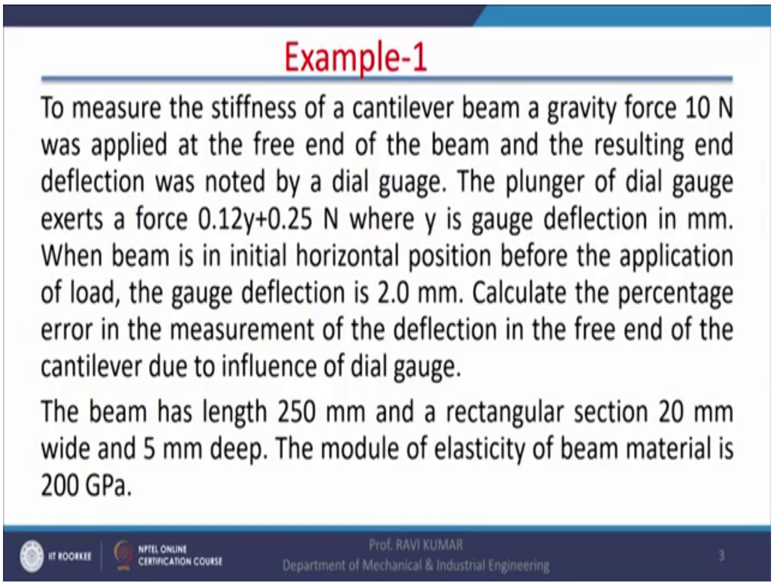
Mechanical Measurement Systems
Prof. Ravi Kumar
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 12
Problem Solving - 1

I welcome you all in this course on mechanical measurement systems. And today's session will be on problem solving. We will solve a few numerical examples based on a statistical analysis of experimental data.

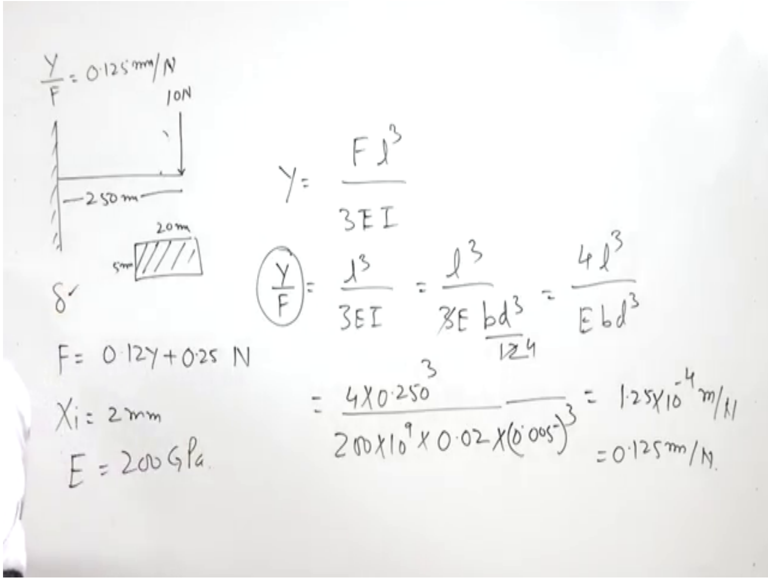
(Refer Slide Time: 00:39)



So, we will start with the example numbeR1 which states that to measure the stiffness of the cantilever beam a gravity force 10 Newton was applied at the free end of the beam. And resulting end deflection was noted by a dial gauge.

So, there is a cantilever it has one fixed end and one free end; on the free end, the load is applied of 10 Newton and deflection is measured with the help of dial gauge right and stiffness has to be measured. So, a stiffness is delta by F deflection.

(Refer Slide Time: 01:03)



Per unit force and sorry here the; it is not is stiffness deflection has to be noted. So, deflection delta has to be noted using a dial gauge right. The pannier of dial gauge exerts afros of 0.12 y plus 0.25 m. So, plunger causes the 4 0.12 y plus 0.1 sorry 0.25 Newton 0.

So, right. So, this is y is the deflection in vertical direction right. So, this is a sort of relation between force and deflection of the dial gauge. Then beam is in initial horizontal position before the application of load the gauge deflection is 2 mm right. So, gauge is installed somewhere here and stylus.

So, very close to this just very close to the load all right and initial deflection is 2 mm. So, x I is 2 mm. So, dial gauge is already compressed initial deflection means it is already compressed. So, when the cantilever will go down the compression in the dial gauge will be relaxed right. So, this is how it is done when beam is in initial horizontal position before the application of load the gauge deflection is 2 mm. calculate the percentage error in the measurement of deflection in free end of the cantilever duo to influence of dial gauge right so, because the dial gauge is compressed initially. So, it is also exerting certain force on the beam.

And this will disturb the original deflection. So, first of all 2 deflection let us calculate 2 deflection of the beam. So, in a cantilever beam y is equal to w l cube by 3 ei this is formula for calculating the deflection right.

