Mechanical Measurement Systems
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Lecture – 11
Uncertainty Analysis

Hello, I welcome you all in this course of Mechanical Measurement Systems and today we will discuss Uncertainty Analysis. And we were we will we will mainly focus on Uncertainty Analysis and will solve certain numerical related with the uncertainty analysis.

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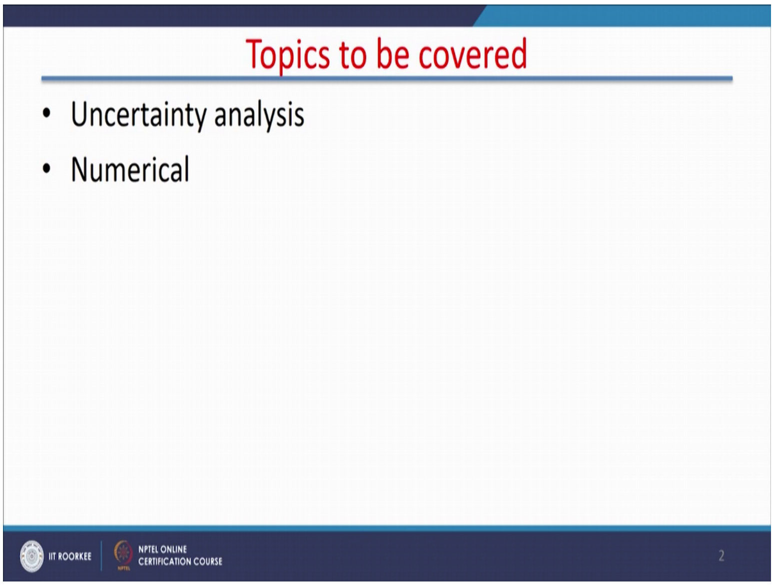

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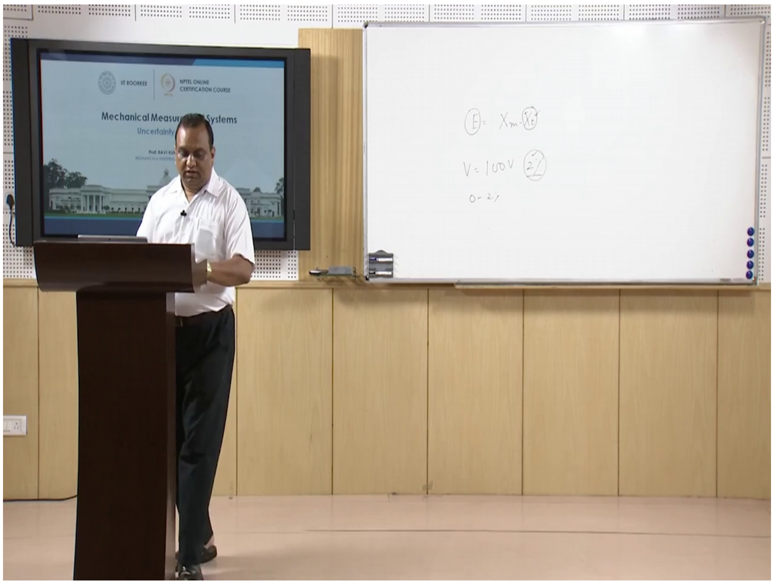

So far, we have gone through the error in measurement and error in measurement is always established as measured value minus true value. In most of the cases, we do not have true values. We are we have the values which are very close to the true value, but exactly exact estimation of true value is difficult right.

So, there is always uncertainty in error measurement right. Now, in uncertainty analysis what we do, we estimate the magnitude of error. Suppose, we are measuring voltage with the help of a voltmeter and the voltage is let us say 100 volts and there is an error in the measurement of let us say of there of 2 percent. When we say uncertainty in the measurement is 0 to 2 percent, it means error in the measurement may lie in this range right. It cannot be exactly 2 percent it may lie in this range.

So, the Uncertainty Analysis it assigns the credible limits to the accuracy of report reported value. So, we report any value. So, it provides the credible limit to the reported value. And when we want to do the uncertainty analysis, there is a little mathematics is involved.

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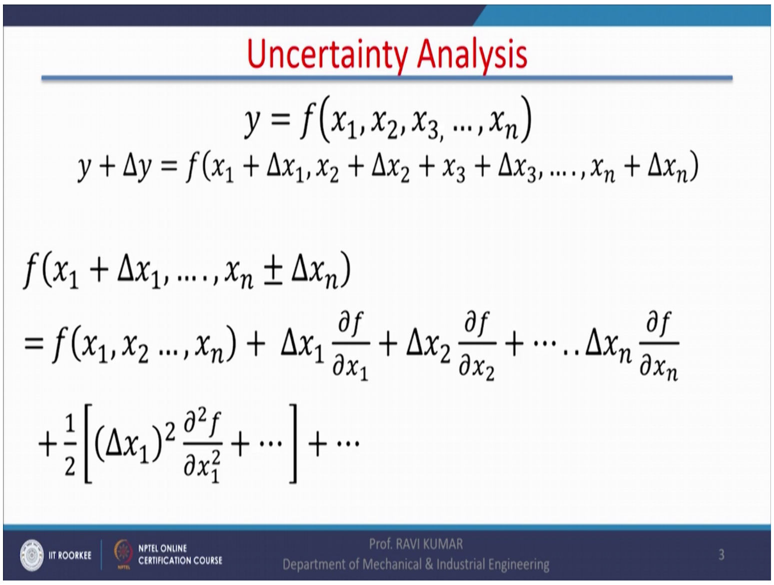

