Mechanical Measurement Systems
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Lecture – 15
First Order System- Step Response

Hello. I welcome you all in this course of mechanical measurement system and today, we will continue to discuss first order system.

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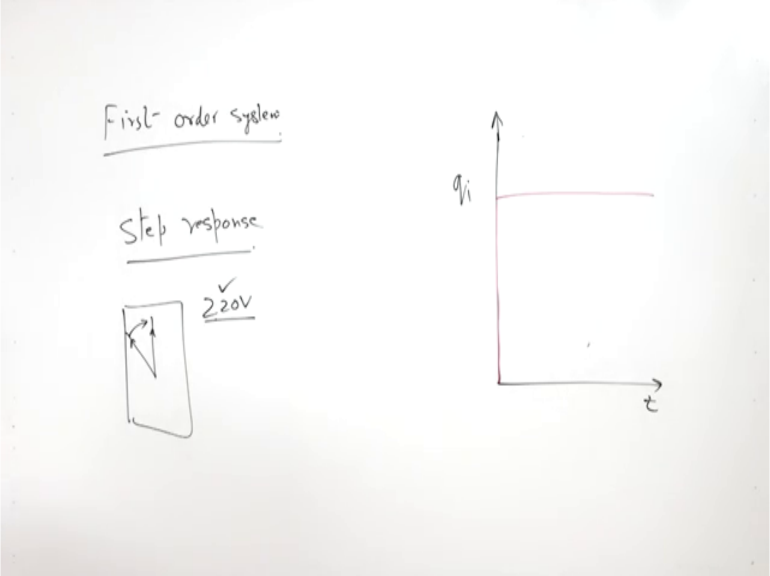

Now, in today's lecture, we will discuss the step response of first order system and we will take a few examples also to have a clear insight of how the system behaves first for the step input. So, first of all, let us start with the step input to the system. So, when there is a step input to the system on x axis it is time on y axis, it is q i input quantity. So, when there is a step input to the system.

The input characteristic curve is something like this at time is equal to 0 the input has increased from 0 to certain value for example, in a voltmeter in a voltmeter when voltmeter is connected to a main line, right and just you when the moment you connect it, right the input is 220 volt, whatever the volt voltage is there in the main line the input voltage is 220 volt right, it is the voltage which is in the main line.

But the voltmeter these analog voltmeters take some time to display that value and this is the this is how we judge the response of the system whether it moves very fast or the needle moves in a very sluggish manner. Another example of step input can be given with the mechanical point of view is like quenching of rods or putting thermometry in a boiling water.

So, if the temperature of the thermometer, thermometer is an atmospheric temperature 25 degree centigrade.

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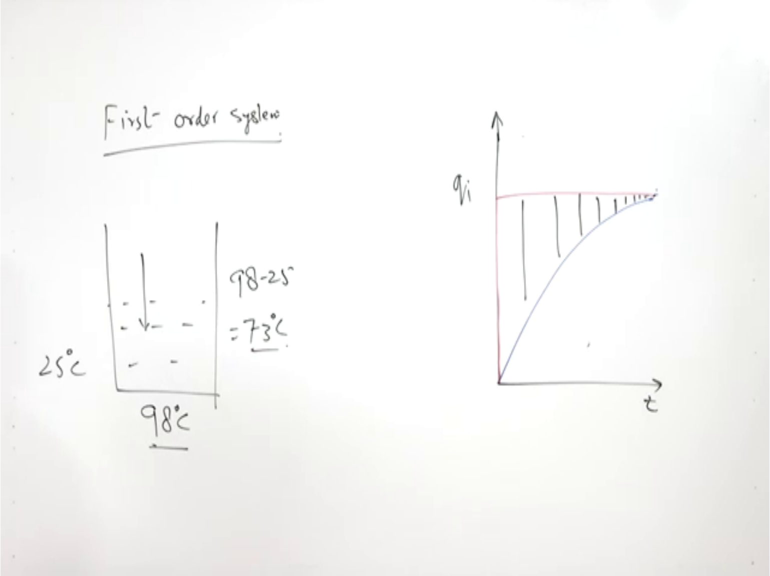

And in the pan there is boiling water. So, the moment the thermometer is dipped in the boiling water, water is boiling at let us say 99 degree centigrade or 98 degree centigrade.

So the moment the thermometer is putting to the vessel, there is a step input to the thermometer and step input of temperature is 98 minus 25, it is going to be 73 degree centigrade. So, mathematically we can say there is a 73 degree centigrade step input to the thermometer and input curve can be shown like this. Now response of thermometer response of the thermometer or the voltmeter in ideal case, it has to be like this it should follow the input curve, but due to the inertia of the instrument and due to many other reasons.

The output of the instrument for the first order instrument goes like this and after certain period it is it becomes very close to the input and if you look at the error in the measurement point of view in this reason the error is high and slowly the error is reducing and a steady state error is at whatever the error is an instrument that will remain.

But this error is diminishing with time this is dynamic error because error is dependent upon the temperature sorry the time. So, this is a dynamic error and it is diminishing now we will do analysis some mathematical analysis of this type of system.

