

Vibration Control
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Module - 5
Design Considerations in Material Selection
Lecture - 2
Design Specification

Hi, this is Dr. S.P Harsha from Mechanical and Industrial department IIT Roorkee. In the course of vibration control, in this module we are discussing about the design considerations in the material selection for suppression of the vibration because this material is basically providing the damping feature in that. So, in the previous class we discussed about the sensitivity analysis because we know that when we are trying to consider the damping feature. If any changes are there because of say the type of loading or because of the environmental conditions or because of you see you know like some specific features which are being added during the running time of machine.

Then, how to relocate or how to redesign the damping features or you know like the material selections in that. Even you see if any there is no change in that, but even you see if we just want to further define like you see as I you know like explained in the previous lecture that if you are considering like the automobile sector. Then we know that every year these automobile companies are just producing the new features of the car.

So, how the new feature are there the main thing is they are just trying to put the new materials or just they just want to refine the existing materials so that they can enhance some of the specific properties of that. So, the same thing is there in the vibration feature also that when we are trying to see that we want to design certain things for certain part of this, you know like we can say these applications part. Then again we need to see that what exactly the sensitivity features are there in selection of the damping part of the material.

So, if any changes are there it is not requiring too much change too many changes you know like in the existing part or else we can say that how the sensitivity part is there especially related to the mass and other stiffness matrices of the materials. So, in this part, now the second category we are just again going to discuss about the design

consideration in the material selection for suppression of vibration, but mainly on the design specifications.

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DESIGN SPECIFICATIONS

The actual design of a mechanism starts and ends with a list of performance objectives or criteria. These qualitative criteria are eventually stated in terms of quantitative design specifications.

Three performance criteria are considered in this section: **speed of response, relative stability, and resonance**. The speed of response addresses the length of time required before steady state is reached.

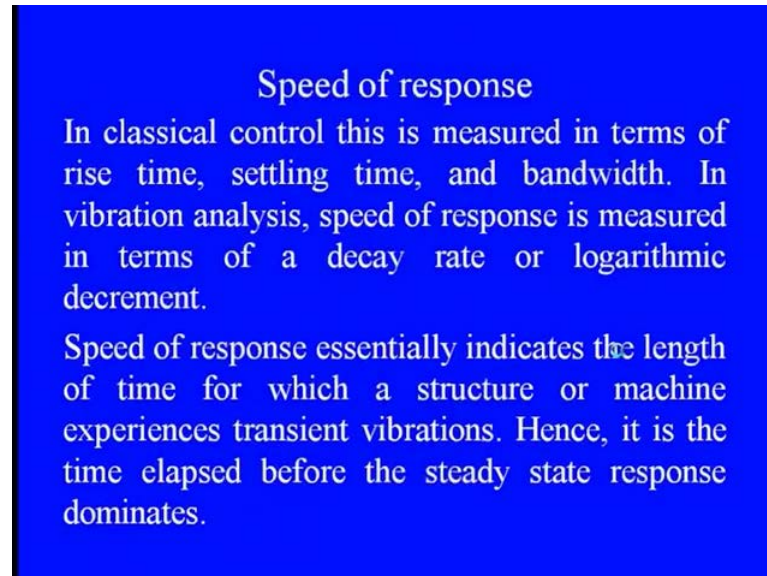
We know that the actual design of any mechanism which starts and ends with the various you see performance objectives or the criteria is requiring a specific specifications. So, without any failure the robustness of the mechanism or the machine will remain and the quantitative criteria which are eventually we know like stated in terms of the quantitative design specification. So, we can actually get the desired performance according to the design of machine or mechanism.

So, these performance criteria when we are talking in terms of the design specification are simply fall in three basic categories the speed of response the relative stability and the resonances. If the speed of response is simply addresses that how what is the length of the time which is being required before the steady state is reached. We know that when the things are just under the dynamic consideration the basic principle of dynamics is saying there is always the transient nature prior to get distance state and this is one of the fundamental law of that.

So, when we are just trying to design say for the steady state feature, we need to just you know critically design or we need to keep the design specification for choosing the material selection for this transient feature. So, this transient feature you know like

requiring that how much is the available time of this transient feature prior to go to this steady state and this time is basically coming under the speed of response.

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Speed of response

In classical control this is measured in terms of rise time, settling time, and bandwidth. In vibration analysis, speed of response is measured in terms of a decay rate or logarithmic decrement.

Speed of response essentially indicates the length of time for which a structure or machine experiences transient vibrations. Hence, it is the time elapsed before the steady state response dominates.

So, when we are trying to say that you know like we are basically interested to locate this the speed of response, so in the classical control theory this is being measured in terms of the rise time. We are generally saying that this is the rise time and then you see you know like it is just coming to the steady state part or even we can say it is a settling time or the bandwidth.

So, in vibration analysis, we can say the speed of response is being measured in terms of the decay rate or the logarithmic decrement for that this is what the analogy feature in classical control theory and the vibration analysis. So, a speed of response essentially indicates the length of time for which a structure or machine experiences the transient vibration.

We know that when we are just going for the generalised solution generalised vibration problem then there are you see the transient and the steady state solutions are there. We need to check it out the nature of transient feature and you see how the variations are being there. So, it is the time, which elapse before the steady state responses are being dominated in the feature, so you see here the speed of response is one of the significant criteria in selection of that.

