Engineering Economic Analysis
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Lecture 07
Problem Solving on Discrete Compounding, Discrete Payment

Welcome to the lecture on interest formulas for uniform gradient series. So we have so far discussed about the single payment series and equal annual payment series.

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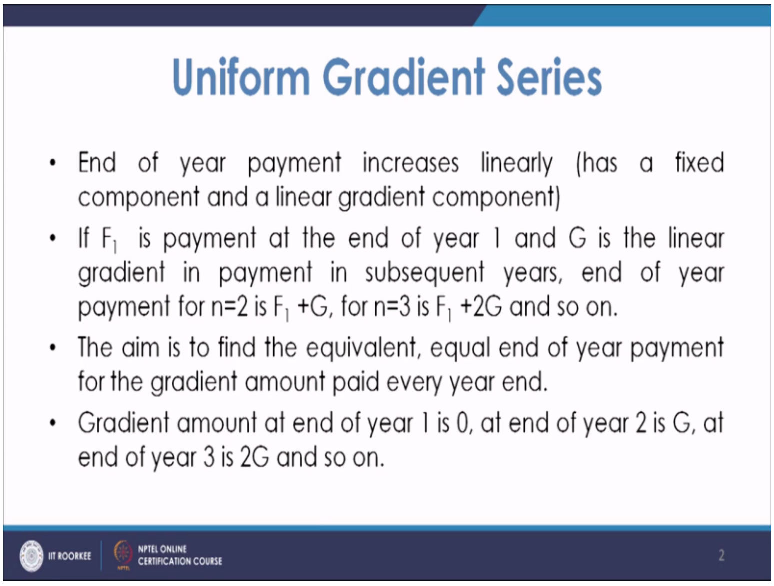

Now this is series where there is gradient but this gradient is linear so this is known as Uniform Gradient Series or Linear Gradient Series. In this series basically end of year payment is increasing linearly means the first year and if you pay F1, the second year end F1 will be increased by a constant value that is suppose G. So this G is the gradient, this G is uniform every year.

So in the second year if it is F1 + G, the third year it becomes F1 + 2G and so on. So in the nth year end we are basically paying F1 + n - 1 into G. Now in this case we have to find what is the equivalent annual series. So the aim is to find the equivalent annual end of yeur payment for the gradient amount paid every year end. So we can see by referring to the table about the gradient series.

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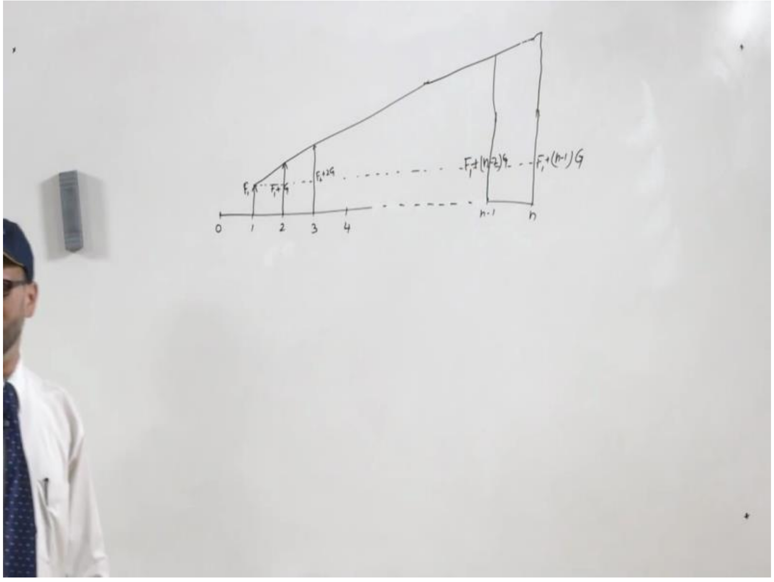

As we know the cash flow diagrams for such series is like this. We have 0, 1, 2, 3, 4, n - 1 and n. Now in this as we have seen, we pay F1 during the first year end and this increases to F1 + G in the second year. So this is F1 and this is F1 + G. So this G is basically the gradient and this gradient will go on increasing.

So in the third year this amount will be F1 + 2G. Hence in the n - 1th this amount will be n - 2 into G and in the nth year it becomes F1 + n - 1 into G. So these type of series are known as uniform gradient series. Now as we see in this series that this F1 is every time for every year it is constant. What is varying is G every year. So here it is G, here it is 2G and this way it is going.

