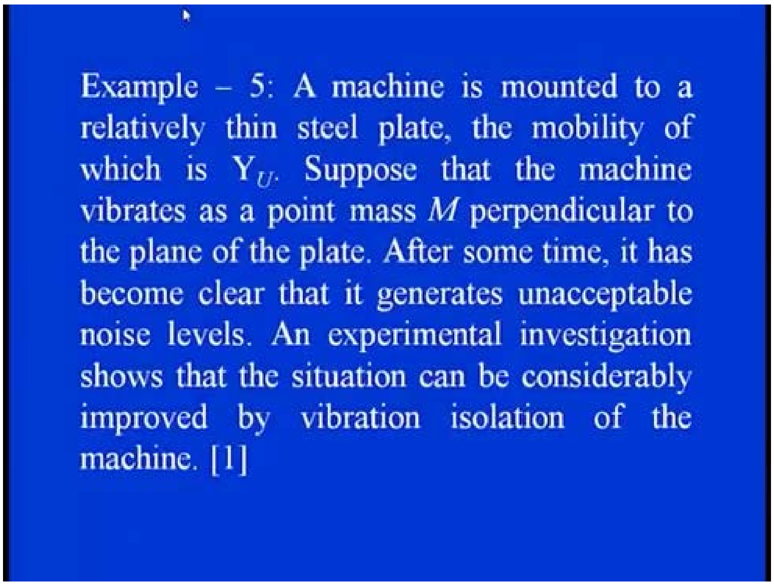

In another numerical you see here, now our thing is a machine is to be mounted on the thin steel plate with the mobility is Y U. And if we are saying that, the machine is vibrating as a point mass M, which is perpendicular to the plane of the plate. Certainly you see, when it is going as, after some time it is unacceptable to find out the vibration level and the noise level. Now, we would like to see that, how we can improve the vibration isolation of the machine.

So, you can see that, there are three figures are there, so that is what you see the machine, which is mounted on the steel plate first, which has the 50 kilogram of mass. The first arrangement in this is given as the spring of say 10 kilo Newton per meter stiffness is being added in between the machine to the foundation. The second feature says that, why do not we add with this, added mass there itself. So, we have a 2 kilogram of added mass to the ground, to suppress the another amount of the energy towards the excitation.

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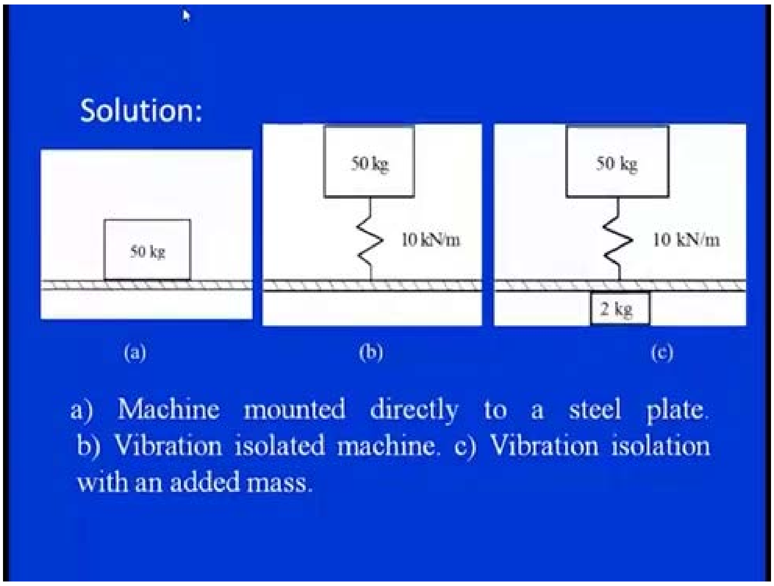

