**Vibration Control**
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**Module - 5**
**Design Considerations in Material selection**
**Lecture - 1**
**Design Sensitivity - I**

Hi, this is Dr. S.P Harsha from mechanical and industrial department IIT Roorkee. In the course of vibration control, we discussed about the basics of vibrations. And also, in the last, in the last module we discussed about the basic isolation feature that you see, when we are just trying to keep, the isolator then what exactly the design strategies are there or what exactly the processor which we need to adopt to effectively control the vibration. And also, you know like then, we discussed about mainly the vibration generation mechanism, in which various categories were discussed.

And then you see here, in the last lecture we discussed about the numerical problems you see then, when we are keeping even the vibration isolator. Then, how the even the losses are being computed in that, how effectively we can produce you know like, the isolator at the specific location or the amount of you see, we can say the isolator properties means, you see the damper or the stiffness or the masses, how we can adopt those things.

So, all these calculations were discussed in the last lecture and we found that, that if we, if we, if we, if we are just controlling the low frequency vibration or high frequency vibrations or at the resonant conditions. Then, how to choose the isolator and the same time you see here, we discussed about that you see even not only, you know like when the unbalanced or any defective features are there in the component then only, the vibration generations are there.

There are various other factors like the self-excited vibration, like you see even when the things are being rotating and some air or some you know like, the flow induced features are being coming then also, you see here the system is excited. And sometimes even in the self-excited vibration, sometimes you see or the system itself is releasing, you know like the energy within that and due to that which we termed as the negative damping, the system almost becomes unstable.

So, what I mean to say here that, there were various you know like we can say, the vibration generation mechanism, which was discussed right from material feature to you know like we can say, the masses or even, whatever the spring elements and all these features are there. In this lecture now, we are going to discuss about the design considerations in the material selection. Because, when we are trying to say that we want reduce the vibration, right at the source.

Then, certainly we need to choose specifically not only from the strength point of view, about the material of the component, but also you see here that material should also provide some kind of damping, towards the vibration excitation. Because you see here, if the material is in the support of the excitation then, certainly you see here along with the forcing frequency and all other factors. We may have some kind of you see, you know like the material phenomena like you see, you know like some kind of which we discussed in the previous lectures about the negative damping in various, you know like various other features, which will be coming out and contributing towards the vibration excitations, so in the first category of the design consideration in material selection, the design sensitivity. So, we need to see that, that when we are designing something then, what exactly the sensitive parameters or the sensitive features of those you know like, the entire phenomena's are in which you see we need to consider in a very prominent way.

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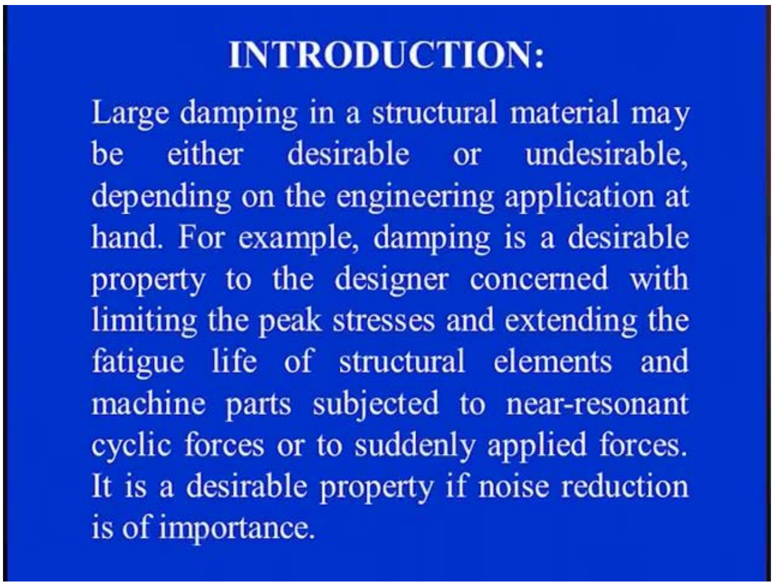

So, in that when we are talking about the large damping in any structural material, we know that either it is absolutely depending on whether the desirable or undesirable feature about the, you know like various excitation part. Because, you see here when we are talking about you know like, all these structures and you know like, the bigger structures, we know that the material damping is always being coming out according to the property of material.

So, sometimes you see here when we are talking about the desirable feature of the damping, as far as the designer is concerned then, we need to see that how we can limit the peak stresses. And how, they can be extended to the fatigue life of structural element or any machine part. And in this case you see here, we have to be very careful that when these excitations or any you know like, such kind of peak stresses are being forming. We need to limit, which are very close to the resonant cyclic forces or even any kind of impact loading. When you see, some abrupt changes are there not only in the magnitude of the forces, but also in the direction where it is to be applied to the system. So, in that case it is the desirable property of a damping, which is supposed to be there along with the material part. So that, we can reduce the vibration excitation and the noise reduction both together, when even you see here, we can put some kind of obstruction for the transmission of these.

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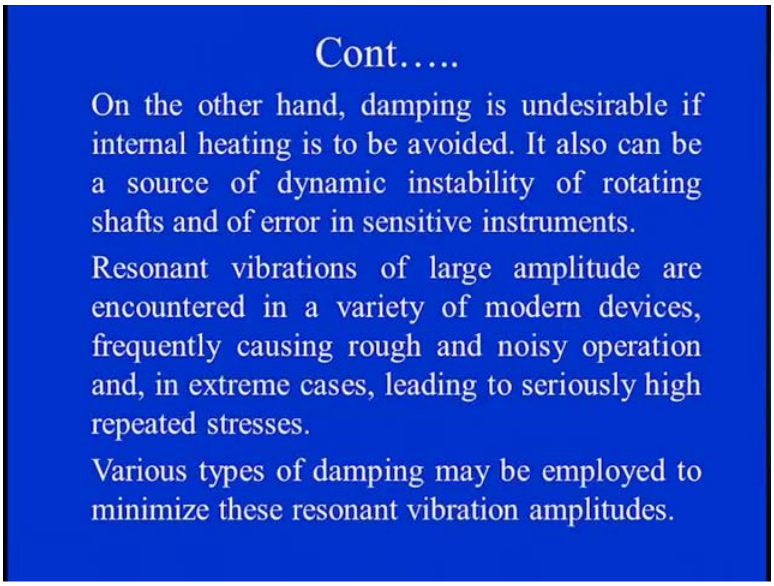

While, on the other hand the damping is sometimes undesirable as well. Why? Because if some internal heating, which we just want to avoid then certainly you see you know like, if such kind of things start are to be avoided, it is undesirable. And also we can say that, when the damping is become the source of instability, in any kind of rotating feature and then certainly, you see here, this is a kind of error generation in the sensitive instrumentation features.

So, when we are talking about the resonant vibration of the large amplitudes, which are always be there in many of the devices. Then, certainly you see here we need to specifically choose the kind of material, which can provide an effective way for dissipation of the energy. And we know that, some frequently causing we can say you know like, whatever the rough or the noisy operations or even in the extreme cases, when it is just going towards the high fatigue cycles or repeated stresses. Then even also you see we can say that, the damping is one of the specific feature.

So, there are various types of damping which can be employed to minimize these resonant vibration amplitudes, right from the intermolecular interactions or even the structural features or even, whatever the fluid surface the interactions are there. So, that interfacing all these you see, either the fluid surface interfacing or even the intermolecular arrangement of the material. They are all you see, are just playing a critical role in putting the overall damping towards the system.

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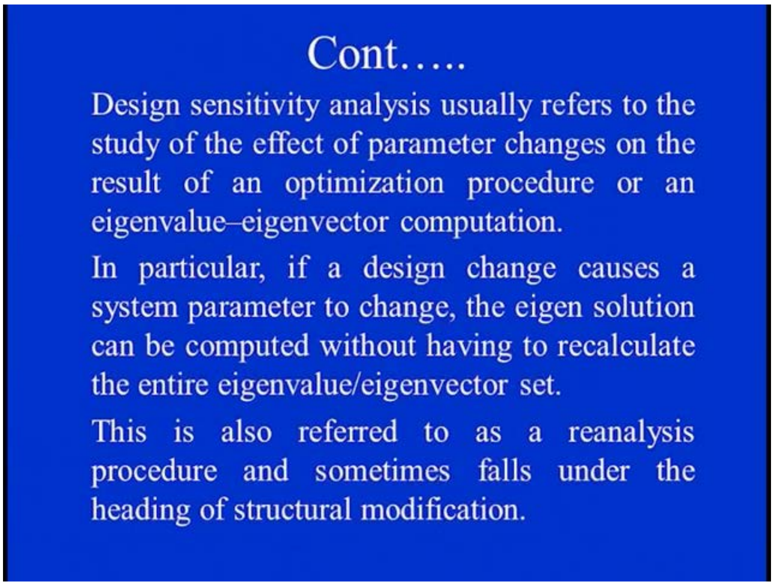

So, when we are talking about the design sensitivity that means, we are just talking about the effect of these parametric changes on the result of an optimum procedure or even, when we are just trying to change something in any of the design parameter. In a system, we need to check it out that how these Eigen values or Eigen vectors are being changes and how we can compute those features, along with the entire system performances.

So, in particular if a design changes and which causes the system parameter to change, the Eigen solution can be computed without having to recalculate the entire Eigen value, Eigen vector set. And if this is there, we can say that whatever the changes are being there, it can be, it is compatible and it can be immediately you see adopted accordingly. And this also you see, referred to reanalysis process or procedure and sometimes you see, it is falling under the heading of structural modification.

So, we need to see that, the how the parametric variations or the changes in these, you know like the whatever the parametric features are there are to be you know, incorporated so that, we can just go for recalculation or reanalysis. And the things are supposed to be more effective in terms of the structural modification.