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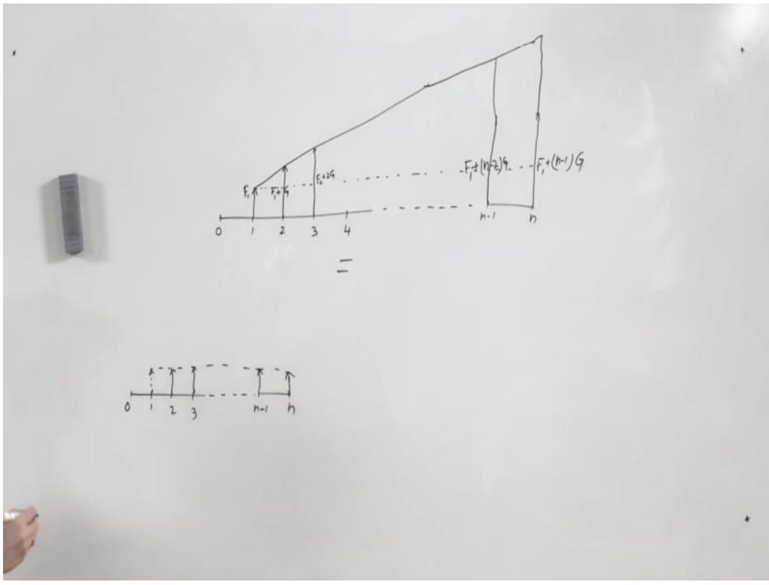

So basically you can represent this cash flow diagram as sum of two cash flow diagrams. So in one cash flow diagram this can be represented as two cash flow diagrams where in one of the cash flow diagram you have equal annual payment series that is equal to F1. So this all amount is F1 + another is its gradient part. So it has two part, one is the fixed amount part, another part is the gradient part.

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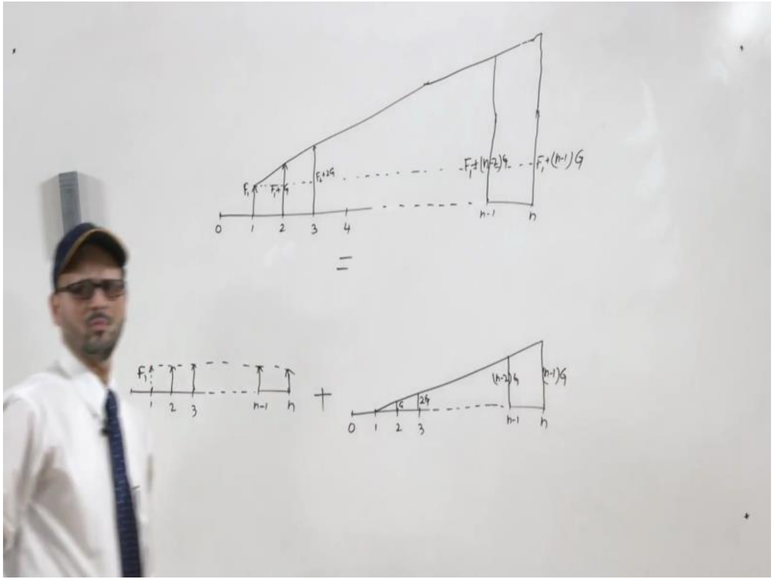

So this part will have the cash flow diagram like this, so in this as we see, the gradient value is 0 in the first year. It starts only from the second year. So in the second year its value is G and then it goes on increasing. So this value from first year, it moves. Here in the second year it is G, in the third year it is to G, in the n - 1th year, it is n - 2 into G and in the nth year it is n - 1 into G.

We have to find an equivalent cash flow diagram which should tell as the annual equivalent value for this gradient series, so that is what our aim is.

