

Vibration Control
Prof. Dr. S. P. Harsha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology Roorkee

Module - 8
Vibration Measurement Techniques
Lecture - 2
Data Acquisition

This is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee, in the course of Vibration Control, we are mainly discussing about the Vibration Measurement Technique, which is the last module in that. And in the previous lecture we discussed about the basics of the measurement techniques, because in any of the rotating machinery we know that, the measurement is one of the specific part. Because, prior to apply any control method, first of all we need to configure rather the dynamic forces.

And accordingly, if you can find out that, this is the basic cause the root cause, we just want to treat the root cause and we just want to cure that, straightaway we can immediately attack on the root cause of these vibration generations. So, we know that there are three basic dynamic parameters like the displacement velocity and acceleration. And all are representing their key we can say roll, in measurement of the forces straightaway and then even when we are going for higher frequency measurement we know that, the accelerometers are the perfect one.

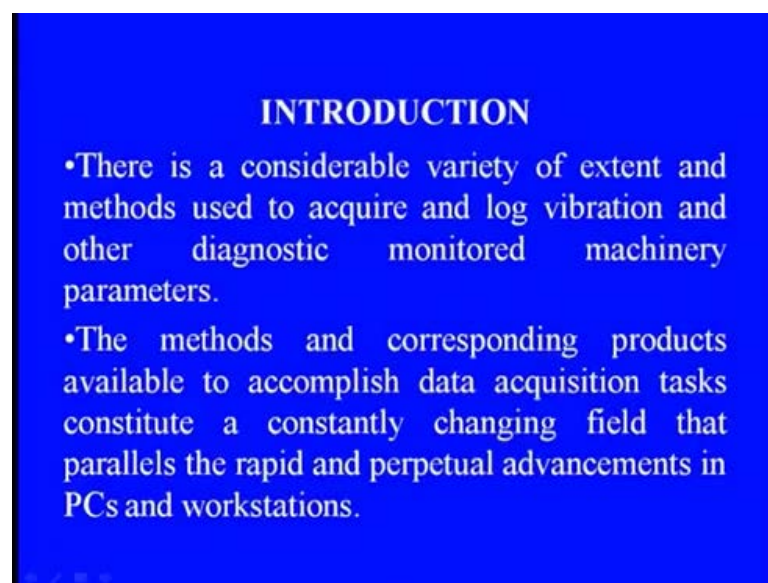
And when we are talking about low frequency measurement well below the 10 Hertz, certainly the displacement measurement is quite good enough for that. So, in the previous lecture, we discussed entire things that how the accelerometers basically work in an, how we can get the accelerations means the inertial forces, and how we can configure that especially when the vibrating masses are being there, at the machine right from the bearings to gears to any kind of the misalignment or the unbalanced rotor forces.

Now, in this lecture we are trying to focus on the measurement technique from the data acquisition point, means now whatever the data whether it is in terms of the time domain or the frequency domain. How we can capture those data's, even sometimes in the last lecture we discussed that, this is really tough to locate the sensor at an accurate position,

because when the things are being rotating, certainly we need to see that where we can keep the closest place the close by area of the sensor.

So, that say if the bearing is rotating shaft, is rotating gears are being under and rotating motion, then what is the accurate location or some adjustable location, so that we can capture almost clear the closest data which we are requiring. So, whether we are talking about the time domain or the frequency domain, we need to check it out that what exactly the data's which are coming, and how we can just process that.

(Refer Slide Time: 03:20)



So, in this there is a considerable variety of the extent and the method use to acquire and the log, we can say vibration and the other diagnostic monitoring machinery parameters. Because, the methods and the corresponding products which are being available to accomplish the data acquisition tasks, just constitute a constantly changing field that is we can say, just either parallel perpendicular advancement in the PC and the workstations. Because, when we are talking about the online fault or online vibrations in nature, we need to check it out that how we can process these data, how we can capture this data and process this data through that.

(Refer Slide Time: 04:03)

- Data acquisition systems are used to acquire, store and analyze vibration data received from sensors. Sensing Systems utilizes state of the art computer based data acquisition systems.
- The equipment is optimized to sample at rates commensurate with the highest frequency expected during testing. Data may be acquired and submitted in different formats for further review and analysis by our customers.

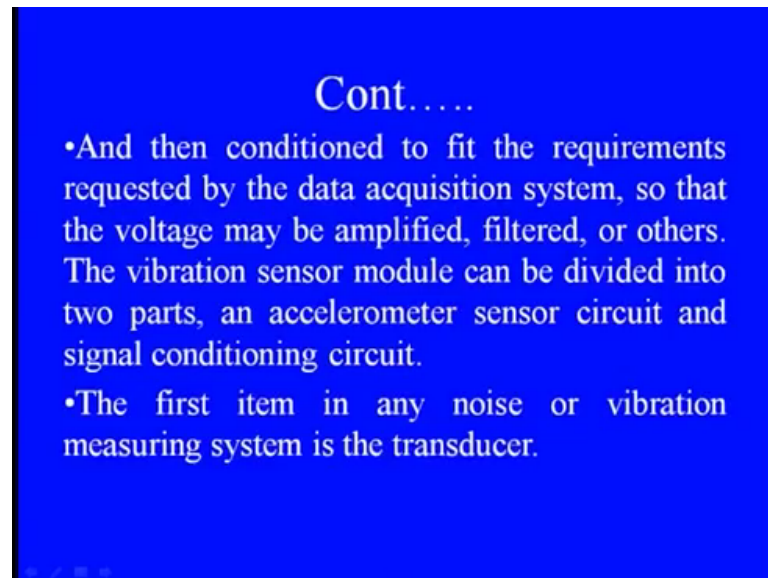
So, data acquisition system, generally we are seeing the DAQ system are used to acquire to store and analyze, the vibration data received from the sensor; and the sensing system utilizes the state of art computer based data acquisition system. So, the equipment is supposed to be optimized to sample at the rates commensurate with the highest frequency expected during the testing. So, that is the one of the key feature that, what is the sampling rate is there to capture the data, and data may be acquired and submitted in different formats for further review and analysis, according to the requirement of or the analyzer.

(Refer Slide Time: 04:46)

- Filtering may be performed during acquisition or digitally following data acquisition.
- Data analysis such as Fourier Transforms may be performed following data acquisition.
- In general, vibration sensor serves to detect mechanical vibrations and convert it into electrical voltage proportionally

So, filtering may be performed during the acquisition or we can say digitally, we can say follow the data acquisition part, so data analysis such as the Fourier transformation may be performed using various, by various data acquisition system in which the inbuilt these transform functions are there. So, in general we can say, the vibration sensor serves to detect the mechanical vibration and convert it into electrical voltage proportionally.