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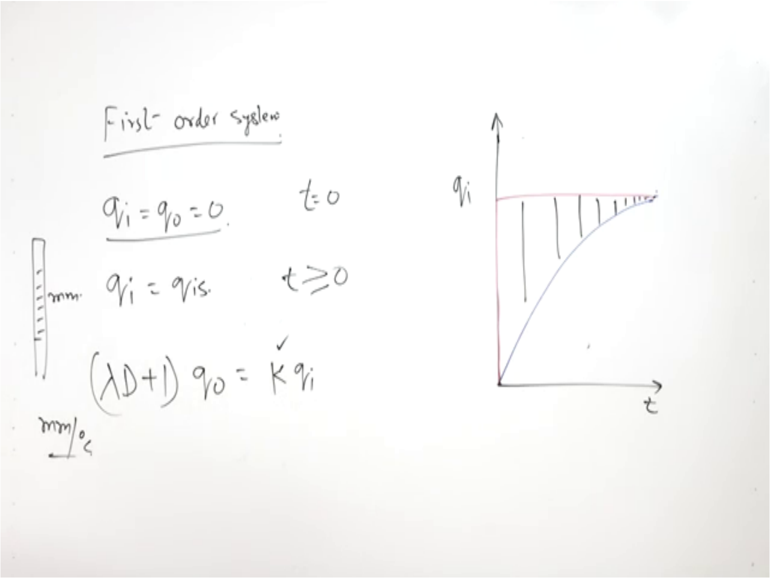

So, for the purpose of mathematical analysis first of all we will start with input qi input or output both are 0 at t is equal to 0. So, when t is equal to 0 initially input and output both are equal and both are 0. So, we are somewhere here now with time input has increased to q i s, but output is not in this qi’s output in start increasing and after certain time interval. it will become close to the input.

So because this is a first order system. So, for first order system because this, this case is t is greater than or equal to 0. Now if you remember the first order equation the first order equation is lambda d plus 1 q o is equal to K q i , this is the first order equation generalized equation. Now, here K is sensitivity because when we are measuring for example, temperature with thermometer input is in degree centigrade output is it millimeter displacement of mercury column in the thermometer and that is in millimeter displacement of mercury column in the thermometer that is a millimeter. So, in that case the sensitivity is the dimension of the sensitivity will be millimeter per degree centigrade mm per degree centigrade. So, K has to be taken into account.

Because we have to measure the dimensions also now here if we try to find solution for this with the help of initial conditions, at t is equal to 0. So, first of all we will take complementary function.

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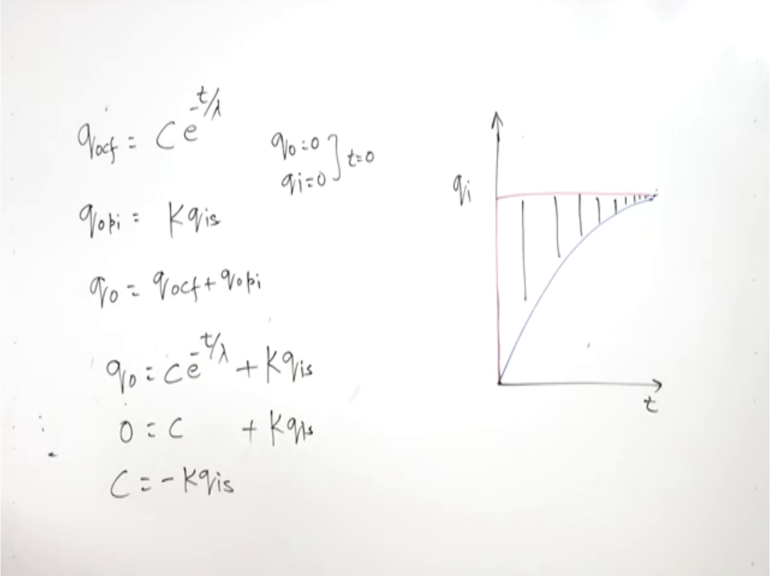

$$q_{ocf} = C e^{-t/\lambda}$$

$$\left.\begin{matrix} q_o = 0 \\ q_i = 0 \end{matrix}\right\} t = 0$$

$$q_{opi} = K q_{is}$$

$$q_o = q_{ocf} + q_{opi}$$

$$q_o = C e^{-t/\lambda} + K q_{is}$$

$$0 = C + K q_{is}$$

$$C = -K q_{is}$$

So, complementary function is going to be equal to lambda d. So, in order to find complementary function, we will take lambda d plus one is equal to 0 and it has one root and the root is minus 1 by lambda for complimentary function, we will solve this equation and we will try to find the roots for d and because it is order of d is 1 and the roots is minus 1 by lambda and the solution for this complimentary function will be C e raised to power minus t by lambda , if you remember, in my earlier lecture then you can easily conclude that complimentary function is going to be like this.

Now, particular integral particular integral

is going to be equal to K q i s this is this is input quantity i s q i s input quantity q i s right now we will add these two and we will find the solution as qo is equal to q o complimentary function plus q o particular integral and this will give the solution of equation as qo is equal to C e raised to power minus t by lambda plus K oh yes sorry K q i s kqis fine. Now, here we will put the initial condition initial condition is qo is equal to

0 when q i is equal to 0 at t is equal to 0. So, this is 0 right and then K q is. So, this is 0 is equal to C e raised to power minus t by lambda plus kq is now here in this case because at t is equal to 0.

So, this term will also become 1. So, C is equal to minus K q i s right and when we are we manipulate this c in this equation we will get the finalized form of the equation as.