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Speed of response

If just a single output is of concern, then the definitions of these quantities for multiple-degree-of-freedom systems are similar to those for the single-degree-of-freedom systems. For instance, for an n-degree-of-freedom system with position vector

$$q = [q_1(t) q_2(t) \dots q_n(t)]^T$$

If one force is applied, say at position m_1 , and one displacement is of concern, say $q_1(t)$, then specifications for the speed of response of $q_1(t)$ can be defined as follows.

You see here this is what needs to be addressed in their specification prior to go to the steady state response and the analyses are being carried out. So, if it is a single degree of freedom system, then the quantities are simply you know like we can say you know like just weld according to the specified orientation, but if we are just trying to go with these quantities for multi degree of freedom system. Then again we need to map these single degree of freedom system variation of individual quantity to entire degrees of freedom which are to be described the described for the multi degree of freedom system.

We have the n th degree of freedom system with a position vector q then the q equals to q 1 of t q 2 of t up to q n th of t with the transpose. So, any property can be there force displacement anything is there you see the linear displacement angular displacement and all. So, if one force is applied say to the mass M 1 and one displacement is there in the single degree of freedom system say q 1 t then the specifications for the speed of response of q 1 t can be defined easily.

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Speed of response

The settling time is the time required for the response $q_1(t)$ to remain within $\pm \alpha$ percent of the steady state value of $q_1(t)$.

Here, α is usually 2, 3, or 5. The rise time is the time required for the response $q_n(t)$ to go from 10 to 90% of its steady state value.

All these specifications pertain to the transient response of a single-input, single-output (SISO) configuration.

Now, there is a procedure for that that you see here the settling time in this is the time required for the response $q_1(t)$ to remain within plus minus alpha percentage of the steady state value of $q_1(t)$. Now, you see what we are trying to set up the relation here in between that steady state time will approach there, but prior to that how much time will be required for the settlement of the transient to the steady state which we always try to put the plus minus alpha percentage of that. Sometimes, you see alpha can be you know like varied right from 2, 3, 4, 5 whatever in that.

Second is the rise time because you see it is just starting from 0 and going up to the peak, so the rise time is the time required for the response to go up to say 10 to 90 percent of its steady state value. So, these specifications with the settling, and the rise time is simply pertained to the transient response of a single input single output configuration. So, we can straightaway configure that you see these inputs are coming and they are in the transient nature. You see what is the nature of the system input accordingly you see you know like the transient features can be settled down with the set with respect to the settling time and the rise time towards going towards the towards going to the steady state feature of that.

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On the other hand, if interest is in the total response of the system, i.e., the vector q , then the response bounds yield a method of quantifying the decay rate for the system. In particular, the constant, called a decay rate, may be specified such that

$$\|q(t)\| < Me^{-\beta t}$$

is satisfied for all $t > 0$. This can also be specified in terms of the time constant defined by the time t , required for $\beta = 1$. Thus, the time constant is $t = 1/\beta$.

on the other hand if the interest is in the total response of the system say you see you know like just generalised solution then the vector q can simply give the respond bounded towards the method of quantifying which needs to be decaying which needs to be consisting of the decay rate of the system. So we can say in particular that a decay rate can also be straightaway consider as q of t the modulus is always being less than whatever the matrix phenomena M is there into e to the power minus beta t , where we can say that it is always being satisfied for all the values in the preceding part t greater than 0. So, that can be specified in terms of the time constant and that you see it is always being required when the beta is equals to 0. We can get the time constant as t equals to 1 by beta because it is what you see the exponential carrier e to the power minus beta t , so you see we can straightaway showed the sensitivity of this beta in by taking one example.

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Example 1

Consider the system. The response norm of the position is the first component of the vector $x(t)$ so that and its norm is $q(t) = (1 - e^{-t} - t)e^{-t}$

$$|(e^{-t} - e^{-2t})| < |e^{-t}|.$$

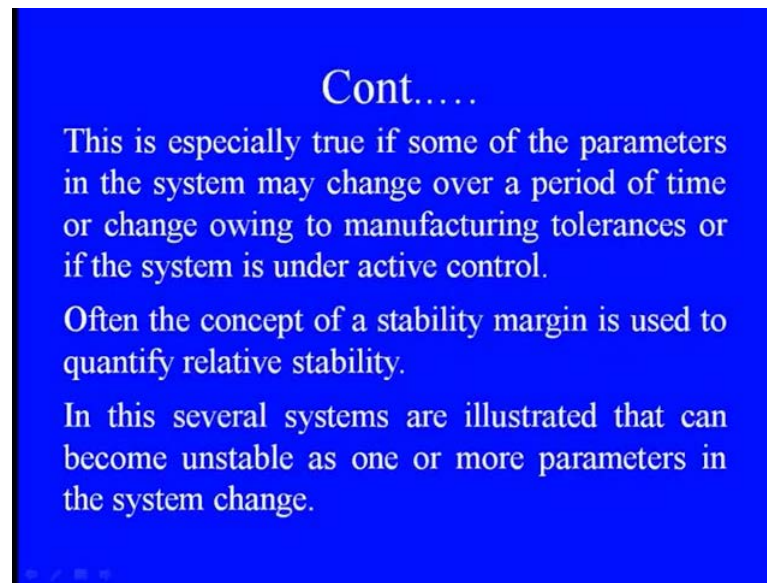
Hence $b=1$, and the decay rate is also 1.