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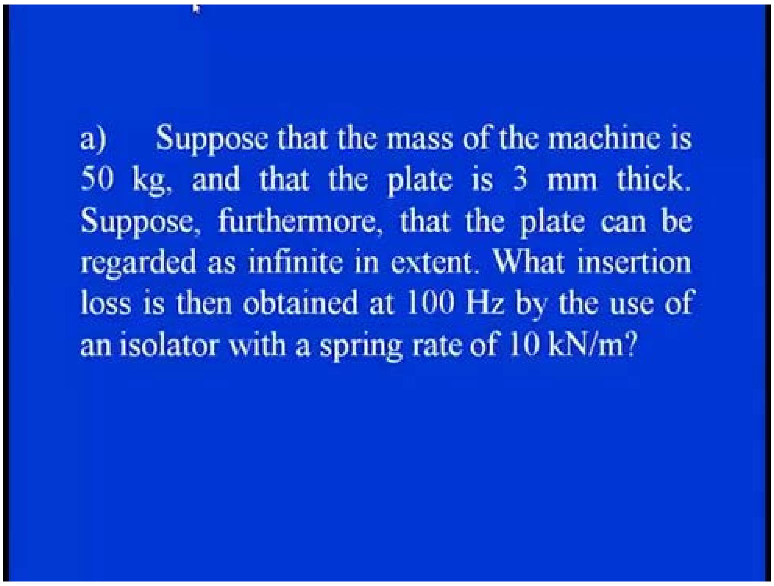

So, when you have these three arrangements, so first question is that, mass of machine is say 50 kilogram and all these plate is 3 millimeter thick and the plate is regarded as an infinite in the extent, what is the insertion loss could be obtained at the 100 Hertz of the exciting frequency with the spring rate is 10 kilo Newton per meter. So, this is what my exciting frequency and I would like to find out the insertion loss.

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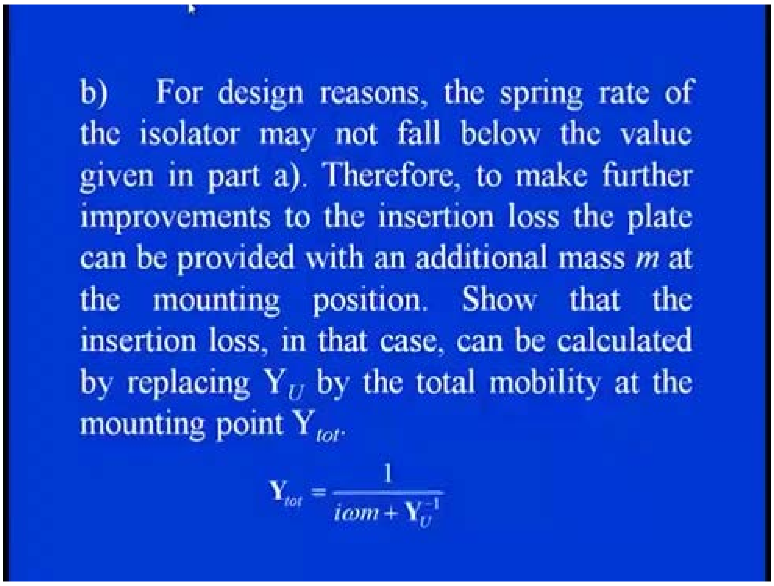

In the second feature with this spring rate say 10 kilo Newton per meter is not perfect isolator, as in the first case. So, to make further improvement towards the insertion loss is of the plate, just an additional 2 kilogram of mass is to be provided at the lower, as we shown in the previous diagram. So, now you see, the insertion losses can be calculated by replacing the mobility Y U by the total mobility as Y total equals to 1 divided by i omega m that is, Y M plus Y U to the power minus 1, that is what you see, we want to show that.

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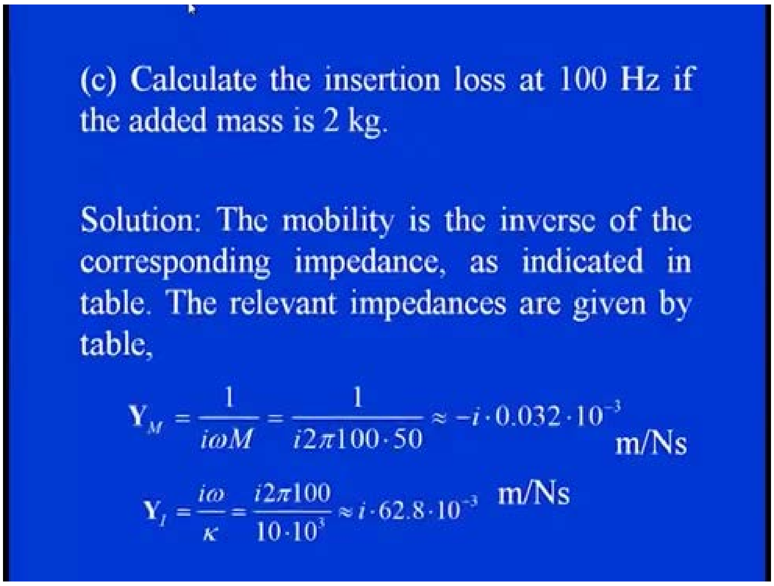

So, now you see here, the first case was this, the second was the mobility replacement in the total and the third case is that, the insertion losses at 100 Hertz frequency if the added mass is 2 kilogram. So, how much you see the insertion losses are there at that exciting frequency. So, the first case, the mobility is nothing but the inverse case of the impendence, as we know that. So, we can calculate the impedance Y M, the impedance equals to 1 by i omega m.