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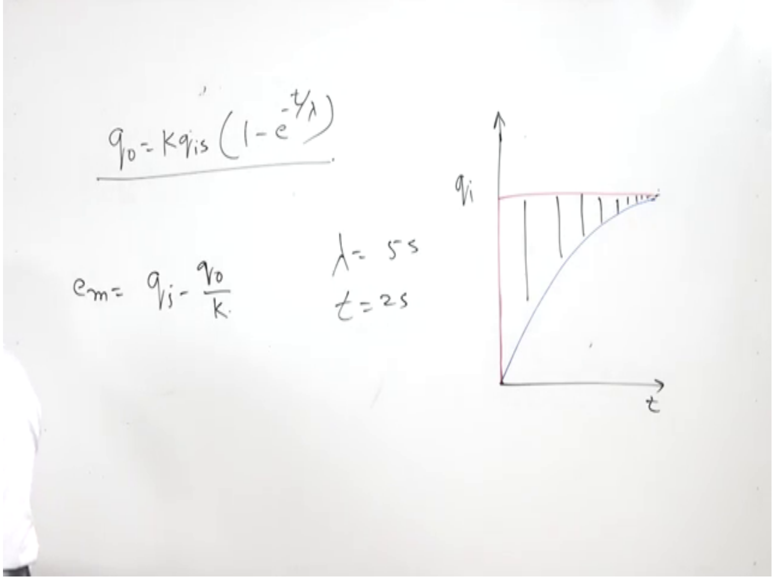

Q o is equal to K q is 1 minus e raised to power minus t by lambda because from here we have derived the value of c and by putting the value of c in this equation because in this equation there is only one known. So, one boundary condition is sufficient.

so in order to find this constant we have just manipulated the values for from initial conditions and this is the final equation now from this equation you can easily judge that for large value of t this expression is going to be this value is going to be 0 or tending to 0 when t is tending to infinity this value is tending to 0 and then qo is equal to K q is ok. Now before bellow for certain value of t, t can be less than lambda also in that case some error will be there in the measurement.

Because lambda is 5 5 seconds and we make measurement of a 2 seconds, t is 2 seconds. So, definitely we will find that there is a significant error in the measurement. So, in order to find in the error in the measurement e m for the dynamic response is q i minus q o upon K that is input minus input we are getting from here q by K.

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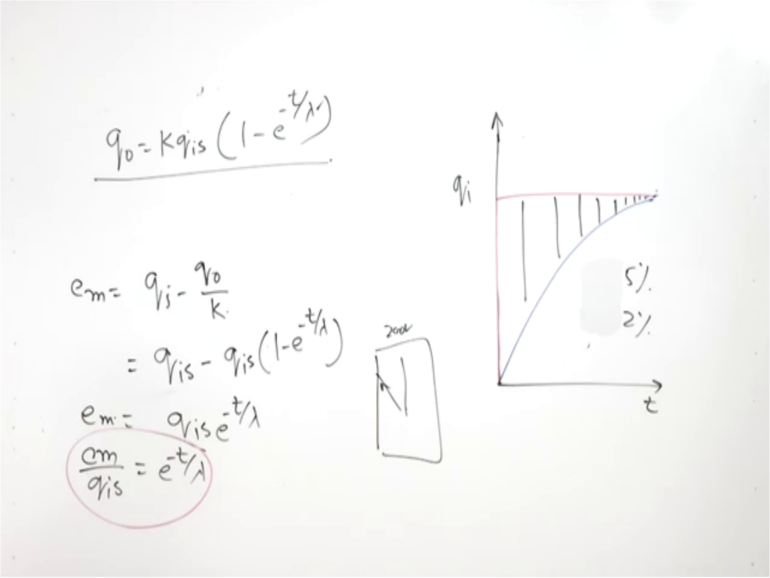

And this will lead to here in this case as q i s minus from here, it is q is one minus e raised to power minus t by lambda and it is going to be equal to q i s e raised to power minus t by lambda this is error in the measurement and definitely it is divided by K and q by K it is already divided.

So, we will not do this right or we can say that error in the measurement divided by input is equal to e raised to power minus t by lambda in many ways it can be written. So, this is a non dimensional form, this one we can say a non dimensional from form of the error in measurement e raised to power minus t by lambda now after this.

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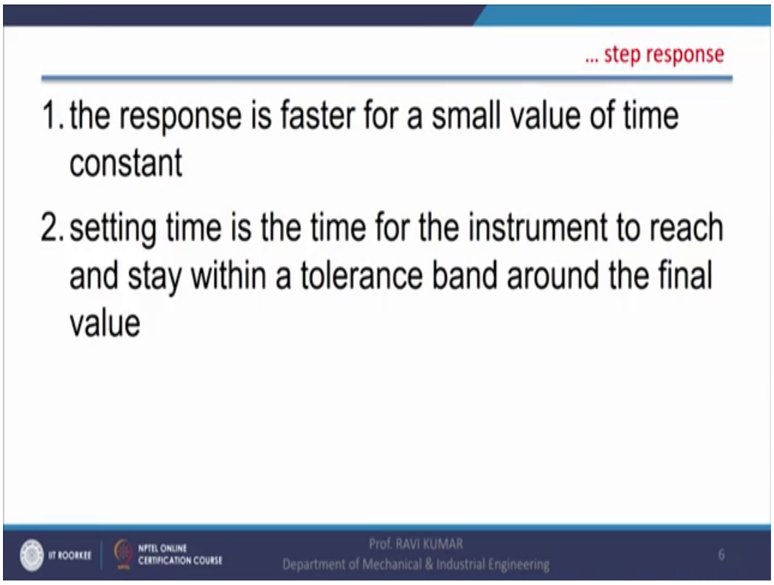

... step response

1. the response is faster for a small value of time constant
2. setting time is the time for the instrument to reach and stay within a tolerance band around the final value

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so response of the system as we can see here response of the system is faster when the value of lambda is small when the value of lambda is small the system will respond quickly right and there is a settling time for the instrument settling time means when input is given the output starts changing right and after certain time interval, it gets settled in the tolerance limit of the measurement that you can say the measurement is in the range of five percent or two percent of the measurement.

