Mechanical Measurement Systems
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Lecture - 14
Zero and First Order systems

Hello, I welcome you all in this course on mechanical measurement systems. Today, we will discuss 0 and first order systems. In today's lecture I will be covering zero order system.

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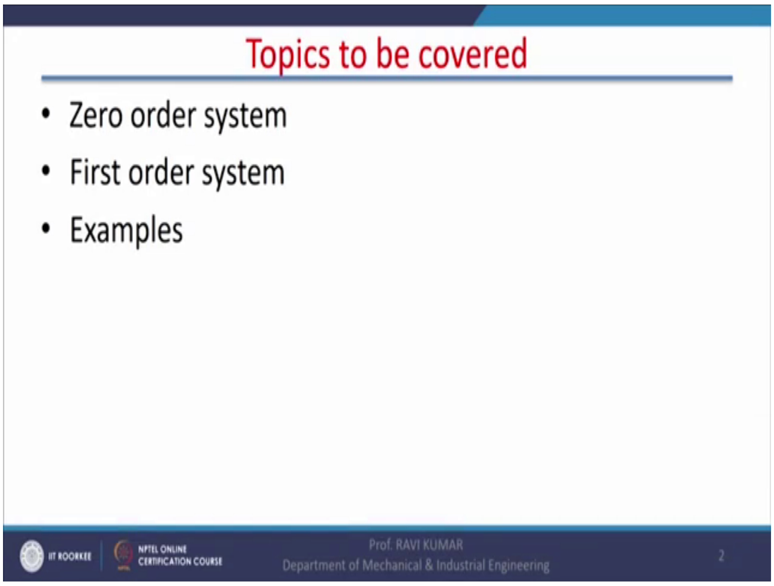

First order systems and I will be giving some example on first order system.

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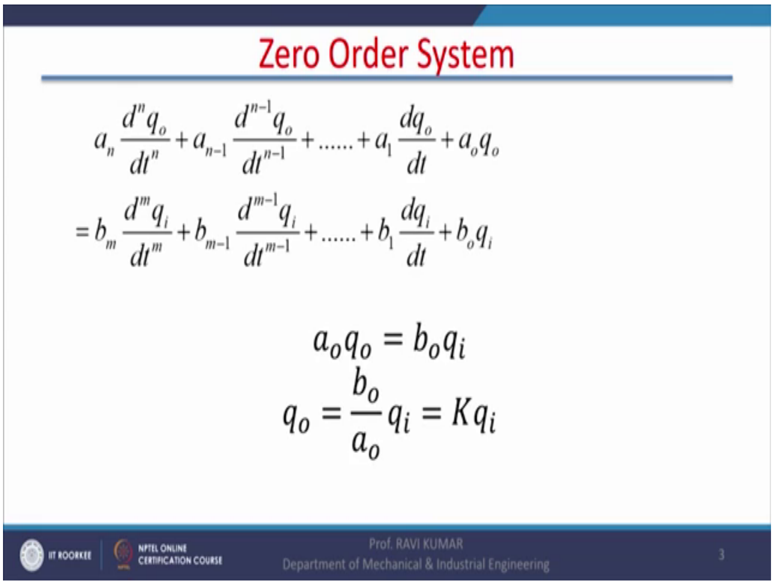

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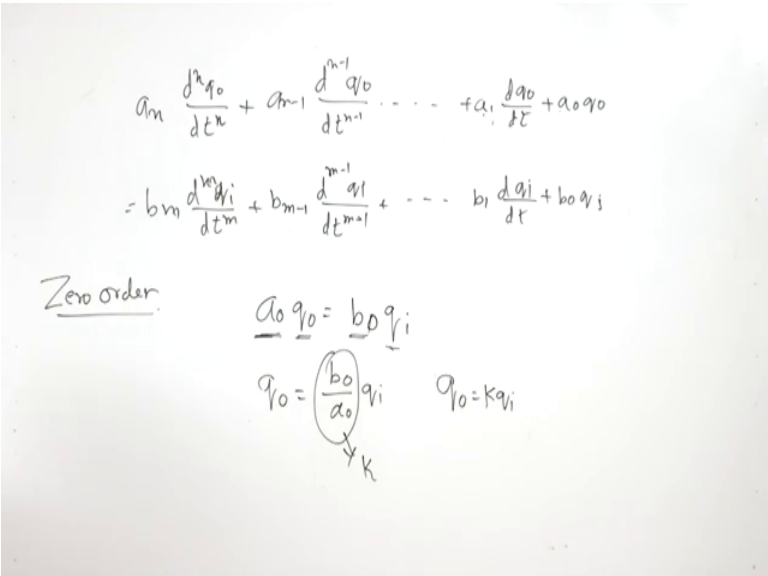