(Refer Slide Time: 05:19)

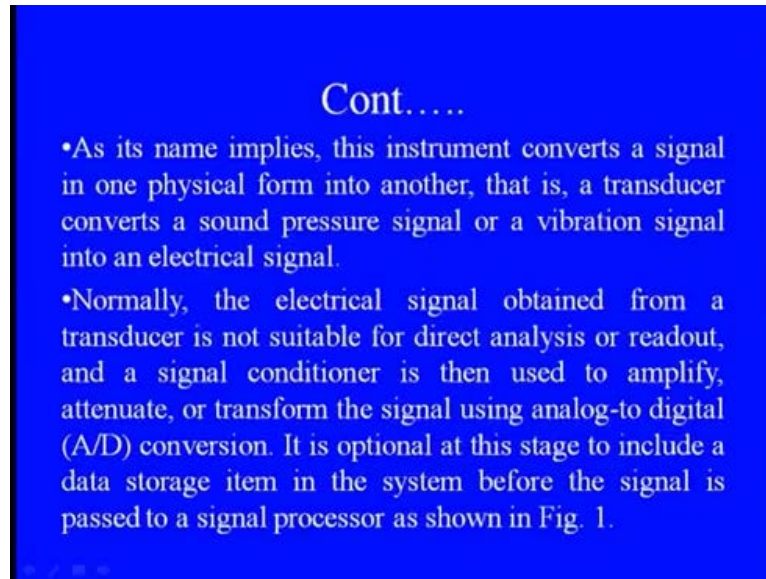


Cont.....

- And then conditioned to fit the requirements requested by the data acquisition system, so that the voltage may be amplified, filtered, or others. The vibration sensor module can be divided into two parts, an accelerometer sensor circuit and signal conditioning circuit.
- The first item in any noise or vibration measuring system is the transducer.

And then whatever the conditions are there to feed the requirements requested by data acquisition system, the voltage may be amplified filtered or the other features are being there with that voltage. So, vibration sensor module can be divided into two main parts, first the accelerometer sensor circuit and second the signal conditioning circuit, there are two first, one whatever the sensor circuit is there based on the accelerometer or anything. And second is what is the signal conditioning, how whether we are just amplifying the signal, filtering the signal, doing various other processes with the signal or what. So, the first item in any noise or vibration measuring is there, is basically just applied to the transducer feature.

(Refer Slide Time: 06:08)



Cont....

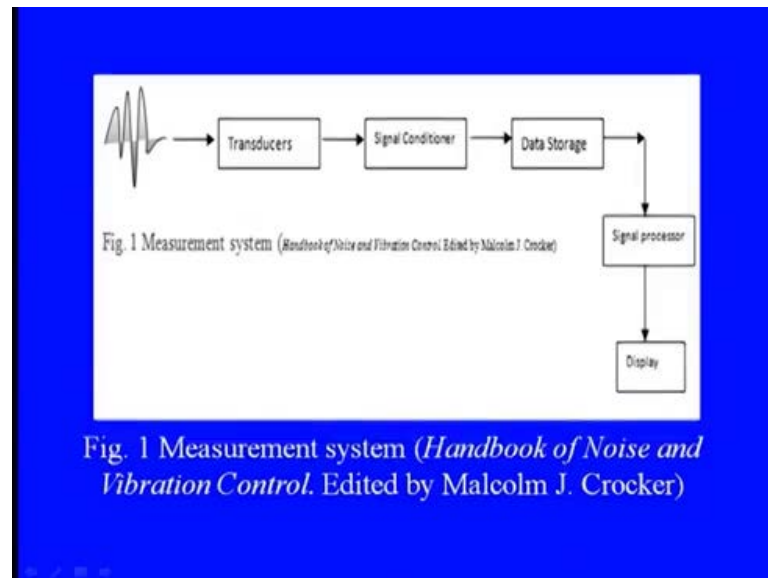
- As its name implies, this instrument converts a signal in one physical form into another, that is, a transducer converts a sound pressure signal or a vibration signal into an electrical signal.
- Normally, the electrical signal obtained from a transducer is not suitable for direct analysis or readout, and a signal conditioner is then used to amplify, attenuate, or transform the signal using analog-to digital (A/D) conversion. It is optional at this stage to include a data storage item in the system before the signal is passed to a signal processor as shown in Fig. 1.

So, as we are using the transducer is nothing but the instrument which converts signal into any from any physical form into another, so this transducer which we can say the sound pressure signal or vibration signal into the electric signal. So, whatever the transducers are there which are being apply directly, they are just taking the pressure or any kind of the things. The mechanical system, we can say vibration signal, pressure signal or anything and they are converting into the electric signal.

And this electrical signal which is being obtained from the transducer, sometimes it is not directly we can see that or what kind of the signals are coming, we need to process that we need to condition. So, signal conditioner is being used to amplify to attenuate or to transfer the signal using this A to D analogue to digital converter. And it is the optional at this stage to include the data storage system, sometimes we are using, sometime the signal processors are there directly and through that is we can do.

So, when we are going for the basic measurement part in this, we can see that this is what the signal which is the raw signal is coming to, whatever the excitation feature is coming, so it is being immediately acquired by the transducer. Now, this raw signal is straightaway sending to the signal conditioner and this signal conditioner, as I told you is doing many of these such kind of applications.

(Refer Slide Time: 07:19)



Then we can rather go for sometimes data storage and sometimes it can be directly go to the signal processor, and through that this displays here. This is the linear path which is being adopted right from raw signal to the display feature, in which the filtering, noise cancelation, various other things the attenuations, various other things are being there right from raw signal to the final display feature. So, either when we are talking about vibration signals or the sound signals.

(Refer Slide Time: 08:12)

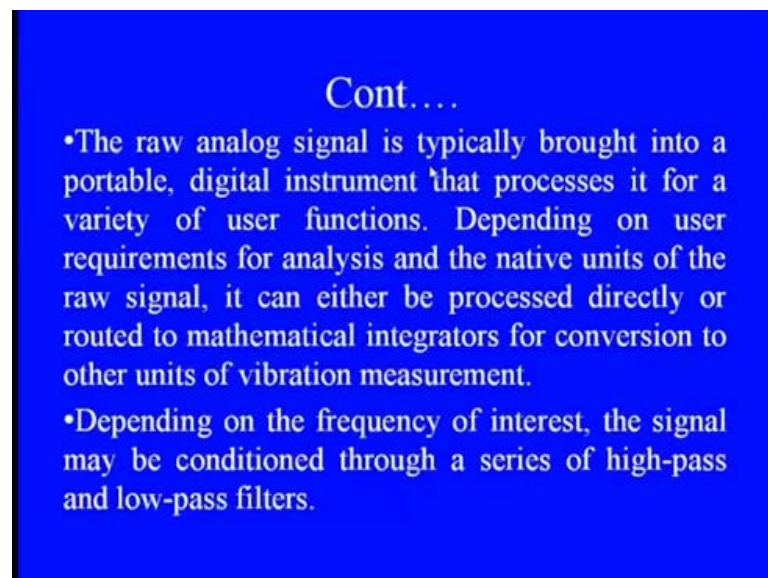
Cont....