So that is the settling time and in first order system there is diver overshoot you must have observed in some of the instrument when input is given , suppose in a voltmeter you must have observed in this some voltmeter also when input is given this happens especially in the electrical instruments right and then the input is two hundred volts , but it will overshoot and then after certain oscillations it will come to restore to the steady state this happens some of the in some of the weighing machines also various prints are used.

The needle will when we put the weight on the weighing machine that is also sort of step input right you are just putting one kg of the object to the weighing machine then needle will start oscillating. So, that is not this is not this kind of response is not. The first order first order system response in first order system when you put the way on the weighing machine the needle will move and it will attain the value in within the tomes limit of the measurement and this is known as settling time.

so in first order system if the weighing machine is a first order system if weighing machine is a second order system then oscillations will take place, for a quick response of any system first order system.

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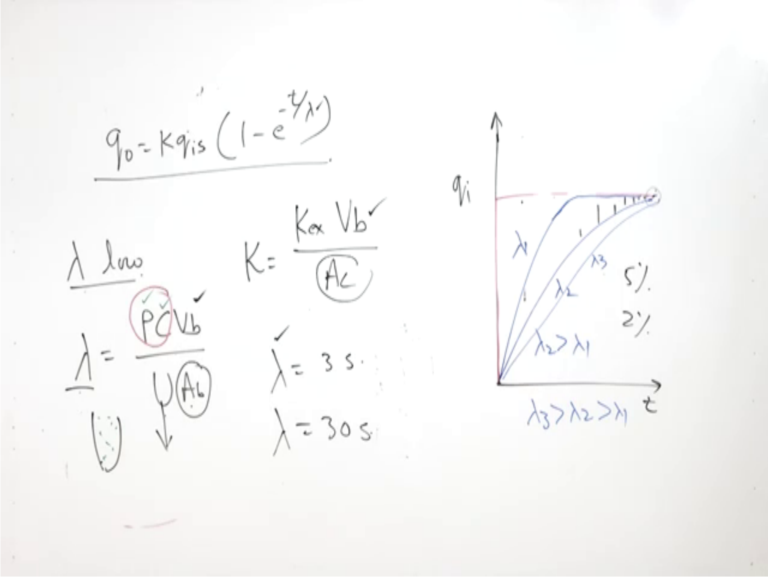

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The lambda has to be low the value of lambda has to be low and. Now let us recall our previous derivation for the thermometer where we calculate the value of lambda as rho c V b divided by UA b right now rho is a specific heat of the bulb fluid which is filled in the bulb sorry rho is the density rho is the density of the fluid which is filled in the bulb there is a bulb in the thermometer.

So, whatever fluid is filled in this bulb the density of that fluid is rho c is the specific heat. So, rho l c has to be high because the these are physical properties of the fluid. So, we cannot control them their physical properties of the fluid whether you are using mercury or some other fluid, but in any case the product of rho l c has to be high as high as possible right.

And then there is a term V b volume of the bulb sorry for lambda it has to be as low as possible sorry, I am sorry it is not as high as possible rho c has to be as low as possible as high possible the it will increase the lambda. So, in order to have low value of time constant this rho c has to be as low as possible and then V b volume of the bulb this has to be as low as possible u as high as possible and Ab as high as possible.

Now here there is a problem the problem is when we talk about this K for thermometer right sensitivity then it is about e x V b and ac this is sensitivity of thermometer, I am talking about liquid in bulb type of thermometer. So, here in order to have high sensitivity we should have high bulb volume right this is the expansion coefficient this is cross section area of the tube where the fluid is rising.

So, now we have to strike a compromise if you want to have low value of lambda then V b should be low volume of the bulb should be low and if you want to have high sensitivity then value of volume of the bulb should be high. So, a strike a compromise has to be strike between struck has to be struck between these two. Now Av of the bulb it should be as high as possible and u overall heat transfer coefficient now this is important we cannot control overall heat transfer coefficient.

But we can always assess the overall heat transfer coefficient suppose the thermometer is measuring some high temperature fluid which is flowing in the pipe in that case u will be high when thermometer is measuring the still air for example, air in this room then u will be low. So, it is possible that the thermometer we are using is the application is flow off to measure the temperature of liquid metal in a pipe liquid metal flowing in a pipe, u will be quite high or flow of hot fluid or hot oil flowing in a pipe, u will be high time constant will be low time constant let us say maybe around three seconds for example, another application of the same thermometer, I want to measure room temperature this room temperature and the air is not moving time constant maybe thirty seconds because time constant will depend upon u.

So there is no fixed time constant for this type of thermometer. So, when you purchase or when anybody purchases thermometer and if you are interested to know the time constant then you must ask the vendor or you must ensure that for which application you are going to use this thermometer suppose. So, it is a specified three seconds, but you are using the air temperature the time constant is thirty seconds that will creep a lot of error in that will result in the creeping of a lot of error in your measurements.