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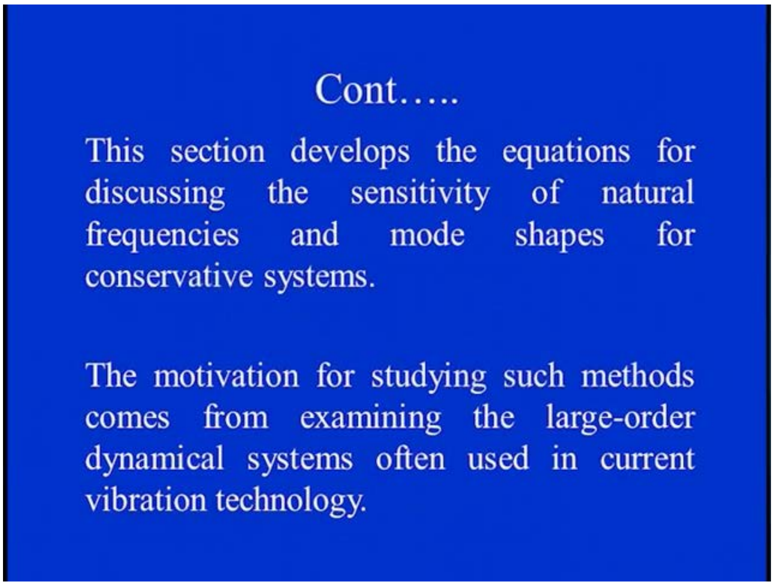


So, in this particular part we need to again generate the equations according to, whatever the changes. And in that you see here, we need to discuss about the sensitivity of these changes on the natural frequency and the mode shape means, the Eigen value and Eigen vector set, for any conservative system. So, such kind of you know like, studies which always you know like, gives the kind of sensitivity are of more importance when you are designing a sophisticated element from the excitation features. And in the modification for studying such method comes under, the large order dynamical system, often used in current vibration technology. So, this is what you see something, which is a need of the recent time when we are going for the designing of the large structure.

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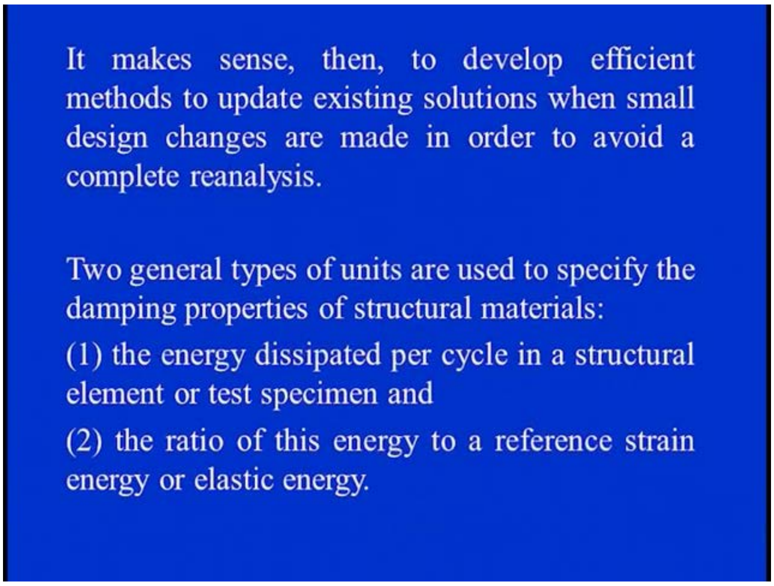


And we need to generate an efficient methodology so that, what are the design changes are there? It can be immediately incorporate without having a significant change in the entire system. Just like you see here, in the recent cars, what are the car modification? Every year you seem these main car companies are changing their entire system like you see here, right from the combustion chamber to the entire engine features. Even the breaking system, even the steering features, even the power transmission features.

And these change, right from the material to the entire structure, it is not that you see every time you will find that there is a significant change. But these minor changes are immediately you know like, it can be accommodate, accommodated in such a way that, it is not affecting the entire system or it is not requiring the complete reanalysis and redesigning of the entire features.

So, this is what you see the recent time requirement and we need to go accordingly so that, we can simply adopt whatever the product designs are there. What are the product you know like, if the design processes are there or even the these production processes are there, it can be immediately accommodated.

So, as far as the damping is concerned here, the two general types of units which are supposed to be specified, in terms of the design properties of any structural material. The first, the energy dissipated per cycle in a structural element or the test specimen. And the second is the, ratio of this energy, which is you see the dissipated energy to the reference strain energy or the elastic energy.

So, these two are the specific damping properties which has to be clearly specified, in that way especially for the structural membranes or the materials in which you see, we need to check it out that how much energy is to be dissipated in every cycle. And how you see the effect of this ratio, which is in between the, this dissipation energy and the strain energy or the elastic energy are to be disturbed with that. So, when we are talking about these two, first of all we need to go with what are the energy units which are available with the damping.

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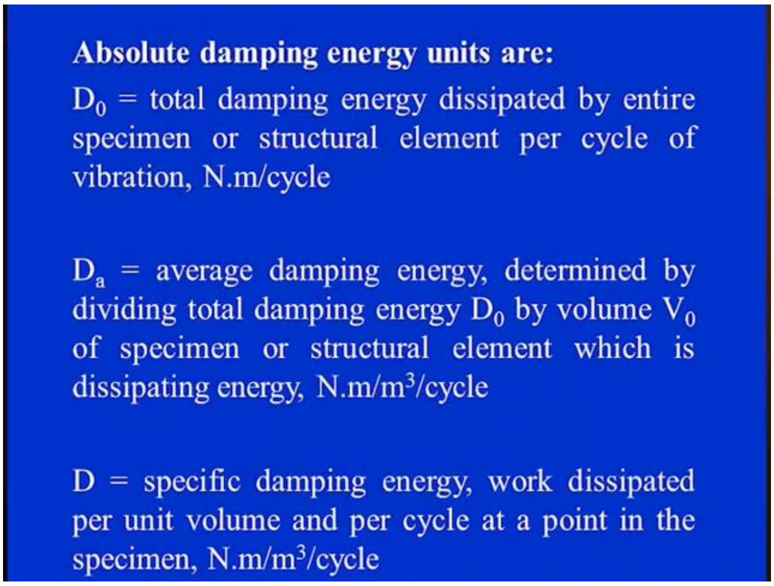


So, sometimes we are saying that the total damping energy dissipated by the entire specimen in every cycle. It is when we are saying that, you know like this, is what the significant energy is being dissipated by the entire structural, which is of very common unit you see here. In general terms we are always specifying this, we are saying that this is D0 say, the Newton meter per cycle whatever. The energy is being dissipated per unit cycle of the excitation of the vibration.

Second, if we are saying no, we are not going with the damping, the total damping energy or which is being dissipated. We are going with the average damping energy and average damping energy can be you know like, calculated using the total damping energy, which is being dissipated by the entire structural element, divided by the volume of the entire specimen.

So, in this case you see here we are basically looking for an average effect because we know that sometimes you know like the moleculars are behaving in the non-linear way. So, to go with the total energy dissipation from entire element, is not giving the specific feature of that. So, in that case you see here, we need to go with the average impact of such kind. And which is nothing but equals the total damping energy of the entire structure divided by, whatever the volume which is being occupied by this specimen.

So, in that case we have the Newton meter divided by the meter cube per cycle and we are saying this is Da. And also you see the third unit, which can also be used sometime is the specific damping energy that means, you see here it has a very specific feature. Where, the work, during the work you see here whatever the dissipations are there of that specific energy, per unit volume in one cycle.

But in this case you see here, we need to take, just like the stress or the force that at a point how much specific energy is being released, the specific damping energy is being dissipated, at that point in a unit cycle, during that entire work part. So, at that point we can say that, the specific damping energy D is nothing but equals to Newton meter, again divided by the volume meter cube per unit cycle. So, these are the three damping features D0 total damping, average damping Da and specific damping D. And of all these you see here, we need to first go with the absolute damping energy unit, which is nothing but you see here the total energy D0 is always one of the specific part of the damping.

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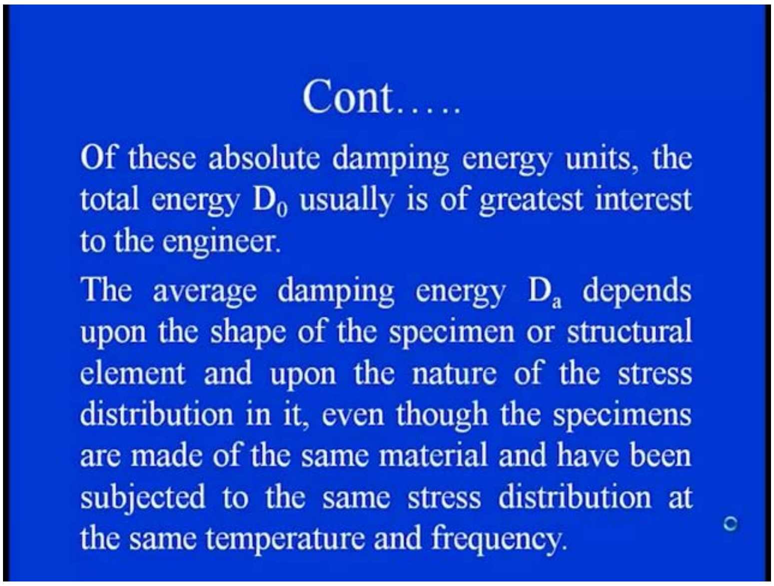


So, as we discussed the three damping's, out of these three damping's the absolute damping energy units, which simply showing the total energy D0 is the main interest of the engineers. And in that you see here, the average damping energy which we were discussing about you know like, the common features is absolutely depending upon what the shape of the specimen. Because you see ultimately, our focus is on the volume the entire volume, which is to be considered.

So, the shape of the specimen or the structural element and also, you see here it is, it is just depending upon the nature of the stress distribution in it. Because, you see sometimes when we are just talking about even the material is homogenous and there is no you know like, the differences are there, but when you see, when it is just subjected by the various stress distribution. And whatever the heat formation is there, with the structural elements or the inter molecular feature and because of this, you know like the inter molecular, this inter molecular we can say whatever the oscillation, some kind of you know like the energy is being carried out and the material cannot be remain as the homogenous one.