Now, as we know that there is a generalized equation for linear system that is an dn qo by dt raised power n plus an minus 1 d n minus 1 qo dt to power n minus 1 and so on plus a 1 sorry a 1 dqo by dt plus ao qo and this is equal to bn d raised to power m qi by dtm plus b, sorry, this is m bm minus 1 d raised to power m minus 1 qi dt m minus 1 and then b 1 dqi dt plus boq.

So, this equation is capable enough to address any type of linear system and it is quite adequate to handle any type of linear system, but in actual practice certain occurrence of certain incidents or certain combination is so frequent that it requires special attention, right and the simplest possible case of such occurrence is we have ao qo is equal to bo this is qi bo qi. This is the simplest combination in this combination, you will find that there is no t as well and other if you go for the higher orders, you will finding this t right, but in this combination, there is not only simply say the constant parameter output constant parameter input.

Now, if you want to have output input relationship then qo is equal to bo by a o qi right and by simply this equation and this bo by ao is nothing, but the static sensitivity k. So, qo is equal to kqi now this type of system is known as 0 order system 0 order because order of differentiation is 0 here. There is no differentiation right for input, we are talking about the input, sorry, we are talking about the output not input output input we can have this equation input, we can have this equation, but here we are yes, we are assuming the simplest case, it is going to be like this only ok. So, qo is equal to kqi.

Now, the best example of such type of 0 order system is can be elivar elivar is also a a 0 order system or a potentiometer very good example of this 0 0 order system is a potentiometer a potentiometer is a device if you remember, it can produce a gradient in emf it means suppose there is a battery connected across the wire.

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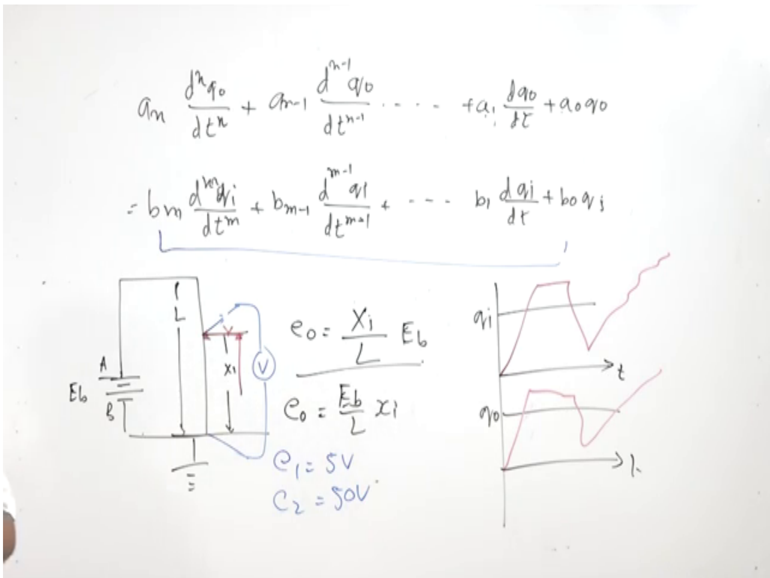

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This is potentiometer wire normally it is arranged in a spiral way I mean they are 5 or 10 or 20 depending upon the sensitivity which is required.

So, all those things will not discuss here. So, this is terminal a this is terminal b this is a potentiometer and let us say this is earth and pointer of potentiometer is at a certain distance x from here right then the output of potentiometer is going to be that is eo is going to be equal to this is x divided by length and eb, this is Eb because the gradient is Eb by L. This is the length of the wire potentiometer wire. So, this Eb by L is a period across this wire because this side it has Eb this side it is earth and this gradient multiply ah. So, the eo is the distance traveled from here so because right and this will give the output of potentiometer.

Now, here the it can be also compared as eo is equal to k Eb, this is static sensitivity no static sensitivity is xi the input is x I input is x I. So, in static sensitivity is going to be Eb by L x i because this is input this is varying right the input is varying and we are getting certain voltage as output. So, this is displacement is input and voltage is output. So, this is the perfect example of 0 order system and input output relationship is not dependent upon type.

