Vibration Control

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Module - 1
Review of Basics of Mechanical Vibrations
Lecture - 3
Free and Forced Vibrations of Two Degree of Systems

Hi, this is Dr. S.P Harsha from Mechanical and Industrial Department IIT Roorke. Today in this lecture we are going to discuss about the Forced Vibration, that you see when the forcing frequencies are there on any systems, then how the system is by periodically excited. So, as we discussed in the previous lecture about the damping, we know that damping is playing a crucial role in controlling the entire exciting frequencies or the vibrations.

So, today you see you know like when the system is by periodically excited then how the damping is again you see know like a critical role we are going to discuss. Second feature which we are going to discuss about, that you see you know when we are talking about a 2 degree of freedom system then how we can frame the equation of motions for that. Because we know that when we are talking about the 2 degrees of freedom system then there is a Newtonian mechanics in which you see we just want to setup the force configurations.

And based on that we can simply generate the two equations which are simply reflecting both degrees you know like of the freedom. We know that there are two you know like the different coordinate system of 2 degrees of freedom. Then you see how we can make the coupled equations and then how we can generate you see here the solution from these two equations.

What are the corresponding we can say natural frequencies or the characteristic routes and then what are the corresponding you see the relative displacement of the masses or something you see here which we are generally saying that the Eigen vectors or the vibration mode shapes. So, you know like in the previous lecture we know that as far as the damping is concerned, there were three modes and all the modes are you know like playing critical role in formation of the complete damping.

So, that is why you see we know that it is really you know like hard to say that this is the overall damping phenomena, because the damping is coming right from the intermolecular motion to the surface rubbing action to the fluid; whatever the fluid is available at the you know like the surfaces. And then what exactly the you know like the total resultant of the damping is it is really hard to calculate.

So, sometimes we are giving you know like a some numerical value it is you see irrelevant to say that this is the overall you know like representation of the damping is because you see the damping is coming out from the as I told you the inter molecular motion and all other aspects. So, again you see here whenever we are talking about the damping we need to be very careful that what kind of material which we are using what kind of you know like the ((Refer Time: 02:51)) which we are using there and what exactly you know like the rubbing actions are there when the dynamic motion is happening. In today's lecture you see we are going to discuss about the free and forced vibrations on the 2 degrees of freedom like on that.

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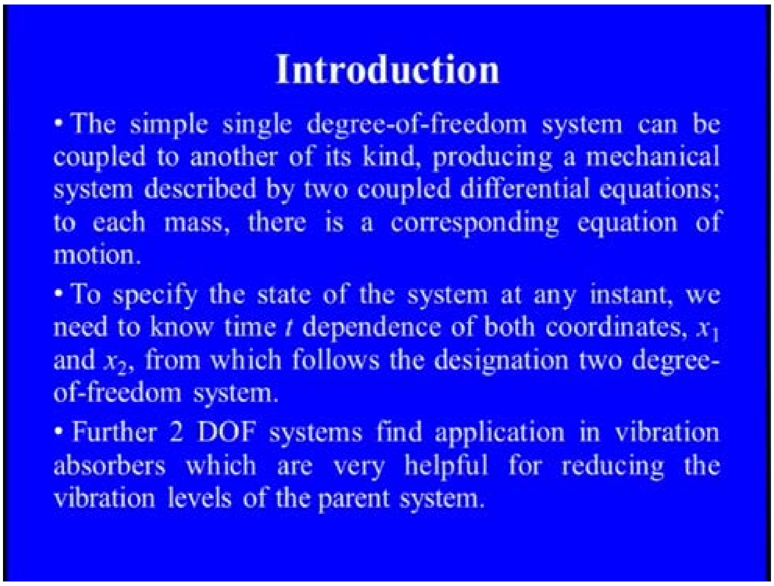

So, as we discussed about the single degree of freedom system that can be easily you know like you know like changed into 2 degree by simply coupling; the similar kind of structure to the another form. So, in that way we can produce a same mechanical system which can describe the two coupled differential equations, since it is you know like a discrete systems, so we have ordinary differential equations there itself. And then you see we can form two equations which can represent the 2 degrees of freedom system.

To specify the state of the systems at any instant we need certainly you see the dynamic parameters with respect to the coordinates x 1 and x 2 in which these two whatever we can see the masses or anything are simply oriented you know like in their configurations. And then you see here based on these two degree of freedom system, we can find out that where we can apply the absorber through, which we can effectively control either x 1 or x 2 of the amplitude of the vibration.

So, that is why you know like this in 2 degrees of freedom system it is a further we can say the refined system in which we can effectively control the vibrations at the root cause of vibration. We can apply this concept to any of the equations or the real application like we have say forging hammer and the anvil on the ground isolators. We know that you see when we are using that, there is you see the contact region in between we can say the hammer and the anvil to the surface. So, wherever you see the contact surface is there we know that this is somewhat a representation of the elastic deformation or we can say we can represent this elastic deformation by the spring.

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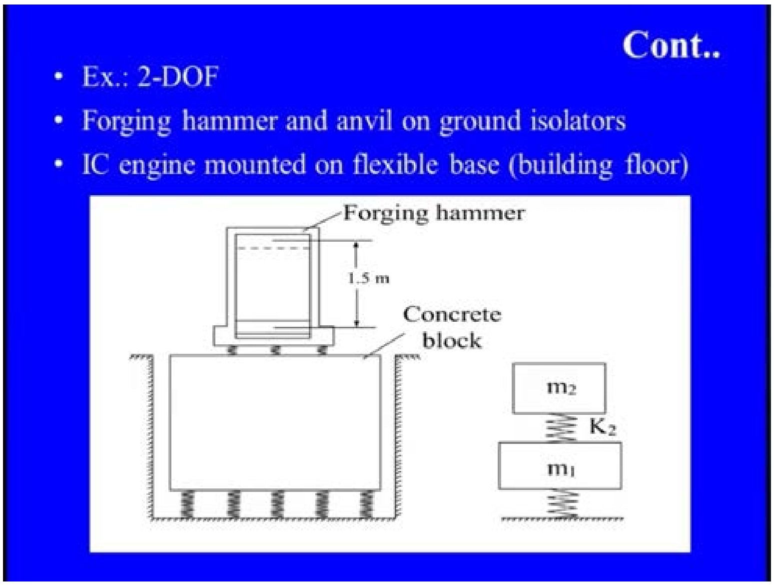

So, as you can see on your screen that what we have the hammer and the an anvil you see the forging hammer is there and there is you see the clear relation between the surface contacts which we are representing the springs at the concrete block and the hammer itself. And then even we can apply the similar kind of concept to the IC engine which is mounted on the flexible base. So, IC engine is representing 1 degree and you see we have a base of which you see the entire system is rotating that is the another degrees of freedom system, we can see that the building floor or anything.

So, this you know like this is a clear representation which you can see that we have this entire you know like the forging hammer and this is my concrete block. I can represent this entire hammer as one mass the discrete mass another discrete mass is basically from the concrete block in between we have the like the deformation that is nothing but the elastic deformation the springs are there we can say right now the spring stiffness is K 1.

The concrete block is at you know like certain surface and they have some kind of you see the deforming features that also can be represented by the springs and that we have you see the K 1. So, right from the base we have first spring K 1 stiffness, then we have mass m 1 then we have you know like the K 2 spring and then mass 2. So, this is you see here we have two independent displacement x 1, which is related to m 1 and x 2 related to m 2, which we are using here we can say this system is absolutely falling into 2 degrees of freedom.

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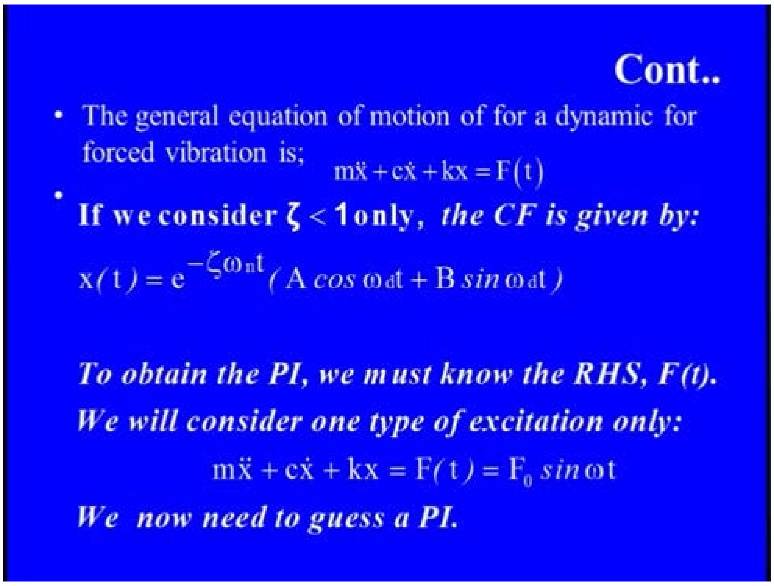

And when we are trying to formulate the equations we know that the basic equation in which you see the system is you know like at the equilibrium position under all the influences of the mass and the forces we can say this is nothing but equals to m x double dot plus k x plus c x dot equals to F of t the forced exciting systems. And if we are considering say we have since the damping is there and that the property of damping is say under damp system is zeta is less than 1.

Then we know that the complementary function which is simply a representation of your free vibration condition is nothing but equals to e to the power minus zeta omega and t. This is one of the exponential d k due to the undammed natural frequency into a cos omega d t plus b sin omega d t. This is you see know like one of the representation of complementary function, which says that you see you have a clear d k of the vibration amplitude with the exponential you know like d k and the sinusoidal features are being there means both the transient the oscillatory term and we have the sinusoidal features together.