Some situations may demand that the relative stability of a system be quantified. In particular, requiring that a system be designed to be stable or asymptotically stable may not be enough.

Say we have a system response with the position and the first component of this vector position x of t is the q of t equals to one minus e to the power minus t minus t into e to the power minus t . So, we can say that now it is absolutely bounded by the e to the power minus t minus e to the power minus $2t$ is less than the e to the power minus t modulus. We are considering here b equals to one and decay rate is also 1 there.

So, in some situations the relative stability of the system can also be quantified in particular that what the system is to be designed to be stable or asymptotically be stable can also not you know like not giving the significant solution in that way. So, here we are saying that you see that how the exponential decay is being featured out and how we can simply get the decay rate in this case, so you know like when we are talking about you know.

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This is especially true if some of the parameters in the system may change over a period of time or change owing to manufacturing tolerances or if the system is under active control.

Often the concept of a stability margin is used to quantify relative stability.

In this several systems are illustrated that can become unstable as one or more parameters in the system change.

It is especially true when some of the parameter of the system may change over the period of time or even they are changing owing to the manufacturing tolerances if the system is under any kind of active control. So, when we are talking about the second factor stability we know that the concept of stability marginally used to quantify the relative stability of the system with respect to the surroundings.

Now, like the various systems are there which are simply showing the stable or become unstable when the parameters are being changes. Even you know like the slightly featured out the slight changes are being featured out and that can be straightaway carried out in this way.

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For systems in which a single parameter can be used to characterize the stability behavior of the system, the stability margin (sm) of the system can be defined as *the ratio of the maximum stable value of the parameter to the actual value for a given design configuration.*

The following example illustrates this concept.

So, for the system in which the single parameter is being used to characterise the stability behaviour of the system, then we are taking the stability margin s_M of the system which is nothing but equals to the ratio of maximum stable value of the parameter to the actual value for the given configuration. So, here we need to check it out that what is the stable value for that system is first with the maximum and what is the actual value incorporated for that kind of changes, so now you see, here we can take some of the example for this.

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EXAMPLE 2.

Consider the system defined with $\gamma = 1$, $c_1 = 6$, and $c_2 = 2$ and calculate the stability margin of the system as the parameter changes. Here, η is being considered as a design parameter.

As the design parameter η increases, the system approaches an unstable state. Suppose the operating value of η , denoted by η_{op} , is 0.1. Then, the stiffness matrix becomes semi-definite for $\eta = 1$ and indefinite for $\eta > 1$, and the maximum stable value of η is $\eta_{imax} = 1$.

We have a system which simply defines say gamma equals to 1 c 1 and c 2 are there and we just want to calculate the stable margin of the system as the parameter changes and we are considering here that one design parameter say this eta. So, as the design parameter eta is increasing, the system is approaching towards unstable feature, and say you see our operating value eta which is being there optimum part say 0.1.

Then, we need to go towards the stiffness matrices and we can simply find out that the stiffness matrices becomes semi definite for all n equals to 1 and they are indefinite when this design parameter is greater than 1. So, we can say it is the maximum stable value for eta is eta maximum equals to 1 because we just want to go with the stiffness matrix variation when the design parameters to be calculated as we discussed in the last lecture.

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Hence, the stability margin is

$$sm = \frac{\eta_{\max}}{\eta_{op}} = \frac{1}{0.1} = 10$$

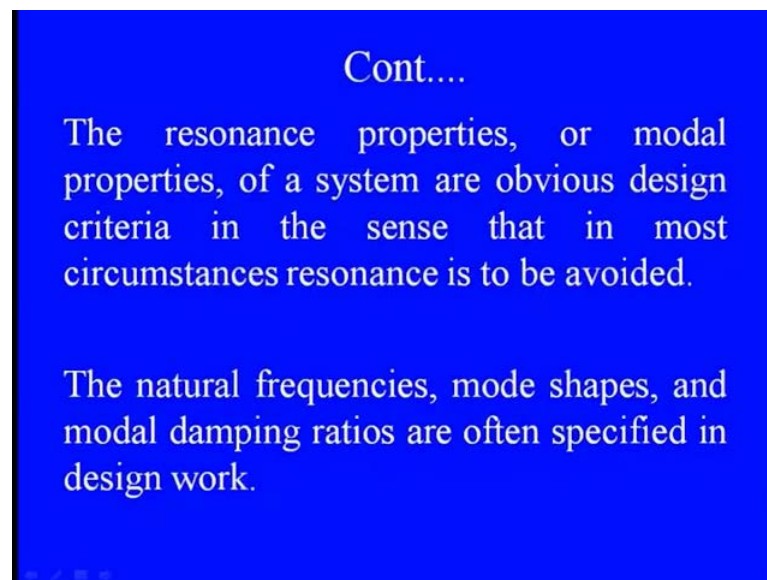
If the design of the structure is such that $\eta_{op} = 0.5$, then $sm = 2$. Thus, all other factors being equal, the design with $\eta_{op} = 0.1$ is 'more stable' than the same design with $\eta_{op} = 0.5$, because $\eta_{op} = 0.1$ has a larger stability margin.

So, with this concept we can say that the stability margin with this consideration of the eta is nothing but equals to the stable s M equals to eta maximum divided by eta optimum. We have 0.1 divided by 1 which is the maximum you know like the eta divided by optimum part is 0.1 which is equals to 10 and if the design of the structure is such that that we have the optimum eta the parameter the sensitive parameter is 0.5.