- Sound and vibration signals produced by transducers are not normally in a suitable form for the study of noise and vibration problems. Frequency analysis is the most common approach used in the solution of such problems.
- Vibration analysis first begins with acquiring an accurate time-varying signal from an industry standard vibration transducer, such as an accelerometer

They are being produced by the transducers that are not normally in the suitable form of the study, so first of all as I told you that, we need to go say that what are the displacement or velocity or accelerations are coming. First the corresponding transducer or the sensor like, the displacement probe, velocity probe or accelerometer is just capturing the things, it is sending to the signal conditioning unit. And then either the time domain or the frequency domain information is coming out.

So, this vibration analyst first begins with the acquiring an accurate time varying signal, as I told you displacement velocity or acceleration, a time varying signal from an, whatever the transducers are there either from accelerometer, velocity probe or something.

(Refer Slide Time: 08:59)



Cont...

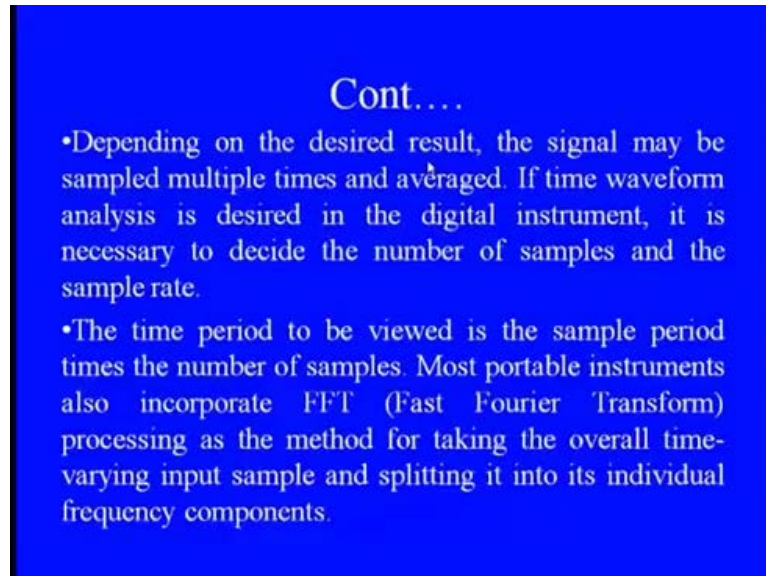
- The raw analog signal is typically brought into a portable, digital instrument that processes it for a variety of user functions. Depending on user requirements for analysis and the native units of the raw signal, it can either be processed directly or routed to mathematical integrators for conversion to other units of vibration measurement.
- Depending on the frequency of interest, the signal may be conditioned through a series of high-pass and low-pass filters.

And the raw signal is typically brought into portable digital instrument feature, that processes it for variety of user functions, and depending on the users requirement for analysis, it is being now absolutely the raw signal is being processed directly or routed to the mathematical integrators for conversion to other units of the vibration measurement. And depending on the frequency of the interest, the signal may be conditioned through various series of high pass or low pass filters.

For basically our interest just to see the high frequency ranges though, we are putting the low pass filters, when we are interested to see, the low frequency analysis then high pass filters are being there. So, I mean to say that what exactly we are looking for we are just

looking the high pass, so it will pass through when we are looking low pass, then it is the low pass frequencies are being going through.

(Refer Slide Time: 10:00)



Cont...

- Depending on the desired result, the signal may be sampled multiple times and averaged. If time waveform analysis is desired in the digital instrument, it is necessary to decide the number of samples and the sample rate.
- The time period to be viewed is the sample period times the number of samples. Most portable instruments also incorporate FFT (Fast Fourier Transform) processing as the method for taking the overall time-varying input sample and splitting it into its individual frequency components.

So, depending on the desired result, the signal may be sampled at the multiple times of the and averaged out, if the time waveform analysis is desired, in the digital instrumentation it is necessary to decide the number of samples and the sampling rate. So, the time period to be viewed is the sample period, in times when we are just going like times to the number of samples, and most portable instrument, we to incorporate the fast Fourier transformation processing. We are always see that what is the varying input sample and is splitting into the individual frequency component and then we are always making the continuous waveforms like that.

So, basically vibration analysis starting from the time varying real world signal, from the transducer or the sensor and this from input this signal is absolutely going from various stages to filtered out or to process this. And it is intended that we need to see that what exactly the internal processing path is there, through which we can simply see that, what the original waveform is coming out and through that, what is the detective frequencies are. So, when we are talking about the time displacement or time, velocity or time acceleration informations are, we need to convert into the frequency part and as I told you that can be done only by the Fourier this series.

(Refer Slide Time: 10:43)

Cont....

- Vibration analysis starts with a time-varying, real world signal from a transducer or sensor. From the input of this signal to a vibration measurement instrument, a variety of options are possible to analyze the signal.

- It is the intent of this paper to focus on the internal signal processing path, and how it relates to the ultimate root-cause analysis of the original vibration problem.

(Refer Slide Time: 11:33)

Fourier series and Transformation:

- Fourier series decomposes periodic functions or periodic signals into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or complex exponentials). The study of Fourier series is a branch of Fourier analysis.

- Most machines emit periodic disturbances to the surroundings, either in the form of fluctuating forces, acting via the machine mounts, or in the form of sound

So, we are saying that the Fourier series and transformation is also one of the important tool, to go the entire signal, to take the entire signal into the fast Fourier transformation or the frequency domain. So, Fourier series decomposes the periodic function and that is why one of the biggest drawback of the Fourier transformation, that if we have the non stationery waveform. In which the various parameters of the wave, the statistical parameters of the wave are being featured out, so if we are talking about the non stationary reform, the fast Fourier transformation is always giving the misleading information.

So, this is the drawback, otherwise the Fourier transformation is the best one to see the dedicated defective frequencies or the frequencies which are being dedicated to the entire exciting zone. So, Fourier series, if we are going for the basics of that the Fourier series decomposes the periodic functions or the periodic signals, into sum of this set of simple oscillating functions. We can say like oscillating means the normal oscillating the sinusoidal or the cos functions.

So, when we are just going with the Fourier this series we are saying that, in the Fourier transformation, all this is the linear summation of these Fourier periodic signals. So, most machines emit the periodic disturbances to surrounding, either in form of fluctuating forces which are being acting via the machine mounts or in form of this sound which is also being the simple harmonic motion.