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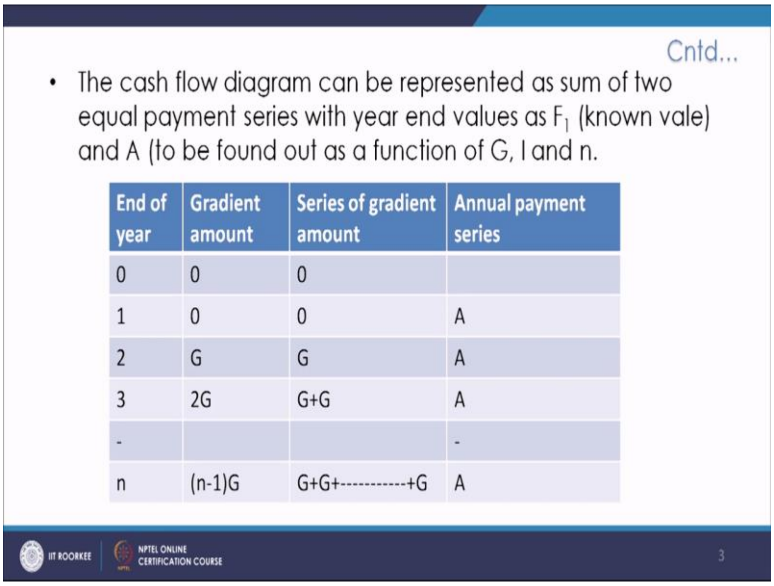

Now as we see that this is the end of year 0, 1, 2 to end. This is the gradient amount as we have seen, this gradient amount is in the first year it is 0. In the second year this is G, in the third year this is 2G, so this way in the nth year it is n - 1 into G. Now this has to be represented by one annual payment series. So basically this series should be equivalent to this series where a series of such type will have the equal year end payment of amount A.

So if you try to find the equivalent, basically summation of all this should be summation of this and this way we can find an co-relationship between A and G. So what we do for this? For this, so now let us see we will equate the total amount. Total amount at a future time will be summation of all this and its equivalent value at this time and this will be nothing but A times the factor F by A I n.

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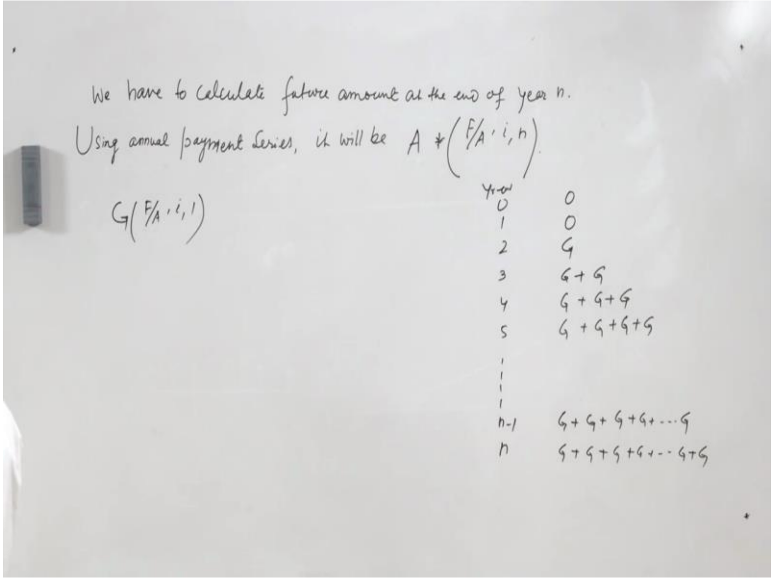

So we have to calculate the future amount at the end of year n. Now what will be the amount at the end of year n? Using this annual payment series, it will be, we have already discussed that when the year end equal amount is multiplied by F by AIM, so this will be A times the factor F by AIM. This amount should be equal to the summation of all this G.

So now the value of this G if you can look at, this is nothing but this G will have a value and its component or its contribution towards F will be G times F by A i 1. Basically what we see here the series goes like this, 0 0 G G + G G + G + G G + G + G + G. So basically we have seen this table, year end 0, 1, 2. And in the nth period it will be G + G + G + G + G + G.

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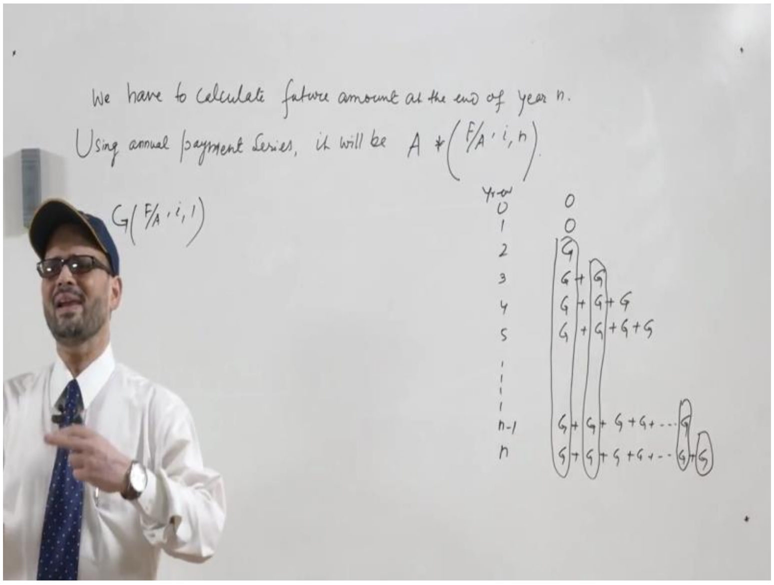