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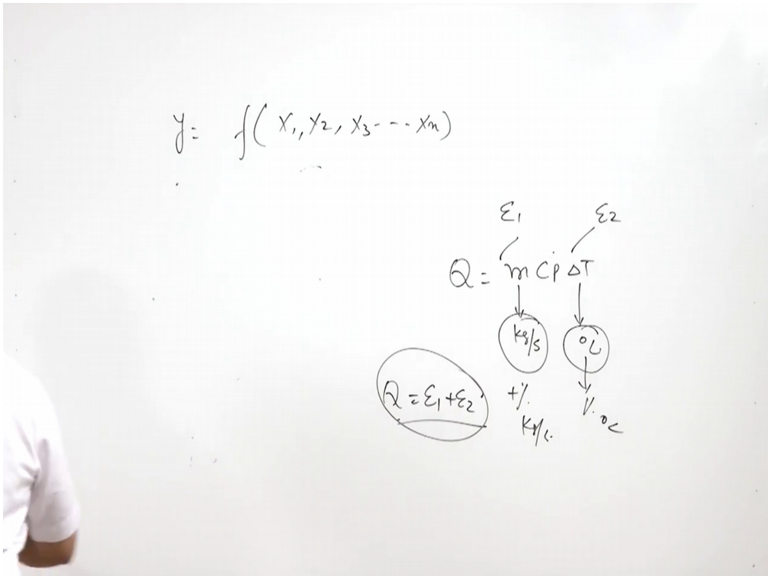

Suppose, there is a function y, x 1 x 2 x 3 and x n right, I will give you an example for example; we are measuring heat carried away by the cooling water in a condenser.

So, that is going to be Q is equal to m Cp delta T. Now, here two things will be measured; one is mass flow rate in kg per second and delta T in degree centigrade. And while doing this measurement, it will incorporate certain error certain error and this will also measurement of temperature where also incorporate certain error in may be expressed in terms of percentage or absolute terms in degree centigrade ok, kg per second.

Now, these two errors will be accumulated. Suppose, this has a 1 error and this has sorry let us say this has e 1 error and this has this one delta t has e 2 error. So, error in Q shall it be e 1 plus e 2, it will not it will not.

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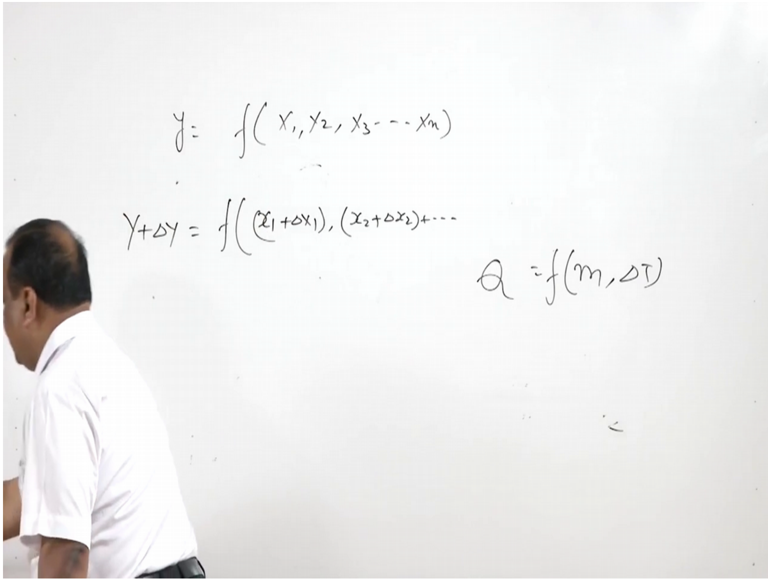

So, in order to find this when y is a function of here because mass flow rate is a function of is a function of sorry not mass flow rate, heat transfer heat transfer is a function of heat transfer rate is mass flow rate and delta T.

Similarly, we have taken here some y is a function of x 1, x 2, x 3 and going up to x n then, Y plus delta Y right. Change in y due to error in a measurement 1, 2, 3 and 4 it has to be estimated right. We cannot determine this. This has to be estimated.

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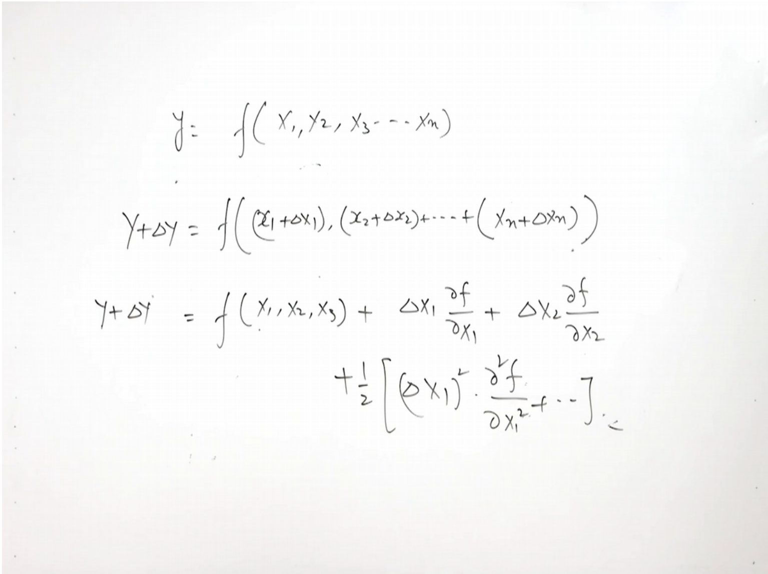

So, this is going to be equal to f x 1 plus delta x 1, x 2 plus delta x 2 and so on and it will go up to x n plus delta x n their cumulative effect.