And to obtain the particular integral which is nothing but the representation of your steady state formation certainly it is subjected by, the forced vibrations the forced whatever the exciting frequencies are. So, we can represent now by this phenomena with this equation m x double dot plus c x dot plus k x equals to f 0 sin omega t. And if you just want to find out this one certainly now we need to see that what exactly the nature of force is.

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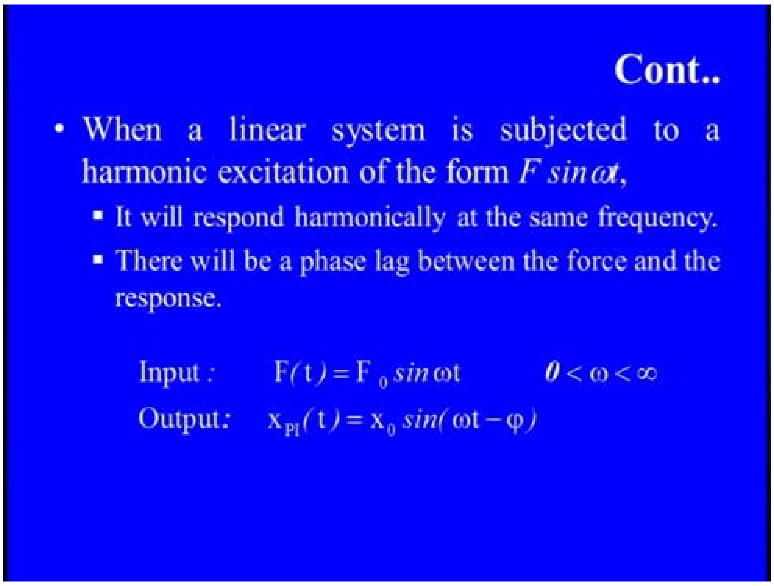

And as you see you know like we have F 0 sin omega t which is nothing but the simple harmonic force excitation with the F 0 the initial force input and omega is you see whatever the forcing frequencies are. And it will certainly you know like respond in the harmonical way at the same frequency which we want, at which you see the subject is excited. And there will be a phase lag in the output and the input part, since the input we are giving f zero sin omega t. So, output which is coming in terms of displacement there may be a phase lag. So, you can see that the if we have the input F equals to F 0 sin omega t then our output may be of x 0 sin omega t plus minus whatever you see it is a phase lag. So, you know like whatever plus minus phi is we can represent by the graph.

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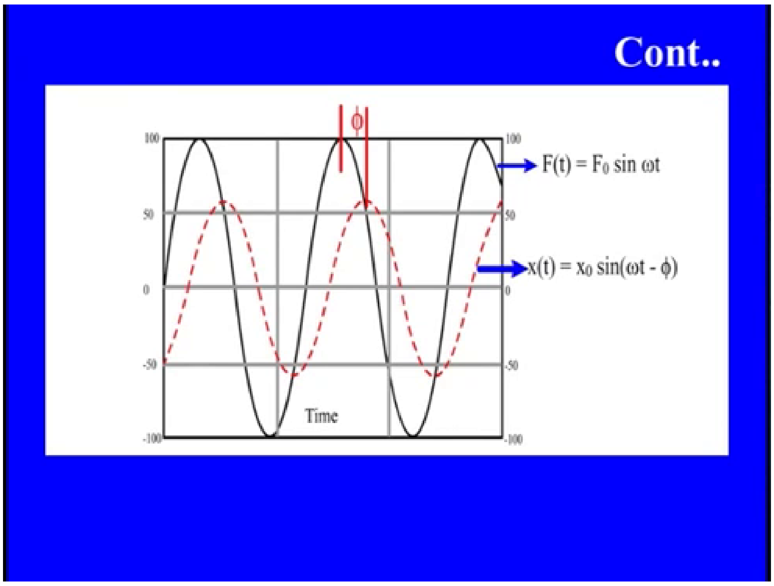

You can see on the screen, this you see here the firm line which is simply showing our input force. The force is simply going at certain frequency say omega, at the same frequency even we are getting the displacement output, but there is a phase lag and the phase lag is this dotted line is showing the output, which has may be you see the less amplitude as compared to the force. But we have a clear phi, you can see it is simply showing by the red feature, there is a phase lag there in that.

And this phase lag is mainly due to whatever the system characteristics are, that how the system is responding as you apply force on the system. When we are now trying to simulate these things in terms of the steady state solution or a particular integral solution p i 1 we can see that the p i for this kind of feature is nothing but x of p i equals to x 0 which we got sin of omega t minus phi.

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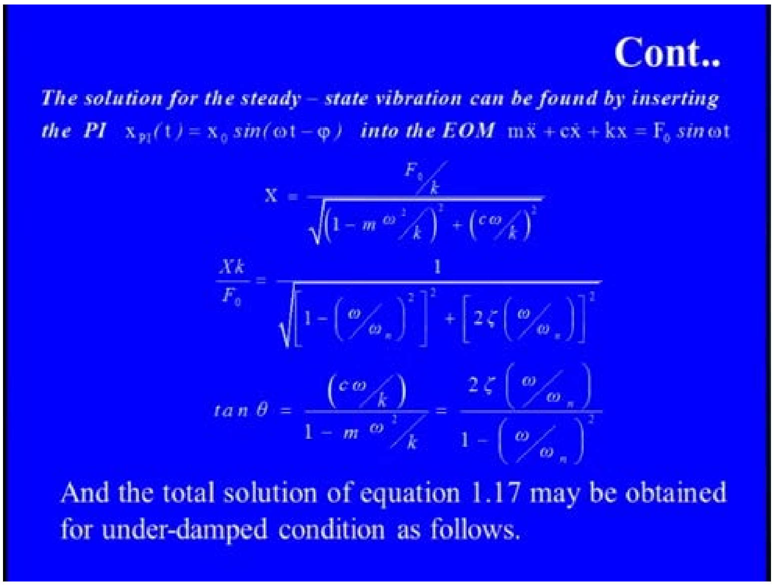

And if we are now going into the basic equation that is m x double dot plus c x plus k x equals to F 0 sin omega t. Then we can get the output x 0 is nothing but equals to the f 0 by k that is my input feature f 0 and k is the systems property divided by 1 minus m omega square by k whole square minus you know like we can say c omega by k. Or else even we can simply show with the various two different features, one we can say that there is you know like a frequency ratio. The frequency ratio sometimes you see here you can represent that what the exciting frequency, the forcing frequencies and what is the exciting natural frequency.

So, if we want to just convert this sometimes in many of the books you will find that we have a frequency ratio r is nothing but equals to omega by omega and omega is your forcing frequency omega n is your exciting natural frequency. And even we can represent this by the zeta that is the damping ratio c by c c or we can say that you see here you know like when you have the omega n which is nothing but equals to square root of by k square root of k by m you can put those things there itself.

So, when you are simply calculating now, what you have when you are simply describing these things x into k in the second equation x into k by F 0 is nothing but equals to 1 divided by you know like square root of 1 minus omega by omega n whole square into whole square plus you see 2 zeta omega by omega in this one.

So, again as I told you omega by omega n you can replace this omega by omega n by the frequency ratio. And then you see here the zeta is already come out, because of your damping is present in the system. So, now, what you have, you have this x by F 0 this is nothing, but equals to your frequency response the outcome is there or we can say it is f r f frequency response function. Because, what you have you have output x 0 you have input F 0, this ratio is simply giving the relation that how your output is affecting, when you have input F 0.

And it is being affected by two main feature omega by omega n that is your frequency ratio and zeta that is nothing but your damping ratio. And then the systems properties are there involving in the formation of r and zeta your damping ratio and the frequency ratios. And then also we can calculate this 10 theta that is nothing but equals to how much you see the phase differences are being coming out in this, you can see that it is nothing but equals to 2 zeta omega by omega n divided by one minus omega by omega n whole square.

So, again you see here either in formation of the phase or in formation of the ratio of output by input, these two the frequency ration omega by omega n and the zeta are playing a key role. And then you see here when we want to design certain things we can simply control these two features as it is. And the total solution if you want to get for you know like this particular forcing frequency including the complementary function and the particular integral for this under damped system.

We can simply see that we have x of t is nothing but equals to one the complementary function which is simply showing the transient response is nothing but equals to e to the power minus zeta omega and t into a sin omega d t plus p cos omega d t that is the one part; and another is x sin omega t plus phi or minus phi, whatever you can simply put that.

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So, here the values of a and b the constants can simply you know like calculate using the initial condition and the boundary conditions for the forcing functions are that how the forcing functions are carried out during the motion. A closure analysis of the above equation yields that you see for a very large value of t means, if you are going for say higher value of the time for which we just want to solve the equations.

We know that the transient response, which is coming out from the complementary function becomes very small, because your forcing factor at that time is playing a key role to carry out the solution. And hence the term steady state response which is coming out due to the particular integral is assigned to a particular solution or we can say that the second term is dominating in the solution, when we are just going for large value of t.

And the value of coefficient for the steady state response or we can say a particular solution becomes very large, when you see the exciting frequency is very close to un-damped natural frequency. Means you see here when we are very you know like when the system is exciting close to your natural frequency we know that the particular solution is really significant at that time.