Now for a step input, if we have different values of lambda, right, suppose the lambda is very low the value of time constant is very low then lambda is very low means the response of system will be like this it will not overshoot, it will go like this not even like this it will be like this. So, suppose this is lambda 1 and this is lambda 2 and lambda 2 is greater than lambda 1, in this case, the response is sluggish, you have another instrument which has again, lambda 3, this is lambda 3. So, lambda 3 is greater than lambda 2 is greater than lambda 1 , right.

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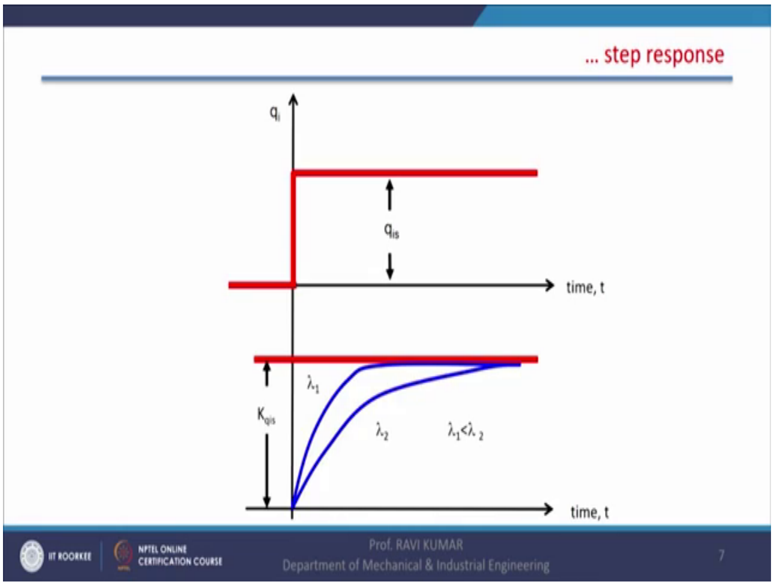

... step response

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So, this is how the system response responds for different time constants.

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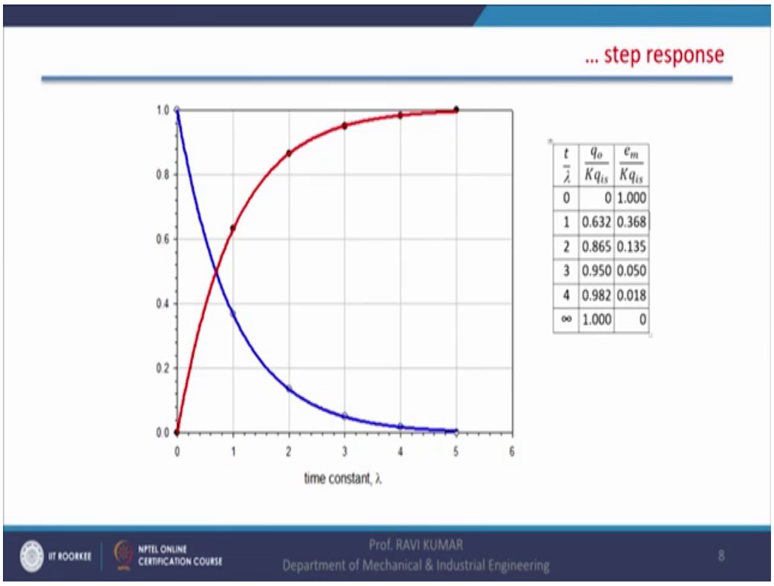

... step response

$\frac{t}{\lambda}$	$\frac{q_o}{Kq_{is}}$	$\frac{e_m}{Kq_{is}}$
0	0	1.000
1	0.632	0.368
2	0.865	0.135
3	0.950	0.050
4	0.982	0.018
∞	1.000	0

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Now, if we draw a curve for different values of time constant first of all.

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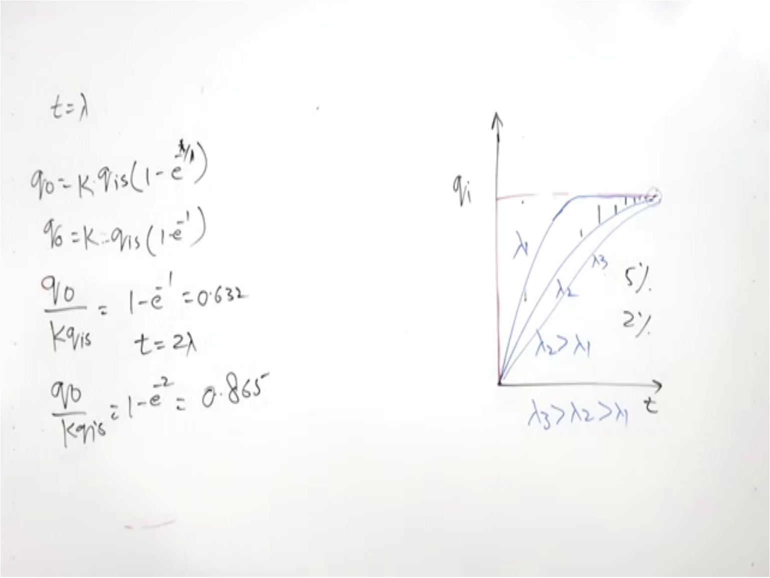

$t = \lambda$

$q_o = K \cdot q_{is}(1 - e^{-\lambda/\lambda})$

$q_o = K \cdot q_{is}(1 - e^{-1})$

$\frac{q_o}{K q_{is}} = 1 - e^{-1} = 0.632$

$t = 2\lambda$

$\frac{q_o}{K q_{is}} = 1 - e^{-2} = 0.865$

Let us say t is equal to lambda when t is equal to lambda in that case q o is equal to q i s 1 minus e raised to power one by one or t by t or lambda by lambda or q o is equal to q i s one minus e raise to power minus 1 right and when we no we we can take we just.