Now, here w instead of w let us take f f. So, y by f y by f for this cantilever is going to be l cube by 3 e I. Now the length of the cantilever is 250 mm and cantilever has certain cross sectional area it is the rectangular cross section area.

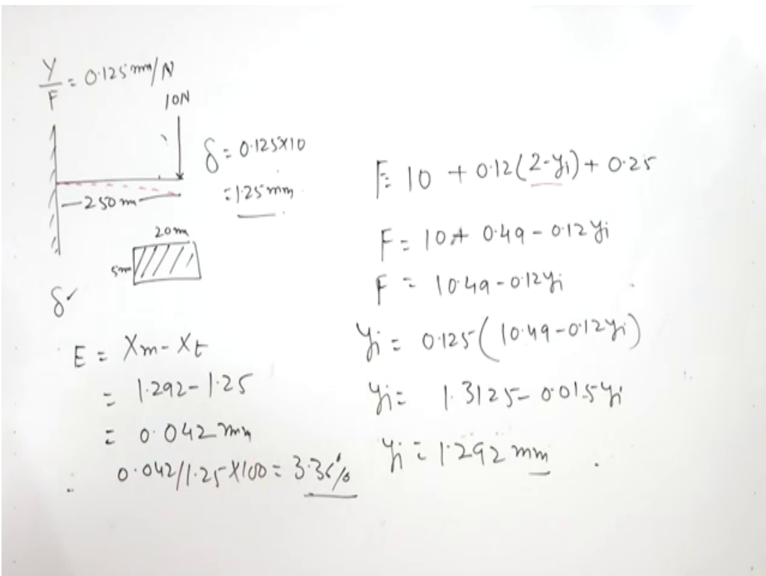
And this is by 20 mm by a 5 mm that is the cross section of the cantilever and the module of elasticity is 200 gigapascal plus we will be using these values here. So, here we can find the 0.25 cube divided by 3 into 200 into 10 to power 9 into I I is. So, it is 20.

So, 0.02 just a minute we can further simplify this is l cube by 3 e. Now I is bd cube by 12. So, bd cube by 12 and this is 3. So, this will be cancelled out 4. So, we will be getting 4 l cube by e bd cube.

Now, here now we can put the values 4 into 0. [Noise] 2 5 0 cube divided by 900 value of e is sorry 200 not 900 200 gigapascal. So, 200 into 10 to power 9 multiplied by b b is 0.02 if you convert this into the meters and 0.5 mmm.

So, it is 0.005 cube. Now this will give the value of y by fs if you simplify this is going to be 1.25 into 10 to power 4 minus 4 meter per Newton or we can say 0.125 mm per Newton. So, y by f for load we have calculated. So, y by f for this load is 0.125 mm per Newton fine now with the help of this. We can always calculate the value of delta delta is this multiplied by the load. So, if y and 2 have delta 2 value of delta.

(Refer Slide Time: 07:28)



So, delta is 0.25 multiplied by load it is 1.25 mm it is a simple as that right. Now total load on this the load is on this cantilever is not due to this concentrated load, but also by the stylus of the dial gauge. So, the total load is f is equal to 10 Newton this is by this. Plus 0.122 minus y I plus 0.125. Why I use the modified deflection? Because when the when the when the cantilever goes down.

Student: (Refer Slide Time: 08:19).

Sorry, yes, it is not 125, it is 0.25 because when the; this when the cantilever is deflected, the stylus of or the spring of the dial gauge is relaxed. That is why we have taken 2 minus yi right. Now, f is again we can calculate 10 plus 2 for 0.49 minus 0.12 yi or we can say f is equal to 10.49 minus 0.12yi this is modified force this is modified force. Now, if we want to calculate yi we have this value.

So, if you want to calculate yi it is f multiplied by 0.125 because we have already have this y by f. So, simply be multiplied f we replace f by yi y. So, we simply yes; so, in order to find yi, we simply multiplied this f by 0.125.

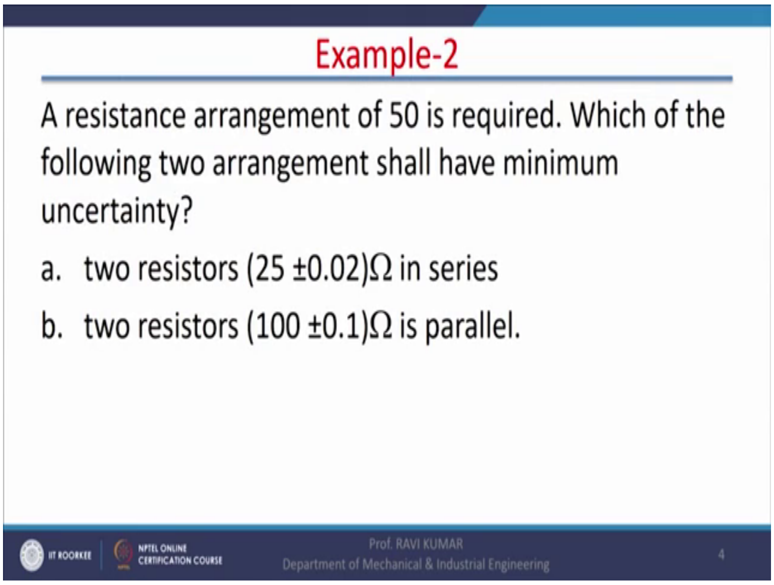
So, we simply this f this is multiplied by 0.125 and we are getting. Now we can solve this and this will give yi as 1.3125 minus 0.015 yi or vi is equal to 1.292 mm. Initially, it was 1, sorry, how much 1.25 mm. Ideally, it should have been, but due to loading this is also a loading effect. As we discussed earlier the stylus is a primary sensing element.