Now, this function can further be modified as f x 1, x 2, x 3 plus delta x 1 del f by del x 1 plus delta x 2 del f by del x 2 right. So, this function is partially differentiated with delta x 1 multiplied by delta x 1, partially differentiate delta x 2 multiplied by delta x 2 plus half we can continue with this series delta x 1 square delta x square f by delta x 1 square plus and so on. So, but these values will be neglected and they; will be rounded off and now, we have y plus delta y.

So, this y and this y f this will be they will be cancelled and we will be getting just a minute.

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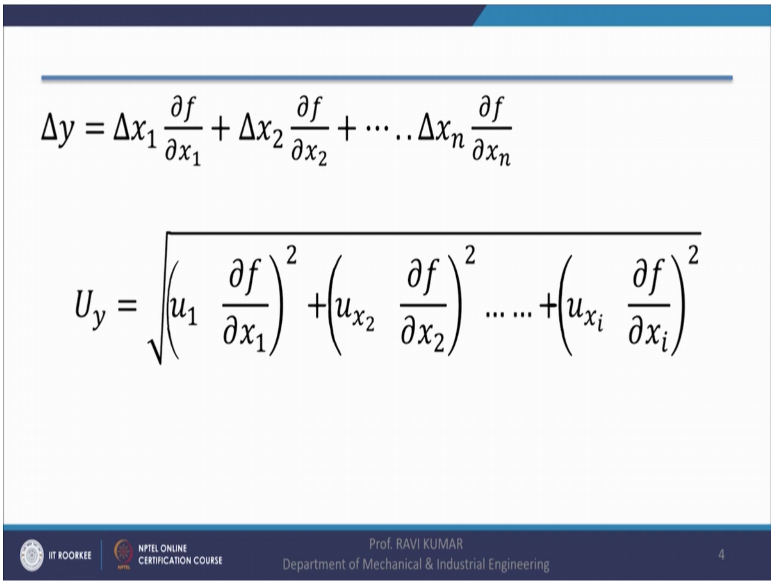

And we will be getting delta y as delta x 1 del f by del x 1 plus delta x 2 del f by del x 2.

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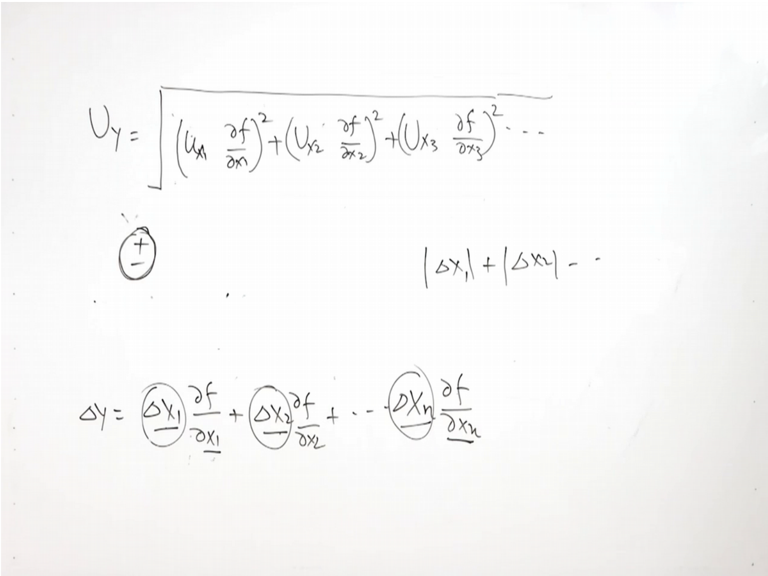

And it will go on up till del x n delta f by del x n right. Now, the question is, now this is differentiation of with respect to the first parameter error in first parameter, with respect to second parameter; error in second parameter and the nth differentiation with partial differentiation with nth parameter this is only error in error in measurement delta x 1, delta x 2, delta x 3.

Now, this error in measurement, we are considering as uncertainty in measurement right and then, we can have the final expression as uncertainty in the measurement of y is equal to under root uncertainty in the measurement of 1 or x 1 then, delta f by delta x 1 whole square plus uncertainty in the measurement of x 2 delta f del x 2 whole square plus uncertainty in measurement of x 3 del f f by del x 3 whole square and. So on up to nth term.

Now, the question is why we are adding squaring and adding or it is known as the Pythagorean sum. Why we are not simply adding these values right. This question may arise and for this purpose first of all the purpose of this is that any plus minus contribution is not accidently canceled. Any contribution in the uncertainty by the virtue of being x 1 delta x 1 positive or negative, they are not accidently canceled. Suppose, this is negative error; this is positive error. This is delta x 2 is positive; this delta x 1 negative, this should not be. In fact, module of these has to be added, module of delta x 1 plus module of delta x 2 like this.

So, in order to ensure that accidentally it is not done, square has been taken. Now, second thing is in this case, large source of error is magnified, large source of error is magnified reason being for example, error in the measurement is let us say there two measurements x 1 and x 2, x 1 the measurement error is 12 percent, x 2 measurement error is 5 percent right.