So, now we have 1 divided by IOTA, the omega is 2 pi into 100 excitation frequency and m is given as 50, kilogram of the mass. So, we have you see, the impedance due to the mass figure, it is minus 10 minus IOTA 0.032 into 10 raise to the power 3 meter per Newton second, just the reverse of the mobility and Y I, the impedance due to this stiffness or the restoring forces is i omega by k, as we discussed already.

So, it is nothing but equals to IOTA 2 pi, the F is given as 100 Hertz divided by 10 into 10 raise to the power 3, that is what you see the 10 kilo Newton per meter, the stiffness variations. So, we have now, the Y Ii equals to 62.8 into 10 raise to the power minus 3 meter per Newton second. So, we have Y M, we have Y I, whatever you see the impedances are there in that.

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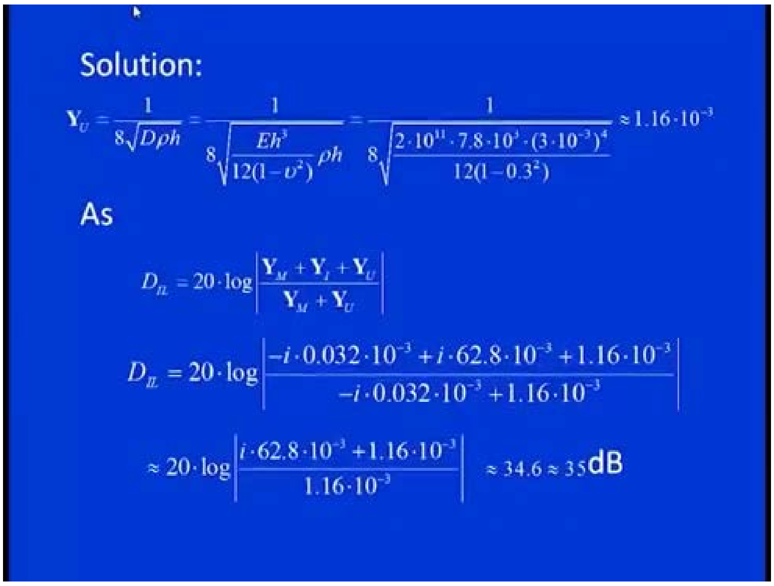

So, we can find out the Y U, that is nothing but equals to 1 over 8 square root of 8 D rho h or we can say it is nothing but equals to 8 E h cube divided by 12 into 1 minus mu square rho h. When you are keeping those things, we have Y U is nothing but equals to

1.16 into 10 raise to power minus 3 meter over Newton second. So, you see when we are calculating this, we can straightaway go to now our insertion losses, that is nothing but equals to 20 log Y M plus Y I plus Y U divided by Y M plus Y U.

And when we are keeping these values there, like you see the Y M is given as minus IOTA 0.032 into 10 raise to the power minus 3. Y I is nothing but equals to IOTA 62.8 into 10 raise to the power minus 3 and Y U is 1.16 into 10 raise to power minus 3 divided by Y M plus Y U then the insertion losses are almost nearly equals to 35 decibels. So, this is what the first case, in which you see, we have direct insertion loss at the 100 Hertz exciting frequency.