Student: (Refer Time: 22:31).

Q o is equal to K qis sensitivity also we have taken into account now if you take qo by K u q i s qo by K q is it is going to be 1 minus e raise to power minus 1. So, if we take this value it is a constant it is 0.632 right.

So, we can say that when a step input is given to the system, we can define the time constant. Now when the step input is given to the system it is the time required by the system to attain 63.2 percent of step input 62, this is a common mistake, not 62.2 percent of final value because now we have started here in this case, we have started step; step input from 0 that is final value minus initial value I will solve one numerical then it will give you the clear insight.

So, we can say that it is time required by the instrument to attain the 63.2 percent of step input now if instead of 1, suppose I take t is equal to 2 lambda, then I will be getting q o by K q i s is equal to 1 minus e raised to power minus 2 and this is going to be equal to 0.8865.

Now we are closing the time is increasing, we are closing to we are going closer to the input. Now again we will take t is equal to 3 lambda.

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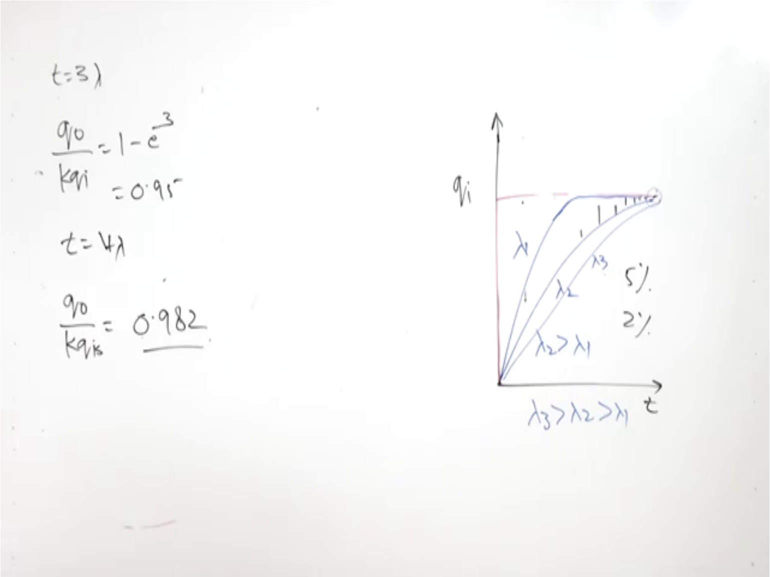

$t=3\lambda$

$\frac{q_o}{Kq_i} = 1-e^{-3}$

$= 0.95$

$t=4\lambda$

$\frac{q_o}{Kq_{is}} = \underline{0.982}$

When we take t is equal to 3 lambda, then we get q o by K q is as 1 minus e raise to power minus 3 and this gives the value 0.95. Now, we have reached the 95 percent bend for 5 percent bend of the final value. Again, we take t is equal to 4 lambda when we take t is equal to 4 lambda, then q o upon K q i s becomes 0.98 to within 2 percent and for 5 lambda, we assume that we have attained the steady state a steady state has been attained and the value has surely reached in the tolerance kind of the measurement.

So, I have shown here in this screen there are two figures the red one is for different values of t and lambda qo upon K qis as you can see the value is increasing the value is increasing for different value of time constant starting from 1, 2, 3, 4 and 5 at 5, you can see it is almost 1 fine. Now for the error in measurement, now error is measurement is if we look at the error.

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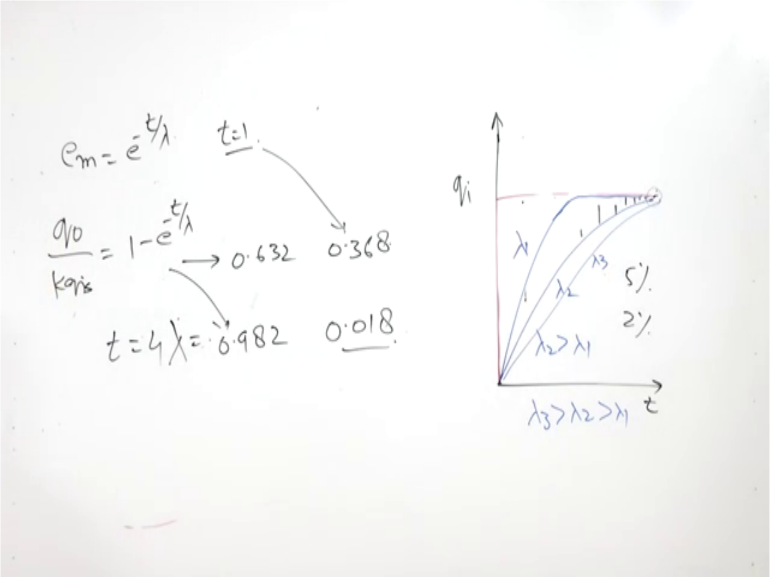

In measurement error, in measurement is error in measurement is e raised to power minus t by lambda.