So, when we are just going for our design part and if, if we are just trying to focus on the average damping energy, Da then we need to not only check it about the shape but also, you see here that, what exactly the type of or the nature of the stresses, which is being distributed all across the material. And then also you see here, it is also we need to check it out that, when the material is subjected by the same stress distribution at the same temperature and the frequency. Then, what exactly the effect of these heat generations are. So, I mean to say that you know like, when we are just going we need to check it out not only about the shape or the distribution or the temperature effect. But also, we need to see that, whether it is convenient to take the average damping energy as the entire you know like, damping property or we need to go with the absolute energy damping.

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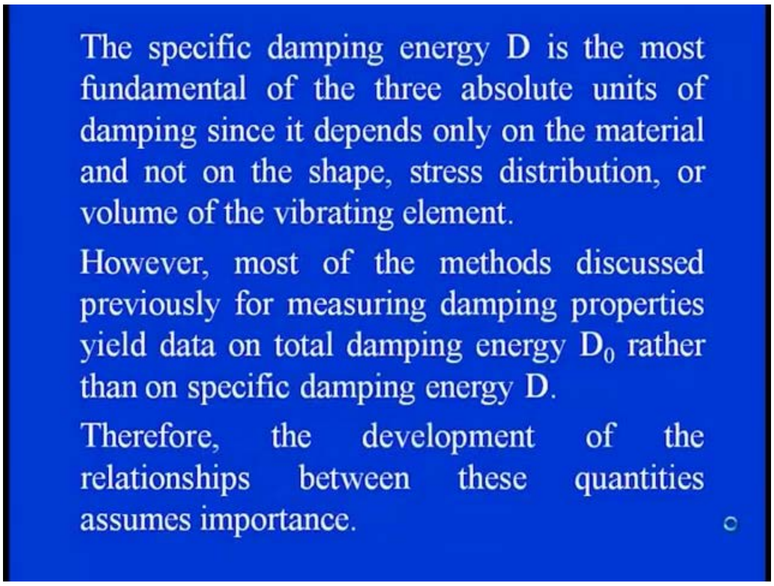


And when we are talking about the specific, the third one, it is the most fundamental of all three units. Because, you see here it depends only on the material, not on the shape or the stress distribution or the volume of any vibrating element. So, sometimes you see most of the methods which discussed you see here for measuring the damping property, it just you know like getting the data or the total energy damping that is the D0, rather than the specific damping energy D. So, you see sometimes you know like we need to check it out that, why D0? Why not D. So, you know like the, there should be a relation which should be you know like, established in order to quantify these things, with that assumptions.

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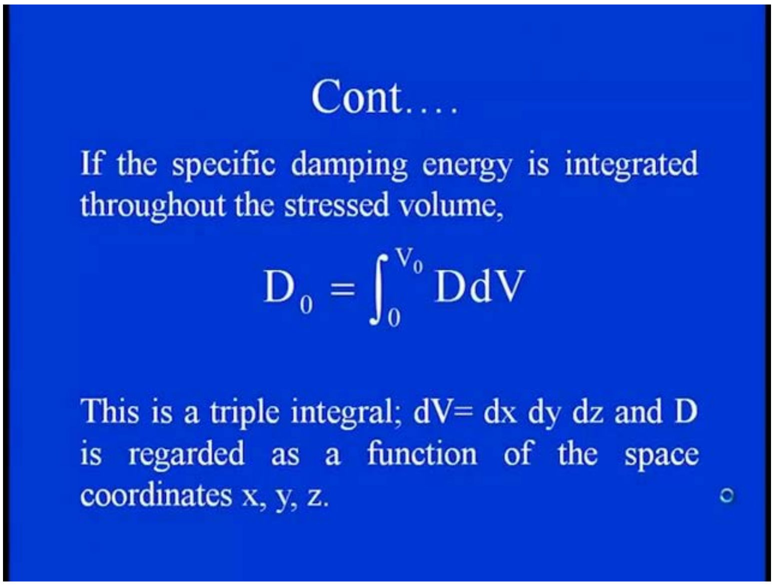


So, if we are saying that the specific damping energy D0 is just to be calculated for the entire element. Then, first of all we need to go with the D that is the specific damping energy. So, D0 is nothing but equals to the volumetric you know like, the volume integral 0 to V0, D into d V. And since it is a volume integral so certainly it is a triple integral with dx, dy and dz, the changes in all three directions.

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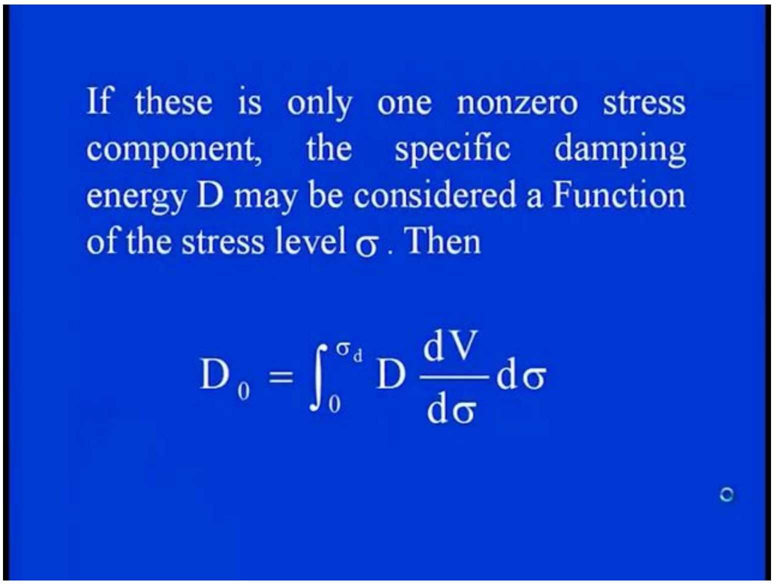


And the D is nothing but the function of x, y, z because you see here this is the specific damping and we know that it can be varied in all three x, y, z direction. So, we need to evaluate if we are just going with the, with the total damping then, we need to evaluate the variation of the specific damping in all three direction and needs to be computed altogether.

And if these is the nonzero stress component just like you see, you know like in the stress the tensor feature, then the specific damping energy D can also be considered as the function of this stress level, which is to be distributed in all across x, y, z feature. So, by D0, the overall damping is the integral of 0 to sigma d, D which is you see you know like the specific damping into d V by d sigma into d sigma. Now you see here, we are just trying to multiply with the entire volume change and the change of this, with the respect to the stresses changes.

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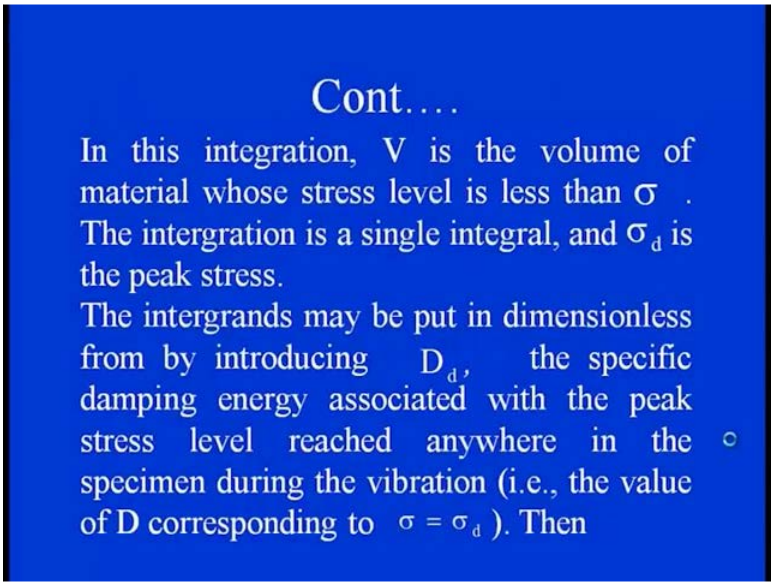


So, in this integration we know that the V is the volume of material, whose stress level is always being less than the sigma. And when we are talking about this, we can say that the sigma d, which is to be there, is nothing but the peak stress and these in this, in these integrands which is just putting there. So, Dd which is nothing but equals to the dimensionless feature of that, is the specific damping and associated with the peak stress level. And which may be reached anywhere up to the specimen, up to the, we can say when the, this extreme level of vibrations are there. That means, the D is nothing but equals to Dd when your stress level is going up to the peak stress features so by the maximum stress.

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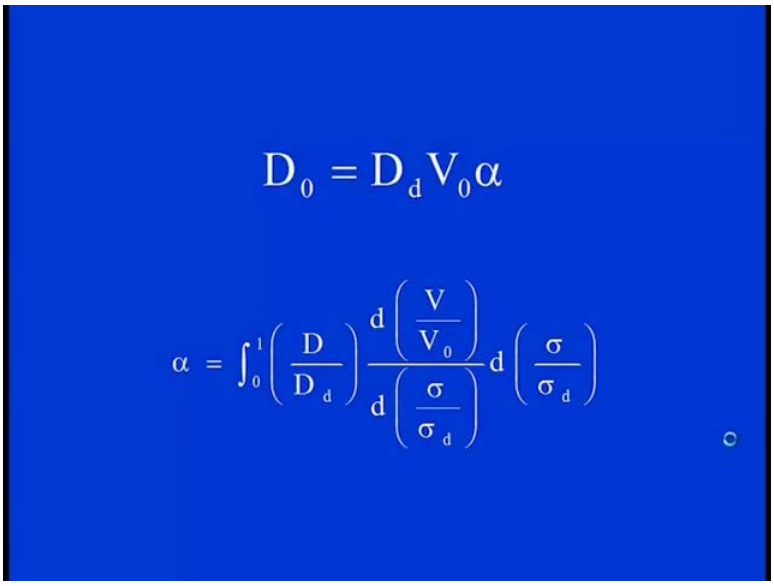


And when this is there, we can say that the D0 is nothing but equals to Dd into V0, the original volume into alpha. Where, alpha is nothing but equals to the coefficient which can be computed right from the integral 0 to 1 D by Dd. The ratio of specific damping to the damping, when it is reaches to the maximum value peak stress into the variation of d V by V0 and d sigma by sigma d.