Then, certainly the stability margin is 2, so we can say that for all the vectors which become equal with this particular specific design then the design with optimum eta 0.1 is more stable. Then the same design with the eta optimum 0.5 because when you have the eta optimum you see you know like a point one then the stability margin is quite more 10

as compared to stability margin with 0.5 is 2. So, from this if you see here we could easily signify that what exactly the impact of these design parameter with the specification feature so that we can get the stability towards the entire system when we are changing this part.

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So, this was the second and the third was the resonance the resonance properties or the modal properties of the system are obviously you see you know like coming under the design criteria in the sense that it is most. We can say circumstances resonance is to be avoided either it is the entire system resonances or even within the you know like the materials resonances because we know that when the system. When the entire material is under you know like the resonant condition the intermolecular interactions are under the severe oscillating feature. Sometimes, even in speed of you know like absorbing the energy with the material damping the damping is just providing the energy to the system which make the system unstable.

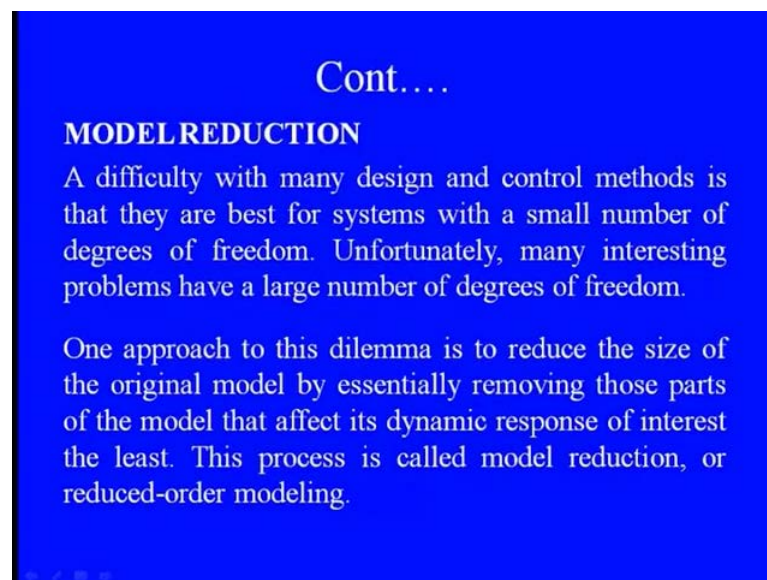
So, the resonance is one of the significant criteria which needs to be avoided by anyway various you see you need to apply the various ways to that to towards that. So, the natural frequencies mode shape and modal damping ratios are often specified in the design work.

So, whenever you see we are designing any system we need to check what exactly the natural frequencies which needs to be considered. According to the degrees of freedom,

what are the corresponding mode shapes with these natural frequencies? Through that we just want to check it out the relative displacements and you see how the variations are then in the model damping ratios with this.

Now, you see here when we are talking about this now our main focus is on the on the model reduction because sometimes you see when we are taking the many degrees of freedom system. Then it is not easy to find out you see the entire natural frequencies and the corresponding mode shapes and the model, you know like these features are there in that the damping features.

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MODEL REDUCTION

A difficulty with many design and control methods is that they are best for systems with a small number of degrees of freedom. Unfortunately, many interesting problems have a large number of degrees of freedom.

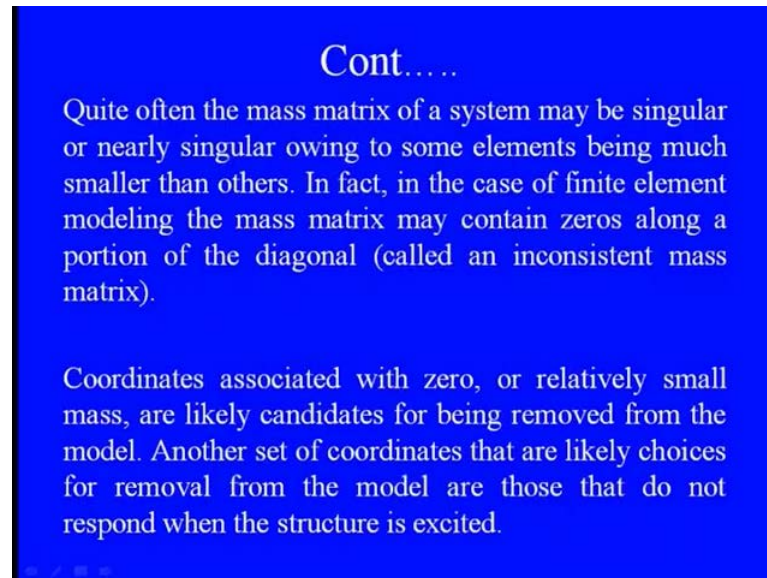
One approach to this dilemma is to reduce the size of the original model by essentially removing those parts of the model that affect its dynamic response of interest the least. This process is called model reduction, or reduced-order modeling.

So a difficulty with many design and the control methods are always you see here you know like we can say a problematic when we are going for the multi degree of freedom system. So, we can simply choose a small degrees of freedom system and then we can effectively apply the control methods and the strategies. Through that you see here we can effectively apply all that control part, but many interesting problem always having interesting problem means the real application features of that. It is always having the large structure with many degrees of freedom system so here we are going to you know like say that how do we do that.

