Mechanical Measurement Systems
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Lecture – 16
First order System - Ramp Response

Hello, I welcome you all in this course of mechanical measurement systems and today we will start with we will continue with discussions on first order system and today we will discuss the ramp response of first order system.

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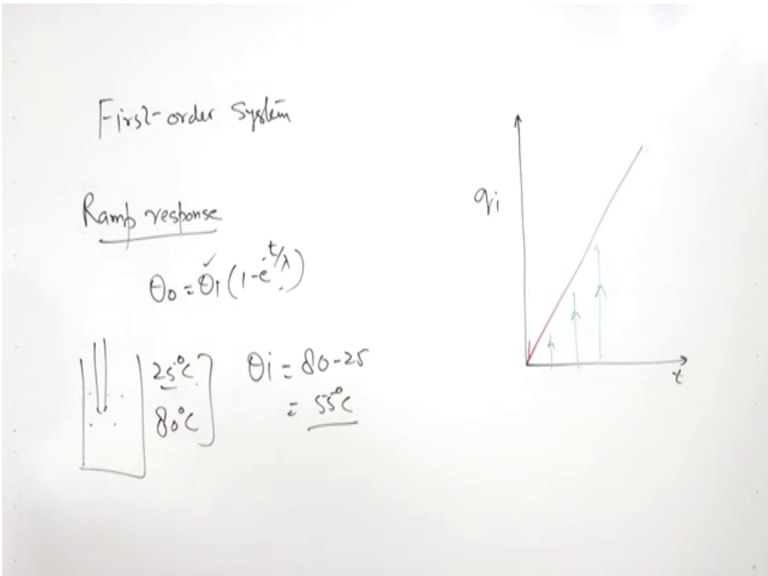

The first order system or systems and we will go with a ramp response. These are the typical outputs like step input, ramp input this is the typical input to any measuring instrument of any order. So, we are right now, we are discussing the first order system. So, ramp input to the first order system. So, ramp input to the first order system is going to be like this is output, this is input time, this is time and this is q i this is not output this is input q i time and then in step input it was like this, but here the situation is different here the input is a ramp input. Ramp input means the input is changing with time there is a ramp. So, for any particular time you will find altogether different input.

Now, I can give you one example of this type of input. Suppose, in a in a vessel in a vessel the water is being heated sensible heating of water. So, water is being heated. The water is available at the room at 25 degree centigrade and then the water in the container is heating the temperature will start rising and in certain time interval depending upon the quantity of heat supplied and quantity of water available in the pan the temperature will rise to let us say 80 80 degree centigrade. The boiling has not yet started the moment the boiling is started the temperature will become constant.

So, during this sensible heating if I am putting in one thermometer, so, the input of the thermometer is changing with time in step input the water was already at 80 the case of a step input is like the water is already at 80 degree centigrade, at possible temperature is 25 degree centigrade the thermometer is taking and it is simply put into the water that is step and here the step is input is going to be theta i 80 minus 25, it is 55 degree centigrade and then we will study the step response of the system by using the formula theta is equal to theta i 1 minus e raised to the power minus t by lambda for a step input, but here in this case this is not constant. This is varying with time this is varying with time right.

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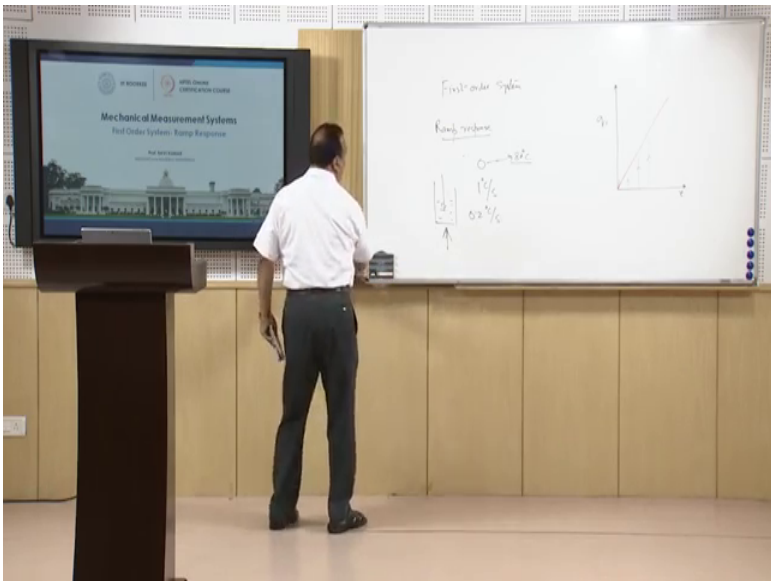