(Refer Slide Time: 13:09)

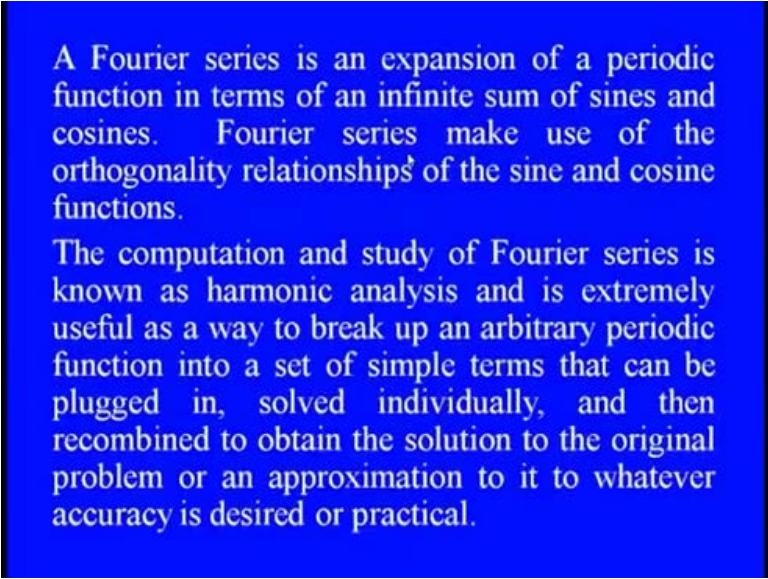
Cont.....

- The reasons for these periodic disturbances can be, to name a few examples, the meshing of gear teeth, imbalances in rotating shafts, or periodic pressure fluctuations that arise in the cylinders of internal combustion engines due to the intake-exhaust cycles.
- To analyze the problem in the frequency domain, a method is needed to divide up a measured signal into its harmonic components, so that they can be individually analyzed. For periodic signals, it is possible to use a Fourier series expansion.

And the reason for these periodic disturbances, maybe we can say the meshing of gear teeth in every rotation, when the problem is there in the meshing of gear teeth, they will again repeated themselves in every period. Imbalances in the rotating shaft, a periodic pressure fluctuation which is being arises in the cylinder of IC engine due to intake or we can say exhaust gases. And in every cycle, they will repeat that kind of sound or the vibration in their motion, to analyze these problems in the frequency domain, a method is absolutely required which is to be divided up to the measured signal into the harmonic components.

Because, that can be individually done, which can be individually analyzed and then linearly sum of that part. So, for periodic signal it is quite possible to use the Fourier series expansion and then we can simply featured out the exciting frequencies there. So, this series the flourier series is nothing but the expansion of the periodic function, in terms of the infinite sum of sines and cosines.

(Refer Slide Time: 14:14)



A Fourier series is an expansion of a periodic function in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions.

The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.

And we can use the orthogonality relations, in making the relation between the sine cosine functions, so the computation and the study of Fourier series for harmonic analysis is absolutely important. Because, through that we can break up the arbitrary periodic function into the set of various simple terms, which can be plugged in and solved individually, and that can be recombined to obtain the solution of the original problem. Or in appropriate we can say accuracy can be adopted for checking out the a clear filtration, or clear excitation peak through this Fourier series solution.

(Refer Slide Time: 14:58)

Cont....

a. Approximation of signals

- As a first step in deriving a method to decompose a periodic signal into its harmonic components, we will study how to best approximate a signal $a(t)$ with a signal $b(t)$.
- We assume that the fitting of the two signals is to take place during the time interval $0 < t < T$. To carry out the approximation in the simplest possible way, we multiply $b(t)$ by a constant β that is varied to adjust the approximation as well as possible

So, when we are doing this the first thing is the approximation of signal, so the first step in deriving the method to decompose the periodic signal into the harmonic components, we need to first approximate a signal, like a signal a of t with signal b of t , and when we are trying to fitting the two signals into just combining together, which is being there in say the, in the time period capital T . Then we need to just see the approximation in simple possible way when we are multiplying some of the constant just to adjust, what are the approximations are there.

(Refer Slide Time: 15:33)

$$a(t) \approx \beta b(t)$$

The error in the our approximation then becomes

$$e(t) = a(t) - \beta b(t)$$

Next, the averaged squared error is computed over the entire time interval $0 < t < T$.

$$\mathcal{E} = \frac{1}{T} \int_0^T (a(t) - \beta b(t))^2 dt$$

So, when we are trying to see this the first thing is coming a of t which is the main signal is almost equal to b of t with the multiplication of that constant beta. And whatever the error is there in our approximation can be nothing but equals to a minus b of t into beta that is what the error is, and we just we want to averaged the squared error which is being computed over the entire time period this capital T. So, we can say the error now, which is the averaged squared error is nothing but equals to 1, for the total time period 1 by total time into integration of to capital T a of t minus b of beta into b of t whole square into d t, that is how we are calculating this, we are computing this error.

(Refer Slide Time: 16:23)

Cont....

- We minimize e with respect to β by differentiating, and setting the resulting derivative equal to zero,

$$\frac{d\varepsilon}{d\beta} = \frac{1}{T} \int_0^T 2(\beta b^2(t) - a(t)b(t)) dt = 0$$

from which

$$\beta = \frac{\int_0^T a(t)b(t) dt}{\int_0^T b^2(t) dt}$$

And if we want to now minimize the error with respect to the coefficient beta, first of all we need to go with the minimum, so minimizing this and putting this entire derivative equals to 0. So, d this error divided by d beta, d epsilon by d beta is equals to 1 by T, now when we are differentiating, you differentiate it out this 0 to capital T 2 betas beta into b squared t minus a of a of t b of t into d t equals to 0. So, we can say this constant which was intentionally being there it is nothing but equals to integration of 0 to T, 0 to capital T for entire time a of t into b of t into d t divided by this b square t into d t for entire time period integrations.

(Refer Slide Time: 17:12)

Cont....

- We now introduce the useful concept of *orthogonality*. If the signals $a(t)$ and $b(t)$ are orthogonal, there is no connection between the two signals, and b equals zero. The orthogonality condition for the two signals $a(t)$ and $b(t)$ on the time interval $0 < t < T$ is therefore

$$\int_0^T a(t)b(t)dt = 0$$

So, now we can say that, this concept which is simply showing the orthogonality, if the signal a of t and b of t are orthogonal, because of the sine and cosine part of the series entire. There is no connection between the two signals b and b equals to 0, the orthogonality condition for the two signal a of t and b of t for in this time periodical part, in which the b this entire 0 to T a of t and b of t into $d t$ is equals to 0 for making this property.

(Refer Slide Time: 17:41)

Cont....