So, this is also causing the loading effect and due to this loading effect. The deflection has in piece for 1.25. So, 1.292 millimeters; now, if we want to have error in the measurement, now, error in the measurement is measured value minus true value measured value is 1.292 true value is 1.25, right.

So, the error in the measurement is 0. What is that value 1.29 to 1.242 0.242 0.042 is equal to 0 .0 4 2 mm. This is error in the measurement and if you want to have percentage error then divide this is equal to 0.042 divided by 1.25 in 200 and that will give the error in the measurement as 3.36 percent. So, error in the measurement of deflection in cantilever by measurement with the help of dial gate is 3.36 percent.

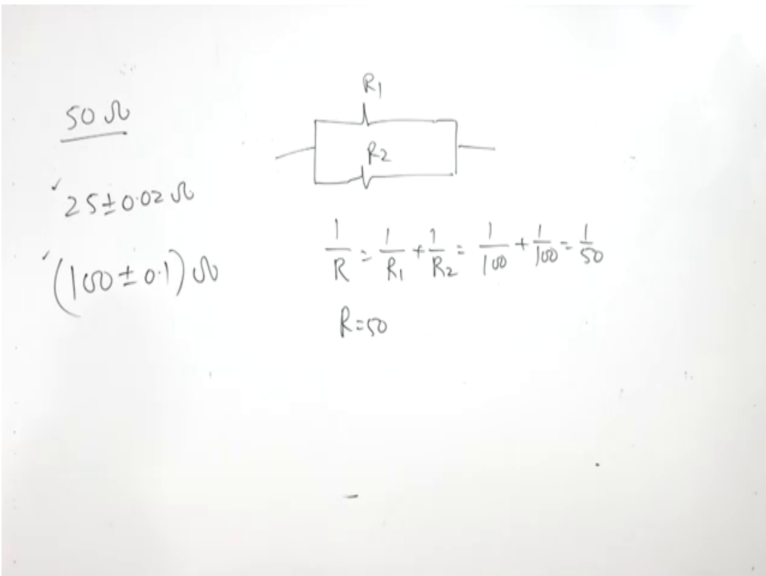
Now, after this we will take one example on.

(Refer Slide Time: 12:10)



Uncertainty in measurements now in registers arrangement of 50 ohms is required a range tens arrangement of 50 ohms is required which of the following 2 arrangements shall have minimum uncertainty 25 plus, minus 0.02 ohms.

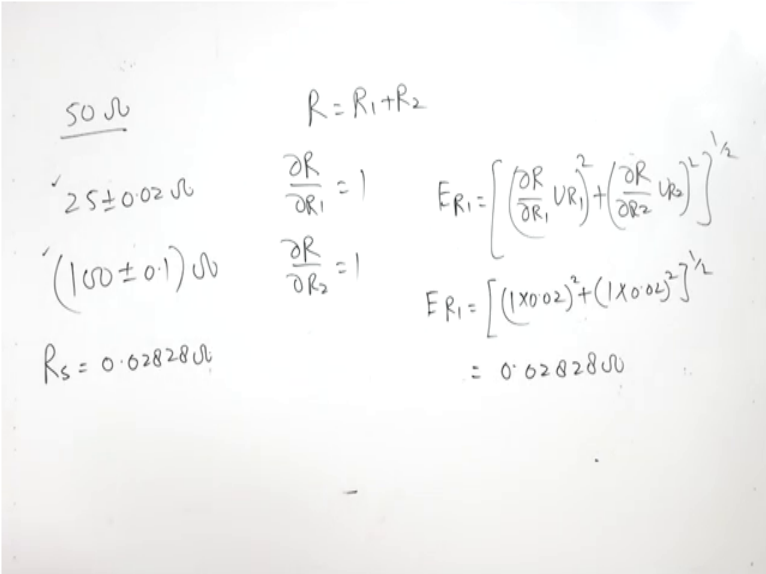
(Refer Slide Time: 12:57)



We have 2 resistors of 25 plus minus 0.02 ohms and 2 resistors off 100 plus minus 0.1 ohm. We have 2 this registers and 2 we have option we can have 15 ohms by putting these 25 in series. So, R1 and R2. So, R is equal to R1 plus R2 is equal to 25 plus 25 is equal to 50 ohms or we have option that instead of putting these 25 ohms in series. If we put 100 ohms in parallel 100 ohms in parallel R1 and R2 then 1 by R is equal to 1 by R1 plus 1 by R2 is equal to 1 by 100 plus 1 by 100 is equal to 1 by 50. So, R is equal to 50 now the issue is which of these 2 arrangements will have minimum uncertainty right.

