Engineering Economic Analysis
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Lecture 14
Comparison of Alternatives: Capitalised Equivalent Amount, Capital Recovery with Return

Welcome to the lecture on comparison of alternatives. We will discuss about different methods, we have anyway so far discussed about the methods like present worth criterion, future worth criterion and annual equivalent criterion. The fourth one which is important is internal rate of return.

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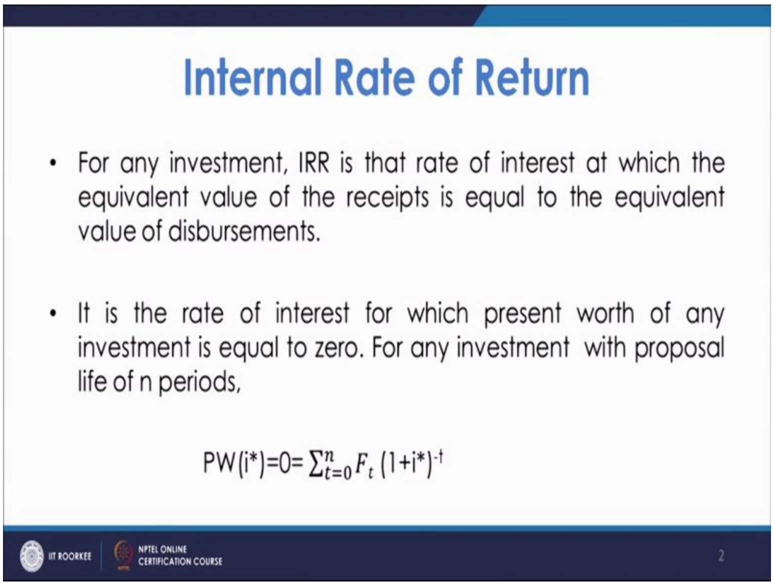

So we have already discussed about it that the internal rate of return is that rate of interest at which equivalent value of receipts is equal to the equivalent value of disbursements. So basically at this rate of interest your present worth value will be equal to 0 for any investment. So this is how the internal rate of return is calculated. So what we see is if you have Ft as the cash flow at any time t.

Then we know that the present worth expression is nothing but summation of t equal to 0 to n Ft into 1 + I raised to the power - t. So the value of I that is I star this I star for this value basically if we calculate the present worth value it is 0. So this I star for which present worth is zero, it is known as the internal rate of return.

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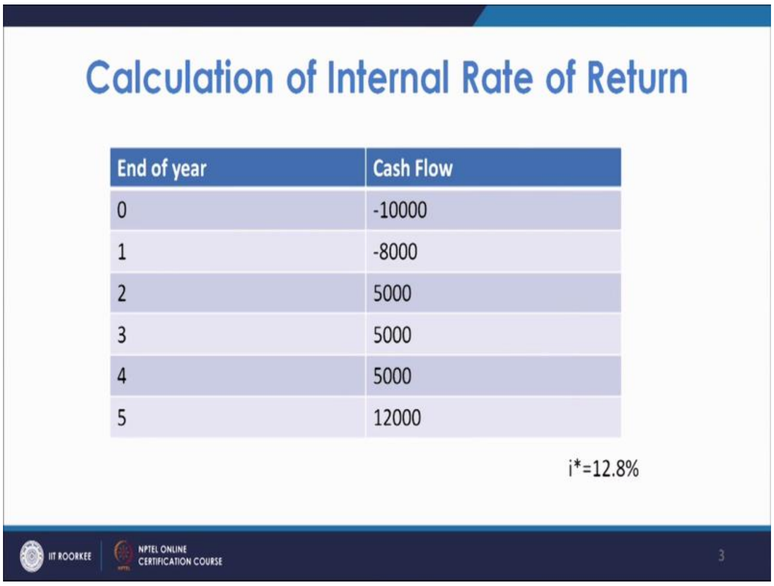

Let us see by an example this is a cash flow diagram and which tells that in the year end of 0 you have a disbursement of Rs. 10,000. In the year end one you have disbursement of Rs. 8000, then from 2 to 4 that is receipt of Rs. 5000 and there is receipt of Rs. 12,000 in the year end 5. Now we have to calculate what is the internal rate of return for this particular investment. Now let us see how to calculate the internal rate of return for this particular cash flow.