So, the moment the input is given input is given qi the same form of output you get width t width t and the same form of output you get there is no distortion in the output right or input is not like this suppose input is like this for example, input is like this output also, you will be getting like this. So, output will follow exactly follow the input and there is no time length time length is 0. So, it appears to be a perfect example of 0 order system, but as we said as we know all of know that all of us know that no physical model can replace the actual system.

So, here also if you I mean look at the system you will find that this system also is not a perfect system reason being when we measure the output for the measurement of output will have to provide a voltmeter and voltmeter none of the voltmeter has infinite resistance. So, voltmeters have certain resistance and definitely due to that resistance they will draw a certain current right. So, for a particular location voltmeter will draw a certain current.

Now, this status is moving right I say suppose this if v v 1 reading suppose e one reading is let us say 5 volts voltmeter will draw certain currents current now e two reading is fifty volts. So, current to the voltmeter will increase now this will cause change in current and theoretically we have assumed that this wire is a pure resistance, but again there is a difference in I mean physical modeling and the actual things because in actually this wire will have certain self inductance or capacitance due to this the signal will be distorted.

So, that is one factor which if we if we close you look at this is not exact depiction of 0 order systems theoretically it maybe for practically there are certain issues there are second issue is loading effect right for loading effect suppose this I want to measure because this 0 order system is normally used for measuring the displacement. So, when we are using the displacement. So, this pointer is loaded this pointer is loaded some displacing device this pointer is loaded with certain displacing device that is how it is the displacement is taking place

Suppose the moment of pointer is in this direction and load is coming in this direction the inertia of this pointer will also hamper the actual or it will disturb the actual displacement. So, input signal will also get disturbed due to inertia of this pointer right. So, in order to curb that the pointer has to be as light as possible this has to be as light as possible this contact device.

So, that minimum energy is consumed in measurement second thing the movement has to be very slow if it is a slow movement in that case the capacitance and this reactance in self inductance in this wire will be minimum. So, movement has to be very slow this has to be very read, but the movement is very quick. So, the effect of this induced effects due to induction and capacitance cannot be neglected right

So, this is about the zeroth order system.

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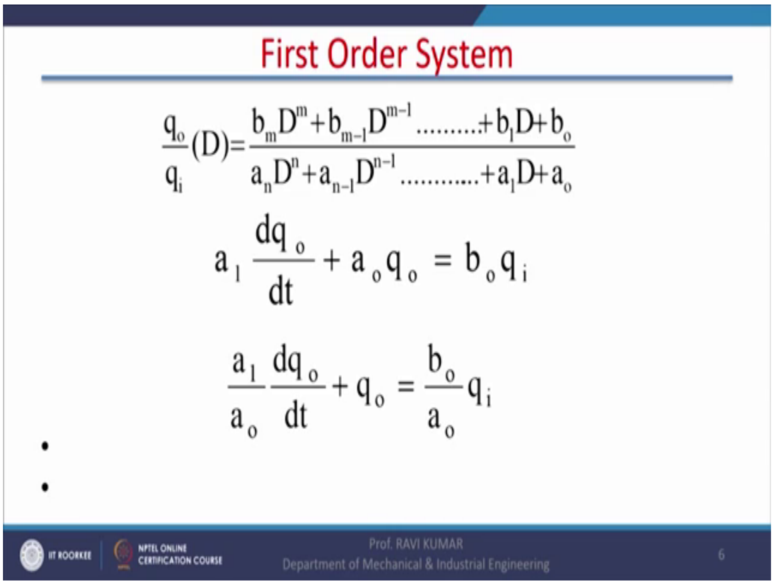

Now, we will start with first order system for first order system first of all.

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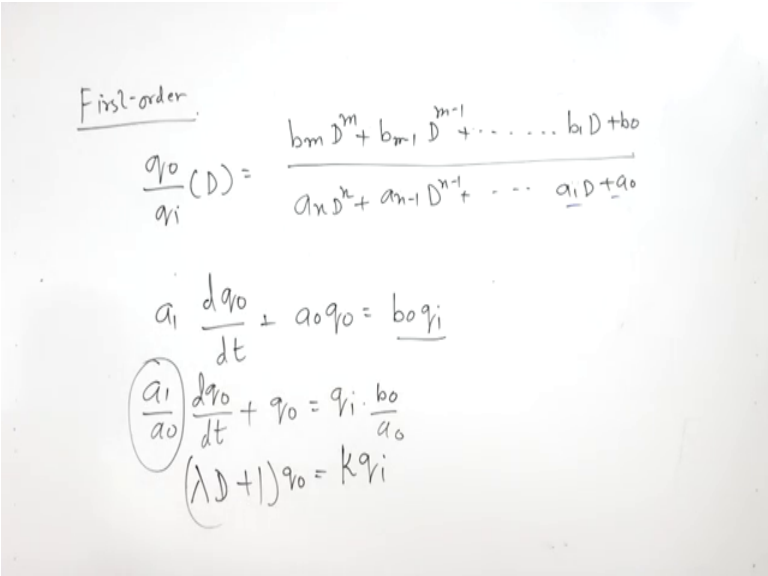