So, we will start with a basic formula for uncertainty measurement.

(Refer Slide Time: 14:28)

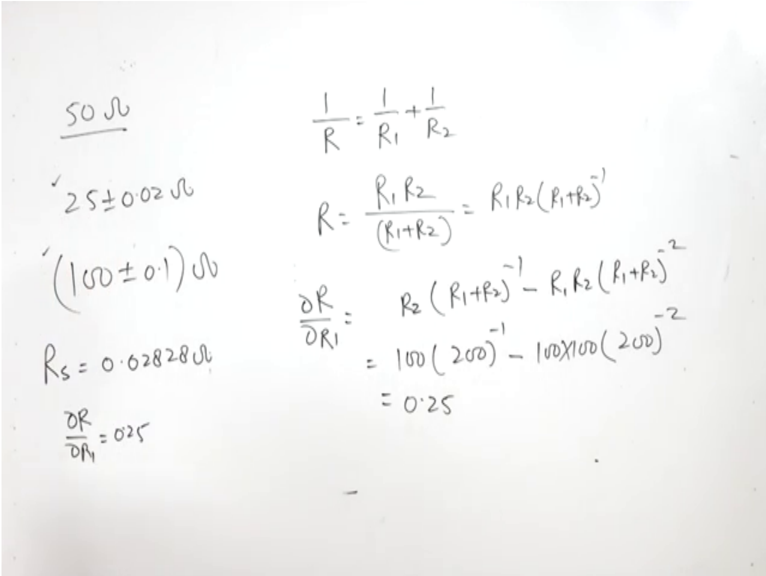


So, for the addition for addition formula R is equal to R1 plus R2, right. So, del R by del R1 is equal to one del R by del R2 is. So, is equal to 1 both are 1. So, error in the measurement of R1 is del R by del R1 by u R1 whole square plus del R del R2 u R2 whole square raise to power 1 by 2.

Now, here because these are have one value. So, error in the measurement of R1 is equal to uncertainty in R1 0.02, right. So, one into 0.02 whole square plus 1 into 0.02 whole square raise to power 1 by 2 and when we further solve it we get error of 0.02828 ohms that is the error when loads are connected in series. So, let us assume R series is 0.02828 ohms. Now second case.

Now, second case is when the resistors are connected in parallel when resistances are connected in parallel then 1 by R.

(Refer Slide Time: 16:12)



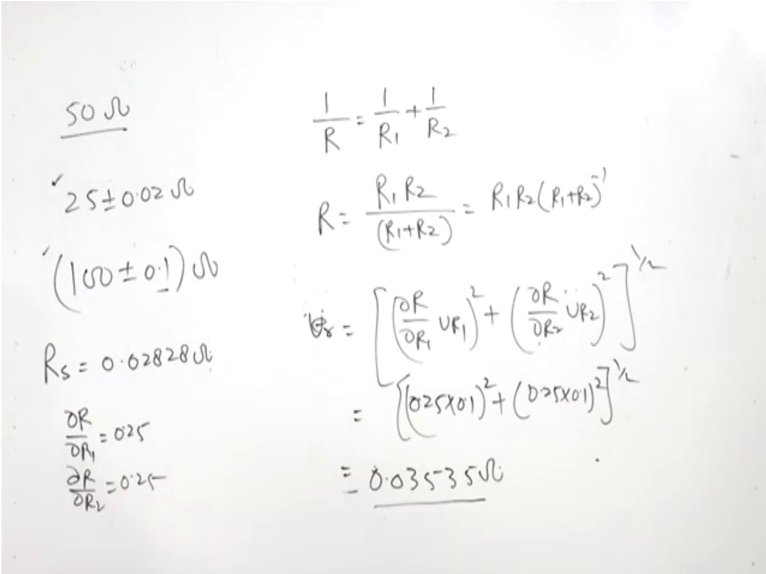
Is equal to 1 by R1 plus 1 by R2 or R is equal to R1 R2 R1 plus R2 or we can say R1 R2 R1 plus R 2 raise to the power minus 1 right. So, now we can take del R by del R1 is equal to R2. R1 plus R2 raise to power minus 1 differentiation of this multiplied by this differentiation of this multiplied by this all right.

So, minus this is R1 when we differentiate it this is minus 1. So, if that is where is coming here, and R2 R1 plus R2 raise to power minus 2 fine. Now, we will put the values 100 200 raise to power minus 1 minus 100 in to 100 200 raise to power minus 2. And this will give the value as 0.25 right.

So, here we will write del R by del R1 in case of parallel is 0.25. Now, doing same exercise shall be done for R2 and R2 also if you look.