So, now this ratio is clearly showing the relation between the volume changes and the stress changes, right from original to the peak value into you know like when, we are computing d into d sigma by sigma d. So, this alpha is a coefficient which incorporates all the type of variations and then it is just you know like, integrated, those variation right from elemental level to the entire feature.

And then, you see with this part when we multiplied with the Dd, which is the peak stress, the damping at the peak stress into the original volume you, we have the entire total dissipated energy. So, the average damping energy in this case can be also formed as we discussed D0 by V0 and we know that it is nothing but equals to Dd into alpha. The relationship between the damping energies like D0, Da and D is absolutely depending upon the dimensionless damping energy integral, that is alpha.

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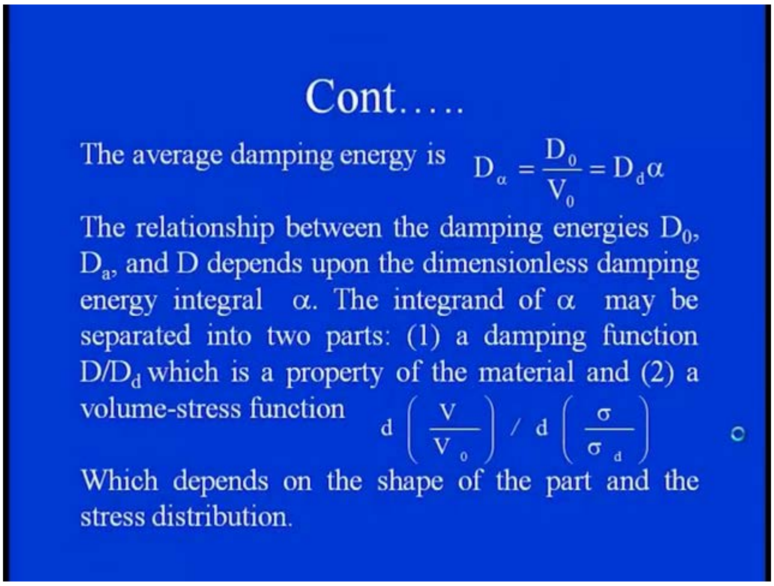


So, this damping energy integral is one often a specific property now, through which we can directly correlate all three types of damping feature. And this integrand alpha can also be separated into two parts as we discussed, first the damping function D by Dd which is, which is simply the property of material. Because you see here, we can see simply that, how the stresses are just going up to the peak level, accordingly Dd is calculated and D is the specific feature. And also second, it is depending upon the volume stress function, because you see here it is also you know like related with the d V by V d, V by V0 divided by d sigma by sigma d.

So, you see this alpha, which is one of the specific property for describing the entire damping feature of the material, which sometimes we are saying that this is something, the damping energy integral in the dimensional way is absolutely depending on both the part. So, we can say that this is you know like the part, which is depending on the shape of part as well as because you see d by Dd or with the material property and this stress distribution in that. So, both material property and the geometrical properties are straightaway incorporating in the alpha and then, based on this alpha we can, you know like set up the relation in between these all the damping features.

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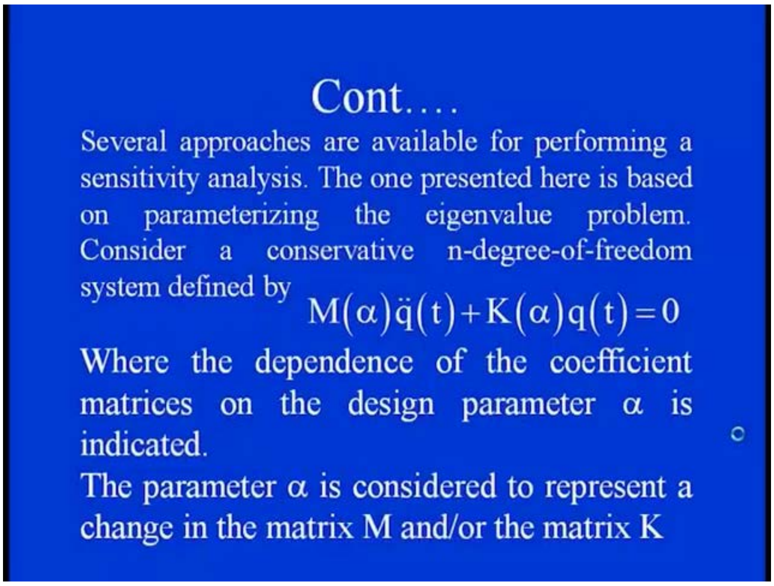


So, you know like the several approaches which are being available for performing the sensitivity analysis for such kind. And in that you see here, if we are just going with the parametric variation in the Eigen value problem. Now, we can go with the n degrees of freedom system like say m into alpha which is, is you know like the whatever the mass features are there of the material M into alpha q double dot t plus K alpha, which is also the property of this alpha, q t equals to 0.

Where, we are just trying to show the dependence of these mass or stiffness matrices on the coefficient of these you know like, we can say whatever, these we can say either the mass matrix in which the coefficients are there, the stiffness matrix the coefficients are there just depending on one design parameter alpha, which is nothing but equals to the you know like, just showing the relation of the damping in all sort.

And this parameter alpha is considered to represent the change in matrix, either m or k in such a way that you know like, whatever the changes are in material or in the shape it can be immediately incorporated in that. So, when we are trying to relate this with our Eigen value problem, then we know that we need to go with the inverse part. So, M inverse alpha into K alpha u alpha is nothing but equals to my Eigen value lambda i alpha into u i, that is my Eigen vector.

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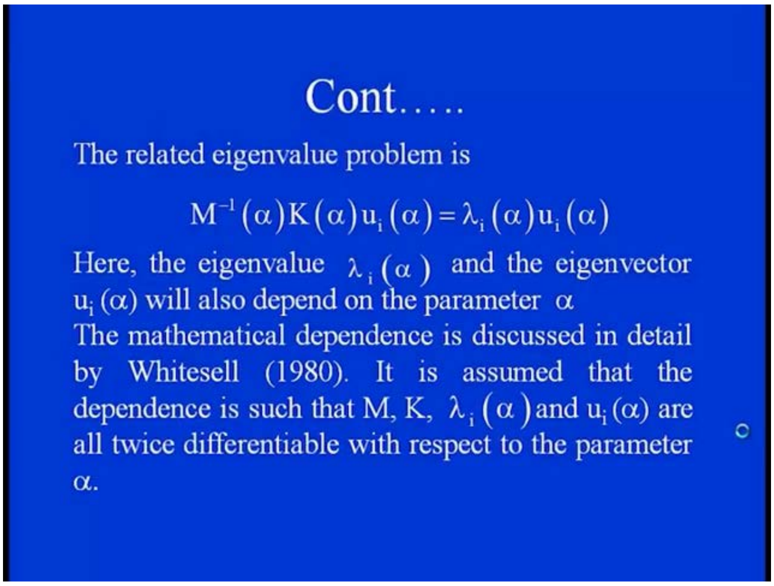


So, when we are trying to relate these things then we can say that, we can easily find out the Eigen value or Eigen vector means, the lambda i and u i. You know like based on the variation of the alpha with related, with in relation to the mass and the stiffness matrix. So, you this mathematical you know like discussion or whatever you see you know like the dependence feature of this entire mass and stiffness matrices on that, is being described by the visual. And then you see in this, we simply find out that when we are just trying to calculate the Eigen value and Eigen vector straightaway, we need to just go where you see the double differential features to calculate the alpha in that.

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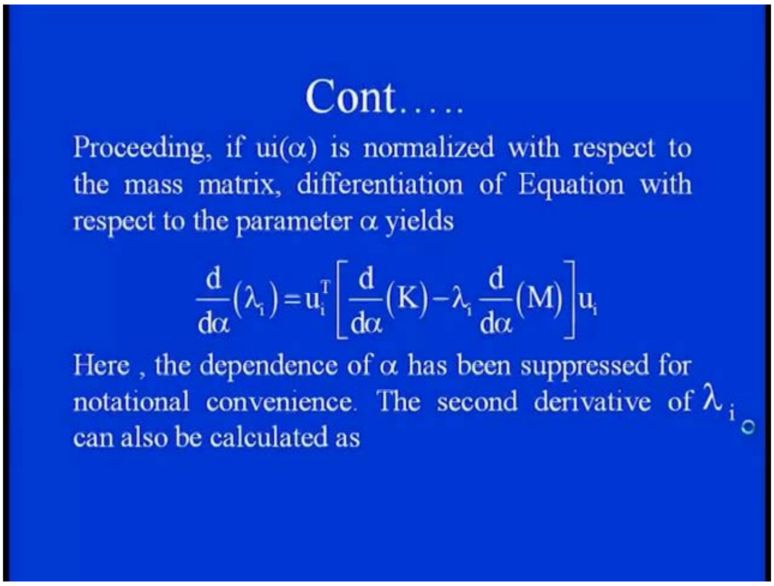

And when we are trying to do this, you see here we could easily figure out the Eigen vector u i alpha you know like, which can be normalized using this you know like, the mass matrix, with differentiation of these you know like, the entire feature with respect to this alpha. So, in that the d lambda by d alpha because we want to differentiate it out this, is nothing but equals to whatever the transpose feature of my Eigen vector means, you know like the u i transpose into d K by d alpha minus lambda i d M by d alpha, with respect to u i. So, both the differentiations are being there with the stiffness and the mass matrixes and they are absolutely depending on the alpha. So, alpha is one of the design parameter and it is clearly showing that if any changes are there, the entire changes are there, not only in the Eigen value, but also in the Eigen vectors. So, it simply shows the sensitivity.