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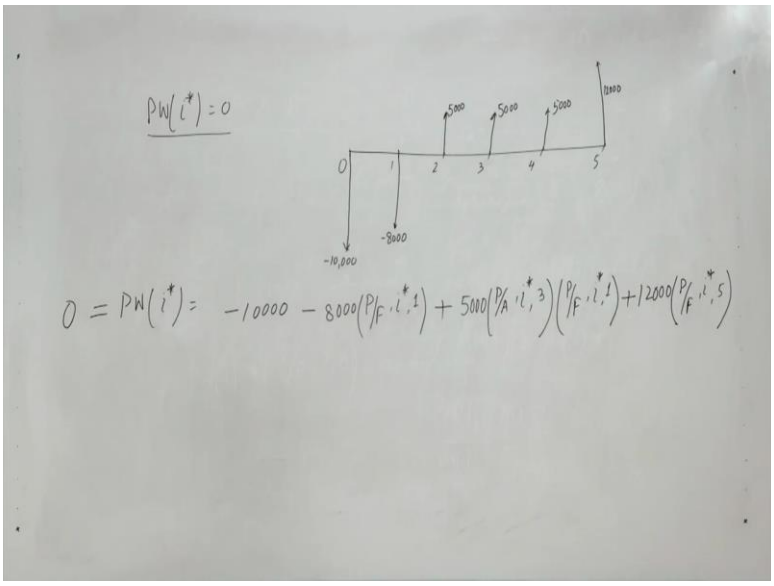

As we have discussed that PWi star should be equal to 0. So if we try to draw the cash flow diagram for this series, we have 1, 2, 3, 4 and 5, so in the 0 year we have disbursement of -

10,000, then we have disbursement of 8000 in the first year end, we have 5000 each during the second, third and fourth year and we have this receipt of 12,000 in the fifth year.

Now for this as the condition tells that the present worth investment should be 0 for that particular interest rate and that particular interest rate will be internal rate of return. So if we try to find the present worth for the rate of return that as I star, this will be nothing but -10,000 because we are basically finding the equivalent value of this at this time itself. Then we will have 8000 multiplied by P by F I star 1.

Then we have 5000 P by A 1 star 3 which will be basically defined at this particular point and this will be for the multiplied at this the factor P by F I 1. So P by F I star 1 because once we have got the equivalent value of these three cash flows using P by A diagram interest factor, this will be defined at this particular time. So from here again we have to multiply with P by F I 1 and that will give us the equivalent value at this time.

Then we have 12,000 multiplied by P by F I star 5. So basically the internal rate of return will be that percentage of interest for which this value should be zero. So we have to equate it to 0. So this is solved by trial and error methods. Now using the trial and error methods, we will find PWi. If we take using trial and error method, so let us start with the rate of interest as zero.

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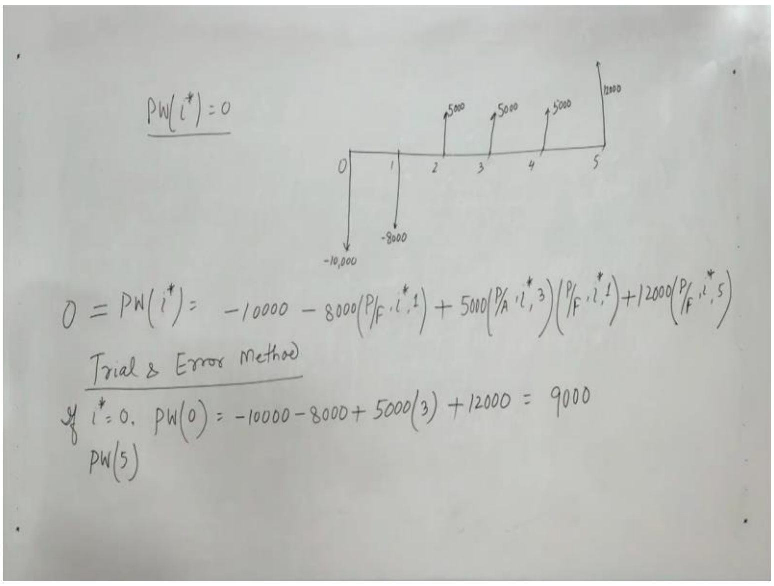