At the values R2 will you are also going to get the same value.

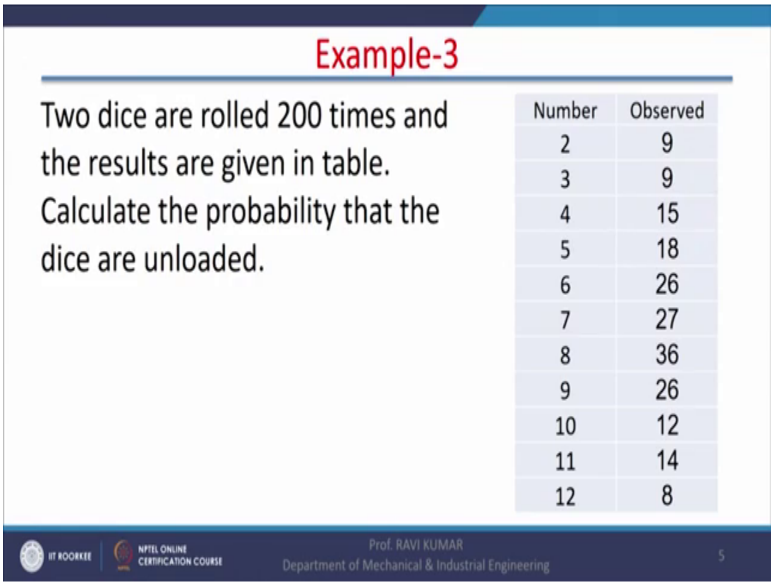
(Refer Slide Time: 18:07)



So, del R by del R2 is also 0.25 right and now error in R del R by del R1 uncertainty in R1 , whole square plus del R by del R2 uncertainty in R2 whole square raise 2 power 1 by 2.

Now, here again uncertainty is 0.1 right. So, 0.25 into 0.1 whole square plus 0.25 into 0.1 whole square raise to the power 1 by 2. And when we solve it we get uncertainty as 0.0353.

(Refer Slide Time: 19:53)

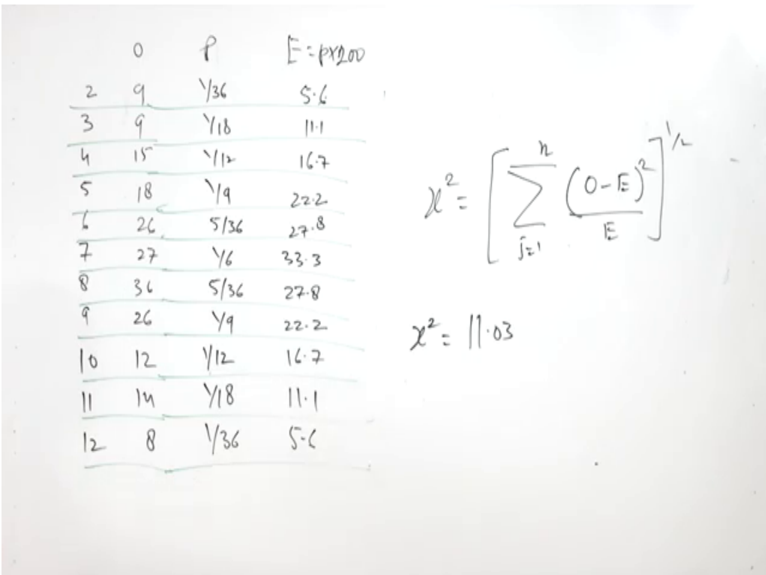


So, error in the measurement or uncertainty in the measurement uncertainty of R uncertainty of R uncertainty in the measurement of R is we are connected in parallel 100 ohms are connected in parallel it is 3.535 process or 0.03535 right. And when they are connected in series, it is 0.02828 ohms. So, definitely this is the better way of getting 50 ohms resistance because error is lesser comparison to the terror arrangement. So, this is about the uncertainty now I will take up one more example, on chi square test this example reads there are 2 dices there are 2 dices. So, I will first prove it off then we will discuss about this ok.

There are 2 dices ok. So, each dice has 6 faces right and they are all 200 times they are prowled 200 times right and 200 times, the results are given in table I will draw the table .

Calculate the probability that a dice are unloaded loaded means there is no unbiased in the dice and it is following the right. Train if dices unbiased then probability of safe for you having one dice and it has bias. So, probability of getting one figure is more than probability of getting another figure. So, here we have to ensure that there is no issues regarding the dice it is not it is not loaded it is unloaded or the the the train is the right train. The train which we are getting order of 200 of samples is the right train. So, the number we will draw first of all I will start from the top. So, 2 so 1 we cannot get in 2 dice.

(Refer Slide Time: 21:32)



So, we will start with the 2. So, 2 3 4 5 6 7 8 9 11 12. So, whenever we throw that to dices we will be getting these numbers addition of;

Student: (Refer Slide Time: 22:12).

2 3 4 5 6 7 8 9 10 11 12 10 11 12. So, we will be getting number out of these 11 numbers now we have thrown dice 2 100 times and we have got .