You will write operator qo by qi d is equal to bm Dm plus bm minus 1 D m minus 1 plus and so on D 1 D plus bo divided by an D raised power n plus an minus 1 b raised to power n minus 1 plus a 1 D plus ao right.

Now, we here we will take output of first order differential first order differential. Now if you take output of first order of deferential then we will get a 1 Dqo by dt plus ao qo is equal to bo qi we could go for higher differential forms, but we have found that for many of the engineering applications bo qi is sufficiently good.

Now, in this case again, we further modify this [FL] a 1 by a o Dqo by dt plus qo is equal to qi multiplied by bo by ao now this a 1 by ao is known as lambda time constant and this can be represented by lambda D plus 1 qo is equal to this bo by ao is static sensitivity qi and further this operational transfer function for.

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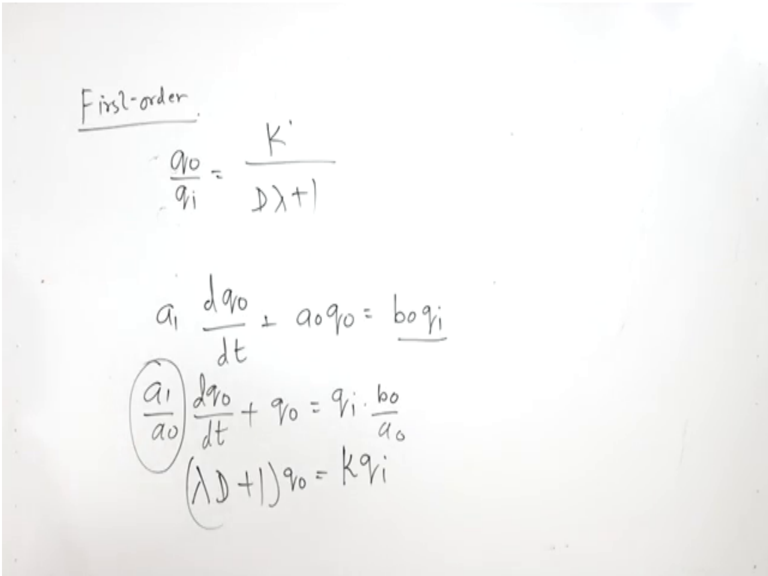

First order system that is qo by q i can be represented by k by D lambda plus 1.

Now, for special cases for example, step input or ramp input for such type of cases we will separately deal with those cases and now we can take some examples of first order system and the best example of the first order system is mercury in glass type of thermometer if we want to go for a mechanical example of first order system that is mercury in glass type of thermometer.

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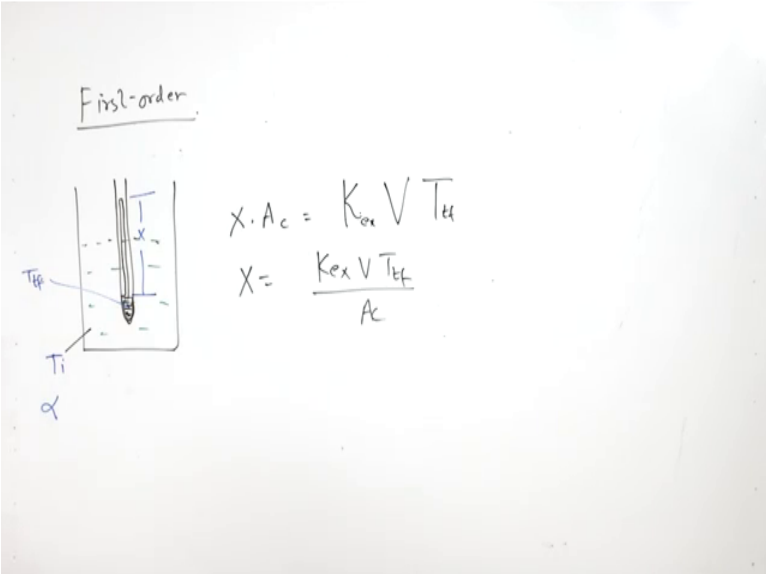