The energy of a signal is proportional to the time integral of the signal squared, thus

$$E_a = \alpha \int_0^T a^2(t)dt$$

in which α is a proportionality constant. Given a signal composed of two parts, $v(t)=a(t)+b(t)$, its energy becomes

$$E_v = \alpha \int_0^T (a(t)+b(t))^2 dt = \alpha \int_0^T a^2(t)dt + \alpha \int_0^T b^2(t)dt + \alpha \int_0^T 2a(t)b(t)dt$$

We can say that the energy of the signal is absolutely proportional to the time integral of the entire signal squared, so we can say E of a which is nothing but the energy of the entire signal is α times of $\int_0^T a^2(t) dt$, where α is the constant which we are using. So, the signal which is composed of $a(t)$ and $b(t)$ we can say that the energy is nothing but equals to α times of $\int_0^T (a(t) + b(t))^2 dt$ or we can say α times of integration $\int_0^T a^2(t) dt + \alpha$ times of $\int_0^T b^2(t) dt$ or α times of $\int_0^T (a^2(t) + b^2(t)) dt$, or else we can say that, when we are expanding this plus α times of $\int_0^T 2a(t)b(t) dt$.

(Refer Slide Time: 18:41)

If the signals are orthogonal, we get

$$E_v = \alpha \int_0^T a^2(t) dt + \alpha \int_0^T b^2(t) dt = E_a + E_b$$

Thus, the energy of the combined signal is the sum of the individual energies. If the signals are not orthogonal, previous equation must be used.

Possibly, the approximation of the signal $a(t)$ given by above equation in combination with first equation gives an inadequate fit. To further reduce the error, we incorporate another signal $c(t)$ with a proportionality constant γ , and write the approximation as

$$a(t) \approx \beta b(t) + \gamma c(t)$$

So, for the orthogonal feature when we are adding into the signal, we can say that the energy associated with the signal is nothing but equals to α times into integration of $\int_0^T a^2(t) dt + \alpha$ times integration of $\int_0^T b^2(t) dt$. So, we have the energy associated with the individual term $E_a + E_b$, so when we are talking about the energy of the combined signal is the linear algebraic summation of the individual energy. And if the signals are not orthogonal certainly the previous equation which we used is absolutely valid.

So, the approximation of the signal can be given by the equation in the combination, as we discussed in the first equation part which is the inadequate. So, to further the reduce the error, we can rather incorporate another signal $c(t)$ with the proportionality constant

g, and we can write you see that this gamma rather. So, a of t is almost equal to beta into b t plus gamma of c t, just to properly feed that part.

(Refer Slide Time: 19:44)

The error in that case becomes

$$e(t) = a(t) - \beta b(t) - \gamma c(t)$$

and the mean squared error is

$$\varepsilon = \frac{1}{T} \int_0^T (a(t) - \beta b(t) - \gamma c(t))^2 dt$$

First minimize e with respect to b ,

$$\frac{\partial \varepsilon}{\partial \beta} = \frac{1}{T} \int_0^T 2(\beta b^2(t) + \gamma b(t)c(t) - a(t)b(t)) dt = 0$$

If we can now choose $b(t)$ and $c(t)$ to be mutually orthogonal, the second term in above equation is eliminated, and gives

So the error can be now straightaway taken as a of t, earlier it was a of t minus beta times of b of t, now further we need to add minus gamma times of c of t. And then we can find out the mean squared error, so mean error in that squared form is nothing but 1 by t integration 0 to T into this all, a of t minus beta times b of t minus gamma times c of t whole square into d t. And we can go with the same procedure when we are trying to minimize this, so we have the differentiation of the error with respect to d beta is nothing but equals to this 1 by t integration of 0 to T to beta b square t plus gamma times of b of t of t minus a of t and b of t into d t equals to 0. So, now you see here, we have the two main parameters b of t and c of t can be chosen like which are mutually orthogonal. And the second term of this feature above term can be straightaway taken away this b of t and c of t.

(Refer Slide Time: 20:45)

Cont....

$$\beta = \frac{\int_0^T a(t)b(t)dt}{\int_0^T b^2(t)dt}$$

i.e., the same expression as in equation 5. If we minimize the error with respect to β we obtain

$$\frac{\partial \mathcal{E}}{\partial \beta} = \frac{1}{T} \int_0^T 2(\beta b(t)c(t) - a(t)c(t))dt = 0$$

If $b(t)$ and $c(t)$ are orthogonal, we can solve for β in the same way as before,

So, when we are doing this we have the beta is nothing but equals to this integration 0 to T a (t) b (t) d t divided by 0 to T b square t into d t that means, we have the same equation which we derived when the c of t was not added. And if we minimize this error, again we have the same thing, so we can say that we can solve this equation for gamma part.

(Refer Slide Time: 21:11)

Cont.....

$$\gamma = \frac{\int_0^T a(t)c(t)dt}{\int_0^T c^2(t)dt}$$

By comparison to equations, the result obtained is the same as what would have followed from the approximation $a(t) = \gamma c(t)$. To improve upon that, we therefore only need to add another orthogonal signal and minimize the mean squared error versus $a(t)$, independently of the other signals included in the approximation.

So, we can have the gamma is nothing but equals to 0 to T a of t c of t divided by 0 to T c square t d t and when we are comparing these things, now we can say that the

approximation function this a of t into c, gamma times of c of t can be straightaway improve. And if you want to improve those things, we need to add the another orthogonal signal, and we can simply minimize the mean squared error versus a (t) which is independently on any other signal part.

(Refer Slide Time: 21:46)

Cont....

- Examples of functions that are orthogonal are $\sin(n\omega_0 t)$ and $\cos(n\omega_0 t)$. They are orthogonal for all integer values of n over the time interval $T = 2\pi / \omega_0$. In the next part, the advantage of that orthogonality property to decompose a periodic signal into sine and cosine components can be taken.
- That gives the desired decomposition of the signal into its frequency components, given by $f_n = n / T$.

So, that is very simple that we can straightaway put the orthogonal signal of the sine $n\omega_0 t$ and $\cos n\omega_0 t$ and then you see here we can do that. So, since we know that the sine and cos are orthogonal for all the integral value of the n , the sine $n\omega_0 t$ and we can say $\cos n\omega_0 t$. So, for the time interval t which is nothing but equals to $2\pi / \omega_0$, we can simply see that how the orthogonality property to decompose a periodic signal into sine and cosine component, and can be straightaway used. So, if you want to go for the desired decomposition of the signal into frequency components, say my frequency f_n is nothing but equals to n / T .

(Refer Slide Time: 22:27)

Fourier series decomposition

Assume that a signal $a(t)$ that is periodic, with period T , and which we wish to approximate with the help of sine and cosine functions,

$$a(t) = \beta_0 + \sum_{n=1}^{\infty} \beta_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} \gamma_n \sin(n\omega_0 t)$$

That is called a *Fourier series decomposition* of the signal $a(t)$. The coefficients b_n and g_n can be calculated separately, and given by equation,

Then straightaway we can simply decompose the Fourier series with respect to again the same a of t , which is the my main signal with the periodic feature and if I just want to approximate these, in terms of the sine and cosine. Now, I have this a of t is nothing but equals to β_0 the β_0 , the initial part at the basic and then you see the expansion plus summation of n equals to 1 to n infinite $\beta_n \cos n \omega_0 t$ plus summation of 1 to infinite $\gamma_n \sin n \omega_0 t$. So, this is the perfect representation of the Fourier series decomposition of the signal a of t , and these β and γ can be straightaway calculated using these equations.