So if I star is taken as zero, in that case present worth for 0% interest will be nothing but -10,000, - 8000 because this will be this is the 0% interest so this will all be 1 + all these 3. In that case we will have 0% interest, we will have these values. So five thousand P by A 0 3 will be 3 and this will be 1 again + this will be again 1 so 12,000. So 12 + 15 27,000 - 18,000 and it is equal to 9,000.

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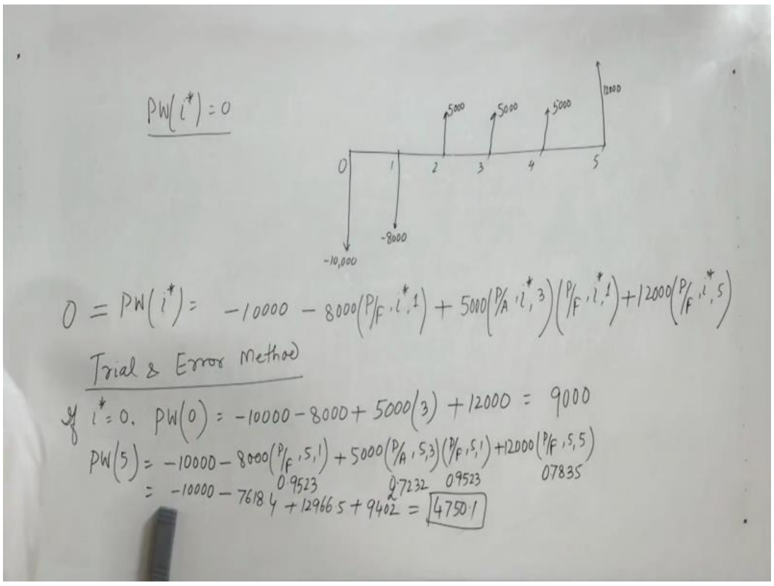

As we go on increasing the value of I this PWi goes on increasing. Let us see we can check the value of present worth at 5% interest. When I start is 5, for that what we **we** have - 10,000 - 8000 P by F 5 1 + 5000 P by A 5 3 multiplied by P by F 5 1 + 12,000 P by F 5 5. So we need to have these interest factors and we can get these factors from the interest table of 5%.

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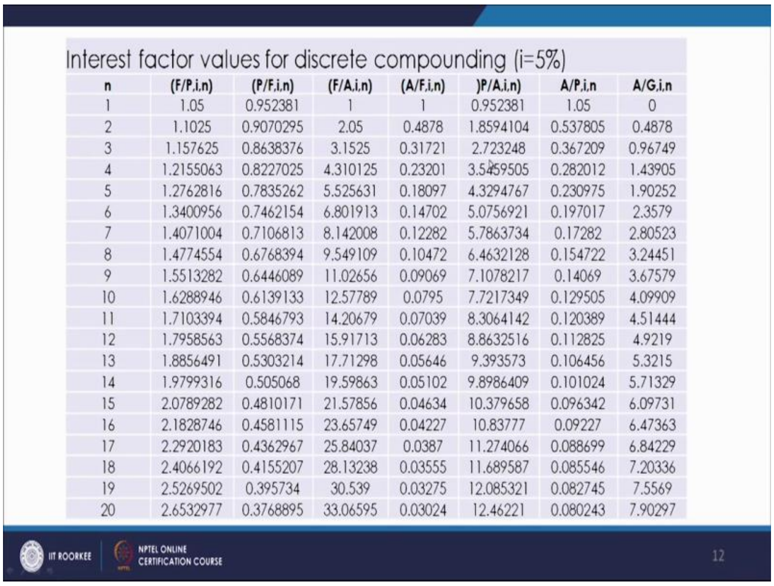