Dynamic error in the measurement, if you refer back , then error in measurement is e raised to power minus t by lambda and here we have measured qo by K q is that is a response of the system e one minus e raised to power minus t by lambda. So, when the response was here for t is equal to one if you remember, for step input, the response was 0.632, this is going to be one minus 0.632.

So, it is going to be 0 point three six eight this error in the measurement and finally, when lambda is equal to t is equal to 4 lambda in that case when this input output input relationship was 982; 0.982 the error in the measurement is going to be 0.018. So, I have drawn both the curves for different value of t and t by lambda starting from 0, 1, 2, 3 and 4 and then output is you can say, it is rising the output of the instrument is riding rising and it is attaining close to the input, here I have written infinite, but if you go for the five lambda or six lambda, it is they are going to be equal and error in measurement is also diminishing and at the lambda is equal to five the error in the measurement is less than one 0.08 percent.

Because at the lambda is equal to 4 t, t is equal to 4 lambda, it is 1.8 percent, right. So, this is the characteristic curve for error in measurement and the output of the instrument.

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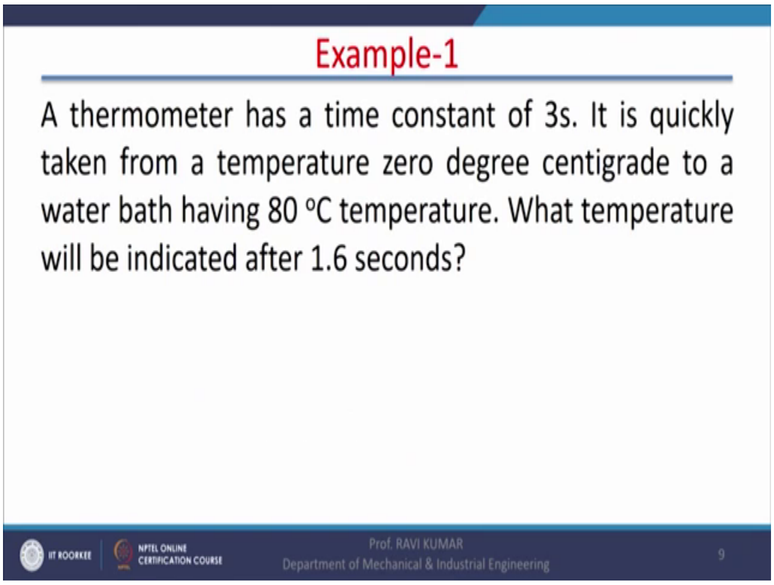

Example-1

A thermometer has a time constant of 3s. It is quickly taken from a temperature zero degree centigrade to a water bath having 80 °C temperature. What temperature will be indicated after 1.6 seconds?

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Now, we can take one example. Now, in this example, it is stated that a thermometer has time constant of 3 seconds. So, time constant for thermometer is three seconds right.

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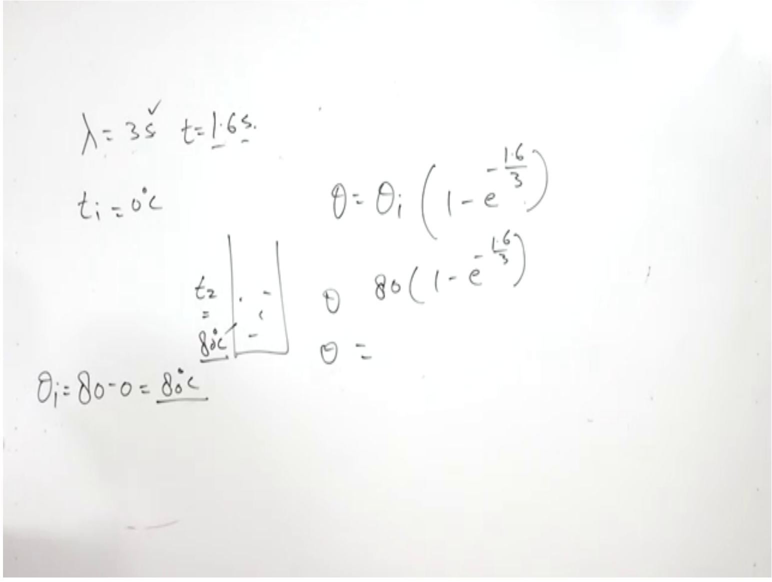

So, temperature initial temperature of thermometer is 0 degree centigrade it is dipped in the melting ice. So, initial temperature is 0 degree centigrade and a there is water hot water in the pan in hot water in a pan the temperature is 80 degrees centigrade this is t t 2 this is this t 1 and this is t 2 when the thermometer is dipped in this hot water the step input is going to be 80 minus 0 is equal to 80 degree centigrade this is input q is input if had it been 10 degree centigrade step input would have been 70 degrees centigrade had, it been 20 degree centigrade.

It would have been 16 degree; 60 degree centigrade. So, input is 80 degree centigrade time constant is 3 seconds. So, what is the temperature it will be indicated by after 1.6 seconds. So, this t is equal to 1.6 seconds. So, t is less than time constant when t is equal to time constant it is only 63.2 percent.

So, it should be less than 63.2 percent, but mathematically we will find here that output is theta this is input theta, let us say theta, I input to the thermometer 1 minus e raise to power minus 1.6 divided by 3. Now theta is 80 and 1 minus e raised to power minus 1.6 divided by 3, if we take this value of theta, then theta is going to be e raised to power minus 1.6 divided by 3 minus 1; 1 minus this multiplied by 80.