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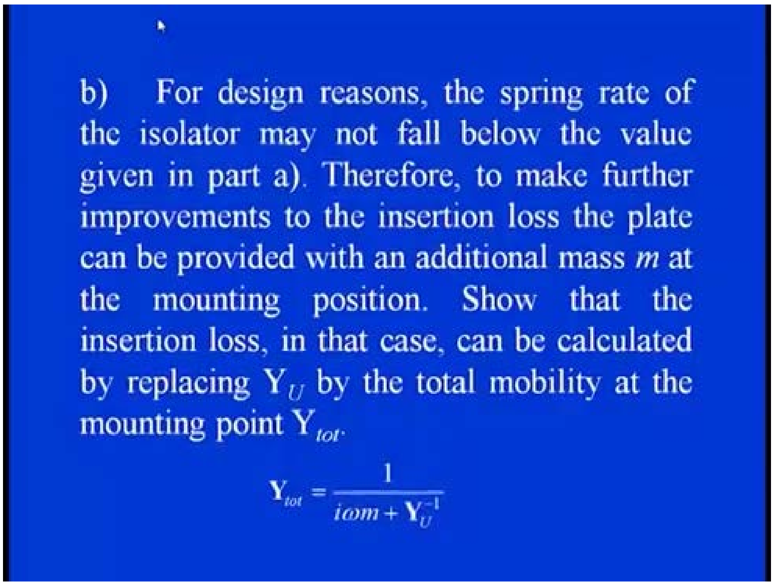

The second case is that, when you have the insertion losses, which we would like to calculate, absolutely when we are just making an additional mass m at the mounting position. And we would like to calculate the mobility at the mounting point, the total mobility in terms of the Y U inverse and i omega m, because of the additional mass m.

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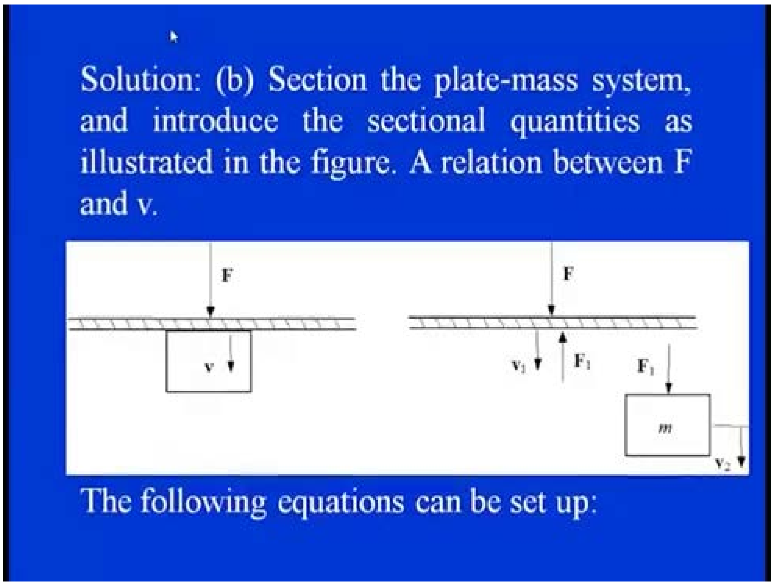

So, we can say that, in this figure, since you have the F, the exciting forces which is being there with the masses there and there is an additional mass, which is being there at the lower end. So, with the force balance we can say that, you have F and F 1, which is being the lowered feature and that this is the velocity, which is being there in between the eccentricity of that part. So, we can say that, in total, we have a velocity v 2 of this mass, v 1 with this foundation and you see the F 1 which is being acted on that on the top side of this mass, equal and opposite direction to whatever the foundation part is, because this mass is being added here.

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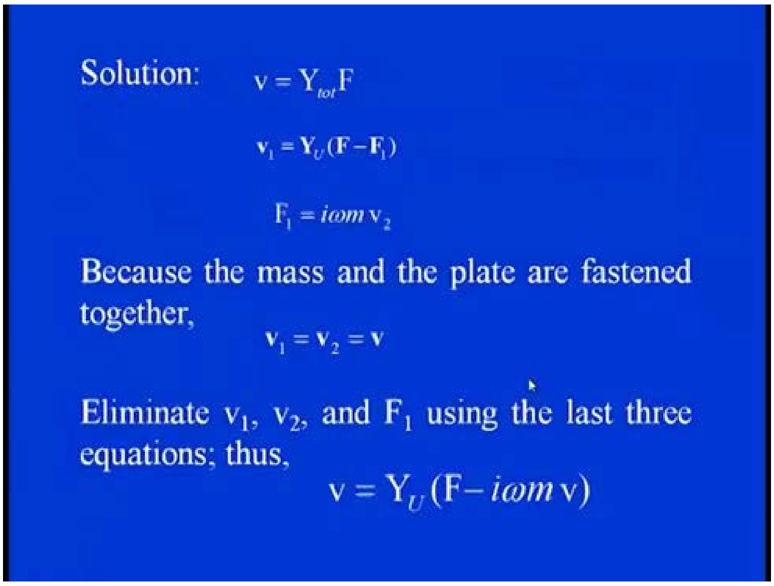