(Refer Slide Time: 23:16)

Cont.....

$$\beta_0 = \frac{\int_{T/2}^{T/2} a(t) dt}{\int_{T/2}^{T/2} 1^2 dt} = \frac{1}{T} \int_{T/2}^{T/2} a(t) dt$$

$$\beta_n = \frac{\int_{T/2}^{T/2} a(t) \cos(n\omega_0 t) dt}{\int_{T/2}^{T/2} \cos^2(n\omega_0 t) dt} = \frac{2}{T} \int_{T/2}^{T/2} a(t) \cos(n\omega_0 t) dt, \quad n=1,2,3,\dots$$

$$\gamma_n = \frac{\int_{-T/2}^{T/2} a(t) \sin(n\omega_0 t) dt}{\int_{-T/2}^{T/2} \sin^2(n\omega_0 t) dt} = \frac{2}{T} \int_{-T/2}^{T/2} a(t) \sin(n\omega_0 t) dt, \quad n=1,2,3,\dots$$

The interval of integration in all above equations is $-T/2$ to $T/2$, but could just as well have been 0 to T . The coefficient b_0 represents the signal's time average.

Where the beta 0 is nothing but equals to like you know like minus T by 2 to T by 2, the entire signal in which you see we have the a (t) d (t) divided minus T by 2 to capital T by this one minus T by 2 to T by 2, this 1 square over d t. Or we can say that when we are just go for this one, we have 1 by T integration of minus T by 2 to T by 2 a of t into d t. Where these beta n and gamma n can be straightaway calculated using this cos and sine parameter, a t cos omega t and n omega 0 t d t sine n omega 0 t d t when we are doing these things we can get straightaway 2 by T of these parameters.

The interval of the integrations in all these above integration is minus T by 2 to T by 2, but could be just well as 0 to T, because you see we know that when it is being varied from minus T by 2 to T by 2. We can assume that you see this variation is of the periodic form, and we can consume all the information for describing these coefficients.

(Refer Slide Time: 24:31)

It can also be shown that corresponding sine and cosine terms can be combined into a single cosine term with a phase angle φ_n .

$$a(t) = \beta_0 + \sum_{n=1}^{\infty} \delta_n \cos(n\omega_0 t - \varphi_n)$$

where $\delta_n = \sqrt{\beta_n^2 + \gamma_n^2}$ $\varphi_n = \arctan(\gamma_n / \beta_n) + m\pi$
 $m = 0, 1, 2, \dots$

where δ_1 gives the signal's amplitude for the first tone, or fundamental tone,
 δ_2 gives the signal's amplitude for the second tone, or the first overtone,
 δ_3 gives the signal's amplitude for the third tone, or second overtone

And it is also shown that the corresponding sine and cos terms, can be combined into single this one by adopting a constant phase angle say phi n. So, we have a of t is nothing but equals to beta 0 plus summation of n equals to 1 to infinite, that we are doing delta n into cos n 0 omega T minus phi n, where delta is the amplitude which can be immediately compute with the composition of both the signals square root of beta square plus gamma square. And phi n is nothing but the phase angle which can be calculated as tan inverse of gamma n divided by beta n plus m pi.

So, the delta which is the basic term here, which we have used here is nothing but the signals amplitude for the first tone or we can say this is what my amplitude of the fundamental tone. Delta 2 can be also be used, where the second tone or delta three is the signal amplitude for the third tone. Or we can see the fundamental tone for delta 1, the first derivative in which we are saying that the first over tone is delta 2, and second over tone is delta 3 is there which is absolute showing the third tone is there.

(Refer Slide Time: 25:41)

Cont....

In order to be able to denote* each frequency component as a complex, rotating vector, which, results in simpler computations and a more compact symbolic expression, a complex Fourier series can be defined,

$$a(t) = \sum_{n=-\infty}^{\infty} \delta_n e^{in\omega_0 t}$$

where the complex coefficient δ_n can be determined from

$$\delta_n = \frac{\int_{-T/2}^{T/2} a(t)e^{-in\omega_0 t} dt}{\int_{-T/2}^{T/2} 1^2 dt} = \frac{1}{T} \int_{-T/2}^{T/2} a(t)e^{-in\omega_0 t} dt$$

So, in order to calculate the each frequency component as the complex, or we can say rotating vector, we need to see that what is the resultant feature of the simpler computations and how we can get the Fourier series out of it. So, sometimes when you are trying to describe these Fourier series in the sinusoidal feature, we have very common part a of t is nothing but equals to all the summation of these sinusoids. In terms of delta n e to the power iota n omega 0 T, where omega 0 is the natural frequency of the system.

And we can make you see you know like the system coefficient say d n is there, then we can say the delta n is nothing but equals to the minus T by 2 to T by 2 a t e to the power minus iota n omega 0 t into d t divided by this minus T by 2 to T by 2 1 square d t or else we have this one. So, when we are trying to compute delta n, we know that we need to have a of t the amplitude of this signal and there exponential features e to the power iota,

this ω_n this $n \omega_0$ $d t$ into $d t$, this is what my exponential decay is there of the entire the sinusoids are.

(Refer Slide Time: 27:01)

In equation, there are components with “negative frequencies”. That is because of a real quantity can be described with the aid of two oppositely rotating complex vectors, each of which is the complex conjugate of the other.

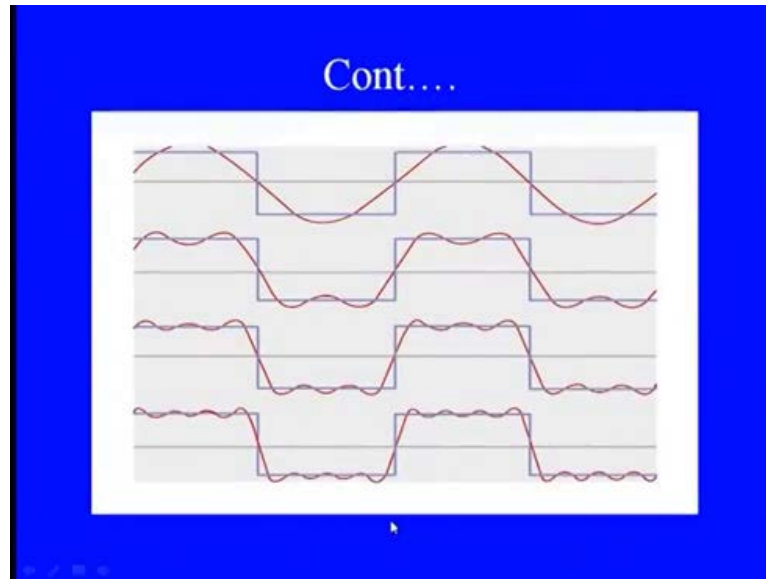
A Fourier series converges to the function (equal to the original function at points of Continuity or to the average of the two limits of discontinuity.)