So, one of the specific thing is the model reduction because you know like there is always a dilemma to reduce the size of the original model by essentially removing those part of the model that effect the dynamic response of the interest. So, when we are trying

to reduce the variation in that by not affecting the dynamic response of the entire structure this process is called the model reduction or we can say we need to reduce the order of the formation of the entire system modelling.

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Quite often the mass matrix of a system may be singular or nearly singular owing to some elements being much smaller than others. In fact, in the case of finite element modeling the mass matrix may contain zeros along a portion of the diagonal (called an inconsistent mass matrix).

Coordinates associated with zero, or relatively small mass, are likely candidates for being removed from the model. Another set of coordinates that are likely choices for removal from the model are those that do not respond when the structure is excited.


So, it is quite often that you see the mass matrix is always we are considering the singular feature or nearly a singular owing to some element being much of this smaller than the other. So, it has to be symmetrical way it has to be singular feature, but in fact in some cases like you see in when we are doing this finite element modelling or something the mass matrix may contain the 0 along the portion of the diagonal part.

So, that means you see here the mass matrix is you know like inconsistent along you know like the diagonal feature or we can say you see sometimes we need to check it out. That is why the variations are being coming out in the mass matrices how to do this particular the singular or just closely singular mass matrices.

In the coordinates which are associated with the 0 or relatively small masses are likely candidates for being removed from the model and through that we can simply go towards the reduction of the model. Another set of coordinates that are that are likely you know like choices for removal of these model are those which are not responding actively when the system is being excited. So, here we can simply you know like check that you see you know like these are some of the damp features which are not even. It is like coming and contributing in not only in the natural frequency. But also in the Eigen

vectors when the system is under exciting feature from the mass or else you see we can say that some of the coordinates, which are even.

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Stated another way, some coordinates may have more significant responses than others.

Consider the undamped forced vibration the Equation $\mathbf{x}(t)=\mathbf{Uz}(t)$ and partition the mass and stiffness matrices according to significant displacements, denoted by \mathbf{q}_1 , and insignificant displacements, denoted by \mathbf{q}_2 .

They are even having more significant responses than other, so we can consider the undamped force vibration say \mathbf{x} of t is nothing but equals to \mathbf{u} of \mathbf{z} t in which you see the partition of the mass and the stiffness matrices are can be straightaway. You know it is configured with the significant displacement and we can simply put the displacement q of i or q of P according to the two features there.

So, what does it means that that now we are trying to see that what are the significant responses in this and in that you see here we can now check the two types of displacement under the excitation of undamped force vibration. One is the significant displacement say we are saying q one and one is the insignificant displacement say q 2. So, when we have these things now we are trying to model the equations of motion based on that, so we have now say that the M_{11} , M_{12} , M_{21} .

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This yields

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Note that the coordinates have been rearranged so that those having the least significant displacements associated with them appear last in the partitioned displacement vector

$$q^T = \begin{bmatrix} q_1^T & q_2^T \end{bmatrix}$$

Now, M_{11} , M_{22} are the basic mass matrix elements and when they are trying to relate with the q_1 double dot and q_2 double dot and the corresponding you see you know like for an undamped system the stiffness matrices are coming with q_1 and q_2 . Then we can say that the coordinates which are being you know like arranged in this fashion can also be rearranged in such a way that they can simply give the least significant displacement associated with that. So, we can say that the partition displacement vector q of t is nothing but equals to q_1 transpose and q_2 transpose and then you see we can consider the potential energy of the system which is nothing but you see you know like this.

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Next consider the potential energy of the system defined by the scalar

$$V_e = \frac{q^T K q}{2}$$

or, in partitioned form,

$$V_e = \frac{1}{2} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^T \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Likewise, the kinetic energy of the system can be written as the scalar

$$T_e = \frac{1}{2} \dot{q}^T M \dot{q}$$
$$T_e = \frac{1}{2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

in partitioned form.

This scalar quantity V of e is nothing but equal to this, whatever the changes are there in q of t which we discussed previously that how the positioned displacement vectors are there so it is q of t into K q divided by 2. We can say that we can simply get the potential energy in form of the matrices, so this is nothing but equals to half q_1 by q_2 transpose the stiffness matrix q K q is K_{11} , K_{12} , K_{21} , and K_{22} with the corresponding displacement q_1 and q_2 .

So, this is one energy which is being available with the variation of the restoring forces with those elements this stiffness like you see we can calculate the kinetic energy of the system the t e which is nothing but equals to half q dot transpose into M q dot. So, we can calculate you see these things like half in the matrix form q_1 dot and q_2 dot transpose M_{11} , M_{12} , M_{21} and M_{22} q_1 dot and q_2 dot. So, when we are just doing the partition in in this particular form then you see we can get both the potential energy and the kinetic energy in such a way that it can be straightaway configured along with this stiffness matrix and the mass matrices.