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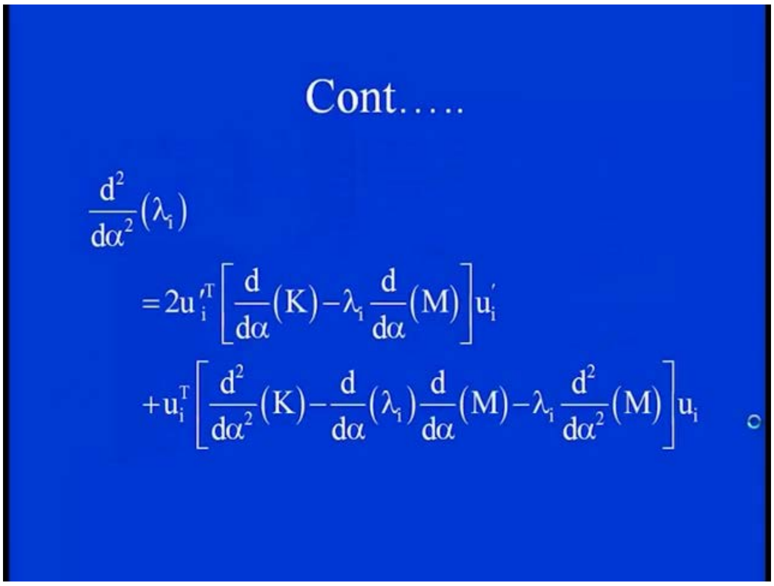


So, if we are going with the double derivative of this lambda, in the differentiation feature because we want to calculate those things. So, it is nothing but equals to d to 2 lambda by d alpha square equals to 2 u dash transpose d K by d alpha minus you know lambda into d M by d alpha, that was there. Plus now, u transpose d 2 K by d alpha square minus d lambda d alpha d M d alpha minus lambda i d 2 M by d alpha square. So, this is what a clear variation in the stiffness and mass matrices, when we are going for the double derivative feature of the lambda, with respect to the alpha.

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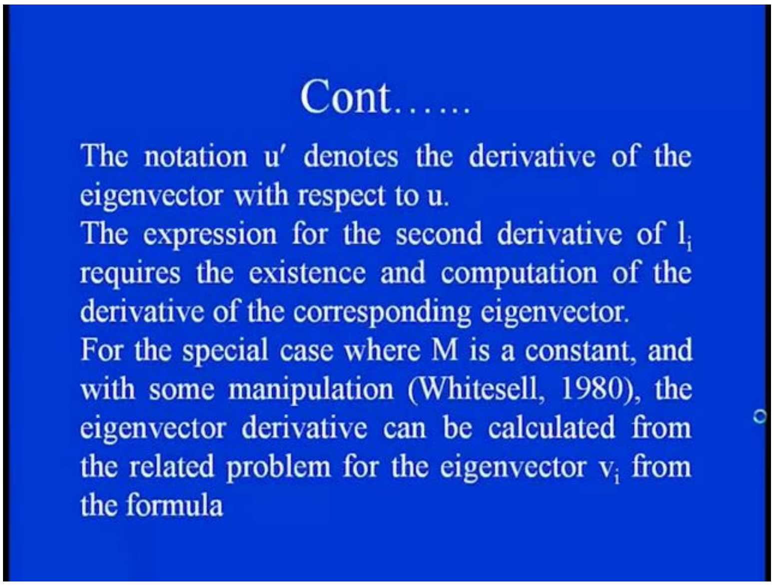


And this you see the, u dash which is simply you know like we are keeping there is nothing but the derivative of my Eigen vectors with respect to the u. So, we can say that the second derivative of this you know like, the lambda is simply showing the dependence of the you know like, all the stiffness and this mass matrices on the lambda. And it, this is clearly showing that the requirement of the existence and the computation derivative is absolutely showing the Eigen vector variation, in the double derivative.

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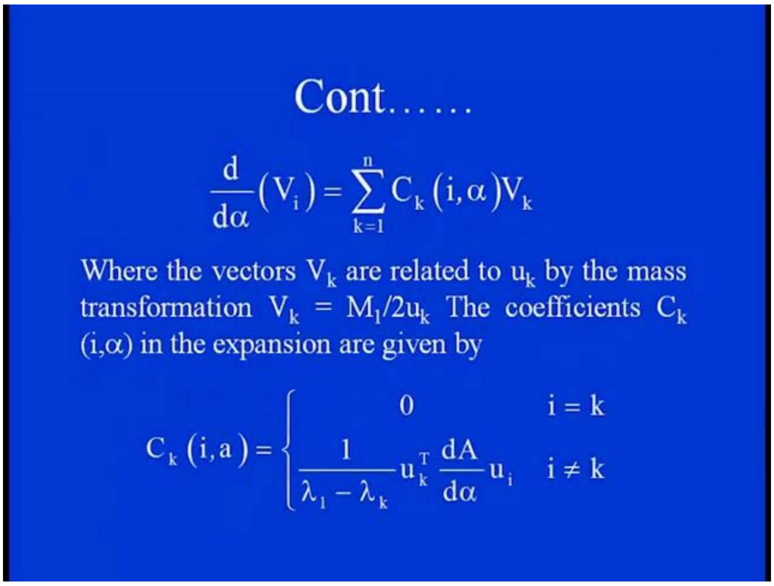

And where you see you know like, when we are saying that the M is constant then, certainly the Eigen value, the Eigen vector derivative can straightaway coming out, with straightaway. You know like, with this particular formula, where the d V i by d alpha is nothing but equals to summation of all these variation, right from 1 to n C, which is again you see here, the coefficient is there in that absolutely you know like depending on the Eigen, the Eigen you know like, value and with the associated you see the matrices.

So, C which is i comma alpha Vk and Vk is absolutely you see the vectors, which is based on the mass transformation. So, we can say that we can calculate Vk as M i divided by 2 u k and Ck can be because if the Ck is you know like, depending on the i comma alpha. So, we can say that it is when i and k both are same means, when there is no variation in that certainly it is 0. So, whatever the variations are there of d V i and d alpha, it becomes 0.

But when you see, you know like if there are not equal then certainly it is showing a difference in the lambda value, lambda i and lambda k. And then you see, you know like we can straightaway calculate by 1 by lambda i minus lambda k, u of transpose dA by d alpha u i. So, the coefficient can be straightaway calculate according to what the variations are there.

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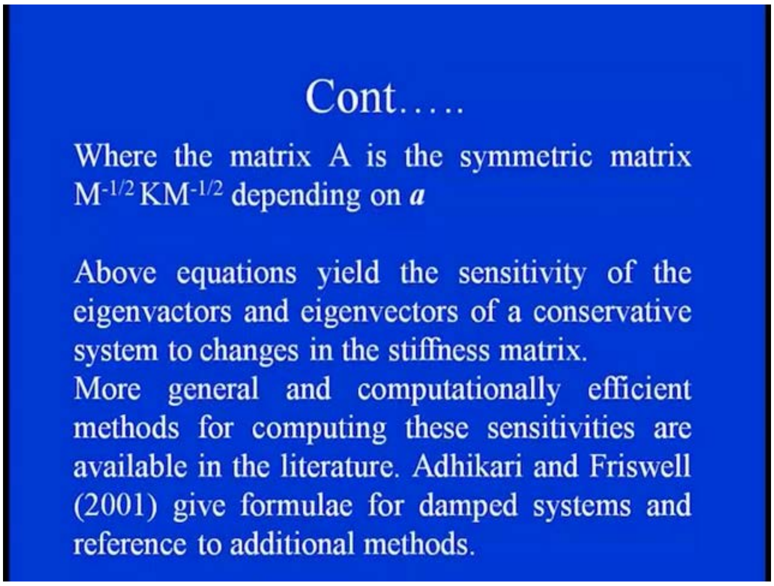


And then you see we can say that, the matrix a which is just showing the symmetric matrix there, is absolutely depending on A and the symmetric matrix is M to the power minus half km M to the power minus half. So, you see here this is what the symmetricity in between the stiffness and mass matrices and accordingly, we can calculate the entire features. So, when we just want to calculate this sensitivity of the Eigen vector and the Eigen value, for any conservative systems then, we need to go that whatever the changes are there in these stiffness matrices. And then means accordingly, we can simply find out the overall effect on this part. So, we can say that when we are just going with you know like, computationally efficient methods for these sensitivities we need to go and you know adopted various things. And in the recent paper of adhikari, they are also you know like calculated all these sensitivity for the damped system. And they simply shows that, how you know like these mass and the stiffness matrices are depending on these design parameters.

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So, when we are talking about the design of damping, there are various ways through which we can calculate like you see, we can take the laminated material. In which you see you know like, some laminated metals are there and they are being simply putting for effective, for effective you know like, control of this one. And this laminated metal material, offers an effective way to increase the inherent level of damping in a simple sheet metal for vibration transmission and the generation.

So, sometimes they are assisting to the product engineer with consideration of these you know like, the laminated material in place of the traditional sheet. And then we need to check it out, that how we can optimize the damping with this. Then, how we can go with the, you know like, these laminated features and then, how we can make the entire composite part.