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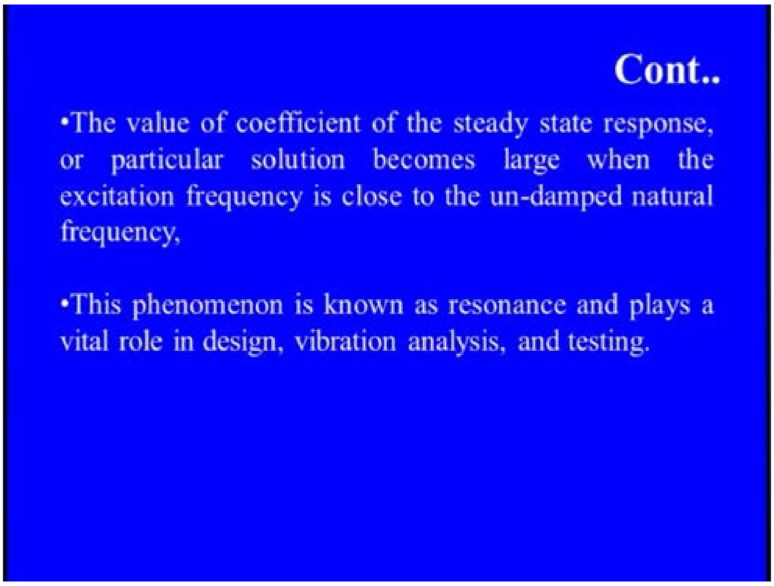

So, this phenomena where the system is going towards your natural frequency is known as the resonant frequency or the resonant concept. And it plays a real good role, because you see at that time you know that the used amount of energy is being explode from the system and we can say if we can control this apart. If we know that you see this much use exciting vibrations are there or we can say the amplitude is there, this feature is really playing a vital role in the design or vibration analysis or in testing of any of the subject. Now, this concept we would like to apply into the numerical problem and how we can play with some numerical data's.

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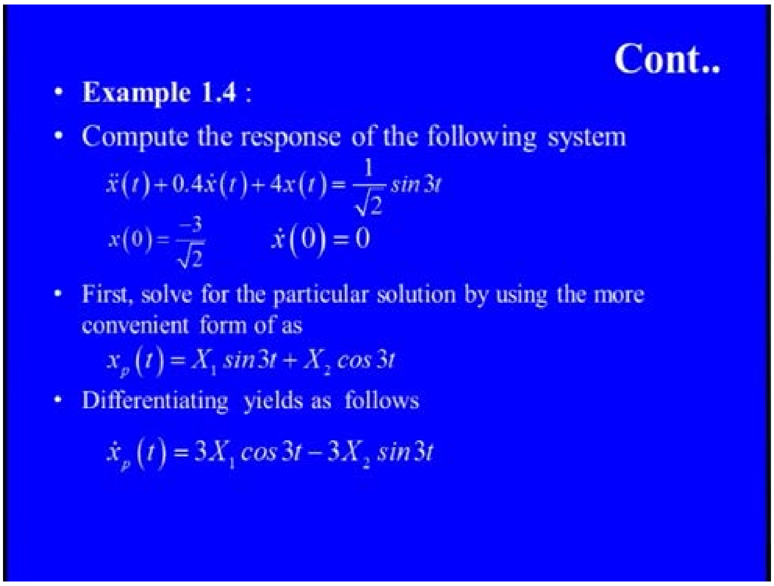

So, here we have the fourth example of this chapter in which we just want to compute that what exactly the solution feature is, what is the generalized solution if the system is constrained by these boundary condition and this equation. So, we have the equation x double dot t means the acceleration plus 0.4 velocity x dot t plus four displacement x of t equals to 1 by root 2 sin 3 t.

So, we have a clear feature of acceleration velocity displacement and 1 by square root square out of 2; that means, you see the initial force and sin this 3 t that is your natural frequency feature the forcing feature from that. The initial conditions are also given to us that x of t x of 0 the t equals to 0 it means a is minus 3 by root 2 and x dot 0 the initial velocity since you see it is a linear term, so it is 0. Now, first this you know like we would like to solve for particular integral or particular solution or p i you see here.

So, for that we can simply assign we know that the forcing factor is coming from sin 3 t. So, certainly we have x of p the particular solution in terms of displacement is nothing but equals to x 1 sin 3 t plus x 2 cos 3 t. And then you see here we can simply find out the differential feature of this displacement; that means, you see here what exactly the loss d and acceleration terms are.

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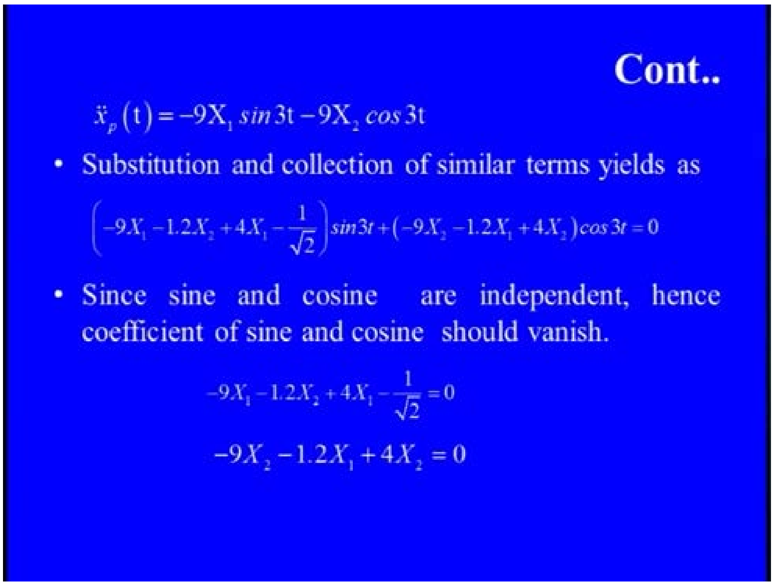

So, when you are just deriving this differentiating with the one derivative that velocity is x dot t is nothing but equals to 3 x 1 cos 3 t minus 3 x 2 sin 3 t. Or else even we can say that if we are simply applying this one to acceleration term then we have final x double dot p for a particular solution of t is equals to minus 9 x 1 sin 3 t minus 9 x 2 sin x 2 cos 3 t.

And now, you see here once you have these, now we have what, we have the displacement we have the velocity, we have the acceleration and if are now applying these both how do like all these terms towards your first main equation, which was x double dot t plus 0.4 x dot t plus 0.4 x t equals to this 1. If you are applying this there then you see we are ending up with this equation this is nothing but equals to minus 9 x 1 minus 1.2 x 2 plus 4 x 3 minus 1 by root 2 sin 3 t. And the same you see here minus 9 x x 1 minus 9 x 2 minus 1.2 x 1 plus 4 x 3 cos 3 t equals to 0.

What we have done here, we have simply configured the entire equation into sin and cos feature. And we know that since the sin and cos features are independent, so certainly you see the coefficient of you know like the sin and cos are should be you know are supposed to be equals to 0. So, you see here we can frame these equations as minus 9 x 1 minus 1.2 x 2 plus 4 x 3 minus 1 by root 2 equals to 0. And the similarly you see the second form minus 9 x 2 minus 1.2 x 1 plus 4 x 3 equals to 0 and you see we have what the two equations, the two unknown coefficients x 1 and x 2 and we can get you see the solution of these equations by simply solving the coupled feature.

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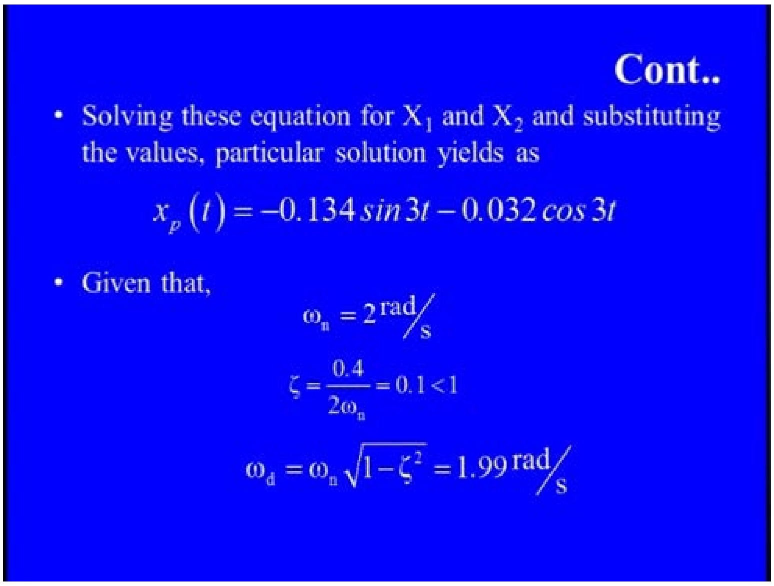

So, what we have the final solution of a particular integral x of p is nothing but equals to minus 0.134 sin 3 t minus 0.032 cos of 3 t. So, this is something you see which we have right now in our feature that we are simply bounded the entire solution with this particular integral, which simply shows that there is a displacement. And the displacement is of this nature in which you see the excitation is there from sin three t and cos three t.

Now, the natural frequency is nothing, but equals to from the first equation x double dot plus you see here 0.4 we can say x. So 0.4 is our k and since x double dot there is no we can say multiplication in terms of mass it is 1, so we have omega n is nothing but equals to two radian per second. And with this now we can calculate the zeta as well the zeta is nothing but equals to 0.4 divided by 2 omega n or else we can say it is 0.1. So, certainly the system is now under damped system and with this you see here we can calculate the damped frequency because there is a damping which is available there.

So, damped frequency is nothing but equals to omega n the square root of 1 minus zeta square because zeta is here less than 1, so probably we can get 1.9, 9 radian per second. So, now, you have both the exciting frequency you have omega, and you have omega d and you have zeta. So, with this particular feature, now we can calculate the other features of the system equations.