So, and rate of heating becomes very important here from 25 to 80 25 to 80 in how much time? in 8 second for 80 second or 800 seconds, that will decide the rate of the loading right and vice versa. If we have the rate of the loading we can find the instantaneous input to the system. Suppose, it is going from for the sake of convenience let us take 0 to 80 temperature is put in the vessel initially the bulb was at 0 degree centigrade water was also at 0 degree centigrade it was all water not ice no ice was there. If ice is there then it is a different story all water at 0 degree centigrade and now, we are start have started heating the water and temperature of thermometer is started rising, right.

If the we are getting at any temperature 8 80 degree centigrade in 80 seconds then input rate is one degree centigrade per second. If we are attaining 80 degree centigrades in let us say 400 seconds. So, input is 0.2 degree centigrade per second if 400, 80 by 400 yes. So, so these two rates are different. For these two different rates of input the response of the system will also be different that is obvious, right. Chances of error in these cases higher than the chances of error in this case, but we have to quantify all these things, by perceptions we cannot take any decision. So, we will do certain mathematical analysis and again we will start with a basic equation.

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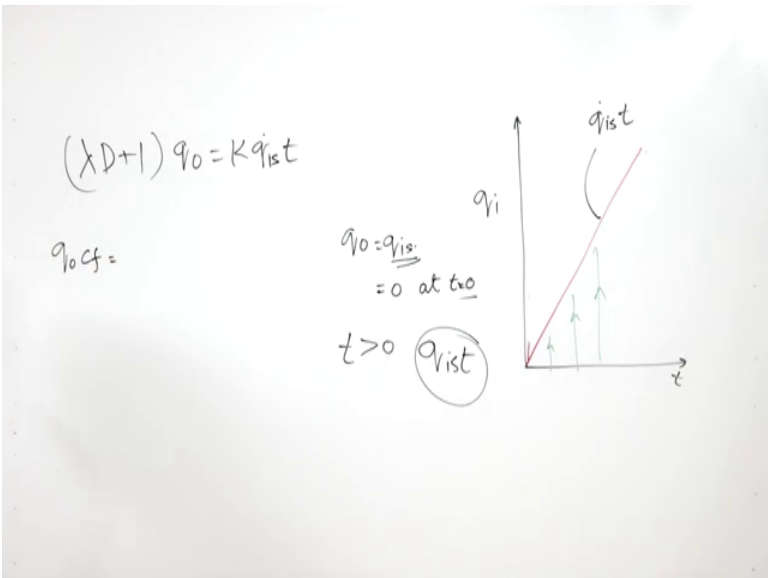

Now, the basic equation is q o is equal to q i 1 minus e raised to the power minus t by lambda. This is the basic equation for first order system. Yes, here we have to multiply by sensitivity also q o is equal to K q i fine. Now, q o is equal to q i s is equal to 0 at t is equal to 0 initially both are 0 right and input to the instrument into input to the instrument is rate multiplied by t suppose rate of input is q i s dot. So, instantaneous input to the instrument is q i s t.

So, for t is greater than 0 we can always say the input to the instrument is q i s t output we do not know output we have to find using this equation. Now, now this is for first order equation this is not for this generalised equation for first order system is lambda D plus 1 q o is equal to K q i now here the q input is instantaneous input this is instantaneous input. Now, here again we have to find first of all this complementary function. So, q o complimentary function is going to be equal to at t is equal to 0. So, is going to be equal to again lambda d plus one is equal to 0. So, D has one root that is minus 1 by lambda.

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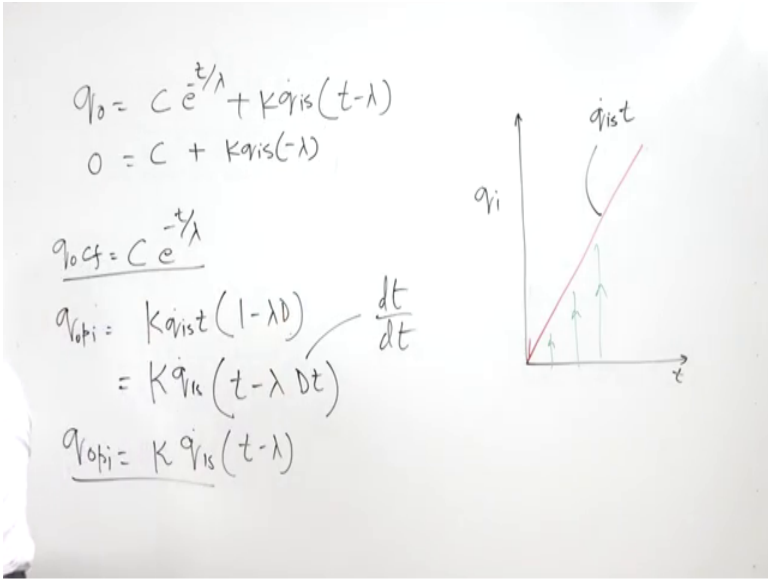