Interest factor values for discrete compounding (i=5%)

n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	)P/A,i,n)	A/P,i,n	A/G,i,n
1	1.05	0.952381	1	1	0.952381	1.05	0
2	1.1025	0.9070295	2.05	0.4878	1.8594104	0.537805	0.4878
3	1.157625	0.8638376	3.1525	0.31721	2.723248	0.367209	0.96749
4	1.2155063	0.8227025	4.310125	0.23201	3.5459505	0.282012	1.43905
5	1.2762816	0.7835262	5.525631	0.18097	4.3294767	0.230975	1.90252
6	1.3400956	0.7462154	6.801913	0.14702	5.0756921	0.197017	2.3579
7	1.4071004	0.7106813	8.142008	0.12282	5.7863734	0.17282	2.80523
8	1.4774554	0.6768394	9.549109	0.10472	6.4632128	0.154722	3.24451
9	1.5513282	0.6446089	11.02656	0.09069	7.1078217	0.14069	3.67579
10	1.6288946	0.6139133	12.57789	0.0795	7.7217349	0.129505	4.09909
11	1.7103394	0.5846793	14.20679	0.07039	8.3064142	0.120389	4.51444
12	1.7958563	0.5568374	15.91713	0.06283	8.8632516	0.112825	4.9219
13	1.8856491	0.5303214	17.71298	0.05646	9.393573	0.106456	5.3215
14	1.9799316	0.505068	19.59863	0.05102	9.8986409	0.101024	5.71329
15	2.0789282	0.4810171	21.57856	0.04634	10.379658	0.096342	6.09731
16	2.1828746	0.4581115	23.65749	0.04227	10.83777	0.09227	6.47363
17	2.2920183	0.4362967	25.84037	0.0387	11.274066	0.088699	6.84229
18	2.4066192	0.4155207	28.13238	0.03555	11.689587	0.085546	7.20336
19	2.5269502	0.395734	30.539	0.03275	12.085321	0.082745	7.5569
20	2.6532977	0.3768895	33.06595	0.03024	12.46221	0.080243	7.90297

So this is the 5% interest table P by F 5 1 is .9523. Excuse me, pardon me this is 0.9523, then P by A 5 3 is 2.7232, P by F 5 1 .9523, P by F 5 5 so P by F 5 5 will be here .7835. So once we do this computation we will get - 10,000 - .9523 multiplied by 8000 so this is 7618.4 + 5,000 multiplied by 2.7232 multiplied by .9523 this comes out to be 12,966.5 + 12,000 multiplied by .7835 so it comes out to be 9402. So if we add these all, it comes out to be 4750.1.

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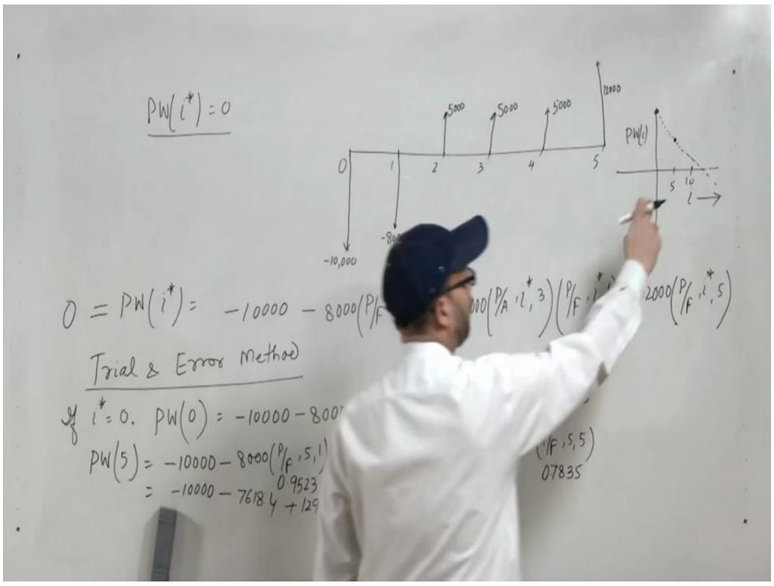

So what we see is for the interest rate, when it is taken as 5% the present worth comes out to be 4750.1. So if we draw basically a curve of present worth value, we have seen the interest rate values, at zero, it was coming as 9000 and then at 5%, it has come to a decreased value. Now this value will decrease continuously and ultimately it has to cross somewhere and this is the point where it crosses this x axis.