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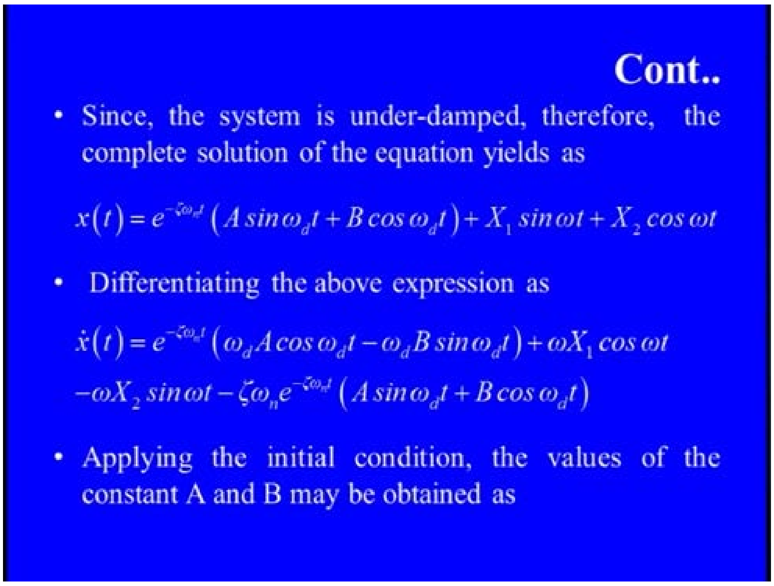

Now, since this is under-damped system, so the complete solution is of the form of both the complementary function which is nothing but equals to e to the power minus you know like zeta omega and t into you know like the a sin omega d t plus b cos omega d t.

And particular integral which we just get the you know like the outcome that is x 1 sin omega t plus x 2 cos omega t, and when you are differentiating this equation with respect to the time.

You have now x dot t is nothing but equals to e to the power of minus zeta omega and t into omega d t A cos omega d t minus omega d B cos B sin omega d t plus; then you see you can further divide this equation into the differential form of your omega. So, omega X 1 cos omega t minus omega X 2 sin omega t minus now once you put entire thing then the zeta, the since it is the exponential term. So, again you see this feature will come into the equation.

So, minus zeta omega n which is nothing but the coefficient of your t in the exponential term will come out and it is minus nothing but you see minus zeta omega n into e to the power minus zeta omega n t into this the same you know like the omega d t coefficients in terms of sin and sin and cos. Now, if you apply the boundary conditions which we have already formed there in the initial feature that you have x 0 and you have x dot 0.

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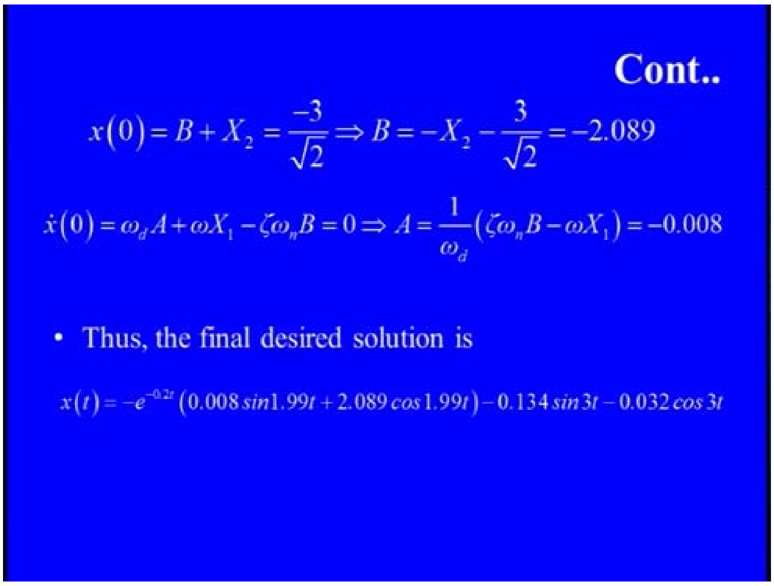

When we applying this equations to their then we have this x 0 which is nothing but equals to you see b plus X 2 equals to this one and we can get this B as nothing but equals to 2.089. Similarly we can apply the same boundary condition x dot 0 which is nothing but equals to 0, when you are applying these things then we have the value of A in the second equation. The A is nothing but equals to 1 by omega d zeta omega and B minus omega x 1 and when we are applying this condition there we have minus 0.008.

So, from these two boundary conditions that is why you see the boundary conditions are very helpful, in finding out the real feasible solution of any equations. Though you see the solution is bounded with particular integral or the complementary function, but the boundary conditions are always giving you some kind of direction, that how the system is really behaving under the situation. So, we can find the final solution that is nothing but equals to x of t is equals to e to the power minus, since you see the it is minus term because there is an d k is there.

So, minus e to the power minus 0.2 t you know like we know that this zeta and omega and both are simply giving the multiplication of 0.2. So, we have this one and then since it is already there the A and B. So, A is minus 0.008, so we have 0.008 sin frequency was there you see the you know like omega 1.99t. So, it is 1.99t plus 2.089 that was you see the coefficient of B into cos of 1.99t. And then you see the previous which we have already calculated for particular integral minus 0.134 sin 3 t plus minus 0.032 cos 3 t.

So, this is how you see we are dealing with this forcing factor and all other factors like omega and you see the zeta and all other things that how they are going to vary for a particular solution. So, this numerical problem is simply giving one feature that when we have you know like when you are just calculating some natural frequency or other frequency phenomena, we have to be very careful that what exactly the boundary conditions are, and how the system is propagating when the vibrations are being there under force forcing factors. Now, we are going towards the conceptual features of 2 degree of freedom system that how the 2 degrees of freedom system is really behaving and how we can put the Newton's law towards you see formation of a equation of motion.

So, there are various steps which are being you know like involving to analyze the 2 degrees of freedom system under the vibrating phenomena to get the natural frequency, because we know that since the system is moving at two different you know like we can say natural coordinates or independent coordinates, certainly we have more than one natural frequency we have more than one mode shapes. So, to resolve these we need that you see whether it is under the forcing factors or what, so when it is you know like the restoring forces are being coming then how the spring is being deflected.

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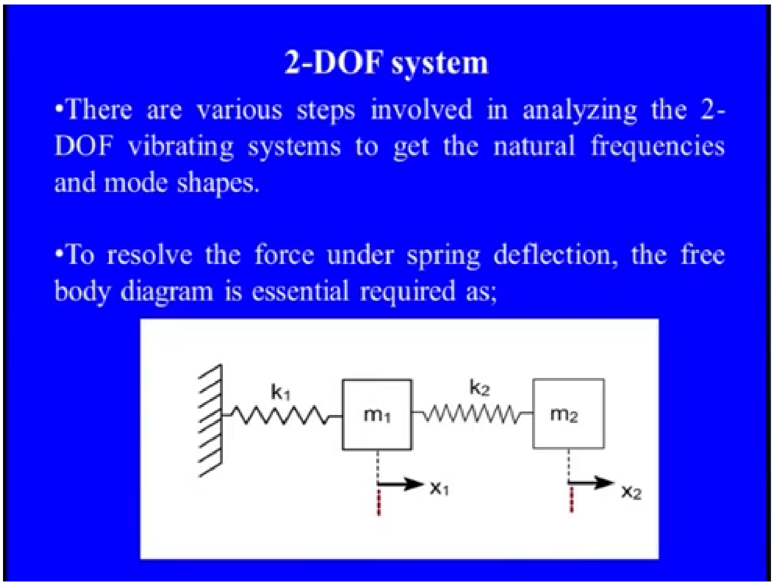

So, you can see that we have a general system the two springs are being simply constrained the two masses and both masses are at the displacement X 1 and X 2. And then we can see that since the springs are there they have the stiffness properties the spring rate you see we can say k 1 and k 2, m 1 and m 2 are the masses X 1 and X 2 are the displacement. With this now first of all we just want to resolve the forces when the springs and the masses are moving relatively.

And when we are doing these things you can see that when the forces are being exerted on that, on top of that the red line is clearly showing that how the restoring forces are being you know like diversify towards other side. So, X 1 and X 2 since they are in this direction the k 1 is just going towards the main rigid frame and X 2 is going towards the free expansion feature. So, this is you see you know like the elastic deformation of the spring according to their stiffness's k 1 and k 2 and in that you see here, if we really see that the deflections are being there in the corresponding motion. So, in other way they may be in the you see the X 1 this k 1 is just going towards the free the free and X 2 is going towards the rigid end.

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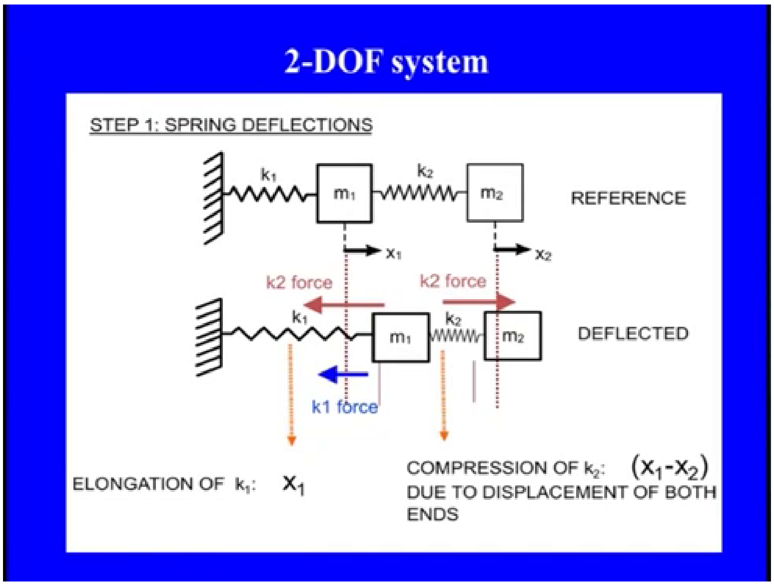