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Since each coordinate q_1 is acted upon by a force f_1 , the condition that there is no force in the direction of the insignificant coordinates, q_2 , requires that $f_2=0$ and that

$$\frac{\partial v_e}{\partial q_2} = 0$$

this yields $\frac{\partial}{\partial q_2} (q_1^T K_{11} q_1 + q_1^T K_{12} q_2 + q_2^T K_{21} q_1 + q_2^T K_{22} q_2) = 0$

Solving the Equation yields a constraint relation between q_1 and q_2 which (since $K_{12} = K_{21}^T$) is as follows:

$$q_2 = -K_{22}^{-1} K_{21} q_1$$

In these each coordinate of q_i is acted upon by the forces f_i because this is the forced you know like forced vibration in the condition that there is no force is to be there in the direction of insignificant coordinate q_2 . So, now you see here, now we are trying to configure those things where we have the significant coordinate because we know that

the significant coordinates are those where there is a direct application of force the insignificant coordinates.

Here, you see the force applications are not directly involved so that you see here they are not putting their contribution towards the excitation features in terms of natural or the Eigen values or you see the Eigen vectors in that way. Now, you see we can get this is the dV/dq_2 because you see this variation is significant is 0. That simply gives that the partial derivative $\partial V / \partial q_2$ of the $q_1^T K_{11} q_1 + q_1^T K_{12} q_2 + q_2^T K_{21} q_1 + q_2^T K_{22} q_2$ becomes 0 when we are trying to resolve the matrix elements along with that deviation.

If we know like if we are solving the equations, then suddenly we have the constrained relations between the q_1 and q_2 as the q_2 is nothing but equals to $-K_{22}^{-1} K_{21} q_1$. Here, we are simply saying that K_{12} and K_{21} transpose are almost same, so when we are trying to put that, now we have the q_2 which is nothing but equals to minus $K_{22}^{-1} K_{21} q_1$.

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This last expression suggests a coordinate transformation (which is not a similarity transformation) from the full coordinate system \mathbf{q} to the reduced coordinate system \mathbf{q}_1 . If the transformation matrix P is defined by

$$P = \begin{bmatrix} I & 0 \\ -K_{22}^{-1} & K_{21} \end{bmatrix}$$

then, if $\mathbf{q} = P\mathbf{q}_1$ is substituted into Equation and this expression is premultiplied by P^T , a new reduced-order system of the form

$$P^T M P \ddot{\mathbf{q}}_1 + P^T K P \mathbf{q}_1 = P^T \mathbf{f}_1$$

This you see expression simply gives that a coordination transformation from all full coordinate system q is to be reduced. Now, towards the coordinate system q_1 and you see this is not you know like a similar simply based on the similarity of transformation, but this is a clear transformation which simply gives the reduction of the model. If this transpose we can say transformation matrix P is there, then the P is nothing but equals to

i_0 minus K_{22}^{-1} and K_{21} as we have simply seen the previous case, so if you are saying that this q which is you see the full coordinate system.

Now, can be substitute in the equation and can straightaway you know like put the expression in the this multi multiplied by you know like whatever you see the things are being there P into transpose part. So, now we have a new reduction order of the entire system with the consideration of full coordinate system q as because you see now we are trying to multiply with the P transpose as q equals to Pq_1 . So, the P of transpose M into P the mass matrix into Pq_1 double dot plus P transpose K P into q_1 equals to P transpose whatever the forces are there so it is f_1 .

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$$P^T MP = M_{11} - K_{21}^T K_{22}^{-1} M_{21} - M_{12} K_{22}^{-1} K_{21} + K_{21}^T K_{22}^{-1} M_{12} K_{22}^{-1} K_{21}$$

$$P^T KP = K_{11} - K_{12} K_{22}^{-1} M_{21}$$

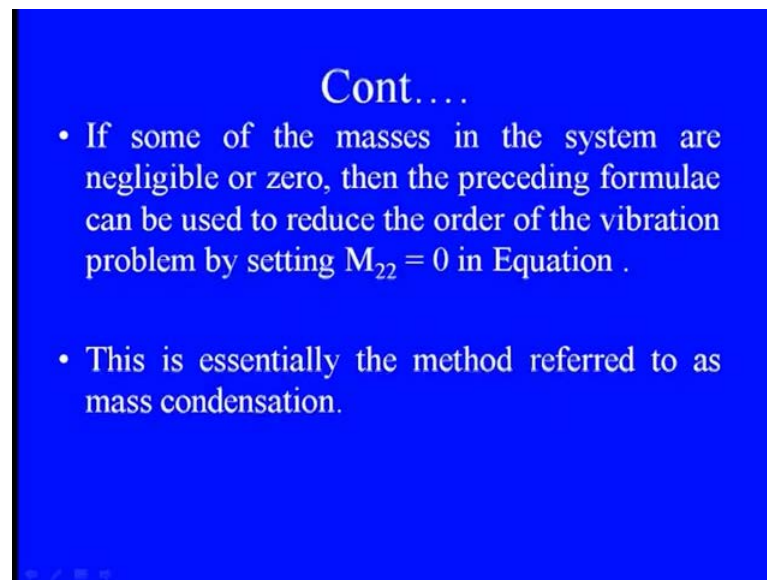
These last expressions are commonly used to reduce the order of vibration problems in a consistent manner in the case where some of the coordinates (represented by q_2) are thought to be inactive in the system response. This can greatly simplify design and analysis problems in some cases.

Now, you see we can simply calculate the P transpose into mass matrix into P that you see now we have a clear feature in this that it is you know like the P transpose M P is giving the variation in the mass matrix. The stiffness matrix is such a way that it is equals to M_{11} minus K_{21} transpose $K_{22}^{-1} K_{22}$ minus the inverse and M_{21} .