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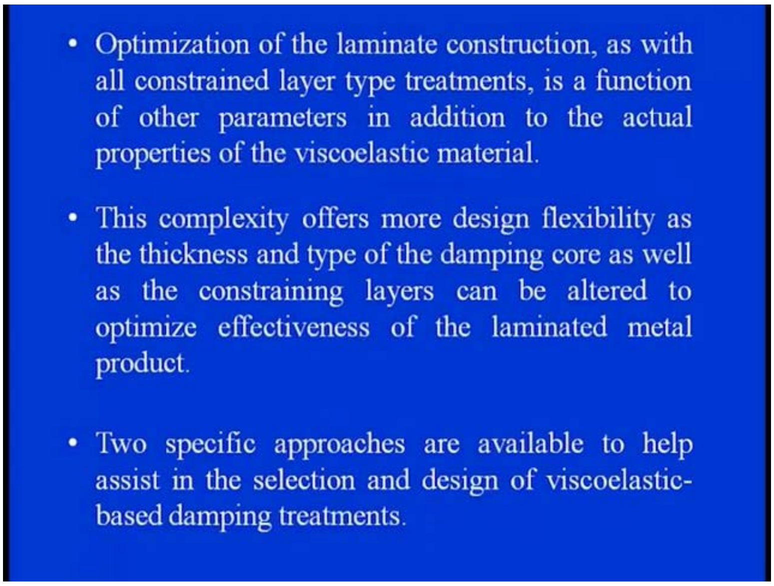


So, this optimization of the laminated construction is absolutely requiring what the parameters, which are being required and how it is being added to the base material. So that, it can provide some kind of viscoelastic material property or other material property, but again you see this kind of feature is always offering a complex solution. And you see here, like the design flexibility, like you see in the previous case we have seen that, just a design parameter is there and sensitivity of the entire feature is depending on that.

But here you see here, we need to check it out for such kind of thing that what is the thickness or the type of you know like, the damping cores are there or even you know like, what are the constraining layers are there, which has to be put in such a way that, we can get an optimum effectiveness of this product. So, you know like there are various approaches are being available in that, according to we can simply select and design the viscoelastic based damping treatment of such problem.

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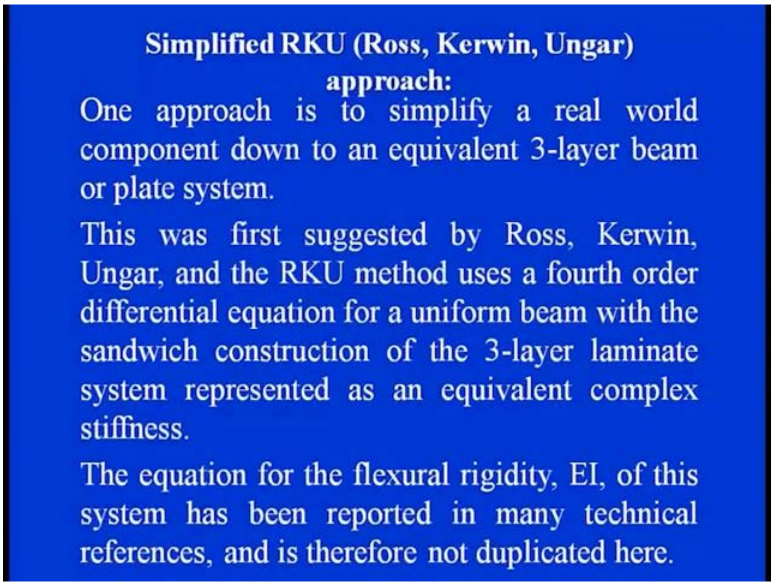


So, the first is the RKU approach, the RKU approach is nothing but you see, you know like the three they simply offered this. The Ross Kerwin and Ungar and they said that, we can simply go with the real world component and we can straightaway make an equivalent three layer beam or the plate for such systems. So, they suggested the method uses simply a fourth order differential equation for this uniform beam, which has you know like sandwich kind of, the sandwich kind of, sandwich kind of feature in the you know like, in this three layer laminated system. And they simple replace this three layer laminated feature, with the equivalent complex stiffness.

So, in that the basic impact is coming on the flexural rigidity EI because sometimes you see here you know like we cannot simply replace this EI. Because E is the young's modulus, the I is the cross sectional feature. So, both material and the cross sectional features, for these you know like the equivalence feature of the three layer laminated part, is somewhat more complex. Because you see here, when you are just going in a uniform way, it is pretty easy to put in a, in a linear algebraic way that just put in a you know, added those part E, E1, E2, E3 or you see I1, I2, I3 added. And we can get, no it is not like that because the real word problems are somewhat different.

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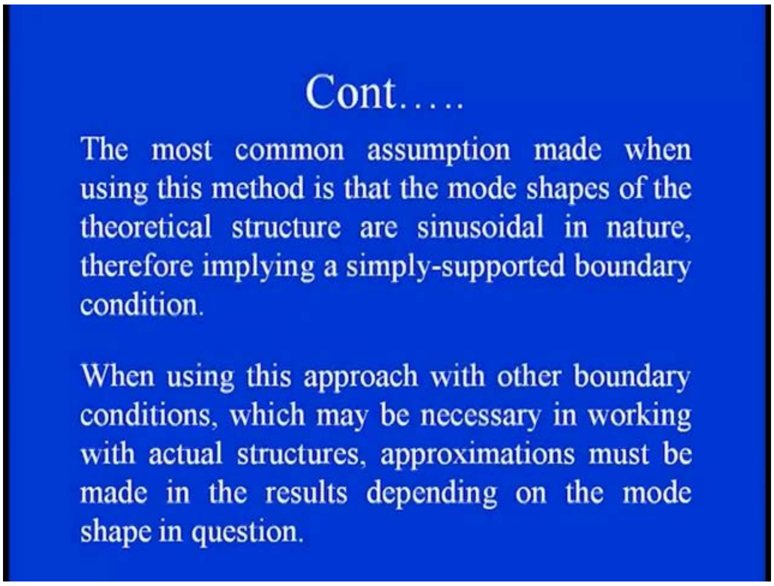


So, we need to make certain assumptions, when we are just going with the mode shapes of this structure. Because, we need to see that when it is a sinusoidal nature then, there is you know no problem with such kind of boundary condition, like simply supported or this. But when you see, some kinds of non-linearities are there in that deviation then, we cannot go with a straight linear assumptions. So, first with this you know like approach, the other boundary conditions which are necessary in the working with the actual, we can say structure, the approximation which has to be there according to the mode shape in this part.

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So, RKU method is really good as a damping indicator when, simply we are not going for the precise damping, but we are just going with you know like, some kind of formation for the real complex part. And the goal is to, is to use this simplified method to develop the design trend, will simply give a selection of damping material, with the constraint layers, the thickness and just to adopt, these things to provide you see here the optimum damping procedure. So, this was the first approach.

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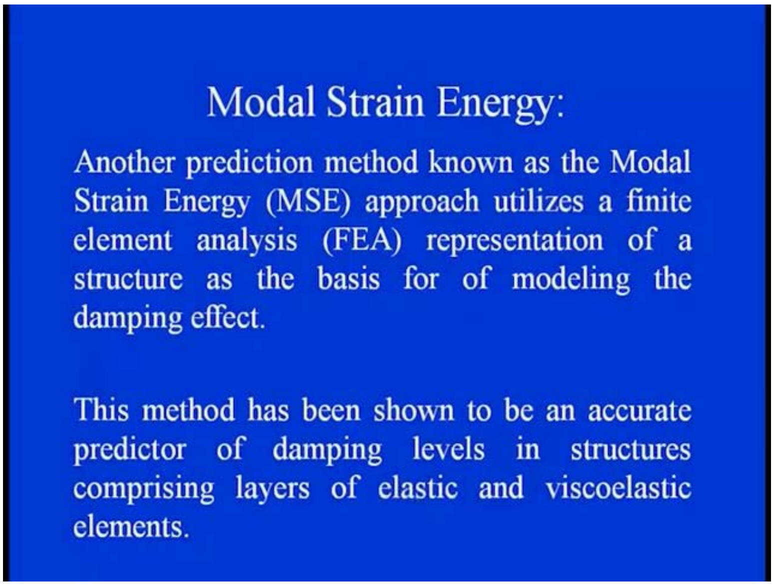


The second approach is the model strain energy. So, the method which is based on the model strain energy, is being utilized for the finite element solution of such problems in which you see here, the entire element is discretized based on you see here, various molecules. And then, this method is just providing an accurate predictor of the damping level, which is simply comprises of the various layers with the properties of elastic and viscoelastic elements. So, this principle states that the ratio of composite system loose factor because you see we know that when you, when we are talking about you know like the viscoelastic feature there is always loss factor is there.

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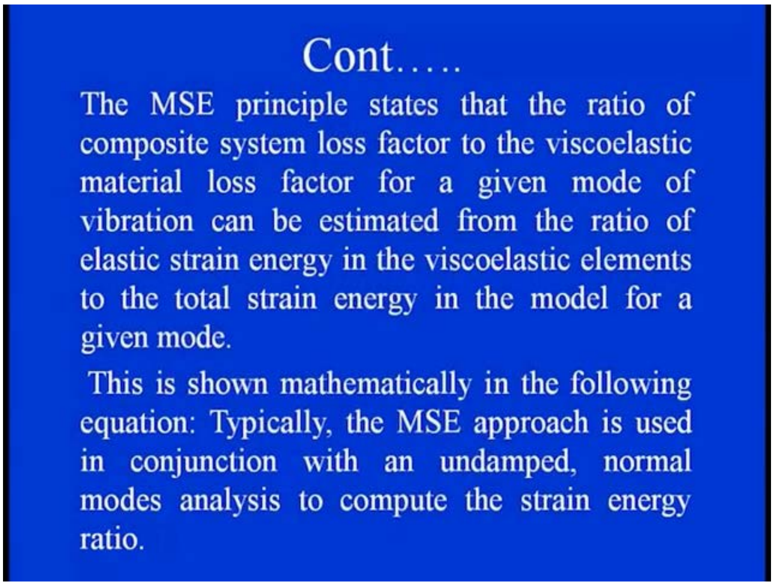


So, this the principle here states that the ratio of the composite system loss factor to, the viscoelastic material loss factor, for a given mode of vibration can be simply estimated with the ratio of elastic strain energy, in the viscoelastic element to the, through the total strain energy in the model for a given mode. So, this is one of the significant feature of this you know like, the strain energy based principle that, we can straightaway go to the elastic energy, elastic strain energy in any viscoelastic element and then, what is the total strain energy is there in the entire model.