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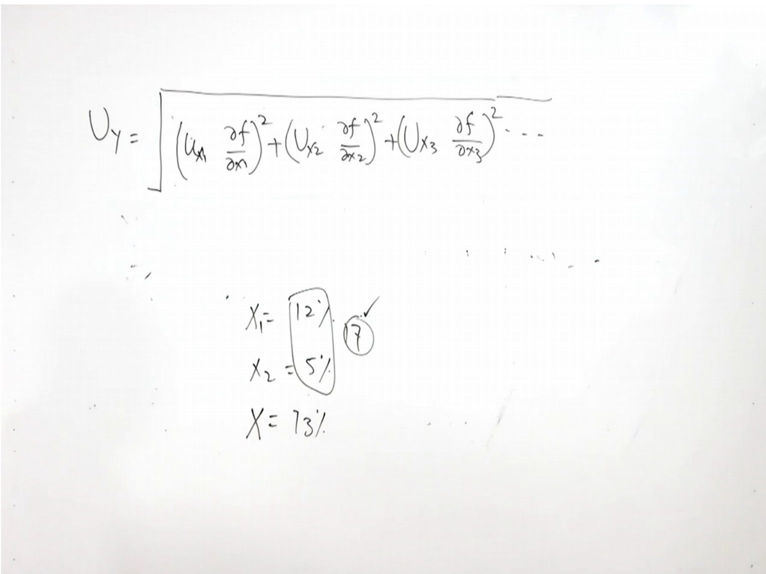

So, now here if we take the Pythagorean sum, you will know that the final error will be 13 percent not the 17 percent that is another reason for taking because this is the worst case scenario. This will rarely happen right. So, this when we take the Pythagorean sum, all these issues are addressed. So, so we never operate in this case, we never operate in extreme worst case scenario. This is the extreme worst case scenario right.

So, for this reason the Pythagorean sum of all the errors all the uncertainty in the measurement is taken. And suppose is now we will take certain cases for example, there is the case of addition.

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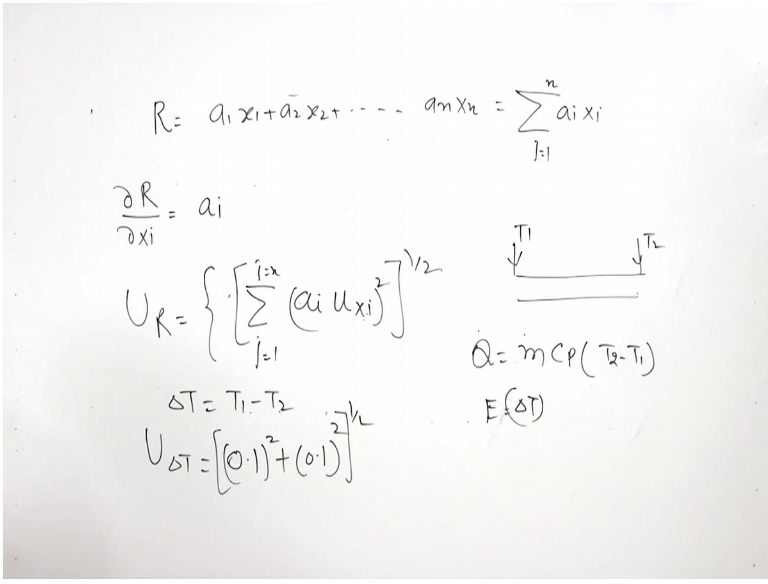

Now, R maybe the some function R is a1 x1 plus a2 x2 plus an xn right or it is sum of ai xi, i is equal to 1 to n fine.

Now, del R by del xi. So, or is equal to ai for any, if I differentiate with the x 1 a 1, x 2 a 2, x n. And so, uncertainty in the measurement of R is going to be the sum of ai u xi whole square raise to power 1 by 2, I is equal to 1 to sorry here. I is equal to 1 to I is equal to n fine. For example, I will give you one example suppose we are measuring the temperature with the help of thermocouples and again the temperature or the heat carried away by the cooling water in a condenser right. Inlet temperature is T 1, outlet temperature is T 2 fine.

Now, Q is definitely heat transfer rate is mass flow rate Cp T 2 minus T 1. Now, error in the measurement of Q or error in the measurement of delta T in the measurement of delta T, so, delta T is T 1 minus T 2. So, uncertainty in the measurement of delta T is going to be as per this. Suppose, error in measurement this is 0.1 and this is also 0.1. So, 0.1 degree centigrade error in the measurement of Ti and T 2 also it is 1 degree centigrade, a 1 ai is 1 in this case, ai is 1. So, we can take 0.1 whole square plus 0.1 whole square raise to power 1.2.

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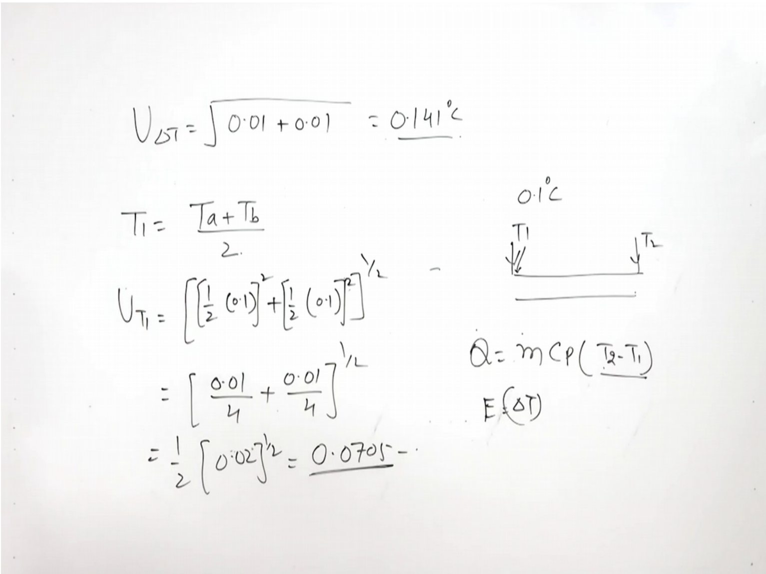

So, when we take 0.1 square, then it becomes the uncertainty in delta T uncertainty in delta T is equal to under root of 0.0 1 plus 0.01. Now, if we take under root of 0.02, it becomes 0.141 degree centigrade right. So, temperature difference individual measurement is 0.1 and 0.1 right. So, it is not 0.2, it is 0.14141 degree centigrade in uncertainty measurement of delta T.