Again, this M_{11} is you know the first element minus K the transpose matrix of K_{21} inverse matrix of K_{22} and mass matrix M_{21} minus M_{12} that is a symmetricity. Now, in terms of M , M_{12} is coming, so another matrix form is the K_{22}^{-1} and K_{21} and then plus K_{21} transpose K_{22}^{-1} is as we discussed in the second term.

Then, it is you see here M_{12} and K_{22} inverse when you are multiplying with K_{21} , so ultimately when we are trying to keep this in the mass matrices with the P transpose and P we can also form the stiffness matrix. With the P transpose K into P is nothing but equals to $K_{11} - K_{12} K_{22}^{-1} K_{21}$ and M_{21} . So, this is what the reduction feature the last expression is simply we can say commonly used to reduce the order of vibration problem in such a consistent manner. Even in case of you know like the coordinate representation in which you see a some inactive features are there insignificant features are there can be straightaway reduced by putting by eliminating q_2 from that. This can be greatly we can say adopt in various analysis problem in you know like this one.

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- If some of the masses in the system are negligible or zero, then the preceding formulae can be used to reduce the order of the vibration problem by setting $M_{22} = 0$ in Equation .
- This is essentially the method referred to as mass condensation.

So, if you are saying that if some of the masses in the system are just negligible or even the 0, then the formula again can be used by simply putting the reduction of you know like entire model feature of the vibration problem by putting the M_{22} equals to 0. So, we need to say that this is something in which you see the mass matrixes are being removed it is called the mass condensation. So, you see here the symmetric feature of the mass is being just eliminated in that, now we are considering one example with the four degrees of freedom system.

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Example 3

- Consider a four-degree-of-freedom system with the mass matrix and the stiffness matrix

$$M = \frac{1}{420} \begin{bmatrix} 312 & 54 & 0 & -13 \\ 54 & 156 & 12 & -22 \\ 0 & 13 & 8 & -3 \\ -13 & -22 & -3 & 4 \end{bmatrix}$$

and the stiffness matrix

$$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

A unit in which the mass and stiffness matrices are being like that, so mass matrix is nothing but equals to 1 by 420. All these elements are 3, 1, 2 along the diagonal 3, 1, 2, 1, 50, 6, 8 and 4. The respective other the elements of the matrices are like in the first row we can say the 3, 1, 2, 50, 4, 0 and minus 13 second 50, 4, 1, 50, 6, 12 and minus 22 similarly, for third and fourth rows are like that.

So, the diagonal is like this and then you see we can simply go with the stiffness matrices for this four degrees of freedom system, it simply show 4 by 4, you know like the elements. So, it is 24, 12, 2 and 4 along the diagonal and then you see you know like corresponding things are there with this say minus 12, 0, 6 and then minus 12, 12 minus 6, 6. So, minus 6 in between is this one and also for the fourth element 2, 4, 4, 4 like this is there so this is a clear presentation of the stiffness matrix with the symmetricity as it is being required for the system with the four degrees of freedom system you see here.

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- Remove the effect of the last two coordinates. The sub matrices of Equation are easily identified:

$$M_{11} = \frac{1}{420} \begin{bmatrix} 312 & 54 \\ 54 & 156 \end{bmatrix} \quad M_{12} = \frac{1}{420} \begin{bmatrix} 0 & -13 \\ 13 & -22 \end{bmatrix} \quad M_{22} = \frac{1}{420} \begin{bmatrix} 8 & -3 \\ -3 & 4 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} \quad K_{11} = \begin{bmatrix} 24 & -12 \\ -12 & 12 \end{bmatrix} \quad K_{12} = \begin{bmatrix} 0 & 6 \\ -6 & -6 \end{bmatrix} = K_{21}^T$$

- These last two matrices form the resulting reduced-order model of the structure.

$$P^T M P = \begin{bmatrix} 1.021 & 0.198 \\ 0.198 & 0.236 \end{bmatrix} \quad P^T K P = \begin{bmatrix} 9 & 3 \\ 3 & 3 \end{bmatrix}$$

When we are now trying to remove the effect of these last two parameters, because we want to reduce this one, so what we are doing here we are simply making now the sub matrices of the equations so that we can easily identify that what are the critical parameters which can be put into the significant or insignificant criteria.

So, when we are saying the mass matrix M_{11} , so certainly it is 1 by 40, the first element that is 3, 1, 2, the first you know like we can say array 3, 1, 2, 54, 54, 1, 56. Similarly, we can go with the subset of M_{12} that is nothing but equals to 1 by 420, 0 minus 13 13 minus 22. Similarly, we can go with the M_{22} as well which is nothing but showing this the entire element in between 141 by 428 minus 3 minus 3 4. So, you see here this is what it is we can go with this this is what my M_{33} was three you see here and accordingly.

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Example 3

- Consider a four-degree-of-freedom system with the mass matrix and the stiffness matrix

$$M = \frac{1}{420} \begin{bmatrix} 312 & 54 & 0 & -13 \\ 54 & 156 & 12 & -22 \\ 0 & 13 & 8 & -3 \\ -13 & -22 & -3 & 4 \end{bmatrix}$$

and the stiffness matrix

$$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

We can choose this and we can choose this with division of 1 by 20, so one either M 1 1 M 1 2 or M 3 3 like you see this can be straightaway calculate according to the requirement of this. So, we can simply go with M 1 1, M 1 2 or M 2 2 in this way similarly, the stiffness matrix the K 2 2 was nothing but equals to the 2 4 and 4 4 like that. So, 2 4 and 4 4 the last element as I discussed in this this what my K 2 2 and then you can see I can calculate the K 1 1 first part as 24 minus 12 minus 12 12 is this 12 is there that is my K 1 1 and K 1 2 can be straightaway calculate 12 minus 6 minus 6 and 2.