So, when we just want to show mathematically then we need to simply go with certain equations, which simply consists the MSE approach in some conjunction of the undamped normal model analysis, which simply gives you the strain energy ratio in that. So, the strain energies can be determined in, in, in relation to the mode shape, relative mode shape and it is being assumed that the viscoelastic properties here, are somewhat linear, in terms of the dynamic strain rate. So, when it is linear the we can say that, the eta s to m, that is you see the, that is nothing but the viscosity is the n VEM into U m VEM divided by U m total.

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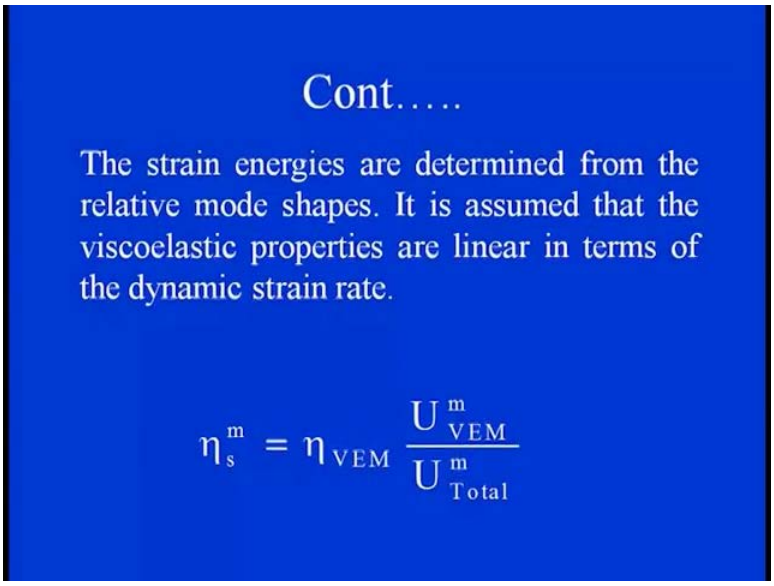


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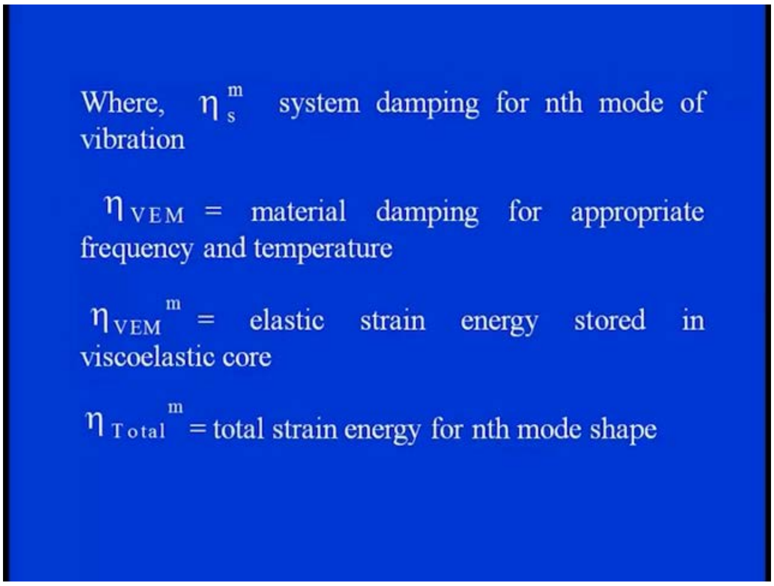


Where we know that this eta is nothing but the damping for the n-th mode of this structure and n VEM is nothing but the material damping for an appropriate frequency and temperature. And VEM into m you know, to the power m is nothing but the elastic strain energy, stored in the viscoelastic core and n total is the total strain energy for n-th mode shape. So, we can straightaway relate the system damping with respect to the material damping at an certain temperature and frequency, the strain, elastic strain energy of the viscoelastic core and the total strain energy for n-th part.

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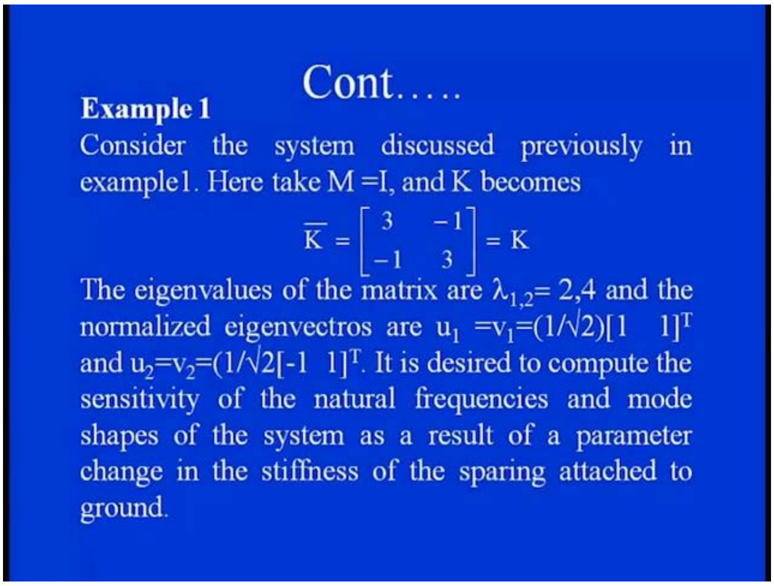


Now you see lastly we are taking one example, in which you see here we are simply considering that, we have the mass m, which is you know like the same example which we discussed in the last chapter, the last numerical was there. We have the stiffness matrix K is 3 minus 1 minus 1 3 so that, you see the inverse and this becomes same.

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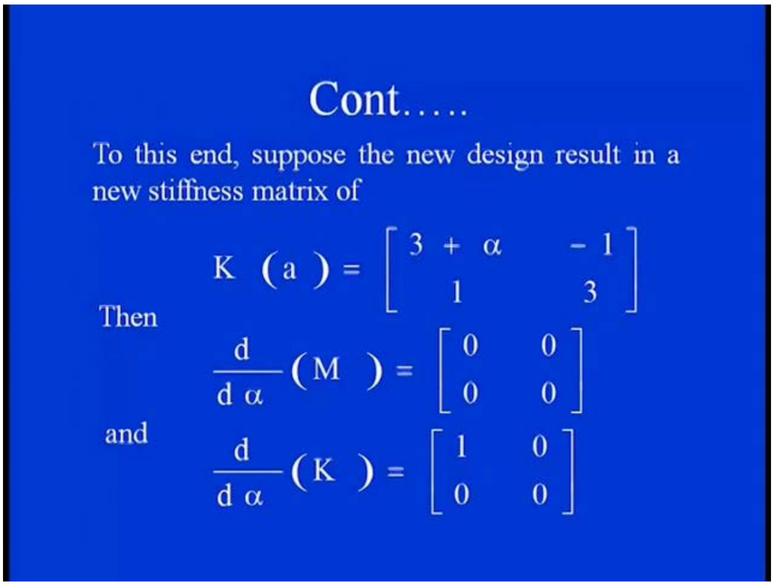


So, we can calculate the Eigen value for such undamped, you know like system. So, it is lambda 1 comma 2 is 2 comma 4 and the normalized Eigen vector can also be calculated as u 1 is nothing but equals to v 1 and we have, 1 by root 2 1 comma o1ne transpose. So, u 1 and u 2, v 1 and v 2 can be calculated and it is desirable to compute the sensitivity based on the natural frequency mode shape, as one of the parametric changes of that.

So, now you see here for this now, we are just taking the stiffness matrix based on the sensitive parameter say a. So, we are adding 3 plus alpha, this is what my design parameter, 3 plus alpha minus 1 1 comma 3. So, we can simply see the mass and stiffness matrix with the variation of alpha, we can get you see you know like the 0 0 0 0 mass and stiffness you see 1 0 0 0.

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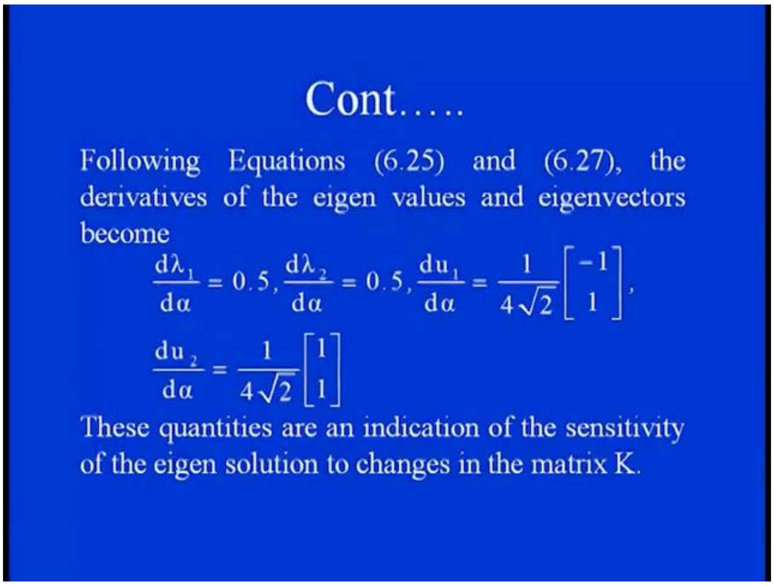


And when we are keeping these things now, we can get the derivatives of the Eigen value and Eigen vectors, the d lambda by d alpha, d lambda 1 or lambda 2 both, with that. And also, we can calculate d u 1 and d u 2 with respect to alpha, in these way and once we calculate these, these quantities are clearly showing the sensitivity of your Eigen value means, Eigen vector and Eigen means, Eigen value problem based on what are the changes are there in the stiffness matrices.