So, with this now, we can calculate the v equals to Y total F, because total you see we would like to find out this one, where the v 1 is nothing but equals to the mobility at the U, at the whatever the U velocity. So, v 1 equals to Y U F minus F 1, whatever the differences are there in between these two forces and where F 1, which is basically there due to the added mass there, is i omega m, this is the added mass m into v 2. And since you see the mass and the plates are just fastened together, we are assuming that, there is no velocity difference is there. We are not assuming any loss of this one, so v 1 equals to v 2 equals to v and when we are keeping those things and we would like to find out the v, that is nothing but equals to Y U F, the total force minus i omega m into v, this is what my F 1 was there earlier.

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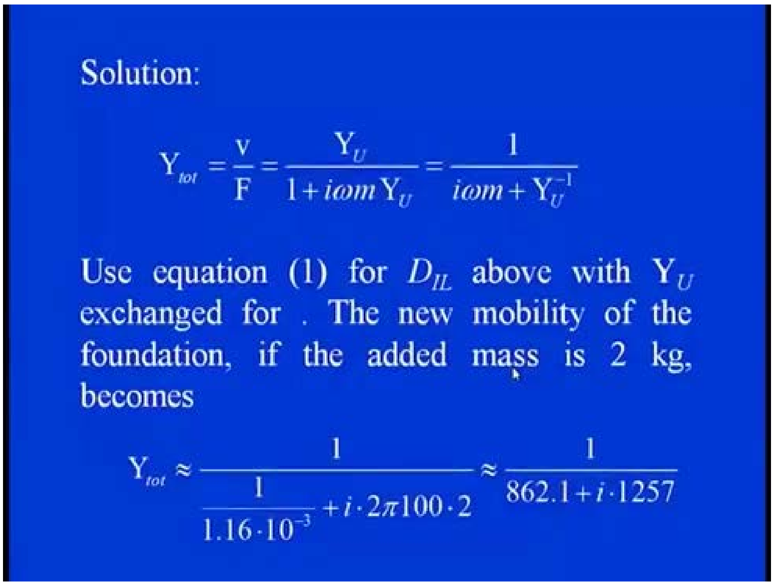

So, now, you see we can straightaway calculate Y U equals to v by F, where it is the Y total we would like to calculate. So, it is Y U v divided by I 1 plus i omega m Y U or else you see here, when we are just dividing these things, it is equals to 1 divided by i omega m plus Y to the power minus 1. So, you see here, the insertion losses for that can be straightaway exchanged when you see, when you are just exchanging Y U with the added mass.

And the new mobility for the foundation, if the 2 kilogram is being added exactly at the 100 Hertz of the frequency, that the Y total is 1 divided by 1 of 1.16 into 10 raise to the power minus 3 plus i omega, that is was omega is 2 pi F means, 2 pi 100 into 2, the added mass is there, 2 pi omega m. So, it is almost nearly equals to 1 divided by 862.1 plus IOTA 1257. So, this is what you see the total mobility, total we can say the kind of impedances, can be calculate with that.

And when we are just putting those things in insertion losses in this is nothing but equals to 20 log Y U plus Y M divided by the Y U. So, when we are putting all these things, it is almost nearly equals to 40 dB or else we can say that, it is 5 dB increment is there at the every stage of this one, in the insertion losses at the 100 Hertz of this one. So, this is you see the insertion loss calculation for any kind of exciting sources, when they are being converted towards the mobility and the impedances features.

In the last example, now we are considering a different kind of the matrices, you have the mass matrix, you have the damping matrix and you have the stiffness matrices together. And now you see, we would like to calculate for multi degree of freedom system, say you see, because the matrices are given. We would like to calculate that, what exactly the damping ratios for that and what the critical damping features are there.