On the other side, inlet temperature is measured by 2 thermocouples. For example, inlet temperature is measured by 2 thermocouples. So, each thermocouple has accuracy of 0.1 degree centigrade. So, T 1 is Ta plus Tb which are a fixed at the inlet divided by 2. So, uncertainty in the measurement of T 1 is going to be equal to half, a a 1 is half and uncertainty is 0.1 square plus again half 0.1 square right and then we get.

Student: Whole square.

Now, we get 0.01 divide by 4 plus 0.01 divide by 4 raise to power 1 by 2 or we can say it is half of 0.02 raised to power 1 by 2 or it is 0.0705. So, you can see; when we are putting tube thermocouples, we have improved the uncertainty in the measurement. Likewise, you can have bunch of thermocouples. If we take the average value the uncertainty the measurement or accuracy of the measurement shall improve.

Now, after this we will take up a case where we have R as product of different parameters R is a product of different parameters.

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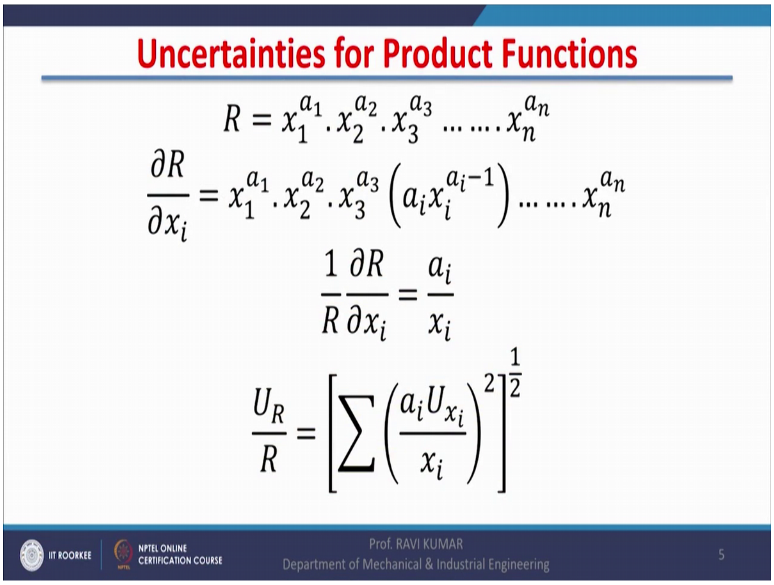

So, that is x 1 raise to power a 1, x 2 raised to power a 2, x 3 raised to power a 3, x n a to power n.

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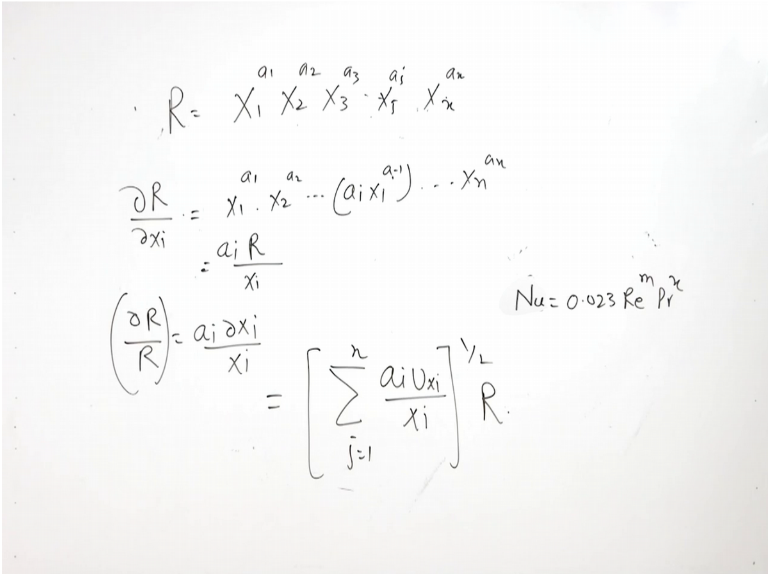

So, these are the cases. For example, Reynolds number those who are mechanical they very well know some constant for example, flow in a pipe 0.023 Reynolds number m Prandtl number n this is 0.8 and this is 1 by 3.

So, such type of relations we have in mechanical engineering. So, R is again del R over del xi because xi here xi is equal to x 1 a a 1 multiplied by x 2 a 2 and when we differentiate xi or this is this we can take xi x n when we take xi and a to power i right, it becomes ai xi a minus 1 and then, we go up to x n right or we can write this as R by xi and this is ai this can be always be written as ai because this if we numerator denominator if you multiply by xi, then again you will be getting this expression. Likewise, you can do for n number of values and ultimately you will find that this R can come down here.

So, del R by R is equal to ai del xi by xi right. And when we say the uncertainty in the measurement, it can be expressed in terms of percentage because it is del R by R. It can be expressed as sigma ai u xi this is uncertainty in xi divided by xi I is equal to 1 to n raised to power 1 by 2 right. So, this is how the uncertainty this will be in terms of percentage. If you want to have extra value, so and absolute value, then it has to be multiplied by R right. So, both the cases we have discussed.

Now, will go for certain numerical; because solved examples will give you the inside of the things.