$$\bar{f} = \begin{cases} \frac{1}{2} \left[\lim_{x \rightarrow x_0^-} f(x) + \lim_{x \rightarrow x_0^+} f(x) \right] & \text{for } -\pi < x_0 < \pi \\ \frac{1}{2} \left[\lim_{x \rightarrow -\pi^+} f(x) + \lim_{x \rightarrow -\pi^-} f(x) \right] & \text{for } x_0 = -\pi, \pi \end{cases}$$

So, in this the negative frequency are simply showing that, that we have the real quantity which can be described with the aid of two oppositely, we can say rotating complex vectors. And each of which is clearly showing the complex conjugate of each other, so we can say that the Fourier series is converges to the function which is equal we can say to the original function at point of continuity. Or we can say to the average to two limits of discontinuities in between that we can say \bar{f} is nothing but equals to if it is we are we are in terms of minus pi 2 pi. Then we can use the limit of \bar{f} this, it is the linear summation, and if you are saying that no it is x_0 is minus pi, then we can straightaway go for what the additional features are there in that.

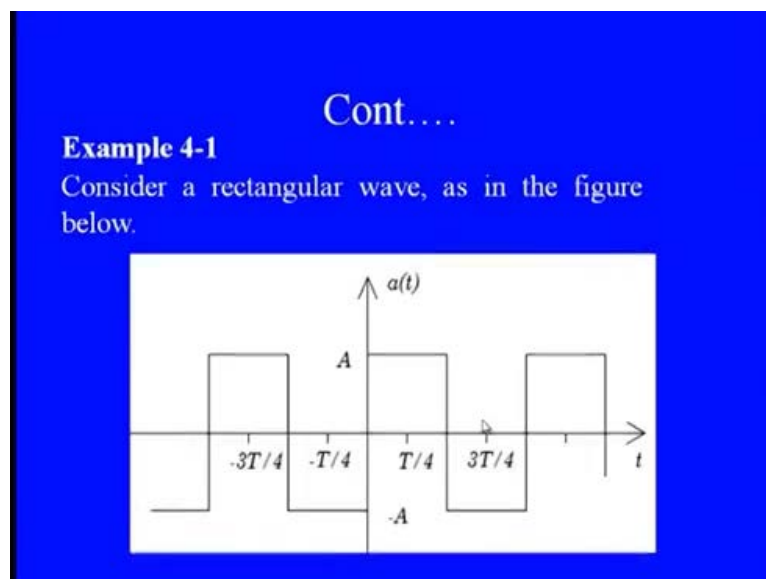
So, this is what the approximations are when you will know that, we have the squared this main information, then we are trying to make this curve fitting, so with the signal sinusoid that is lots of errors are there in that. So, now then we are adding to, in that you see here we are almost try to approximate minimize the error in that, but again we know that lots of these blocks are there which are being left out. Then we are going for more number of nodes at the individual component, so we have the three featured at this.

(Refer Slide Time: 27:49)



So, you can see that now, we are almost in the close proximity of the actual a of t , this whatever the signal is and when we are going for the number of n is becomes more there now. So, then this is absolutely what the required, the signal strength is there and how we are decomposing this part, now we are using one of the numerical in which we have a rectangular wave.

(Refer Slide Time: 28:55)



And this rectangular wave is clearly showing with a of t with the time, now you can see that at this point the total amplitude variation is a to minus a, and this total period right

from 0 to T by 2, so in between T by 4, 3 T by 4 minus T by 4 and minus 3 T by 4 is there. So, this is you see the square wave which is simply varying in that particular time period and the amplitude part; now we would like to calculate the coefficients there for this rectangular wave, so we have the beta 0.

(Refer Slide Time: 29:29)

From equation

$$\beta_0 = \frac{1}{T} \int_{-T/2}^0 -A dt + \frac{1}{T} \int_0^{T/2} A dt = -\frac{A}{T} [t]_{-T/2}^0 + \frac{A}{T} [t]_0^{T/2} = -\frac{A}{2} + \frac{A}{2} = 0$$

i.e., the time-averaged value is, as expected, equal to 0.
Equation yields

$$\beta_n = \frac{2}{T} \int_{-T/2}^0 -A \cos(n\omega_0 t) dt + \frac{2}{T} \int_0^{T/2} A \cos(n\omega_0 t) dt =$$

$$= -\frac{2A}{T} \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_{-T/2}^0 + \frac{2A}{T} \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_0^{T/2} = -\frac{A}{n\pi} \sin(n\pi) + \frac{A}{n\pi} \sin(n\pi) = 0,$$

So, as we discussed already the beta 0 which is nothing but equals to 1 by T integration of minus T by 2 to plus T by 2 was there; now here we need to go up to minus T by 2 to first 0, in which minus a is there.

(Refer Slide Time: 29:43)

Cont....

Example 4-1
Consider a rectangular wave, as in the figure below.

If you look at this, it is suppose to start from minus T by 2 to it is what it is there to when we are going up to 0, we have the minus a.

(Refer Slide Time: 29:56)

From equation

$$\beta_0 = \frac{1}{T} \int_{-T/2}^0 -A dt + \frac{1}{T} \int_0^{T/2} A dt = -\frac{A}{T} [t]_{-T/2}^0 + \frac{A}{T} [t]_0^{T/2} = -\frac{A}{2} + \frac{A}{2} = 0$$

i.e., the time-averaged value is, as expected, equal to 0.
Equation yields

$$\begin{aligned} \beta_n &= \frac{2}{T} \int_{-T/2}^0 -A \cos(n\omega t) dt + \frac{2}{T} \int_0^{T/2} A \cos(n\omega t) dt = \\ &= -\frac{2A}{T} \left[\frac{\sin(n\omega t)}{n\omega} \right]_{-T/2}^0 + \frac{2A}{T} \left[\frac{\sin(n\omega t)}{n\omega} \right]_0^{T/2} = -\frac{A}{n\pi} \sin(n\pi) + \frac{A}{n\pi} \sin(n\pi) = 0, \end{aligned}$$

So, minus A into d t plus 1 by t 0 to T by 2 means the upper side A into d t, so when we are now trying to do this integration feature, we have minus A by T into whatever the T function is there, it is minus T by 2 to 0 first, and second is also plus A by T, we can simply apply this 0 to T by 2. So, when we are doing this our beta 0 is 0 that means, you see the time average value of the signal which is obviously, it is a squared value, so time average value is equals to 0. So, then we can go with the beta n and beta n is nothing but equals to minus 2 by T integration of minus T by 2 to 0 minus A cos n omega 0 T into d t plus 2 by T. Now, 0 to T by 2 integration a cos n omega 0 n t into d t, and when we are now making equal to this, so now we have the minus A divided by the n pi into sine n pi plus A by n pi sine of n pi.