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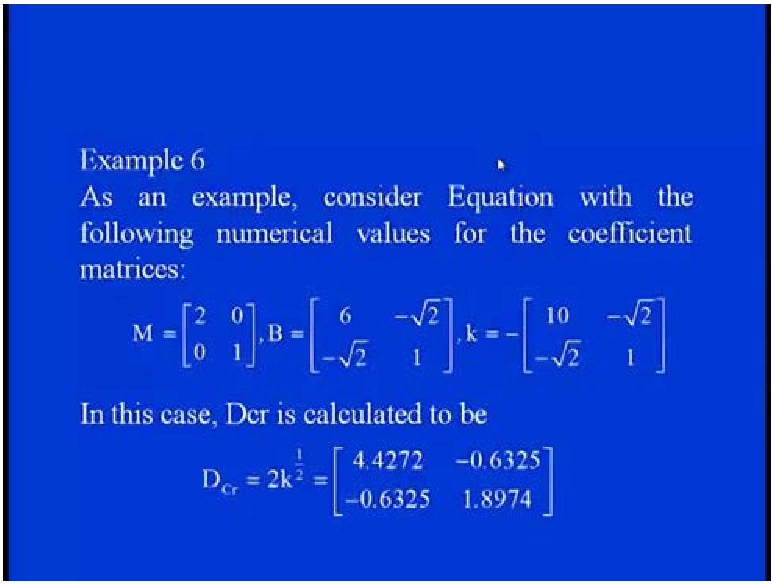

So, critical damping is nothing but equals to D critical equals to 2 k to the power half, so we can say that, this matrix is nothing but equals to we can simply square root of this stiffness matrices. So, we have 4.42 minus 0.63 minus 0.63 and 1.8, so this is what my critical damping matrix.

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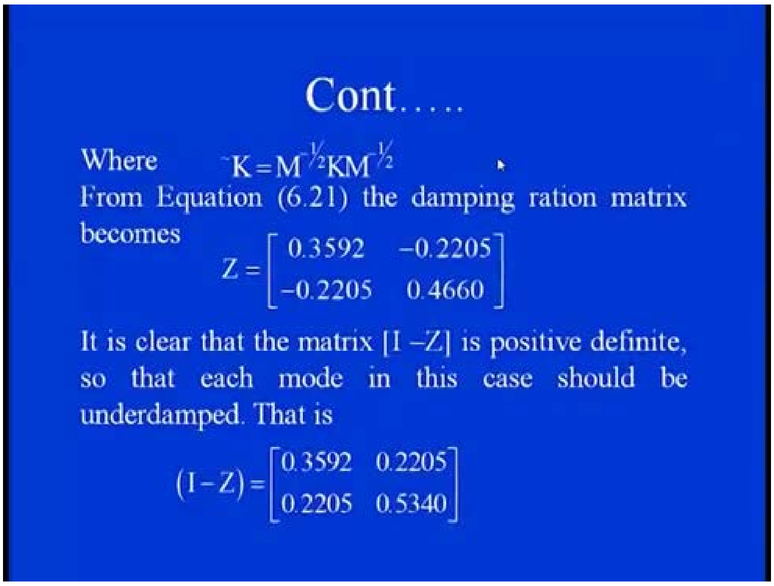

And then when we are putting this matrix to the main equation of K, which is nothing but equals to M inverse half K M inverse half or even M inverse minus M minus 1 K M 1. So, it is what you see the balanced equation, which we discussed in the last chapter of that. So, the damping matrix ratio, whatever you see the damping ratios are there in the matrix form, it is nothing but equals to 0.35 minus 0.22 minus 0.22, that is what the symmetricity and 0.466.

And it is clear that, the matrix I minus Z is positive definite and you see here, we can straightaway calculate that, this is what the case of the under damped system, where the zeta the damping ratio, which is nothing but equals to even the damping critical and available or the frequency excitation, the damped and undammed is always under damped system. So, that is I minus Z, we can calculate 0.35 0.22 0.22 and 0.53 can be straightaway calculate. So, with this you see here, we can say that, we can simply find out the determinant of this and it is you see, the less than 0.1432. It is a positive definite, so we can say that, this is what the underdamped system.

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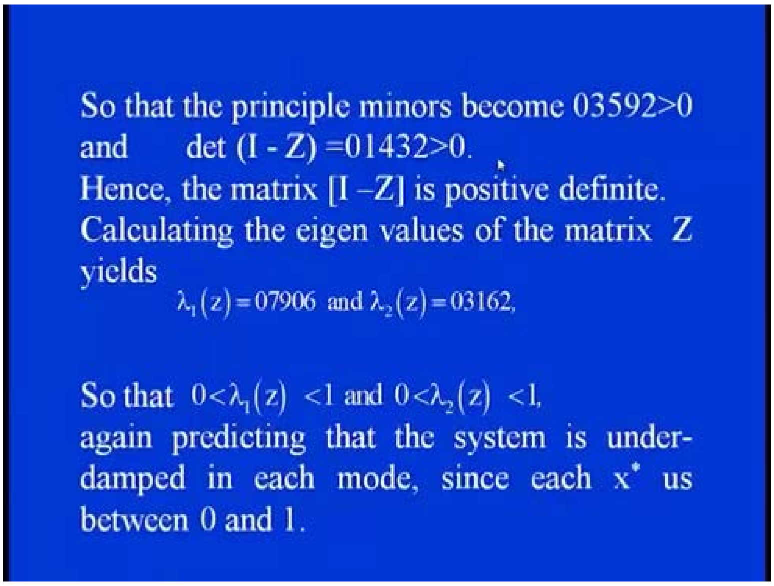