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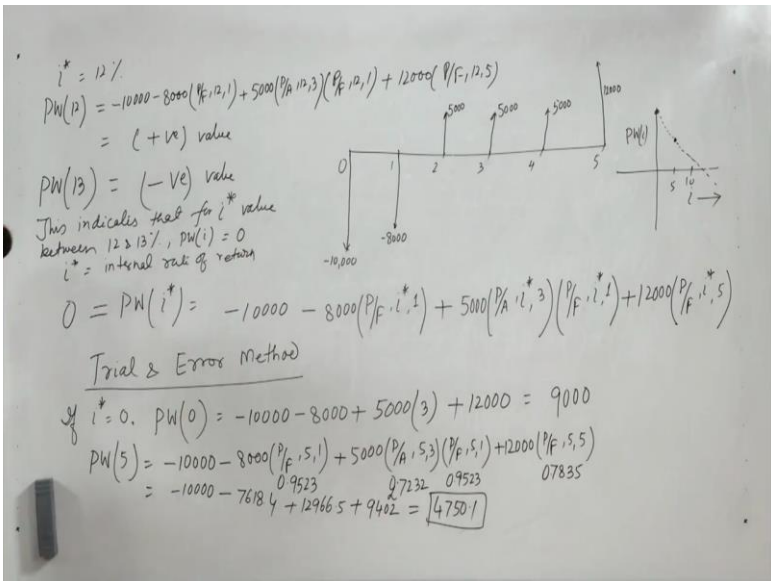

This point tells the interest rate value for which the present worth value will be zero and this will be the internal rate of return. We can further check by taking another interest factor. So if we take I star as 12%, what we get? We get - 10,000 - 8000 P by F 12 1 + 5000 P by A 12 3 into P by F 12 1 + 12,000 P by F 12 5. So basically this term will be a positive value. This we can complete when we solve the problem in the next lecture.

Incidentally when we calculate for 13% interest, this eventually comes out to be a negative value. This indicates that **that** for I star value between 12 and 13% PWi will be equal to 0 and this I star will be the internal rate of return. So basically we will solve it in our next tutorial class when we will solve this finally and we will see how we can get this I star by using the interpolation methods.

So basically once we have solved these, it has been found that the value of I star is coming out to be 12.8%. So this is how the internal rate of return is calculated.

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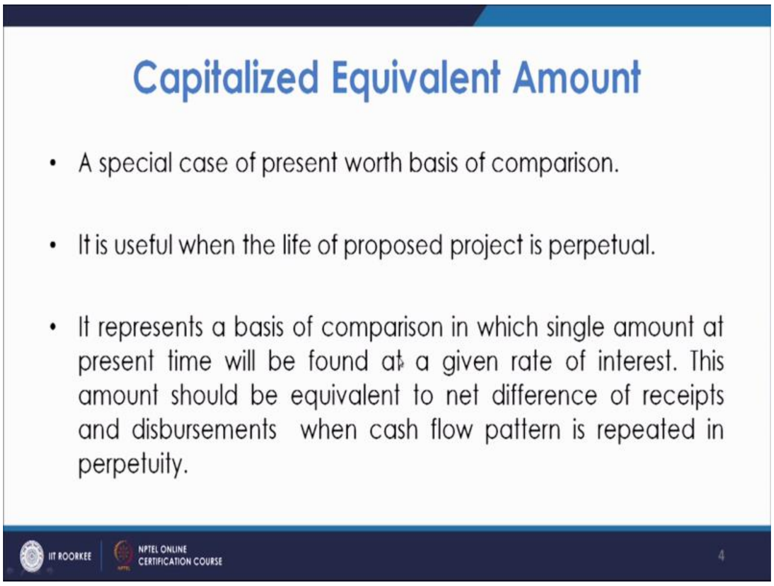