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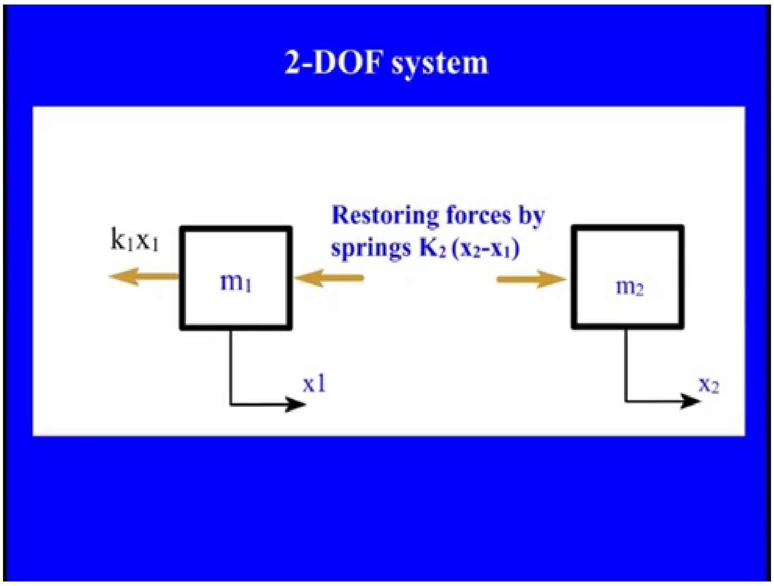

So, these are the two possible combinations which can be there in terms of the restoring forces according to their stiffness's. Or else if we just want to generally for generalize this thing then we have on m 1 on one side we have k 1 X 1, because the spring is well connected to this part and it has you see the restoring properties towards that. So, restoring force for this is k 1 x 1. And on other side of the m one we have the restoring forces, which is basically due to the relative displacement of this spring from k 1 and k 2.

So, we have k 2 into X 2 minus X 1 and for m 2 you see here since it is a free end, so other side it is nothing and since they are in the balanced condition. So, in between the m 1 and m 2 the same force k 2 into X 2 minus X 1 is applied in you know like the action and reaction feature according to the Newton's law, since it is a well balanced criteria

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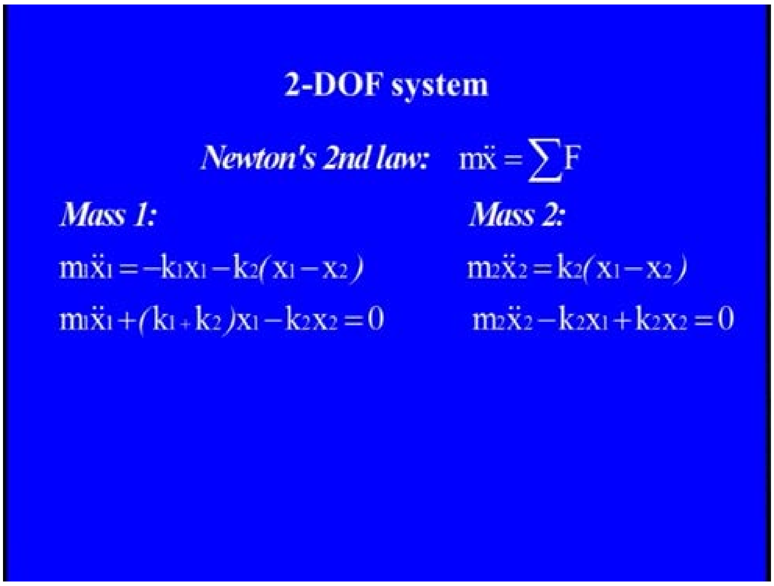

When we apply these conditions to get you know like this equations of motion we can straight away go first to m 1 mass and for m 1 mass we know that the forced balanced equation says that the inertia force due to the mass rotation m 1 x 1 double dot is nothing, but equals to minus k 1 x 1 into minus k 2 x 1 minus x 2. And with this you see here we can frame the equation of motion as you can see on your screen m 1 x 1 double dot plus k 1 plus k 2 x 1 minus k 2 x 2 is equals to 0.

Similarly, we can apply the similar concept to mass m 2 where the forced balance is just with m 2 x 2 that is the inertia force and there is a restoring force which is coming out in between m 1 and m 2 because the spring is connected to the both the side. So, we have k 2 into x 1 minus x 2. So, the equation is pretty simple that m 2 x 2 double dot minus k 2 x 1 plus k 2 x 2 equals to 0. So, we have a coupled equation of second order ordinary differential equation and you can see that in this we have both, in the first equation we have a clear representation of x 1 and x 2. In second equation we have a representation of x 1 and x 2 so; that means, it is a coupled equation.

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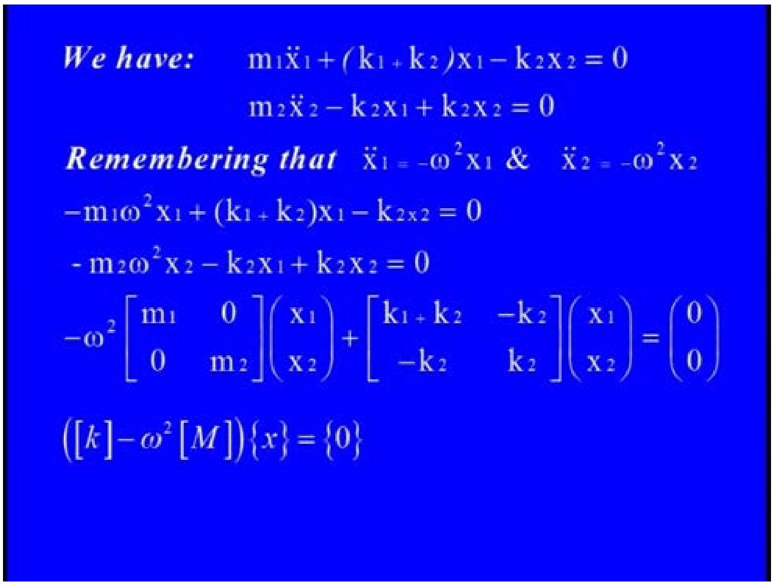

And in that you see here we can simply you know like form a state we can say steady space form or rather you see since we are only dealing with 2 degree of freedom. We can write independent with these equations as you can see on your screen m 1 x 1 double dot plus this and m 2 x 2 double dot plus this one. And we know that this equation is simply representing a simple harmonic motion where you see the input parameter and the output parameters are just showing the periodic solution.

So, we can say that x one double dot is nothing, but equals to minus omega square x 1 this is a well known theory you see because we know that when you are just taking say the simple harmonic motion. So, x 1 if you are saying it is a simple harmonic motion, so x 1 is nothing but equals to A sin omega t then x dot 1 means the velocity is nothing but equals to you know like omega will coming out. So, omega A cos omega t and x double dot the double derivative is nothing but equals to minus omega square A sin omega t since A sin omega t which we have already you know like consumed that it is a x 1.

So, we can replace this x double dot is nothing but equals to minus omega square x 1 this is a well known feature of any sinusoidal excitation. So, we can replace this now acceleration x 1 double dot is equals to minus omega square x 1 and x 2 double dot is minus omega square x 2. And when we are keeping these things, so what we have now we have the equation minus m omega square x 1 plus k 1 plus k 2 you see into x 1 minus k 2 x 2 in first equation and when we are replacing this x 2 double dot from minus omega square x 2.

So, we have minus m 2 omega square x 2 minus k 2 x 1 plus k 2 x 2. We can frame the simple equation with this particular omega square in the matrix form, because you see the two equations are there and they are representing with the same you know like independent coordinates x 1 and x 2. So, we have now minus omega square m 1 0 0 m 1 0 m 2 this is my mass matrix into x 1 x 2 these are my state space feature plus.

Now, when we are taking this right from in the first equation what we have we have k 1 plus k 2. So, the first element in my stiffness matrix is k 1 plus k 2 then minus k 2 from this side and in the lower equation we have minus k 2 and k 2 in this x 1 x 2 is equals to 0. So, in a broader manner now we can write the same equation in k minus, since it is you know like we have the stiffness matrix. So, k stiffness matrix minus omega square mass matrix into the independent coordinate x 1, x 2 that we are saying that the displacement the relative displacement in between the masses m 1 and m 2.

So, we have x factor equals to 0. So, this is you see the standard equation for showing more than 1 degree of freedom system when you see the system is excited with the various degrees of freedom, this equation can be shown to 2 degree, 3 degree, 4 degree or any multi degree of freedom system, where we have some of the properties.

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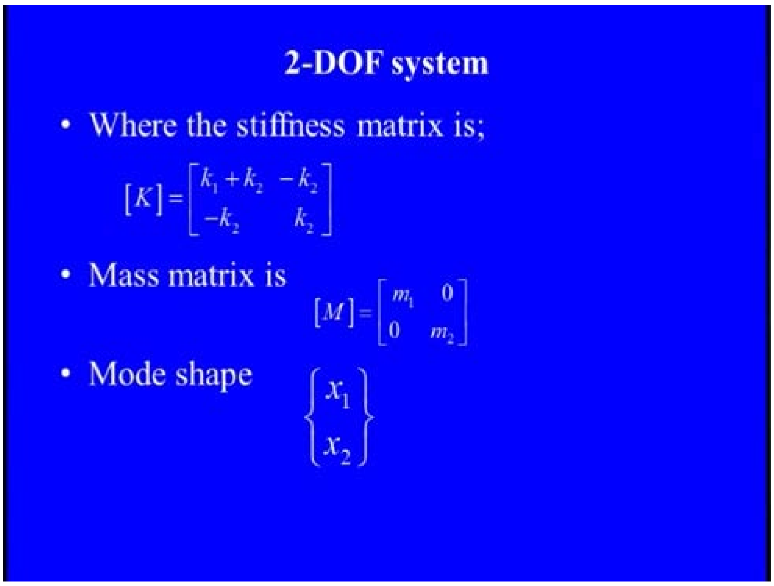

And this is something sometimes we are saying that this is the cross check also. So, the stiffness matrix k is k 1 plus k 2 in the first element, minus k 2 as the second element, minus k 2 on the lower side third element and k 2 as the fourth element. Mass matrix is m 1 0 0 m 2 mode shape, which is nothing but equals to the relative displacement of the masses they are simply x 1 and x 2. And when you see you know like we are calculating the Eigen value that is the characteristic root of the equation they are simply giving the natural frequency square root of this one.