So it is going to be 33.1 degree centigrade, initial temperature is 0. So, we can always say the temperature indicated by thermometer is 33.1 degree centigrade. Now, for t is equal to lambda, this temperature would have been 63.2 percent of step enough. So, it is less than 63.2 percent also 33.1 degree centigrade.

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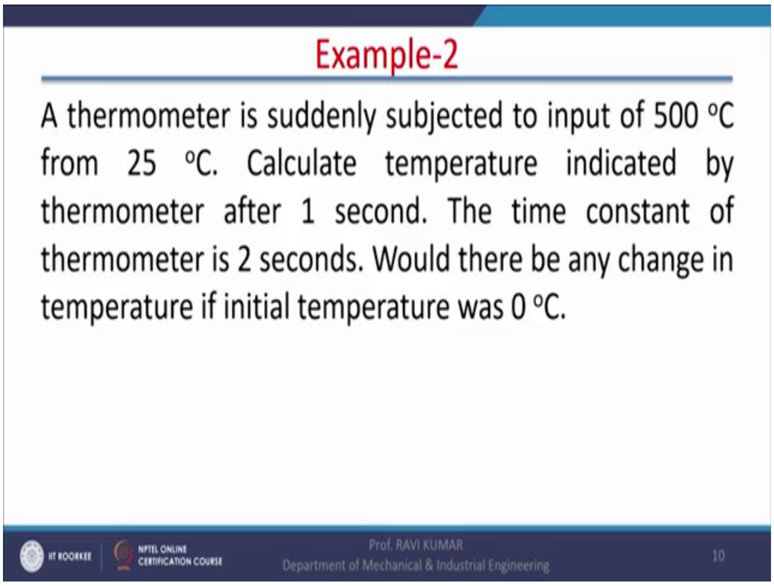

Example-2

A thermometer is suddenly subjected to input of 500 °C from 25 °C. Calculate temperature indicated by thermometer after 1 second. The time constant of thermometer is 2 seconds. Would there be any change in temperature if initial temperature was 0 °C.

Now, we can take another example also. Now in this example the thermometer is suddenly subjected to input of 500 degree centigrade from 25 degree centigrade.

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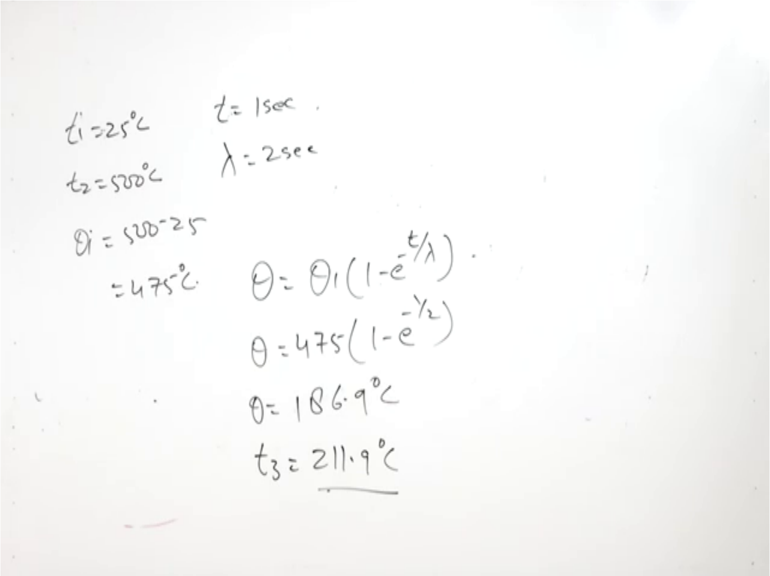

$t_1 = 25^\circ C$ $t = 1\,sec.$

$t_2 = 500^\circ C$ $\lambda = 2\,sec$

$\theta_i = 500 - 25$

$= 475^\circ C.$

$\theta = \theta_i\left(1 - e^{-t/\lambda}\right).$

$\theta = 475\left(1 - e^{-1/2}\right)$

$\theta = 186.9^\circ C$

$t_3 = \underline{211.9^\circ C}$

So, t 1 is 25 degree centigrade t 2 is 500 degree centigrade. So, theta e is equal to 500 minus 25; 475 degree centigrade, right calculate the temperature indicated by thermometer after one seconds. So, t is equal to 1 second time constant 2 seconds time constant is two seconds. Now, here in this case comfortably, we can take theta is equal to theta i 1 minus e raised to power minus t by lambda and now theta is theta is 475; 1 minus e raise to power minus 1 by 2 and when we solve this, we get the value of theta as 186.9 degree centigrade.

Now, in order to find temperature this initial temperature will be added. So, add here initial temperature and then we will get the value of t 1 t 1 sec one sec or t 3, we can get t 3 as temperature after 1 second as 186 plus 25; 211, 9 degree centigrade , right. So, simply we can predict the temperature after one second actual temperature is 500 degree, we are far away from actual temperature.

Because at 500 degree centigrade temperature , if it is a steel the color is you can see the color I mean through color you can also judge the because earlier days, there was no thermometry or this temperature is scanning of the surfaces, but the gold smiths, they are very expert through the color of the object only they could identify that what is the temperature of the object though those estimates are not very accurate I mean that value when I in a range of plus minus 25 or 50 degree centigrade, but they could get fairly good idea about the order of temperature , this is all for today. So, in the next class, we will discuss the ramp response of first order system.