Now next method of comparison is capitalised equivalent amount. Many a times it has been seen that basically the type of cash flow is perpetual means it goes for a very long period. So sometimes we need to have a deposit so that certain amount you can get for the whole of the life or for a very large duration. Means in these cases the n is tending towards infinity. So basically it is a special case of present worth basis of comparison.

It is useful when the life of proposed project is perpetual and it represents a basis of comparison in which single amount at present time will be found. So basically your job is to find the present worth present time at a given rate of interest so that this amount should be equivalent to net difference of receipts and disbursements when cash flow pattern is repeated in perpetuity. So basically you see that certain cash flow diagram is going for ever or continuously.

For an example, when we construct a bridge, the bridge is to be maintained in course of time. Now in that you need a fund from where you can get certain amount for the maintenance purpose of this bridge and that may be required for years continuously, maybe 50 or 100 or so. So basically you need a fund and that is known as a capitalised amount or capitalised equivalent amount.

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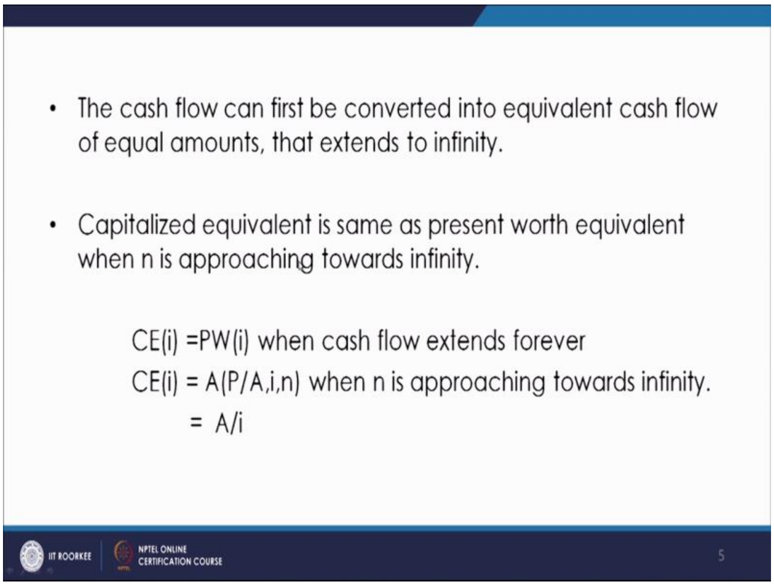

So it means the cash flow in this case is basically converted into The Equivalent cash flow of equal amounts and basically this equal amount has to go for infinity. So it is same as present worth equivalent when n is approaching towards infinity. So basically it is the present worth amount for which you are getting certain equal annual payment but n is moving towards infinity.

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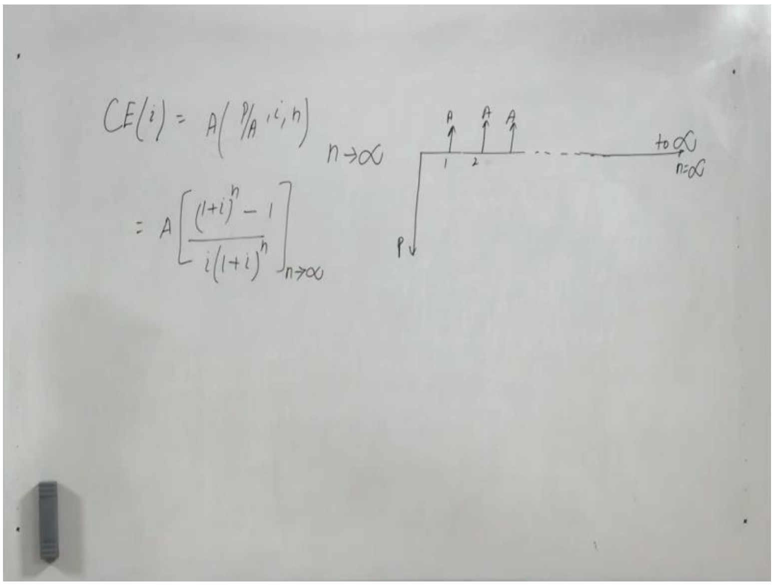