Now as we see here, so we can see that this is your future time, in your future time one is this G, in this, this will be coming, so this way. And here this component will come. So like that you have to find the equivalent value of all these quantities at the year end of n. Now this G remains a G itself, so this G is multiplied by F by A i 1. F by A i 1 is 1 itself, so this G remains as G + this 2G.

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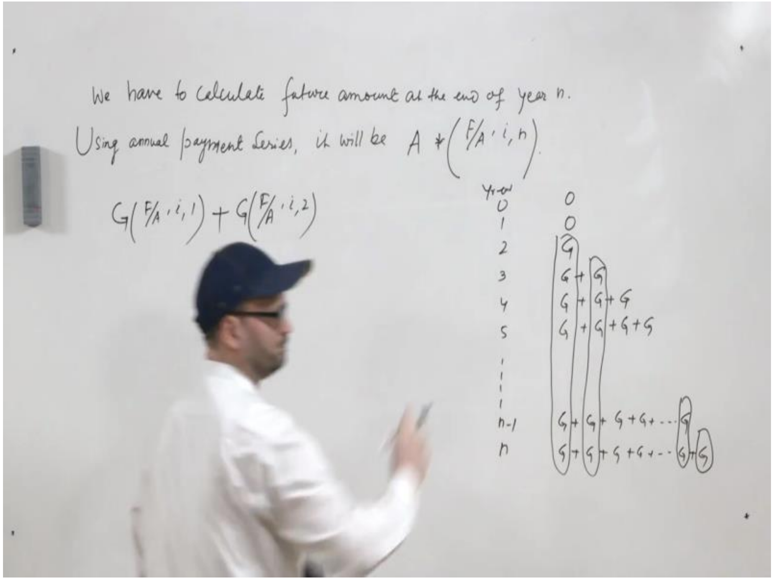

G + G is nothing but its component at the future will be G times multiplied by F by A i 2. So this way it will move and it will come up to here. So it will go up to G times F by AIM - 1.

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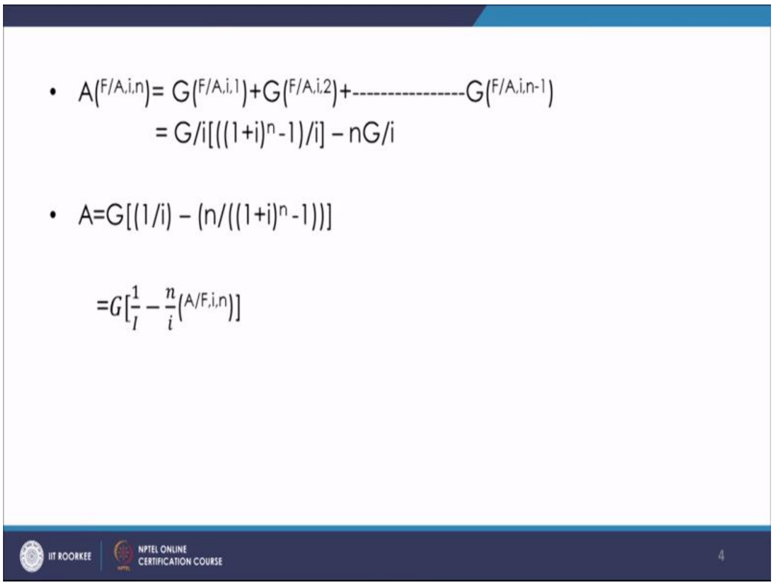

So what we have seen, this is the expression what we get A into F by A i n should be equal to G times F by A i 1 that is single G. So you can write here, this is nothing but G times F by A i 1. This G will have component G times F by A i 2 like that, So this is moving and it is going up to G times F by A i n - 1.