The frequency 9 9 15 18 26 27 36 26 12 14 and 8, right. So, in order over to any confusion I draw the oriental lines.

Now, this is the actual data we are getting observed value. Now what should be the actual value ideally? How many times the 2 should come? Probability first of all we will find the probability, what is the probability of getting 2 combination of 1 and 1? So, and total combinations are 36, so, it is 1 by 36.

Probability of getting 3 probability of getting 3 3. We can have how many combinations we can have 2 combinations 2 1 and 1 2. So, dice 1 1 dice 2 2 and 2 1. So, 2 by 36 2 by 36 is 1 by 18, right.

So, we will further explain these lines otherwise right. Now, what about 3rd one 4. How many combinations we have 4 4 2 plus 2 2 2 3 1 1 3. So, 1 by 12 the 3 by 36 is 1 by 12 n then 5.

How many combinations for 5 4 1 1 4 3 2 2 3 4? So, 1 by 9 so, likewise we will write probability of each and every 5 6 6 is 5 by 36 7 is 1 by 6 5 by 36 1 by 9 1 by 12 1 by 18 and 1 by 36.

So, 7 is the highest probability here 7 is the highest probability and followed by 8 and 6 right. Now, this is the probability total number of throws are 200. So, if you multiply probability by 200 that is a true value expected value.

right. So, expected value is going to be this is this is expected value. So, this is the probability and expected value will be probability multiplied by 200. So, if we multiply this by 200 we will be getting the expected as 5.6, 11.1, 16.7, 22.2, 27. No 8 right, 733.3 27.8, 22.2, 16.7, 11.1, 5.6. So, these figures we have got just to multiplying probability with the 200.

So, this is the expected value it means the expected number of times 5 will appear is 22.2, do practically. It is not possible either will have 22 or 23, but frequency has to be

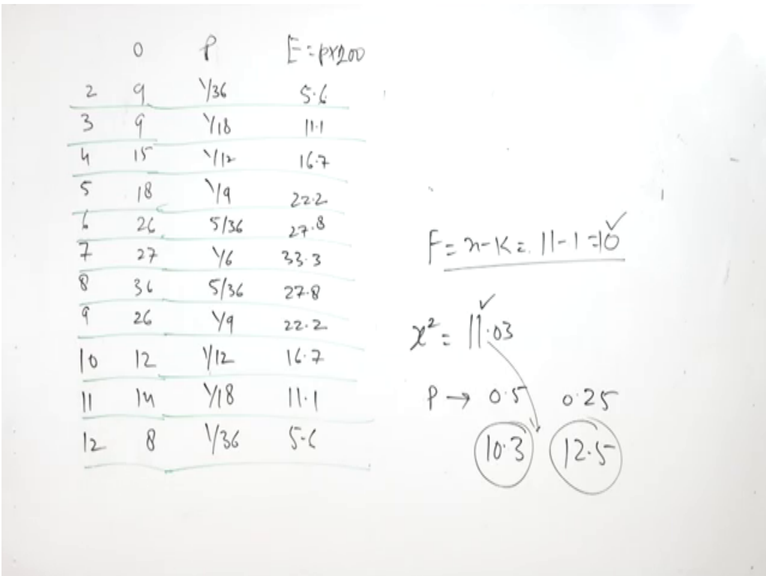
22.2, but we are getting only 18 for. Let us say 7 it should have been 33.3 times, but we are getting 7 only 27 times.

Now, we this is these are the observed value and these are the expected values. Now we can simply do the chi square test. Now, in order to chi square test simply we have to calculate the value of just chi square.

Right and it is a again some of if you remember I is equal to 1 to I is equal to n. Observed value minus expected value whole square divided by expected value raise to power 1 by 2 right and when we do for all these. I will not going to the detail calculation whether it is simple arithmetic you will be getting the value of chi square s 11.03 right. Now regarding the degree of freedom the degree of freedom here the total how many 12 12 minus 1 11. So, there is only one restriction that number of trials are I mean.

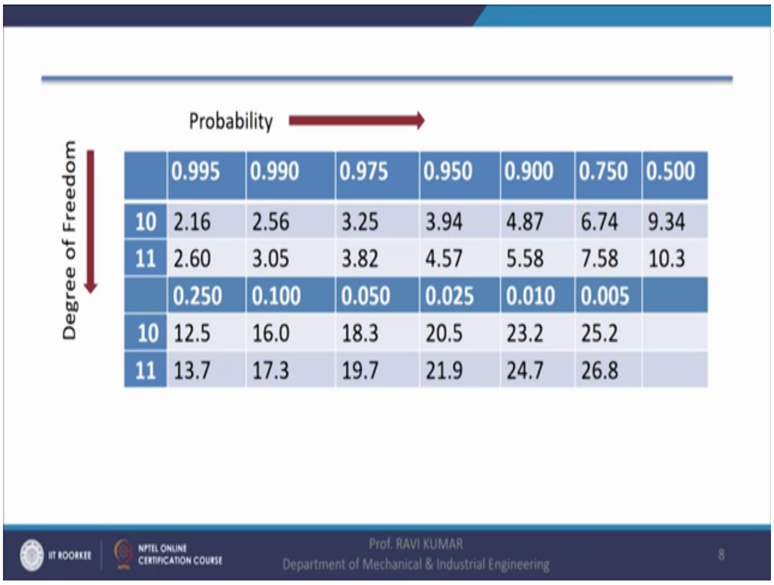
These numbers are restricted.