Supposed there is a vessel right and vessel is filled some water some water at a particular temperature, right and this water temperature is measured with the help of a mercury in glass type of thermometer it has a bulb the thermometer has a bulb right and the water temperature is the water temperature is ti initial temperature of measurement the this temperature is Ttf right and alpha is expansion coefficient because the mechanism of the temperature measurement is the energy from the fluid surrounding fluid enters the bulb and the fluid in the bulb expands and when the fluid in the bulb expands the level of the fluid rises in the capillary deep inside the thermometer.

And because when the fluid rises there is a lag also inertial lag also, but this inertial lag is negligible if you compare the inertial lag in transmission of heat from the fluid to the bulb and bulb to the inside fluids. So, this is often neglected x is the, height of the fluid in the capillary in these thermometers, it is also ensured that the volume of capillary is negligible in comparison the volume in the bulb. So, it is always assumed that the fluid in the bulb the mass of the fluid inside the bulb is constant.

Now, in this thermometer the height of capillarity of x x is multiplied by cross section area is the volume of the fluid inside this capillary right and this change in the volume by virtue of change in temperature is going to be equal to expansion coefficient k expansion right multiplied by the volume multiplied by T tf right and this T tf is 0 when x is equal to 0.

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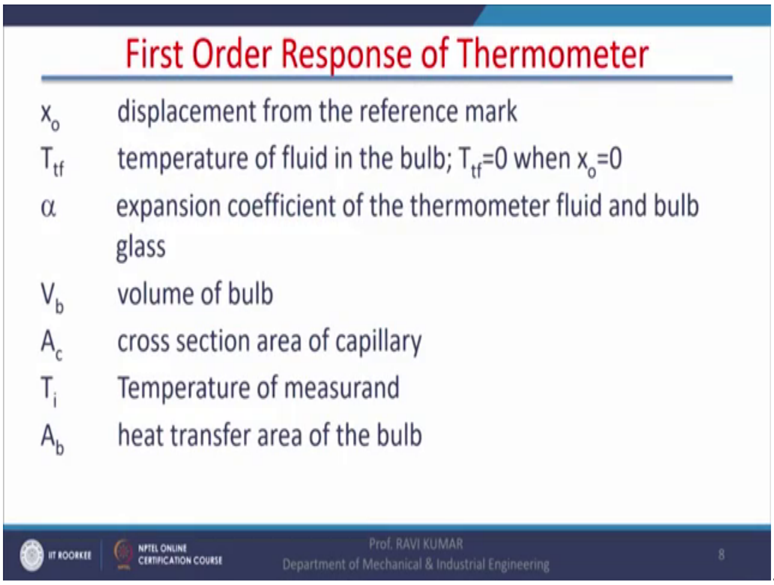

So, this is the T Ttf is change in temperature right and then from here we can find the value of x as Kx v Ttf by cross section area right.

Now, we will do the energy balance energy balance means the energy entering the bulb the energy which is entering the bulb is overall.

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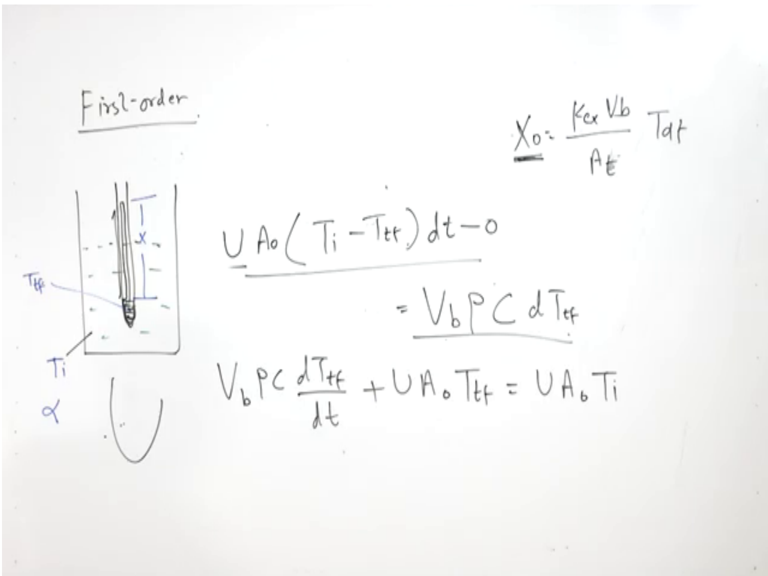