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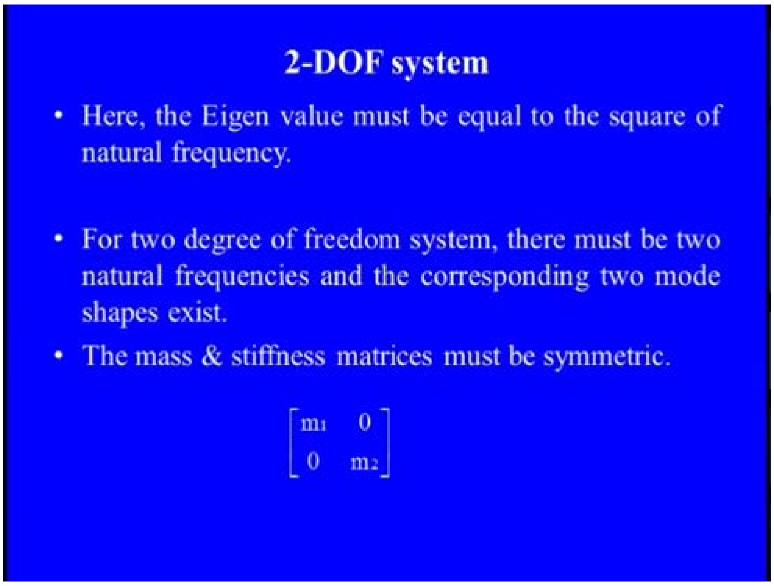

So, first 2 degree of freedom system there must be the two natural frequencies because two equations are there and two masses are moving with the two stiffness's certainly we have the two different exciting frequencies with these terms. And then you see since the two natural frequencies are there every natural frequency is corresponding to the mode shape. So, we have two mode shapes in that case and one of the important or we can say the significant term in this, is the mass and it is stiffness matrices must be symmetric.

If they are not in the symmetric way you cannot get the equilibrium feature in the any dynamic situation. So, this is in reverse way we are saying that this is the cross check for that. So, the mass matrix it is showing on your screen is m 1 0 0 m 2. So, you see the diagonal all the masses which are playing a critical role in formation of the equation or the exciting frequencies they are always coming as in the diagonal form.

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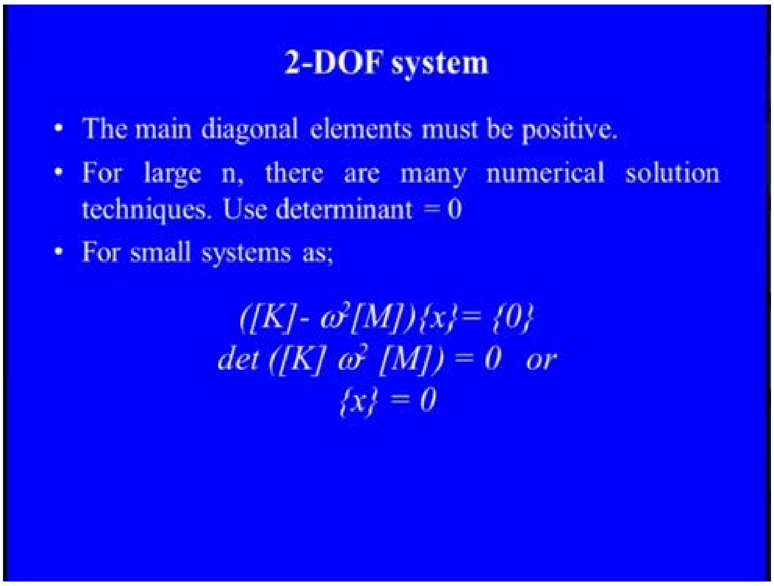

The main diagonal element is always be positive and for any large number n degrees of freedom system the many numerical you know like the techniques are there; for that we can resolve the issues according to time integration techniques analytical formulae or anything even the discretized techniques as well; like infinite element finite volume anything like that. And in that you see what we are trying to see if you want to calculate this we are just saying that we just want to find that the determinant of the matrix which must be equals to 0. So, when we apply to this concept to any of the matrix form we have k minus omega square m into x equals to 0 if we want to find the solution then we need to go with the determinant. So, determinant k the matrix k minus omega square m equals to 0 or we can say the x equals to 0 the more shapes they are in the well balanced feature.

And in this particular case now we can say that for the non trivial solution, we have the determinant k 1 plus k 2 minus omega square that is one term, minus k 2 was another term minus, k 2 was the third term in the lower. And then k 2 minus omega square m 2 equals to 0 or else you see if we resolve this issue, we know that we have k 1 plus k 2 minus omega square and k 2 minus omega square minus k 2 square equals to 0.

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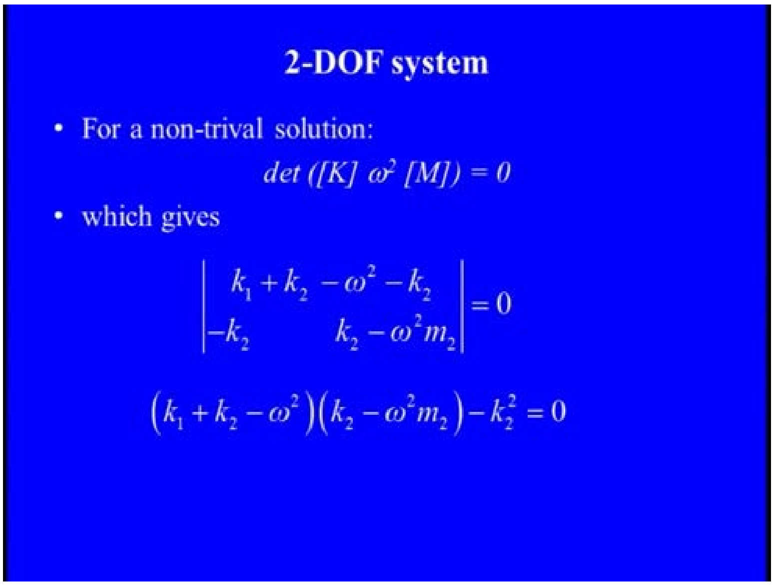

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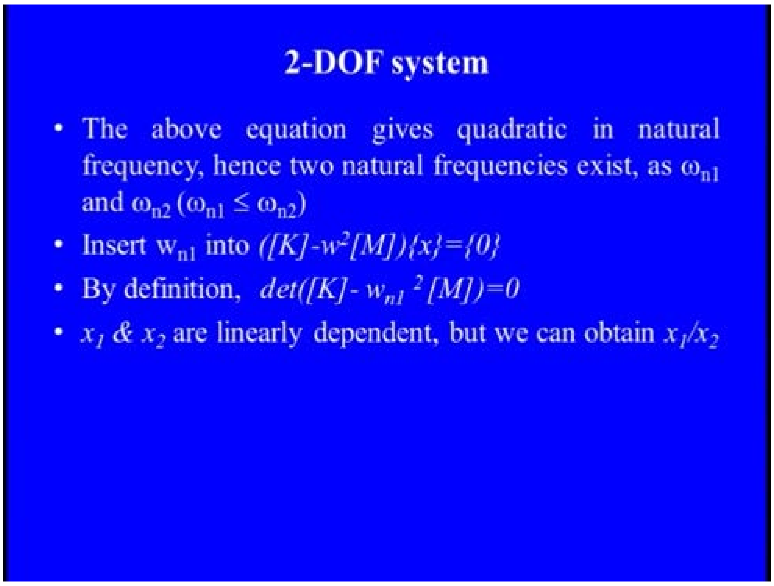

When we are trying to solve these equations we know that the above equation is always in the quadratic form. The omega to the power four in the natural way, so two natural frequencies are there one is omega and one which corresponds the first mass omega n two which corresponds to the another mass of this with this stiffness value. So, the first natural frequency and the second natural frequency, the nature of the natural frequency existing with any kind of system is like that.

The first natural frequency must be less than or equal to second natural frequency it cannot be greater than that, because as it is you know like moving further we know that the higher order excitations are creating more exciting frequencies. And when we are inserting these natural frequencies into the basic equation, say one or two then we can simply find that the determinant k minus omega n square m equals to 0. And determinant k minus omega into square m equals to 0 and correspondingly we know that we can get x 1 and x 2, because they have the linear a linearly dependent parameters on these feature. So, the x 1 and x 2 the relative displacement is giving some value on that.