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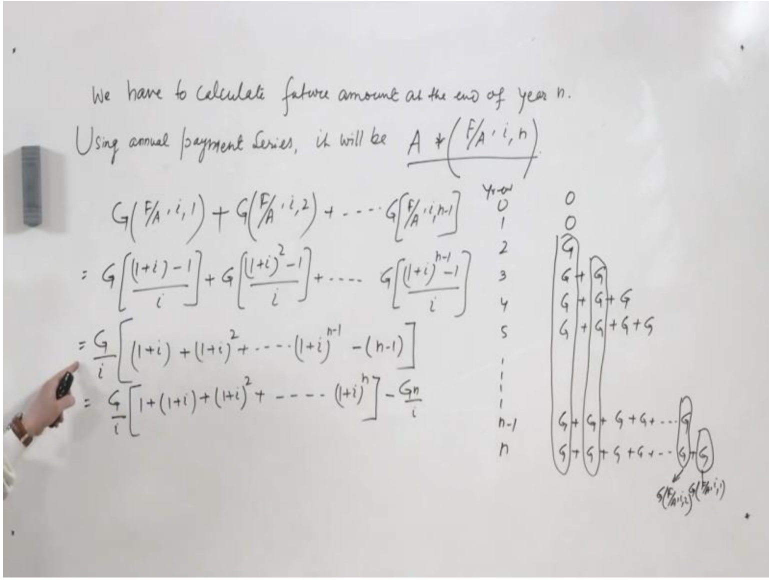

Now you will find the expression and you have to equate them. So as we know F by A i 1 F by A i 1 is nothing but G times 1 + i - 1 by i + G times 1 + i raised to the power 2 - 1 by i. So this will go on G times 1 + i raised to the power n - 1 - 1 by i. This amount basically the expression which we get that will be equated to this that we will do later.

Now let us find the expression for this so this will be G by i we can take out 1 + i + 1 + i 2 + 1 + i upto n - 1. We have n - 1 terms at n - 1 terms this 1 comes, so this comes as - n - 1. After this you can write this as G by i this 1 will come here, 1 + 1 + i + 1 + i 2 up to 1 + i raised to the power n. This will be one term - G n by i.

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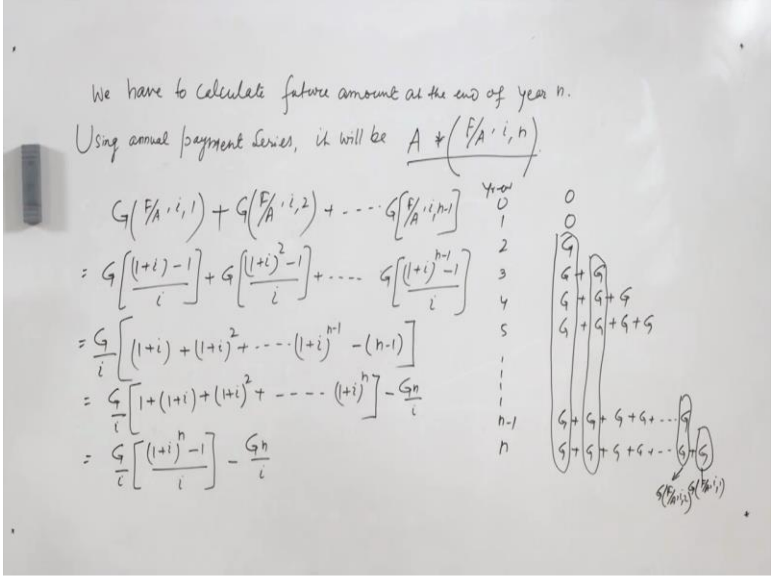

So we have separated now this term with n outside and the term which is inside the bracket this is a geometric progression term and summation of the GP series term will be used. So this will be written as G by i 1 + i you have n terms n - 1 upon i. That is in the denominator we have 1 + i - 1 so which is i - G n by i. Now we have to find the expression for A so this term will be equated to this term.