So, this is what you see here it can be straightway putting here and we can get the transpose of you see this one so K 1 2 which is nothing but this element is the K 2 1 transpose feature. So, these two last matrices are simply showing the resulting feature of the reduced model the reduced order model of this structure.

So, we can say now we can go with the two main part the P transpose mass into P matrix which is simply giving. Now, when we are trying to multiply this with this is 0.1, 1, 2, 0.198, 0.198 and 0.236 and the same time we can calculate the P transpose K and P. It is giving the 9, 3, 3, 3 like that, so these are you see you know like the two main we can say the reduced order model. These entire matrices are there which are clearly showing that how the reductions can be done with the significant and insignificant features of that.

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It is interesting to compare the eigenvalues (frequencies squared) of the full-order system with those of the reduced-order system, remembering that the transformation P used to perform the reduction is not a similarity transformation and subsequently does not preserve eigenvalues. The eigenvalues of the reduced system and full-order systems are

- $\lambda_1^{\text{rom}}=6.981, \lambda_2^{\text{rom}}=12.916$
- $\lambda_1^{\text{rom}}=6.965, \lambda_2^{\text{rom}}=12.916$
- $\lambda_3^{\text{rom}}=3.833\text{E}3, \lambda_4^{\text{rom}}=230.934$

So, now if you are comparing the Eigen values of the full order system with those with the reduced order system we can see that you see there is a clear transformation feature the P matrix transformation matrix P can be used to perform the reduction. It is not again you see you know like the similarity transformation and then we cannot go that you see here whatever you know like the preserved value of the Eigen values are there.

We can straightaway use and we can simply transform based on the similarity transformation. So, here the Eigen values of the reduced system and the full systems can be straightaway compared like this. We have the lambda 1 rom which is 6.981 and the lambda 2 rom whatever you see you know like this feature is 12.916.

Similarly, you see here when we are trying to put this we have now the lambda 3 and lambda you know like four for these kind of you know like the feature. So, when this is the reduced ordered model rom means the reduced ordered model. You can simply see that there is a clear feature that you see we have the 3.83 into e to the power 10, 10 to the power 3 and 230.934 is my lambda 3 and lambda 4 in comparison to the full scale model. So, there is a clear feature that you see here when we are trying to find out the Eigen values of the reduced order model and the full model. There is a clear comparison that you see we can get you see the lambda 1.

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- Where the superscript 'rom' refers to the eigenvalues of the reduced-order model. Note that in this case the reduced-order model captures the nature of the first two eigenvalues very well.
- This is not always the case because the matrix P defined in Guyan reduction, unlike the matrix P from modal analysis, does not preserve the system eigenvalues.

Here λ_2 , λ_3 and λ_4 in such a corresponding way that it can be easily replaced with this. So, as I told you the rom refers to the reduced ordered model and in this case you see that the reduced ordered model captures the nature of the first two Eigen values very easily. There is now you know like a problem in calculation of the significant features of that. You see this is not always the case because you see the transformation matrix P is always been defined by you know like this by consideration the given reduction. So, unlike the matrix P from the model analysis it cannot preserve the Eigen value problems as just like you know in the similarity transformation.

They are always preserving you see the Eigen values of the Eigen values of the system this property with the full system analysis. So, here in this case you see we need to check it out that how much perseverance is there of the real Eigen values of the full system. So, when we are applying in the model reduction system means in which you see you know like there is a reduction of the model parametric feature. Then we need to check it out that what exactly the compatibilities there in between that and accordingly because this is beyond reduction for formation of this transformation matrix. It is always being considered in such a way that that it should be compatible and whatever the values the Eigen values are coming though.

It gives you a clear and easy part of this Eigen values, but it needs to be preserved in such a way that there is there should not be significant change in this. So, in this design

specification feature of you know like the material consideration. Today, we discussed about mainly the three main part of that the speed response because the speed of response is clearly showing the dependence of the entire responses on the transient to the steady state in between that. Second was that when you are you know like checking this, what is the stability feature of that because the specifications can be changed of the material for suppression of the vibration.

It is a not you know like allow the system to go in the unstable manner just like you see in the speed of response when we are changing the specification we are not trying to allow those. You know like the transient feature becomes more dominant as compared to the steady state and the third was the resonance, so when we are trying to refer the resonances. We need to check it out that whatever the specifications are being changed or the modification then what is the criteria to avoid the situation of the resonance.

So, these things were discussed in that and then we discussed about how to reduce the model because ultimately in the specification when we are going for the material considerations. When you have a multi degree of freedom system then it is not that easy to find you see entire things and the changes in the multi degrees of freedom systems.

So, then we need to reduce the order of the model and then we can straightaway apply the sensitivity if those things were the significant features are there either in the mass or the stiffness matrices in that. So, in the next lecture also we are going to discuss about the design consideration of the material selection and we will try to focus on other you know like issues. So, when we are trying to design the material for reduction of the vibration, then we can simply configure those properties and we can simply put you see these orientations towards the designing feature of the materials.

Thank you.