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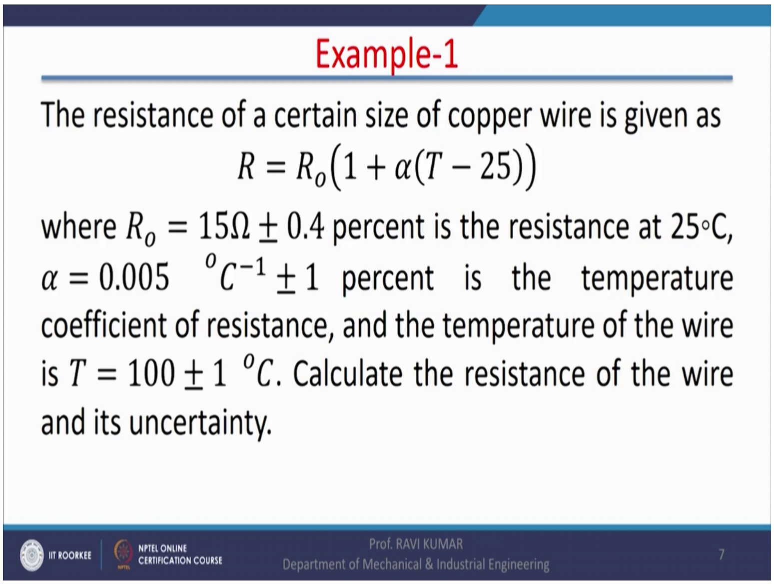

Let us start with the solved example number 1. Now, in solved example number 1. It states the resistance of a certain size of copper wire is given as R is equal to R o 1 plus alpha T minus 25.

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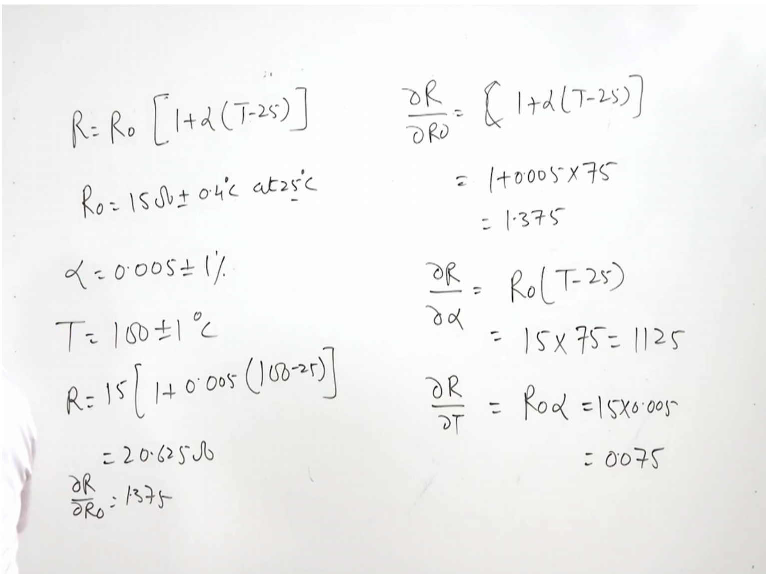

R is equal to R o 1 plus alpha t minus 25. This is the variation of resistance with temperature ok. Where R o is 15 ohm plus R o is 15 ohms plus 0.4 degree centigrade plus minus 4 degree centigrade at 25 degree centigrade right. Alpha is equal to 0.005 plus minus 1 percent. It is in terms of percentage right. Temperature coefficient of resistance in the temperature of wire is T again 100 100 plus minus 1 degree centigrade right. Calculate the resistance of the wire and it is uncertainty, so, the resistance of the wire at 100 degree centigrade.

So, it is very simple. R is equal to R o, R o is 15 1 plus 0.005 T minus T is 25, sorry T is 100 100 minus 25 and this will give the final resistance and it is going to be 0.005 into 20.625. So, it is 20.625 ohms. Now, we have to find the uncertainty in the measurement.

So, we will differentiate R with respect to R o, R with respect to alpha, R with respect to T right. So, del R by del R o is equal to 1 plus alpha T minus 25 right. So, is equal to 1 plus 0.005 into T minus 25 is 75 and this that will turn out to be 1.375. Let us confirm this 75 into 0.005 plus 1 1.375. Now, delta del r alpha. When we are differencing this differentiating this with alpha will be getting only R o T minus 25. So, R o is how much 20.625 into 75. So, you will be getting 15 into this is T minus 25 is 75. So, 15 into 75, it is going to be equal to 1125 15 into 75, 1125.

Now, del R by del T with respect to temperature. Now, when we are doing due to the respective temperature, it is only R o alpha right and that is going to be equal to 15 into 0.005 and that is equal to 0075 sorry not 0.075 ok. So, now, we have partial differentiation of R with respect to R o alpha and T.