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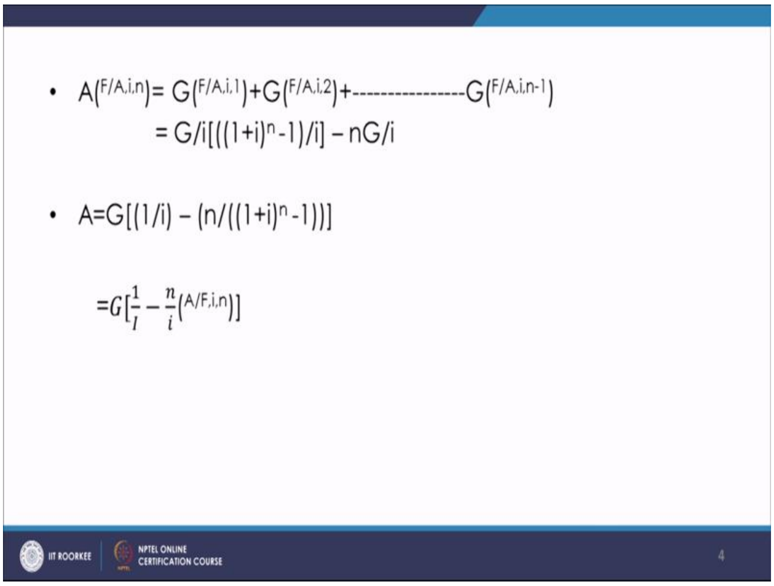

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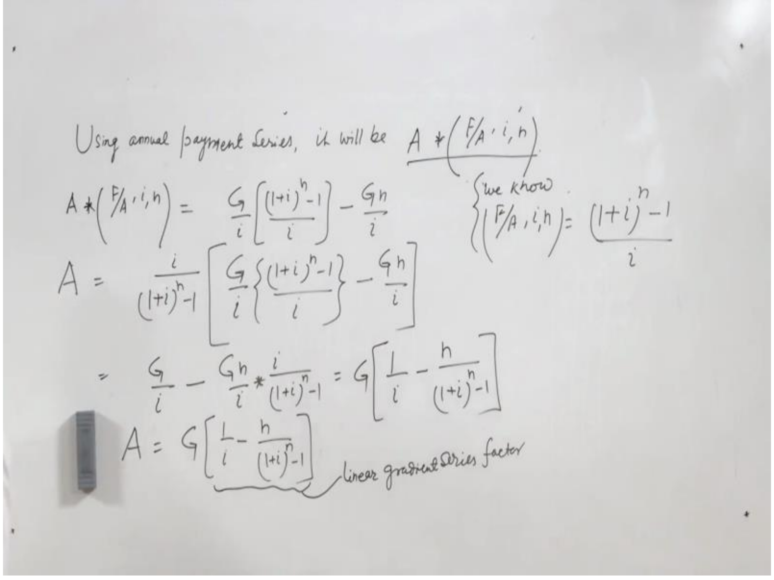

So what we will calculate now, A into this factor F by A i n will be equal to G by i 1 + i n - 1 upon i - G n by i. As we know we know F by A i n this factor is nothing but 1 + i raised to the power n - 1 upon i. So we will put this factor here in that case A will be equal to 1 by 1 + i n - 1. In place of this 1 we will have i the reciprocal of this term multiplied by G by i into 1 + i n - 1 by i - G n by i.

Once you get this now you have to again multiply these terms one by one so you get G by i because this and this cancels - G n by i times i by 1 + i raised to the power n – 1. So the expression is if you look at you can write this as G multiplied by 1 by i - n by 1 + i raised to the power n – 1. So finally we are getting A equal to G into 1 by i - n by 1 + i raised to the power n – 1.

This is the factor which when multiplied with G the constant gradient amount will give you the equal year end amount equivalent value. So this is a factor which is known as linear gradient series factor. Now this amount is basically corresponding to the gradient amount. Now this gradient value can be either positive or negative. So once you get this gradient value you have another fixed component that is F1.

So ultimately, your net equal annual equivalent value will be F1 + this A which you obtained through this value because these are the known components with you, you know the G, you know i and n. So from here you can calculate the net value of annual equivalent payment annual payment either it will be F1 + A or - A depending upon the sign of A you get.