That is why it is written capitalised equivalent I at interest rate I is nothing but it is the present worth value when cash flow extends forever. So n is moving towards infinity, so you can write as CEi and A times P by A I n. Now what you see is, capitalised equivalent at rate of interest I, it is nothing but the present worth. You need A P which will give you certain A forever. So n is infinity.

So you need this present amount the equivalent present worth when n is moving towards infinity. So once you know that A, A can be multiplied with P by A I n and n is tending towards infinity. You need certain equal annual payment that has to continue forever, what we see here. So this A, its present worth equivalent will be A times P by A I n and n is moving towards or approaching towards infinity.

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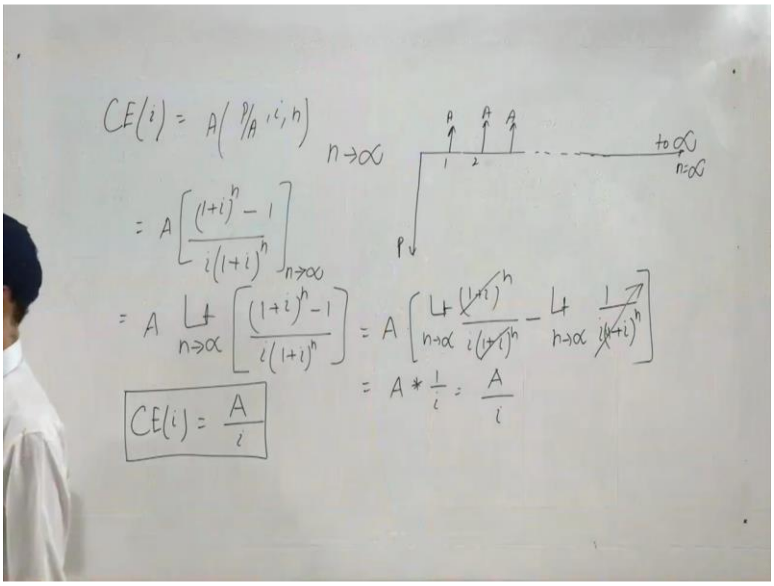

You can write A into as we know P by A I n so it is nothing but and this is the case when an is approaching towards infinity. So we can write as A times limit n tends to infinity 1 + I n - 1 upon I into 1 + I n. So it can further be written as A times limit n tends to infinity 1 + I n upon I into 1 + I n - limit n tends to infinity 1 upon I into 1 + I n. So as n tends to infinity this will vanish and hear this and this term will be cancel so you get 1 by I.

So A times 1 by I that is A by i. So capitalised equivalent amount is nothing but the annual equivalent amount by I. So this is how the capitalised equivalent amount is calculated. Now we will look towards meaning, it is nothing but the investment or an initial deposit which will earn certain interest and this interest amount will be given to you whatever. So A is nothing but P times i.

So basically it is that amount which is the interest earned on certain deposit and you are likely to get it forever, so it is known as capitalised equivalent amount. Next is capital recovery with return, now capital recovery with return it is under those cases where you have two types of expenditures, capital expenditures. One is the initial cost of the asset and the another type of cost is the return on this asset when you are disposing it at the end of its life.