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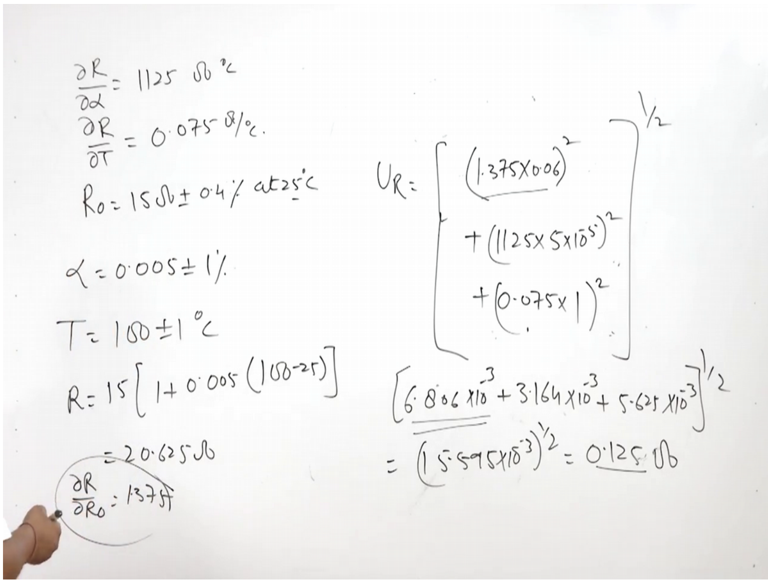

Now, using this information let us note I am somewhere let del R by del R o is equal 1.375. Del R by del alpha is equal to 1125 and unit we can right ohm and alpha is the degree centigrade and this is going to be unit less delta R by del T is equal to 0.075 ohm per degree centigrade. Now, uncertainty in the in the determination of R not measurement of R; in the determination of R is uncertainty in the determination of R is partial differentiation R with R o right and error in the measurement of R o.

So, partial differentiation is 1.375, error in the measurement of R o is 0.4 percent of 15. 0.4, this is percent not degree centigrade 0.4 percent. So, it is going to be 0.06. 0.4 percent will square this plus again next alpha it is 1125 multiplied by uncertainty in this. So, 5 into 10 to power minus 3, 5 into 10 to power minus 5 so, it is going to be 5 into 10 to power minus 5 whole square plus temperature.

So, uncertainty in temperature 0.075 so, this partial derivative of temperature is 0.075 and uncertainty in the measurement is 1 degree centigrade right. So, it is 1 and square is taking and will take under root of entire expression. So, this is going to be like this. Now, this value is 6.806 into 10 to power minus 3 plus 3.164 into 10 to power minus 3 plus 5.625 into 10 to power minus 3 raise to power 1 by 2 right.

And here, if you see that highest contribution in uncertainty is coming from here and this is delta R by delta R o right. And we further resolve this and it is going to be 15.595 into 10 to power minus 3 raise to power 1 by 2 and it is 0.125 ohms. This is the estimated uncertainty. We are not doing any measurement of R. So, uncertainty in the measure determination of R is 0.125 ohm. If you want to express in terms of percentage, you simply divide this.

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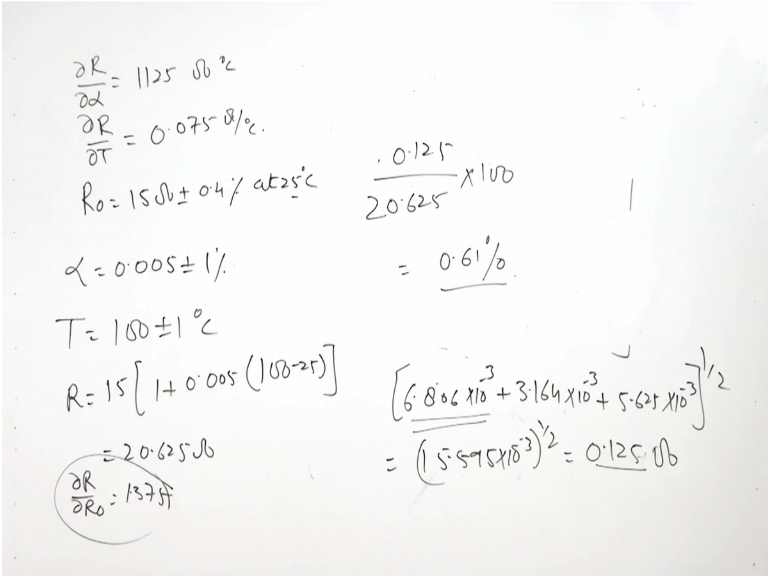

Simply divide this 1250.125 divided by this 20.625 multiplied by 100. This will give you uncertainty in the measurement in terms of uncertainty in the determination of a in measure of in terms of percentage.

So, this is one example regarding determination of uncertainty in the measurement.

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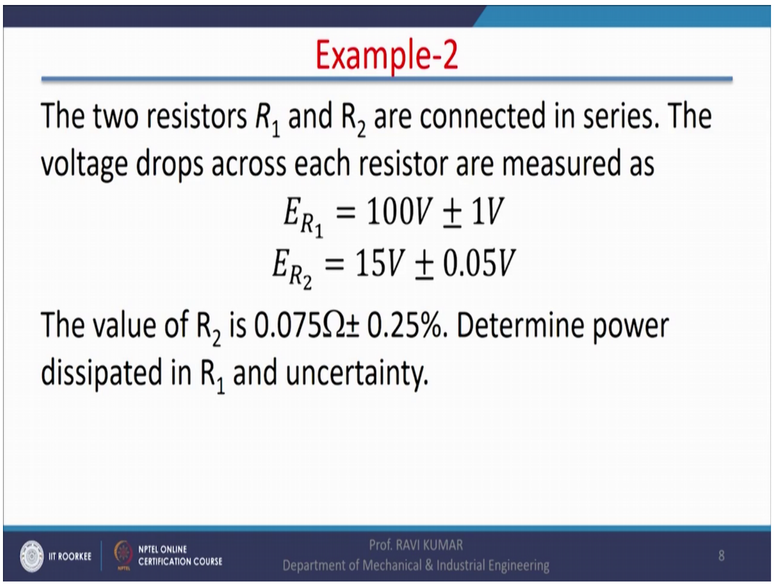

Another example is, if 2 resistors R 1 and R 2, they are connected in series. So, I will rub this one. There 2 resistors R 1 and R 2, they are connected in series. This is R 1 and this is R 2 right.