(Refer Slide Time: 27:54)



So, f is equal to n minus k 11 1 also a 0 sorry 10. So, for degree of freedom 10 this is square let us find out the probability and once we look at the chart for 10 right. So, for 10, it is 11.03.

(Refer Slide Time: 28:11)

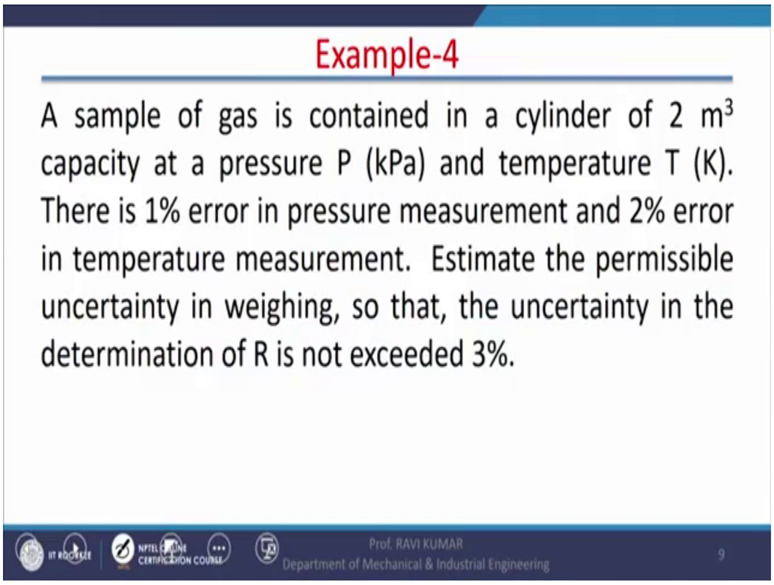


	0.995	0.990	0.975	0.950	0.900	0.750	0.500
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3
	0.250	0.100	0.050	0.025	0.010	0.005	
10	12.5	16.0	18.3	20.5	23.2	25.2	
11	13.7	17.3	19.7	21.9	24.7	26.8	

So, it is between 0.5 and 0.25 because at 0.5 it is 10.3 0.5 the probability is 10.03, 10.3 and 0.025.

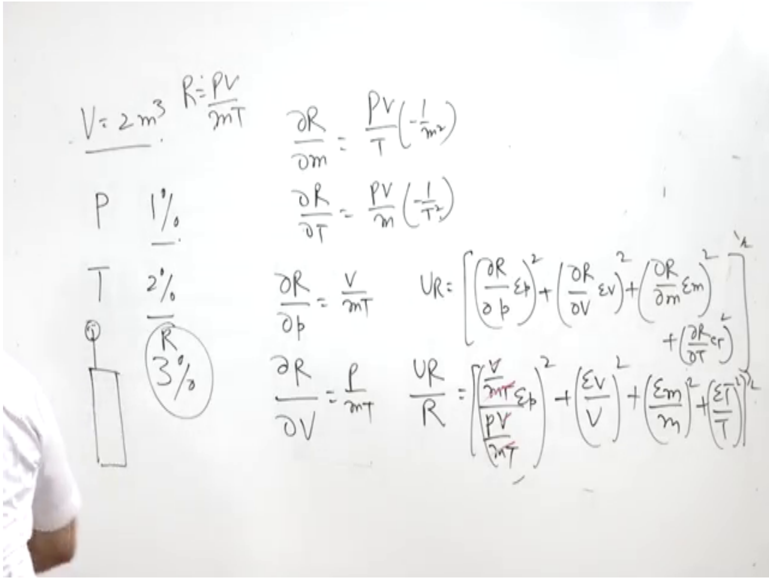
Sorry 0.25 and 0.25 it is 12.5. So, P and this is the value of chi square. So, definitely this will lie in between of in between these 2 values. So, these values these numbers they are following the right train and there is no bias in the dice right.

(Refer Slide Time: 29:26)



Now, after this we can take one more example, that is about again related with the heat transfer. Now in this example there is a cylinder having the volume of 2-meter cube there is a cylinder having the volume of 2-meter cube.

(Refer Slide Time: 30:12)



That is given which has a certain pressure in temperature it there is a certain pressure is not known and certain temperature gas is filled and there is a dial gauge on the cylinder which reads pressure with 1 percent accuracy and temperature like 2 percent accuracy after value of the displayed value.