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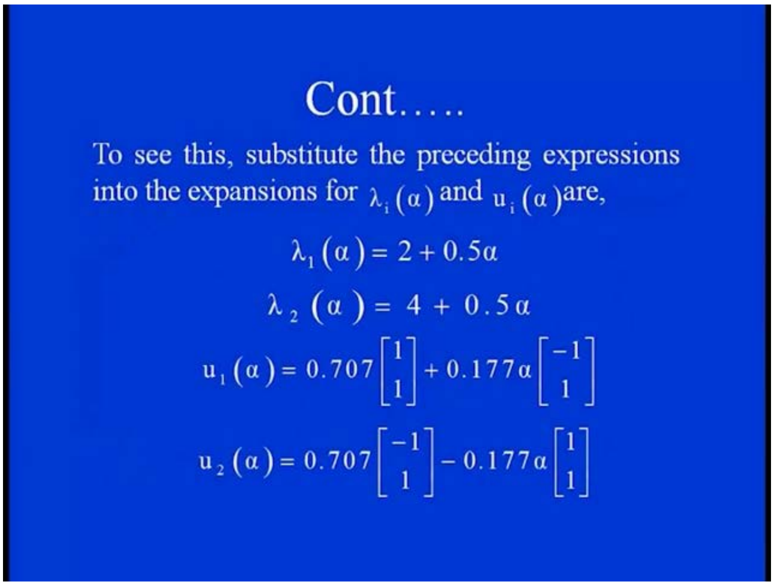


So, correspondingly you see we can straightaway put those things to get the final value of lambda 1 alpha and lambda and u 1 alpha like, lambda 1 alpha is 2 plus 0.5 alpha and lambda 2 is 4 plus 0.5 alpha. So, you see whatever the changes are there in the alpha, corresponding changes are there in the Eigen value or the natural frequency. Similarly, we can get the u 1 alpha and u 2 alpha proportionate. So, it is nothing but equals to 0.707, the square root is there. So, 1 comma 1 plus 0.177 alpha minus 1 comma 1 and similarly, we can get the value of u 2.

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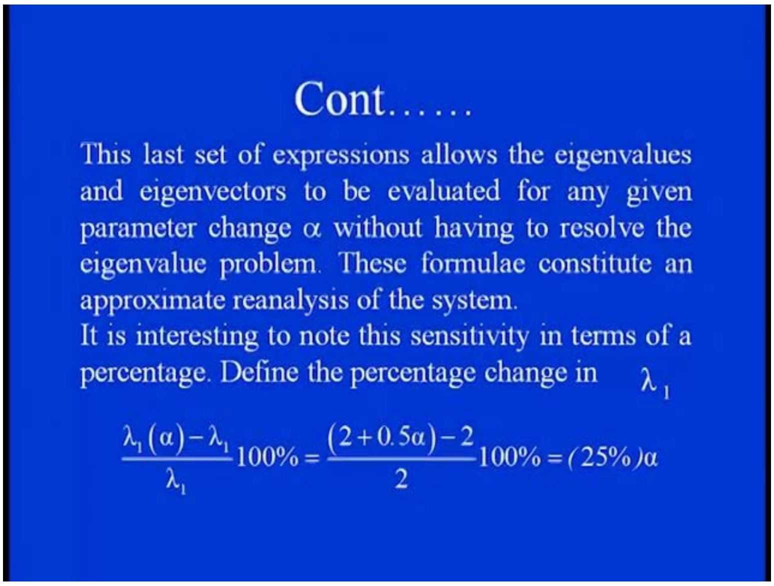

So, the, in this numerical problem you see here, we could easily find out that, when we are trying to get the Eigen value and Eigen vector for that, the, whatever the changes are there in the parameter can be immediately put without changing in the entire system features. And these formula, can be simply give some kind of you know like, the reanalysis feature only not redesigning feature. So, we can say that the sensitivity in terms of percentage can also be calculate like, lambda 1 alpha minus lambda 2, the lambda 1 alpha minus lambda 1 divided by lambda 1. So, it is giving 25 percent of this part alpha, in terms of lambda 1.

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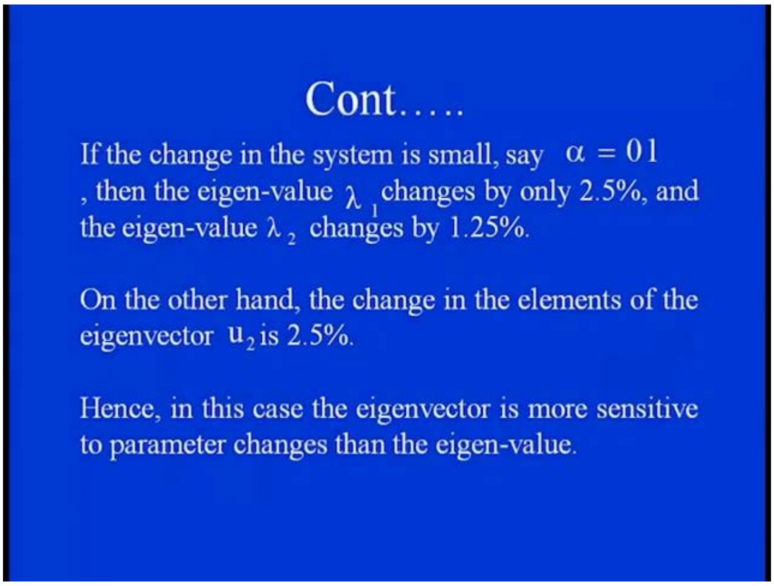


So, if the changes in the system is small say, alpha equals to 0.1 or something then, Eigen value changes are 2.5 percent and Eigen value lambda 2 is also being changed by 1.25 percent. On other hand, you see we can go with the Eigen vector, that is also being changed by 2.5 percent. So, in this case we can say that, we can straightaway calculate that Eigen vector, which needs the change in percentage is 2.5 percent more than that, is more sensitive towards the design parameter as compared to the Eigen value. So, we can say that the relative displacement in the entire structure, when we are changing any you know like, this alpha or that kind of feature is become more significant, as compared to exciting frequency.

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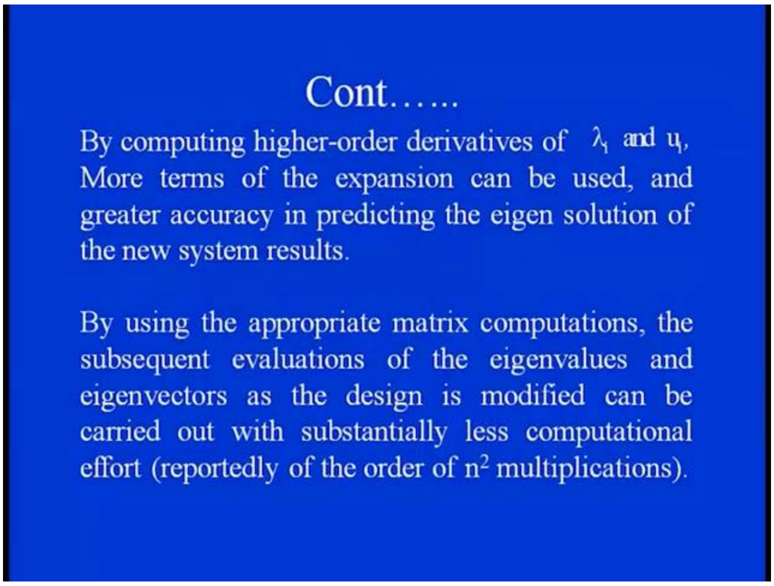


So, in this numerical you see here we could figure out that, what the lambda means the Eigen value and Eigen vector are absolutely depend, you know like depending on this, this design parameter. And we can see the changes in that, as we are simply changing the design parameter changes. So, you see with the using of this approximation matrix composition, the equations for Eigen values and Eigen vectors can be again re-modified, according to the changes in that. And you see here you know like, these multiplications or the manipulations can be done as per our requirement, as per the applications and as per the conditions the constraints, which are being available, according to the sensitivity analysis.

So, in this you know like chapter, we discussed mainly about the sensitivity analysis, in this, which has you see a significant part on, not only on the Eigen vector and Eigen value based. But also you see here, whatever the damping features are there in that, how do we calculate the overall damping feature, which parameter will give you the real reflection of that. And in that, even in the damping what exactly the sensitive features are there, not only right from the material chosen, but also from the application part in that and what exactly the excitation features are there.

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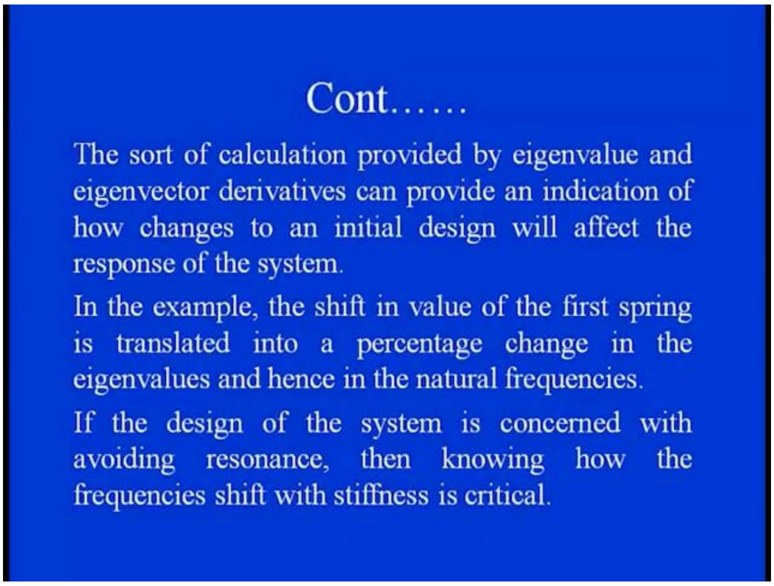


So, if the design of the system is you know like concerned with the avoiding resonance then, we need to see that how the frequencies are being you know like, shifted according to the stiffness. And then, you see here what exactly the critical parameter is there, which can be, which we can choose as the design parameter, to see the effect on the Eigen value and the Eigen vector. So, this was the first part of the you know like, design of the material chosen or you know like the material consideration, based on the sensitivity analysis.

In the next part you see here, again we are just trying to see the design of the material selection means, what exactly the design concept is there for the material selection, just to suppress the vibration you know like, when it is being excited. So, there are lots, lot more parameters are there which we are going to consider for design of, for consideration of the material selection based on some design features.

Thank you.