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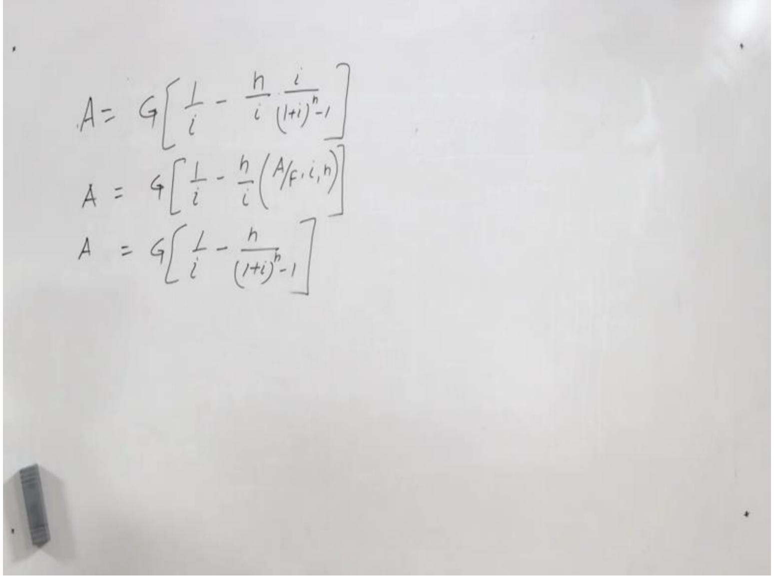

The expression can further be written in other form you can also write this expression what you have received G 1 by i - n by i into i by 1 + 1 n - 1. And if we recall this is basically a factor this factor is nothing but A by F i n. So you can write it as G 1 by i - n by i A by F i n.

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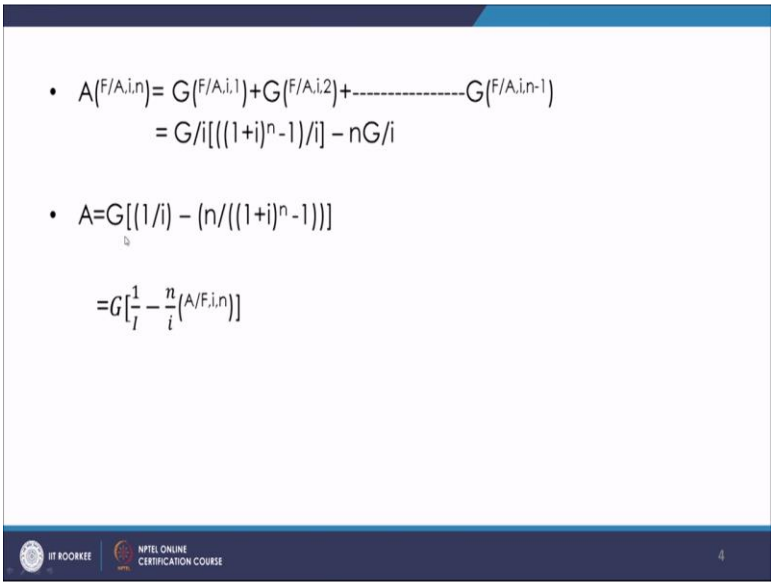

So what we see, we have basically derived this expression G A equal to G 1 by i - n by i A by F i n and this is known as the linear gradient series factor. Now let us discuss a problem based on these type of gradient series factors. So what we have seen that you have A as a function of G 1 by i - n by i A by F i n or you can also write it as G 1 by i - n by 1 + i n - 1.

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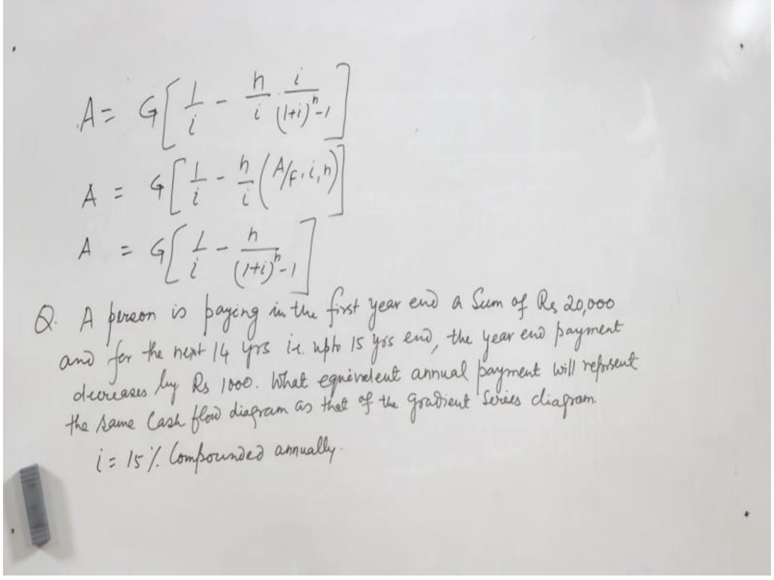

Now let us see, you have a question, an example tells that a person is paying in the first year end a sum of Rs 20000 and for the next 14 years that is up to 15 years end the year end payment decreases by Rs 1000. What equivalent annual payment will represent the same cash flow diagram as that of the gradient series diagram?