So, there is a cylinder and on the cylinder it has pressurized gas and there is a dial gauge which shows the pressure and the accuracy is 1 percent and temperature sensor is also fixed which has the accuracy of 2 percent. I mean 2 percent of error in the measurement and pressure gauge has one percent error of measurement estimate the permissible uncertainly in the weight. So, that uncertainty is in determination of R does not exceed 3 percent right. So, that is in R. So, pressure vessel has certain gas filled at a certain pressure filled at certain temperature.

Error in the maximum possible error in the measurement of pressure is one percent maximum possible error in the measurement of temperature is 2 percent we have to determine the value of R. So, what we can do? We can way this cylinder. Now, permissible error in the measurement of cylinder way is mass is required. So, that uncertainty in the measurement of R is not exceeded 3 percent.

So, simply we will use R is equal to sorry m is equal to PV over RT that is idle gas equation. So, R is equal to PV by mT. Now, we have to find uncertainty in the measurement of R. So, uncertainty in the measurement of R is del R by del p is equal to v by mT del R per del V is equal to P by mT now again del R by del m because R is equal to PV is equal to RT. Sorry R is equal to PV is equal to mT py mT.

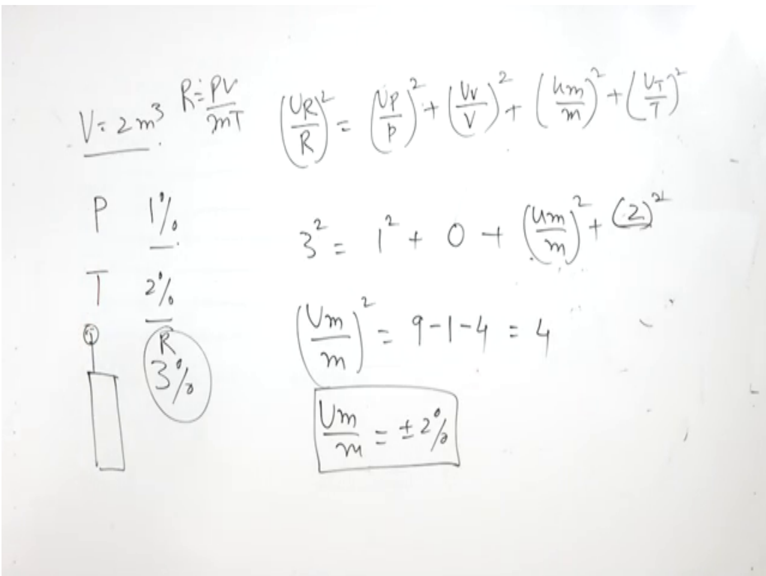
So, when we say del m then it becomes PV over T minus 1 by m square. And del R by del T is equal to PV by m minus 1 by T square.

Now, uncertainty in the measurement of R is going to be equal to del R by del P error in the measurement of p whole square plus del R by del V error in the measurement of V whole square plus del R by del m. error in the measurement of mass whole square plus del R by del T error in the measurement of T whole square and raise to the power 1 by 2. Now, error with the measurement of pressure is not known, but delta P by P is known to us right.

So, so, we can always take uncertainly the measurement of R divided by R. Now, both sides are divided by R right. Now, this side if you take this side delta RO delta p is equal to V by mT divided by R. It is going to be PV by mT n.

So, here mT mT will be cancelled out v will be cancelled out. And we will get this error in P divided by P whole square. Similarly, here we will get error in V divided by V whole square and similarly here we will get error in m divided by m whole square and error in T divided by T whole square raise to power 1 by 2.

(Refer Slide Time: 35:41)



Because the old expression is divided by PV by mT, right and now, if we further simplify this will be getting uncertainty in R divided by R. Let us say, it is whole square then is equal to this error in your or uncertainty in P by P whole square or we let us take. Plus uncertainty in V by V whole square plus uncertainty in m by n whole square plus uncertainty in T by T whole square. Now, this is uncertainty in the measurement of R is 3 percent. And so, permissible uncertainty in the measurement of R is 3 percent. So, it is 3 square is equal to this is one percent. So, one square plus volume is given, I mean the volume is not measured.

So, this is going to be 0 this is going to be 0 uncertainty in the measurement of mass we have to find out uncertainty in the measurement of temperature is 2 percent. So, 2 by; so, 2 2 2 percent. We are not divided by in this because both sides we will be dividing by 100 square, we will getting the same values.

So now uncertainty in the measurement of mass whole square is equal to 9 minus 1 minus 4 is equal to how much? 4 or uncertainty in the measurement of mass is plus minus 2 percent already in the permissible arrange the measurement of mass is only 2 percent.

So, a cylinder if you want to find the gas constant R for a gas using the idle gas equation PV is equal to RT mRT. Then error in the measurement of pressure is 1 percent error in the measurement of temperature is 2 percent.

If you want to restrict the determination of R by 3 percent error in the measurement of mass should not exceed 2 percent. Right that is all for today from the next class. We will start with the dynamic response of measuring systems.

Thank you.