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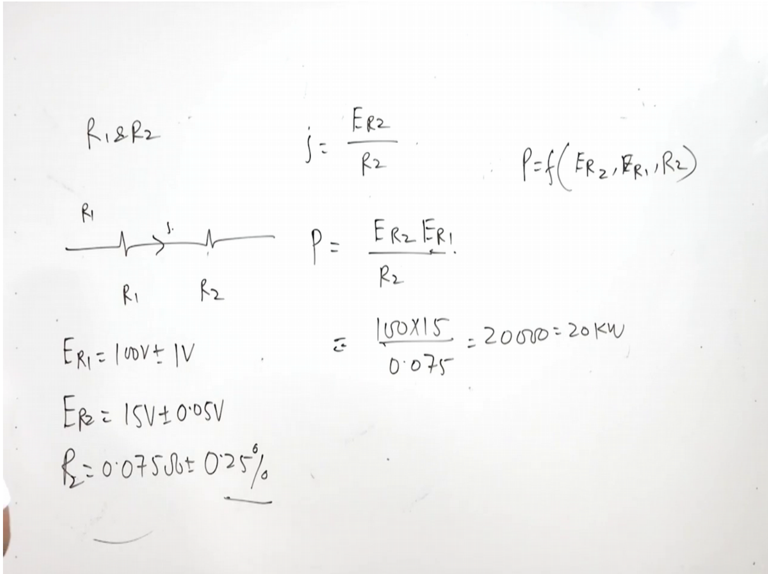

And voltage drop across the each registered is measured as R 1 voltage drop across the R 1 is so, E R 1 is 100 volts and plus minus 1 volt and voltage across R 2 is 15 volts plus minus 0.05 volts right.

So, voltage we have measured which one voltmeter here, one voltmeter here, this voltmeter is showing 100 volts this is showing 15 volts with some error in the measurement. Now, the R 2 value is given R 2 value is 0.075 ohms plus minus 020.25 percentage. This is the percentage of measured value it is always if it is written like this, it is always measured.

So, dissipate find out determine the power dissipated in R 1, how much power is dissipated is R 1 and uncertainty in the determination of the power. Mathematically it is very simple because once R 2 is given a voltage is given. We can always measure the current and current is going to be same in both the cases. So, current is R 2 divided by R 2. And once we know the current and then we can always measure power in dissipated in the resistor one that is I R. So, I is E R 2 by R 2. This is this is current multiplied by the voltage will give the power P is equal to Va.

So, that is E R 1. Now, we have power as a function of R 2. This is this is a voltage in R 2 voltage in sorry voltage in R 1 and resistance R 2. So, power is calculated 100 into 15, but product of the voltage divided by 0.075 and it is going to be 20,000 or 20 kilowatt.

Let me check, now, uncertainty in the measurement of power, so, uncertainty in the measurement of power because now, this is a case of products right.

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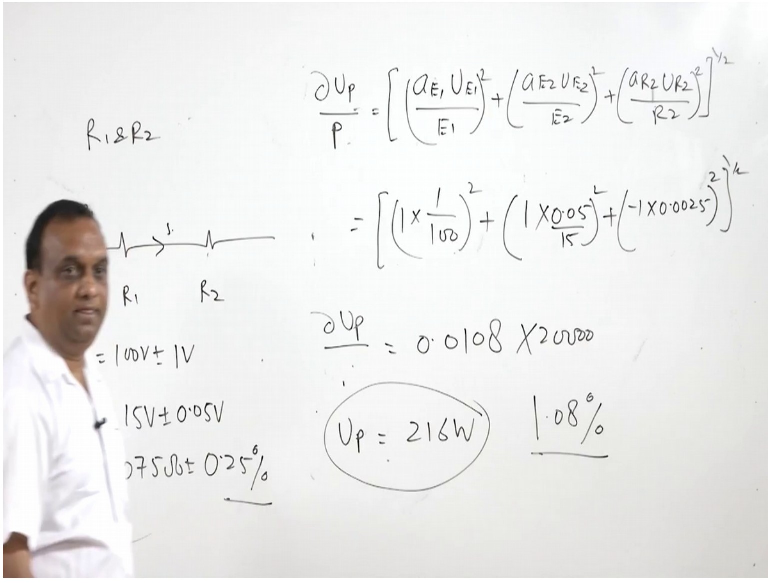

So, uncertainty shall be calculated as uncertainty in power divided by power is equal to a E 1 uncertainty in measurement of E 1 divided by E 1 whole square plus a E 2 uncertainty in E 2 divided by E 2 whole square plus a R 1 sorry R 2, a R 2 uncertainty in R 2 divided by R 2 whole square raise to power 1 by 2.

Now, here this values R 1 because we have because this is this a E 1 is 1, a 2 is 1 and this is minus 1 right. So, here we will calculate uncertainty in power divided by power is equal to 1 into error is 1, 1 by 100 whole square plus again a E 2 is also 1. So, 1 into 0.05 by 15 plus here it is minus 1 right. And then, it is 0.0 into 0.0025 it is given in the terms of percentage; so because it is given in the terms of percentage right.

So, this is the mathematical expression for delta U. So, here, we get if we solve this will get del U P by P as 0.0108 or error in the measurement of uncertainty in the measurement of power is 216 watt. When we multiply this by 20,000 right, then we get 0.0108 multiplied by 20 216 watt. So, this is the uncertainty in the measurement of power, it is 2 one 6 watt or if you want to express in terms of percentage, it is 1.08 of the measured value right. This is all for today.

Thank you.