Now let us see because many a times we deal with certain situations where the person has paid certain amount in the first year end and there is a change in that amount. In this case it is decreasing, so for this case if you look at and also you will be given i as 15% compounded annually.

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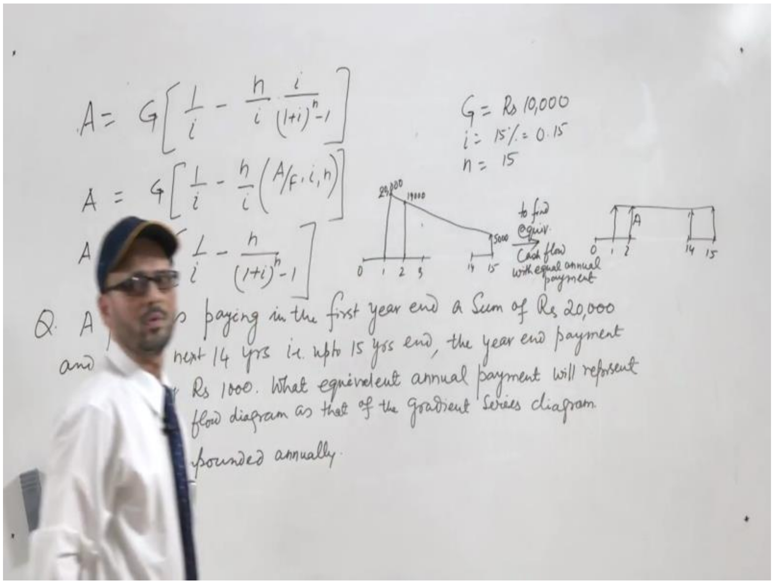

So if your problem is like this it means you are given G as Rs 1000, i you know as 15% n is 15. So your job is to find so your basically cash flow diagram the first cash flow diagram where in the first year he is paying 20000, second year he pays Rs 1000 less so he will be paying 19000 like that it will come and in the 15th year he will be paying 5000.

Basically your job is to find equivalent cash flow with equal annual payment and that will be like this 0, 1, 2, 14 and 15 and basically you have to find what is this A. So basically whatever value you get here that will be deducted from Rs 20000.This is, sorry this is only 1000.

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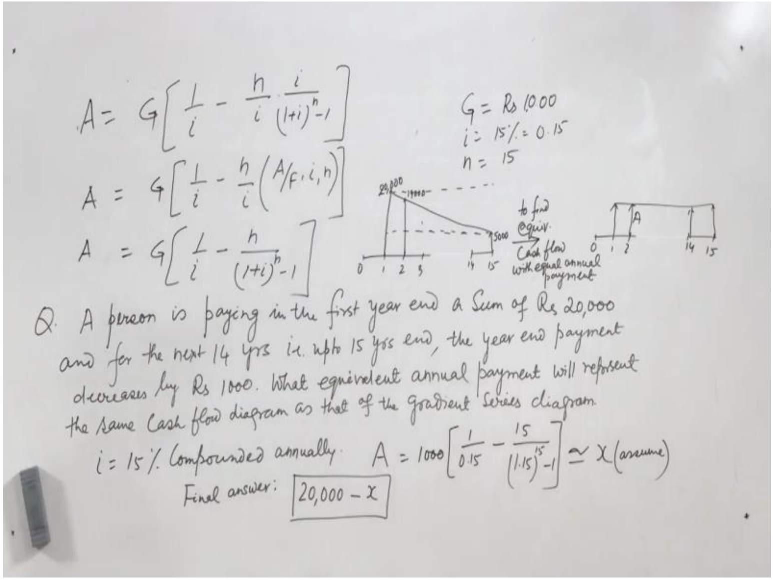

So what you do is you use this series so what you will do you will find A as G is given as 1000 multiplied by one by i 1 by 0 point 15 - 15 by 1 point 15 to the power 15 - 1. Whatever comes here suppose it comes as X then your final answer is because this is the equivalent to this gradient series, this gradient series is X. This X amount will be basically deducted from this 20000 so your final answer will be 20000 - X.

So this way you solve such problems. Here the gradient amount is a negative amount that is why we have subtracted it If it is a positive amount we will add it here it will be 20000 + X because this line will go like this. So this way we calculate the equivalent annual amount for a linear gradient series factor. Thank you.