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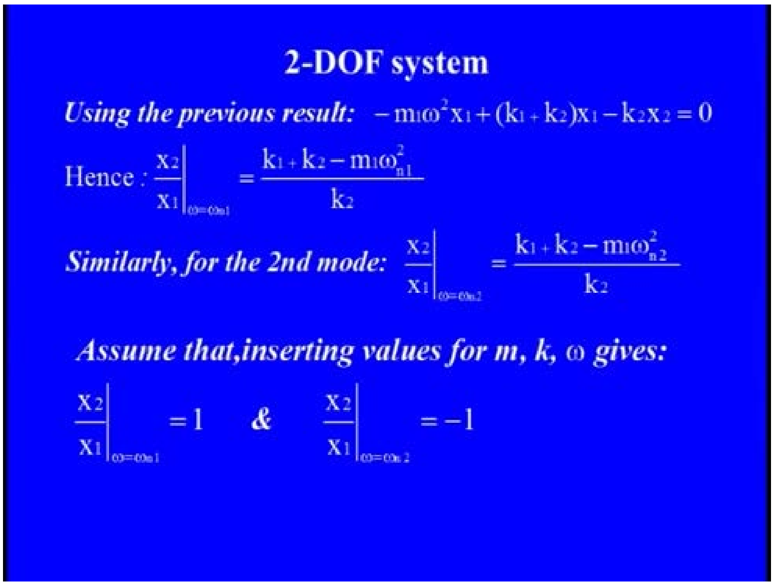

So, now, if we are using this what we have first minus m omega square x 1 plus k 1 plus k 2 x 1 minus k x 2 that was our first equation. And if we are applying this we have x 1 by x 2 at the first natural frequency is equals to k 1 plus k 2 minus m omega n 1 square divided by k 2. And similarly for the second mode means at you see here omega equals to omega n 2, we have this one.

And since you see we are taking the relative displacement right now we are saying that x 2 by x 1 that is nothing but the relative displacement of output by input is equals to 1 for one part and another for minus 1. So, 1 and minus 1 are just showing the phases in between the first and the second and in the diagram we have shown that the springs are the restoring forces are moving towards one direction, towards and or free part. Or in the other diagram also it was just you see you know like in case of repulsion. So, in both the case you see the displacements x 2 and x 1 are just 1 or minus 1.

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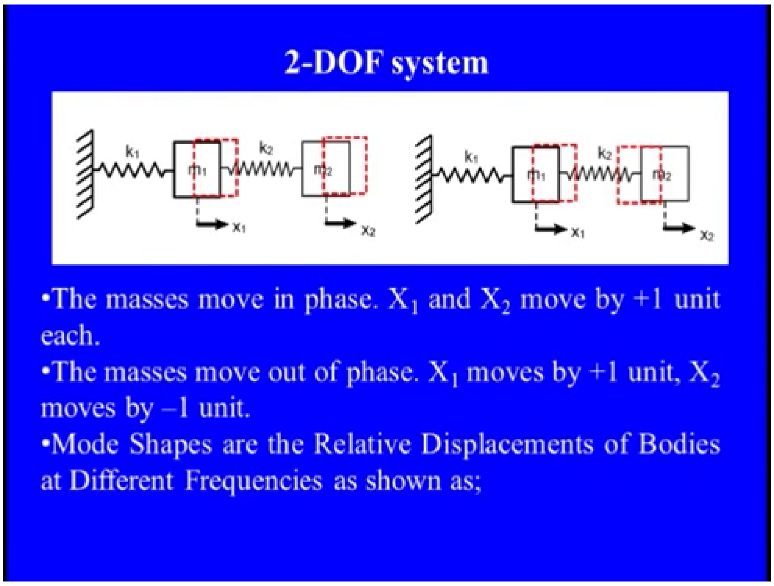

As you can see here in this also we have seen you know like the red feature they are showing either in first phase they are just going in a similar phase part both are in the extension feature. Or in the other part you see here it is simple you see one part is going forward other part is just coming towards that side, so we have minus 1. So, this is first case is showing one feature the masses which are moving if they are in the phase they have plus 1 x 2 by x 1 the masses.

When they are just in the outer phase means x 1 is just moving positive and x 2 is moving minus 1 we have always in the opposite phased feature and mode shape are nothing but as we already discussed they are the relative displacements of the bodies at the different frequencies. We can show these things that how you see since we have the two masses how we can say that they are in phase or they are out of phase. So, you can see that in this.

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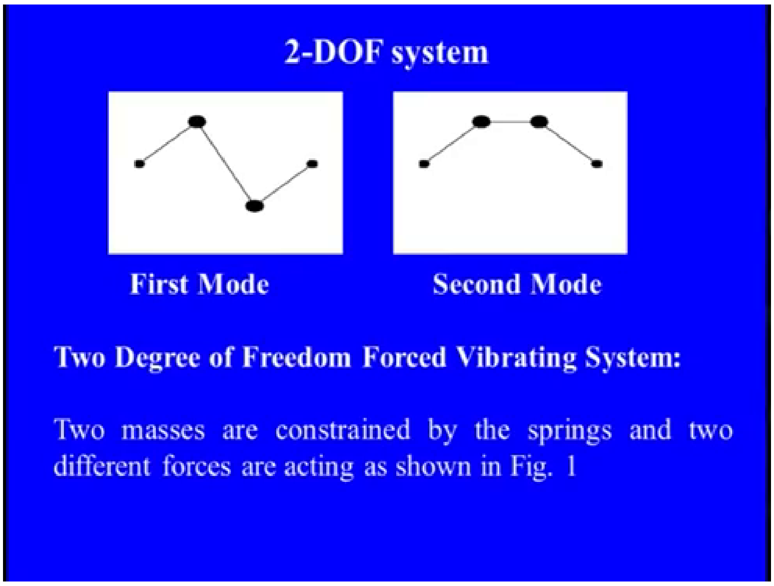

This first mode when we are saying that they are absolutely in the opposite phase you can see that both are just in you know like diagonal to each other. One is going down, one is going up, one is going up, one is going down in the first mode, while in the second mode you can see that it is they are in phase means both are coming together downward direction or upward direction.

As you see these are the two masses connected to two springs they have the same relative movement in the second mode or in first mode irrespective of whatever it is. So, accordingly you see we can say that x 2 by x 1 are 1 or minus 1 according to their relative motion. So, 2 degrees of freedom system showing two natural frequency two mode shapes and both mode shapes are simply the relative displacement of their masses towards that.

And two masses which are being constrained by the spring certainly we have two different you know like the restoring forces on that and we can show that these things that you see how the masses are being you know like encountered towards that. So, please remember that whenever we are discussing about like the more degrees of freedom they are simply reflecting that the masses which are being connected by the springs or any of the constraints they have relative displacement towards that. And say if you are going towards three degrees of freedom certainly we have the masses sometimes in phase or out phase together.

May be you see out of three masses the two masses are in phase one mass is out of phase or even all three masses are in the phase in that we are saying that this is the orthogonal condition. So, sometimes you see when you are going towards the higher orders generally we are saying multi degrees of freedom system the orthogonality is one of the key property in the mode shapes. And we can calculate the mode shapes means, we can calculate the relative displacement of all the masses according to their exciting frequencies. And that is why you see here if we are saying 3 degrees of freedom we have three natural frequencies, if we have 4 degrees of freedom four natural frequencies are there or n degrees of freedom simply giving you the n degrees of freedom systems.

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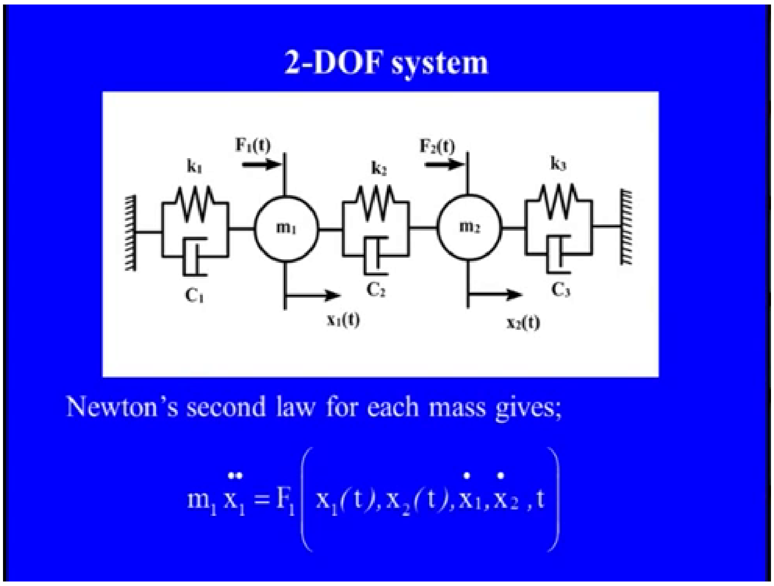

Now, you see here we are applying the similar kind of concept to another feature that if you see the end is not free if the end is even constrained there. So, you look at that what we have we have m 1 and m 2 these are the two masses which have the displacement the displacement is nothing but equals to x 1 of t and x 2 of t. These two masses have a clear constraints right from the beginning to the end. So, we have in the beginning the k 1 and c 1 the stiffness and the damping in between these two now we have k 2 and c 2 and in the last at the constraint feature other at the outside we have k 3 and c 3.

So, this system is now having the three you know like the damping feature and three spring features the stiffness feature. And now we are applying the force to introduce x 1 and x 2. So, for m 1 the force is which is being applied the forcing factor is f 1 t and x 1 t is corresponding resultant at f 2 t is being applied to m 2 and corresponding x displacement is x 2 t we know that under the forced excitation the system is well established the equilibrium feature.

So, we can find out that you see the inertia of forces which are being generated due to the force application on the masses m 1 x 1 double dot is nothing but equals to f 1, which is being applied x 1 by t. This is one part means you see what you have you have displacement x 1 t you have displacement x 2 t and then you see you have the velocity component x 1 t and x 2 t because the damping is there. So, velocity is also one of the important feature in dissipation of the energy as the viscosity it is there.