Heat transfer coefficient bulb surface area initial temperature this is the initial temperature and this is the Ttf final temperature of the fluid in the bulb and because this is energy transfer rate multiplied by dt this is the energy going into the bulb no energy is coming out of this control volume. So, this is 0 and this energy is used in changing the interval energy of the fluid mcp delta t. So, mcp delta t is going to be volume of the bulb density of the bulb. So, energy going to the bulb no energy is leaving the bulb and this is the energy stored in the bulb right and this energy stored in the bulb will ultimately change the volume of the bulb fluid in the bulb and the level of the fluid will rise in the tube.

Now, there are certain assumptions in this analysis and the assumptions are keep bulb wall this is the bulb wall both sides, it is a pure resistance that is the first thing and there is no heat storage capacity in the film u is constant.

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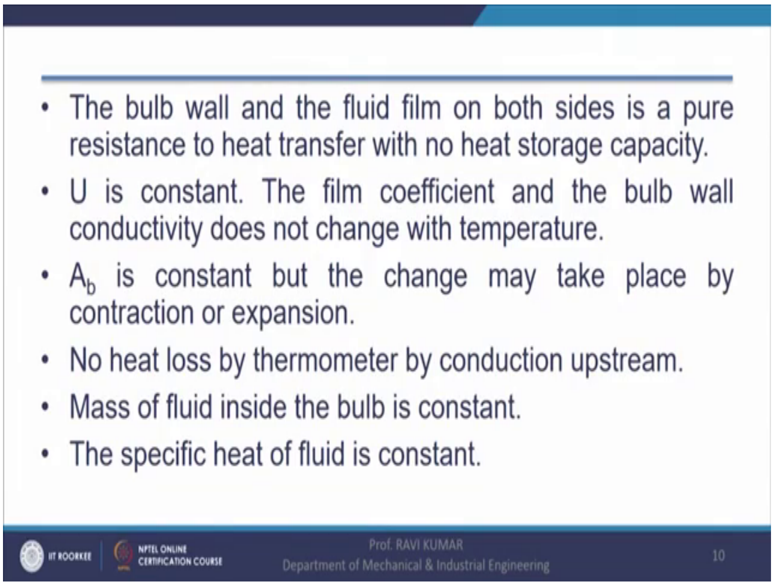

We are assuming here u s constant this is these are all assumptions any of the bulbs any of the bulb this, this is also constant right, but this may also change because when the bulb is surface is heated it may explain or when the temperature is reduced the surface area may reduce.

There is no heat transfer in upstream direction all the energy coming to the bulb is stored in the bulb itself. So, that is another assumption another is mass in the bulb it remains constant. In fact, it does not remain constant, but mass moving out of the bulb is negligible if you compare the mass of the fluid inside the bulb and specific heat of the fluid is constant. So, every derivation has certain assumptions. So, this derivation has also certain assumptions.

Now, if you remember the previous expression that xo is equal to expansion volume of the bulb divided by area area of the tube and Tdf right then we can have a equation in terms of xo also now in order to have equation in xo we will this equation this equation can further be written as now this equation is modified Vb rho sorry Vb rho C d Ttf by dt plus UA area of the bulb Ttf is equal to u U ao and T i this is arranged in a different form because this dt has been taken we have taken the first order this D Ttf by dt right.