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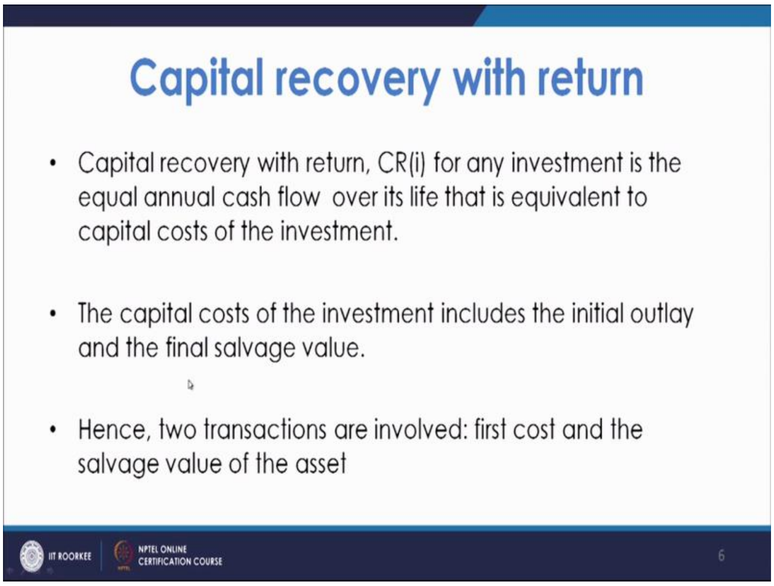

So at the end of its life what you are getting is known as the salvage value. So basically this is known as Cri, so it is represented by the term CRi. At particular I rate of interest for any investment is the equal annual cash flow over its life that is equivalent to capital costs of the investment. So capital cost involves the initial cost and also the return you get after selling the final asset and you get the salvage value of it.

So capital cost includes initial outlay and the final salvage value. So you have 2 transactions involve, first cost and the salvage value of the asset. So let us see how the cash flow diagram is represented in the case of capital recovery with return.

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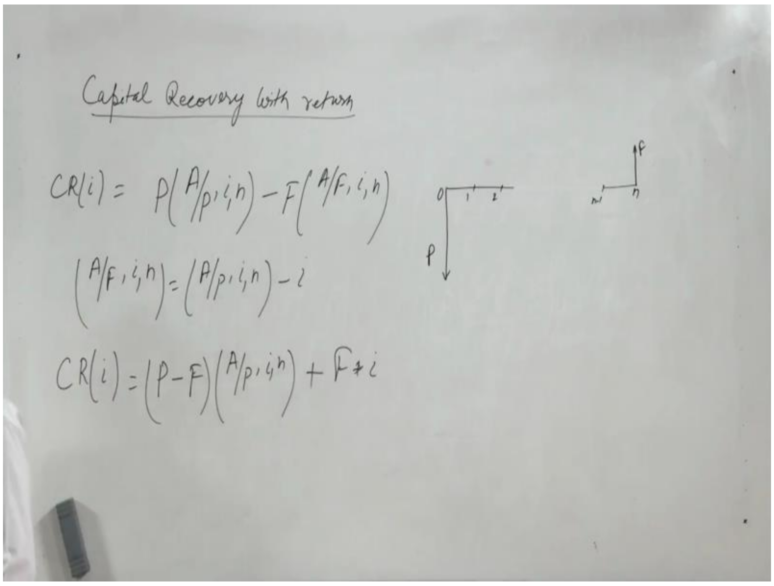

So what we see is in this case your, in this case the cash flow diagram is this, you are investing certain amount now and at the end this is your salvage value. So initially you have put in this amount and this is the final salvage value which you get it while selling all the property which you had, this is known as the final salvage value. So this is basically the equivalent annual amount.

So capital recovery with return is the basically the equal annual payment which is corresponding to the equivalent of this cash flow. So you can say for this the equivalent annual cash flow will be P times A by P I n because once you multiply this P with factor A by P I n, it will tell you certain A which is basically a disbursement.

Then this F also multiplied with A by F I n will give you certain annual equivalent value, so this is to be subtracted with A by F I n. Now as we see that A by F I n is nothing but A by P I n - I, so it comes out as P - F A by P I n + F times I. So this way we can calculate the capital recovery with return where you initial outlay as well as the final salvage value and this formula is used for finding the capital recovery with return.

So this is how these equivalent values are calculated, we will discuss about the problems in the next lecture to come. Thank you.