So, please remember that if the system is simply a mass and a spring the displacement is enough to find out the characteristics features of the system, but when you have the damping involved there. Then we need to check that how the frequencies are being affected by this damping due to this dissipation of energy; that means, in other way we need to consider the displacement and the velocity together. So, we have you know like right now the system is integrated with the spring and the damping here. So, we can say that it is f 1 x 1 t x 2 t x 1 dot t x 2 dot t and the t because the time is the because it is a dynamic features, so we have the time, so this is for the first mass.

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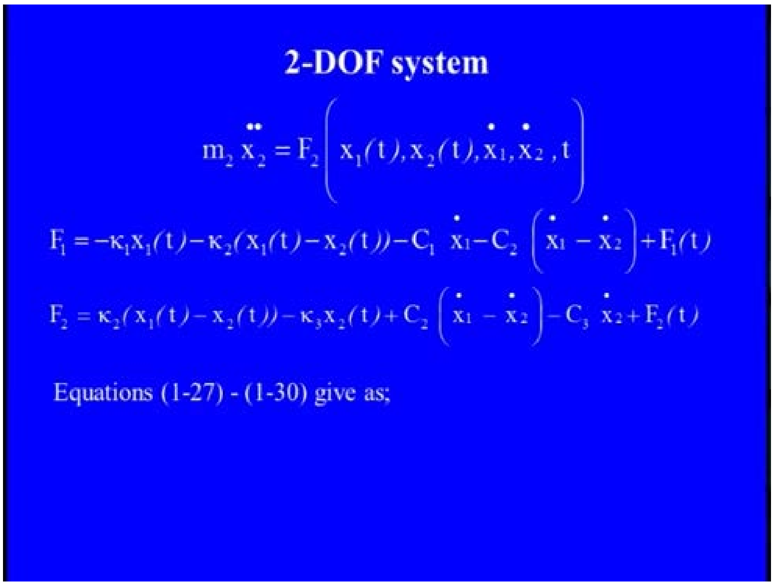

We can apply the similar things to our another mass m 2 x 2 dot which is nothing but equals to f 2 or we can say you know like this particular function of x 2 t x 1 t and then x

1 dot t and x 2 dot t. So, this is you see either the inertia force at first mass inertia force at another mass certainly you see this is a function of both the displacement and the velocity component together.

And now you see if you are applying the balanced condition there itself then what we have we have f 1 t, f 1 which is you know like the forcing factor there on the first mass is nothing but equals to minus k 1 x 1 t, because you see the restoring forces which is just coming out from the base. So, minus x 1 you see here is the displacement and k 1 is multiplication. So, restoring force minus k 2 x 2 and you see when we are talking about the k 2 this k 2 is in between the displacement x 1 and x 2.

So, certainly we have the relative displacement, so this k 2 is influenced by x 1 t minus x 2 t. Then you see a if you are going towards the viscous damping again you will find that from the base end the c 1 into x 1 dot t; that means, you see we have c 1 velocity minus c 2, because you see the c 2 is also coming in between the two different displacement x 1 and x 2.

So, certainly you see there is a velocity difference which is coming out and which or else you see which has you know like direct influence on the damping forces formulation for the first mass balanced feature. So, you see we can say it is c 2 into x 1 dot minus x 2 dot in that and plus you see here whatever you see the forces which are being applied to the system f 1 t.

So, this is the total force which is being there and we can simply formulate the equation of motion for the first mass similarly we can go to the another force that is f 2 on the another mass. So, what we have see straightway we can go to the k 2, so k 2 since it is coming in between x 1 and x 2 in the previous figure. So, k 2 into x 1 t you know like minus x 2 t that is a relative displacement feature and then you see we can say that minus now we have the k 3, k 3 is nothing but the spring stiffness which is just you see on the extreme side of mass m 2.

So, since this is coming as the independent coordinate of your x 2. So, we have minus k 3 x 2. Similarly, we can say that when we are simply playing with this c 2 which is in between you see your x 1 and x 2. So, certainly the velocity differences are being there as x 1 dot minus x 2 dot and then you see c 3 which is you know like the extreme end your damping feature. So, c 3 into x 2 is your damping force at the extreme end plus whatever the force which is being applied to mass. So, when we apply these equations both f 1 and f 2 equations the balanced equation then what we have, we have the equation of motion like that.

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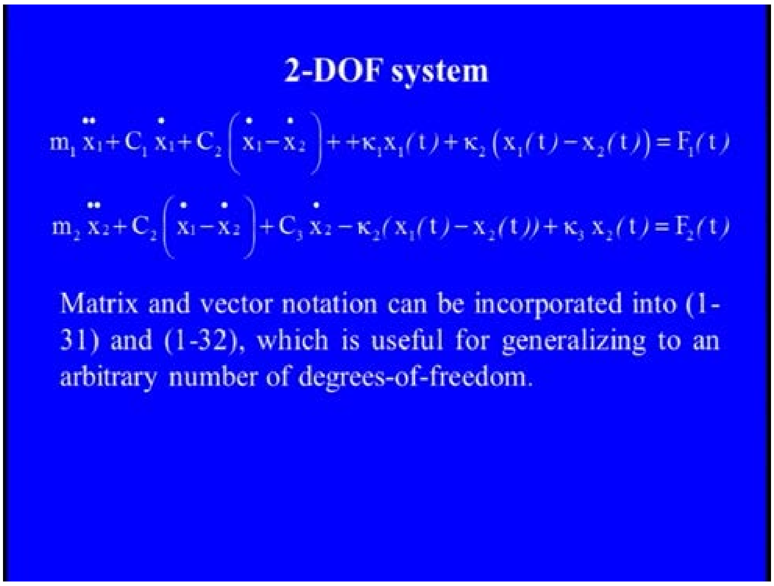

So, equation of motion is m x double dot the inertia forces for first displacement second we have the damping forces c 1 x 1 dot plus c 2 x 1 dot minus x 2 dot. So, this is the total damping forces which is being straight way applied to both the side of the mass just to make the balance or the energy dissipation, plus you have the springs; since this mass m one is connected by both the side spring one is k 1 and one is k 2. So, k 1 is affected by x 1 and k 2 affected by x 1 minus x 2.

And when we are doing these things then it is equals to whatever the force which is being applied to the system external force. So, this is the well balanced equation from the Newton's law and we can say that we can apply the similar kind of situation to our another equation of motion m 2 x 2 dot is you see into c 2 x 1 x 1 dot minus x 2 dot plus c 3 x 2 dot. So, again you see here this is the total damping of the mass m 2 on left hand side we have relative displacement x 2 and x 1 and x 2. So, you see the velocity component is coming in formation of damping force on left hand side in x 3 x 1 and x 2 and on other side we have x 3 dot which is nothing but you see since it is a independent coordinate with the c 3. So, we can calculate this x 2 dot into c 3.

And similarly, you see for restoring forces on the left hand side of m 2 we have relative displacement between x 1 and x 2 on other side on right hand side of mass m 2 we have a independent coordinate of x 2. So, we can calculate you see the k 2 and the k 3 formation accordingly the restoring forces and then you see the f 2 which is being applied force on the mass m 2.

So, these two equation of motion is simply shows that how you see the damping restoring forces and you see the inertia forces are being well balanced even when it is being applied by external force f 1 or f 2 on m 1 and m 2 masses then you see again we need to if you want to solve this then again we need to apply the same that you see since these forces which are being applied to the system they are harmonical forces.

So, we can simply assume that even f 1 is nothing but equals to f 0 sin omega t f 2 which is nothing but equals to f 0 sin omega t in other way and then you see you have the two forcing factors the entire system is under periodically exciting free figure. Or we can say simple harmonic motion you can apply you see same that x 1 since you see you have the 2 degrees of freedom system with the x 1 and x 2 displacement you can say x 1 double dot is nothing but equals to minus omega square x 1 x 2 double dot is nothing but equals to minus omega square x 2.

And when we are applying these conditions we can frame the same equation as the matrix k in this particular feature matrix now we have the more number of elements because what we have since you see it is a 2 degrees of freedom system, but there is a clear interaction of three springs. So, we have you see in our elements we have k 1, k 2, k 3 interacting together and in the same you see we have the damping figures.

So, now there is a damping features which will come out because now the system is also exciting with this particular phenomena. So, we have what we have un-damped natural frequencies and the damped natural frequencies we can simply calculate that whether the system is under damped critically damped or over damped. And accordingly you see we can frame the equations then itself for the solution and then we can find out that what is my outcome is there.

So, x 0 or you see here x 2 by x 1 according to the mode shapes. So, in simple way that simply you know it is like 2 degrees of freedom system we can resolve all these issues by assuming the simple harmonic motion input excitation and output feature x 0 in the simple harmonic motion. So, the frequency response functions can be calculated mode shapes can be calculated or the natural frequency can be simply you know like obtained using these equations omega n 1 and omega n 2.

So, matrix and vector notations can be incorporated into these equations which is useful for you know like generalizing an arbitrary numbers of degrees of freedom. So, we can use the similar kind of steps to solve even the n degrees of freedom system. So, in today's lecture most of our discussion was you know like that if the system is under forcing factor or the system is of 2 degrees of freedom. Then how we can resolve the two degrees means the two coupled equation, how we can give the input features of the simple harmonic motion or how the boundary conditions or the initial conditions are being straight away applied to get the final numerical solution.

In our next lecture now we are going to discuss because you see till now you see we were discussing about this 2 degrees of freedom. So, we would like to now finish up to we will go up to certain higher degrees and we just want to see that how the forcing factor the free and forced vibrations are there for more than 2 degrees of freedom generally we are saying that the multi degrees of freedom because as we know that the theme of this course is basically the vibration control.

So, control should be applied at the main exciting frequencies. So, from these you see you could figure out easily that how you know like the exciting features are being taken place at the system which part is exciting you know like more; or what exactly you see you know like their phases when all the masses are being you know like under the exciting frequencies.

Thank you.