And we can calculate the Eigen values for this Z matrix, which is nothing but equals to 0.7906 and you see the lambda is 0.3126. So, you see here in this case, whatever you see the lambda 1 and lambda 2 are absolutely greater than 0, but they are going towards the less than 1, because they are all 0.79 and 0.31. We can say that, this is absolutely a undamped system, in which you have in phase and out phase of both the masses, as the matrix are just showing just the four elements in the two rows.

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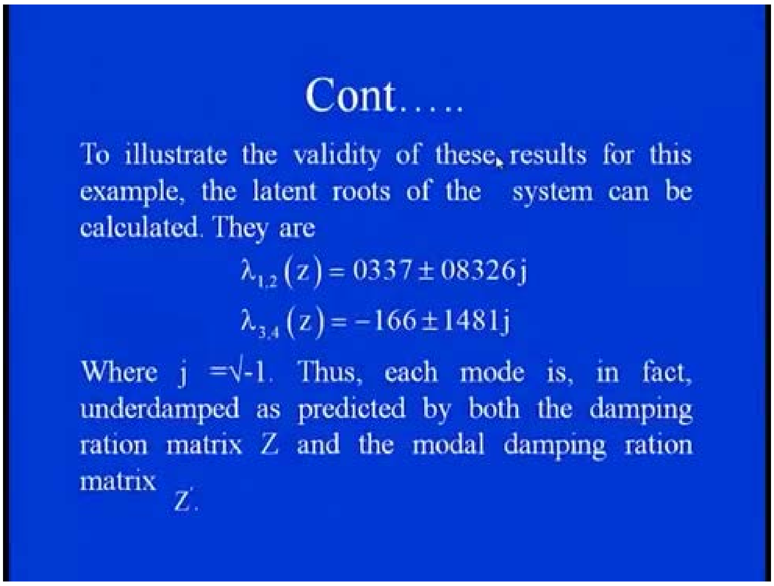

So, certainly you see we have the two degrees of system and both the masses are in and out phase. And in between we can say, the mode shapes are at 0 and 1, so we can say that, in the undamped system, this can be easily calculated. So, you see the lambda 1 and lambda 2, because we know that, we just want to find out these total amplitude. So, it is nothing but equals to 0.337 plus this 0.8326 j or else we can say that, the another feature is minus 166 plus this one. So, in this case we can say that, each mode, underdamped system is predicted by both the damping ratio and modal damping ratio, Z 1. So, we can say that, in such cases, where the underdamped systems are there, we have exploration decay in the oscillatory feature with sinusoidal features are there.

So, you have both coefficients lambda 1 comma 2 and lambda 3 comma 2 with this damping ratio matrix in this form, where you have 0.33 plus IOTA times of plus minus IOTA times and 166 plus IOTA times of these things. So, the first lambda 1 comma 2 is showing in phase and lambda 3 comma 4 is showing out phase, as minus 166 plus minus of this. So, these are the numerical problems, which are closely related to the insertion losses when the exciting frequency of the vibration and the sound propagations are there.

In the how much, when we are keeping isolators in various ways then how we can interrupt the path or interrupt even at the source itself, when the insertion losses are being there in relation to mobility or the impedance of that. And if the large difference is there, certainly we can say that, the isolator has to be check according to the insertion losses. The last numerical was there related to the multi degree of freedom system, where we can calculate the damping ratio for underdamped or critical dammed feature.

So, these numerical problems are just showing along with whatever the theory, which we discussed the internal mechanism or the physical sense of the those problems along with this. Now, in the next lecture, we would like to discuss about the another module, where you see here, we are going to discuss about the requirement of the vibration isolation features in these, when the materials or the damping or any other features are there related to the vibration control part.

Thank you.