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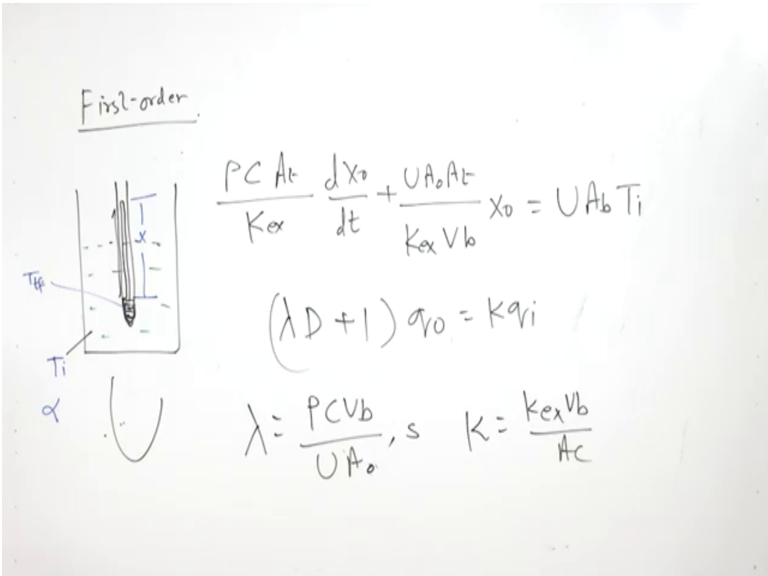

Now, in this equation if we replace this differential of this with differential of this, now, D Ttf here we will get expression in terms of dxo and the final equation may emerge as rho c a this is cross section area of the tube minus k expansion coefficient t xo by dt plus UA Ao at by kx vb xo plus u sorry is equal to u Ab

Now, if we look at here you will find that if you if you compare with this one lambda d plus o1e into qo is equal to k qi right. Now, qo is output xo right and qi is temperature of measurement

So, now we if you manipulate this equation again you will be getting a time constant lambda s rho Cv b divided by UAb seconds your time constant is always in seconds and k is equal to static sensitivity s kex Vb by Ac now this is going to be the time constant and this is going to be the value of k if you want to have minimum time constant then U and Ab has to be or area of the bulb or Ao has to be high and these values have to be low if you want to have a static sensitivity high then this has to be high this has to be low.

Now, regarding this, we can take one numerical.

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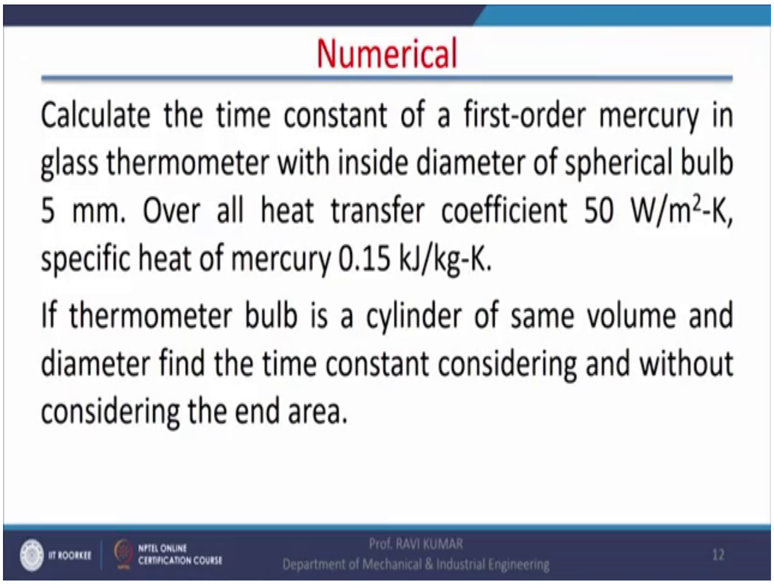

And this numerical says that calculate the time constant of first order mercury in glass thermometer with inside diameter of a spherical bulb 5 millimeter overall heat transfer coefficient is given. So, inside diameter of a spherical bulb is 5 millimeter.

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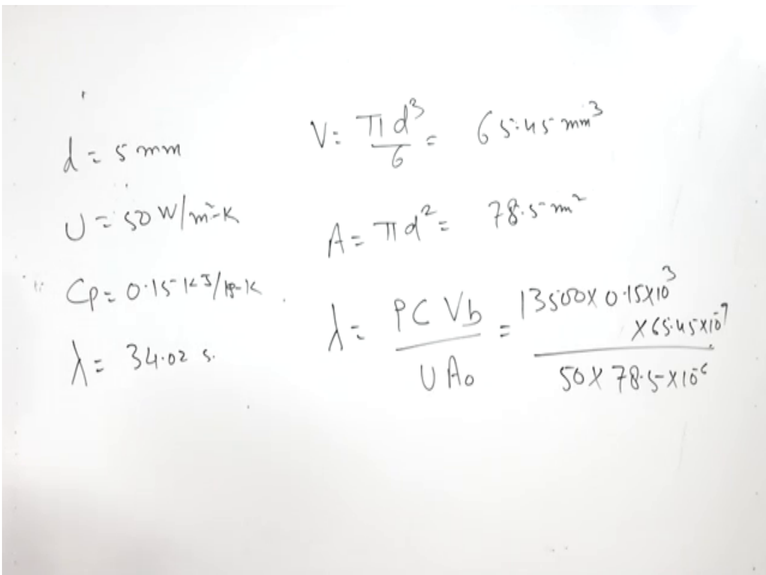

Overall heat transfer coefficient is given 50 watt per meter square Kelvin, right and specific heat of mercury is given 0.15 kilo joules per kg Kelvin.