(Refer Slide Time: 31:11)

and equation gives

$$\gamma_n = \frac{2}{T} \int_{-T/2}^0 -A \sin(n\omega_0 t) dt + \frac{2}{T} \int_0^{T/2} A \sin(n\omega_0 t) dt$$

$$\gamma_n = \frac{2A}{T} \left[\frac{\cos(n\omega_0 t)}{n\omega_0} \right]_{-T/2}^0 + \frac{2A}{T} \left[-\frac{\cos(n\omega_0 t)}{n\omega_0} \right]_0^{T/2} = \frac{2A}{n\pi} (1 - \cos(n\pi)) = \begin{cases} \frac{4A}{n\pi} & n=1,3,5,\dots \\ 0 & n=0,2,4,\dots \end{cases}$$

So, again it is the beta 0 which is even making on that is also 0, so when we are now going to the additive this part gamma n, the gamma n is nothing but equals to 2 by T minus T by 2 to 0. Again same minus A sine omega n 0 t d t plus 2 by T integral 0 to T by 2 A sine n omega 0 t into d t, and when we are making this now, we have the gamma is nothing but equals to see the two main value. When we have the number of odd waves 1, 3, 5 and all, we have a clear amplitude 4 A by n pi, and when we are using the odd terms 0 to 4, the even terms 0 to 4 it is 0 and when we have the odd terms, it was four a by n pi.

(Refer Slide Time: 31:59)

Cont.....

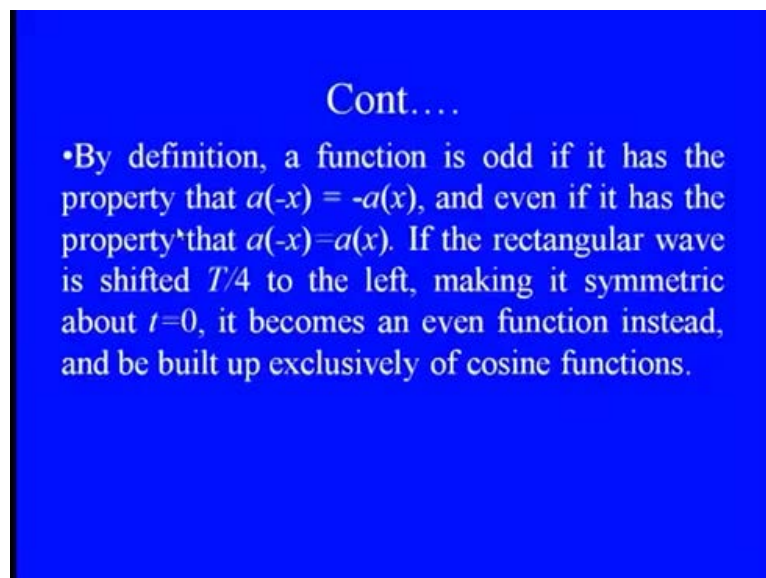
Thus, the Fourier series can be expressed as

$$a(t) = \frac{4A}{\pi} \left[\sin(\omega_0 t) + \frac{1}{3} \sin(3\omega_0 t) + \frac{1}{5} \sin(5\omega_0 t) + \dots \right]$$

It only consists of the odd sine components. That only sine components appear in the Fourier series is because the rectangular wave is an odd function; odd functions can be decomposed into sine components, since the sine function itself is odd.

So, when we are decomposing this now, it is very clear that it only consisting the odd sine wave, so when we are try to now make the Fourier series, through this decomposition. We have a of t is nothing but equals to the amplitude $4A$ by π into sine of $\omega_0 t$, n is now 1, so $\omega_0 t$ plus 1/3rd of sine $3\omega_0 t$ plus 1/5th of sine $5\omega_0 t$. So, that means, the odd terms are clearly showing their presence and the only sine component which appeared in the Fourier series, is because of the rectangular wave is an odd function. And odd functions can be decomposes into sine components, since the sine function is itself is odd, so that this is what the clear expression of the basics of these Fourier series are.

(Refer Slide Time: 32:57)



So, by definition we can say that a function is odd, if it has the property a of minus x is minus a of x that is what we already done in our basics that, if we are talking about the odd property, certainly the dependency of a of any function minus x is certainly beat this one. And even if it has the property of this a of minus x equals to a of x, then also we can say that, these things are being clearly termed out. So, if the rectangular wave is shifted to $T/4$ to the left, making it symmetric about the t equals to 0, it becomes an even function instead and can be built up exclusively for cosine term. So, you see this is what a clear representation is there of the sine and cosine with the relation of the $T/2$ to $T/4$.

(Refer Slide Time: 33:51)

Cont.....

- The raw analog signal is typically brought into a portable, digital instrument that processes it for a variety of user functions. Depending on user requirements for analysis and the native units of the raw signal, it can either be processed directly or routed to mathematical integrators for conversion to other units of vibration measurement.
- Depending on the frequency of interest, the signal may be conditioned through a series of high-pass and low-pass filters.

So, the raw analogue signal is typically brought into the portable digital instrument, and process it for this variety of the user functions. And depending on the frequency of the interest, the signal may be conditioned through the series of this high pass and low pass filters, that is what we discussed. And then in this decomposition we need to check it out that what is the time frame, in which the both the components sine and cosine can be included, and wrote the orthogonality conditions are there, when we are doing the decomposition of the Fourier series.

So, in this lecture, we simply decomposes whatever the time domain informations are there which we are measuring using all these transducers, whether it is like in terms of the displacement, velocity and acceleration. And then we will try to convert to convert this into Fourier series and as I told you one of the unique feature in that, if it is the stationary wave terms are there, in which a clear wave form it is in the form of we can say the periodic wave, simple harmonic wave. It can be easily decomposes, because it can be divided into sinusoidal and cosine feature, and the orthogonal feature, and that can be added according to the Fourier series component the sinusoids.

But, if it is the non stationary part is there, then we need to take that, what is the nature of this non stationary feature of the signal, where the abrupt we say, if bump is coming or any abrupt changes are there in that. So, if that can be analyzed using the short term Fourier transformation STFT, Hilbert Hung or we can go for the wavelet transformation

by taking the small part and analyze for the non stationary part there itself. Using some statistical features of this entire wave part, where it is being the non stationary in the nature. So, in this lecture that all the information was discussed, in the next lecture now we are going to discuss about the filters. Because when we are talking about the signal processing, the filter part is very important that, how we can adopt the low pass, band pass or means various types of filters. Because, this is also one of the important part, prior to go for signal processing how we can adopt those filters, how we can design those filters there itself.

Thank you.