If the thermometer bulb is a cylinder of same volume and diameter of 500 time constant considering without considering the end area. So, volume of the bulb is pi d cube by 6 and if you are putting the value of d as a 5 millimeter the volume is 65.45 mm cube is the volume of the bulb area is pi d square and again the area we are getting 78.5 mm square. So, here this is this is 78.5 ok. So, lambda is time constant here in this case is rho c Vb divided by UAb or area of the bulb this we have already derived.

So, density of the mercury is thirteen thousand 5 hundred kg per meter cube this is a specific heat of mercury volume we have taken 65.45 into 10 to power minus 9 when we will go into the meters ah, then divided by overall heat transfer coefficient overall heat transfer coefficient is 50 multiplied by outside area and that is 78.5 into 10 to power minus 6, if you simplify this, you will be getting the time constant for a spherical bulb as 34.02 seconds.

Now, instead of having a spherical bulbs, we want to have a cylindrical bulb, if for the same volume the bulb is made cylindrical when the bulb is made cylindrical volume is same. So, volume we are maintaining same that is volume is 65.45 mm cube right.

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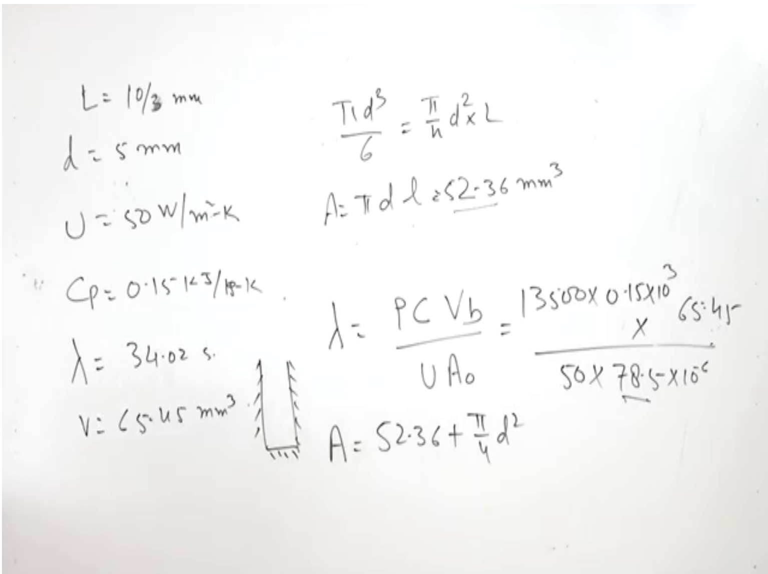

Now, when it is a cylindrical we have to find the length of the cylinder. So, length of the cylinder is pi d cube by six is equal to pi by 4 d square into l and from here the l is the length of the cylinder is 10 by three sorry 10 by 3 mm. Now, in this case, the area of the surface area of the bulb will change volume will remain same. So, density cp and volume the volume will change and if you volume if you calculate the volume by pi dl right and then in that case 52.36 mm cube is the volume. So, 52.36; this 65 point this will be right. So, this is not volume this is surface area and this is volume. So, volume is pi r square right. So, volume is same. So, volume we are maintaining same. So, volume is 65.45 65.45 and this surface area will be replaced by the surface area that is it and we will get another time constant.

Now, if one side is considered, now we have considered only outside area of the cylinder. Now if we end at the end of the cylinder also, then again the time constant will change and that because the volume in both the cases is same. So, area is going to be equal to 52.36 plus this area will also be added, that is it. So, this area is pi by 4 d square and again, we will replace this area here volume is same rest of the things are same and we will be getting another time constant. So, the surface area and volume ratio of cylinder or the bulb also as causes also causes the change in time constant and ultimately response of first order system detailed analysis of first order system will start in subsequent lecture.

Thank you very much.