So, again it is going to be C e raised to power minus t by lambda as we did in the case of step response. Now, regarding the particular integral particular integral particular integral is going to be K q i s t 1 minus lambda D ok. Now, we can further manipulate this equation K q sorry q i s this is rate and here t minus lambda this Dt. Now, this Dt is Dt by dt. So, this is going to be equal to 1. So, particular integral here in this case going to be K q dot i s t minus lambda.

So, now, we will add these two complimentary function and particular integral and we will get the final expression the final expression is q o is equal to C e raised to power minus t by lambda plus K q i s dot t minus lambda. Now, here again we will put the initial condition the initial condition is at t is equal to 0 output is 0. So, this is 0 when we are putting t is equal to 0 this will become C plus K q i s and then it is minus lambda because here t is 0.

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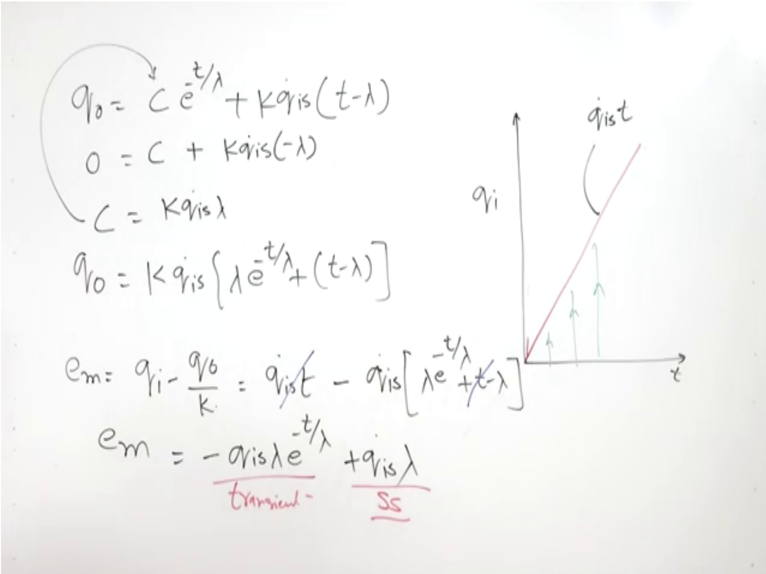

So, we will be getting the value of C as K q i s lambda. Now, while putting this value C here in this equation we will be getting q o is equal to K q i s lambda e raised to power minus t by lambda plus t minus lambda. This is the repose of response equation for ramp input to first order system again for this we will have to estimate the error also because the instrument which is responding to this input shall not follow this line shall not follow this line it is obvious, but here we have to quantify.

So, in order to quantify that we will calculate error in the measurement. Now, for doing this error in the measurement we can take q i input minus q o by K. This is input, actual input minus input indicator because ultimately we are output of the instrument is also showing input to the instrument right. So, this q o by K will give you the actual input. Now, when we are doing this we will get q i is q i s t right minus this is q i s and this is also dot this is also dot right. So, it is equal to q o by K. So, minus q i s dot lambda e raised to power minus t by lambda plus t minus lambda.

Now, here this q i s t will cancel this and we will get the final expression as error in the measurement is equal to minus q i s lambda e raised to the power minus t by lambda plus q i s lambda. This is error in the. So, this error has two component one component is this one which is time dependent another component is this which is not time dependent which is dependent only on the constant time constant and rate of input that is it it is not dependent on anything. So, this is time dependent or it is transient error because it is a function of time and this error will change with time if the value of t is infinite this is going to be 0. So, after certain time interval this error will diminish, but in any case we are going to have this error through the entire measurement process right and if we draw the response of this step input sorry, the ramp input response of ramp input then we will get a curve like this initially.

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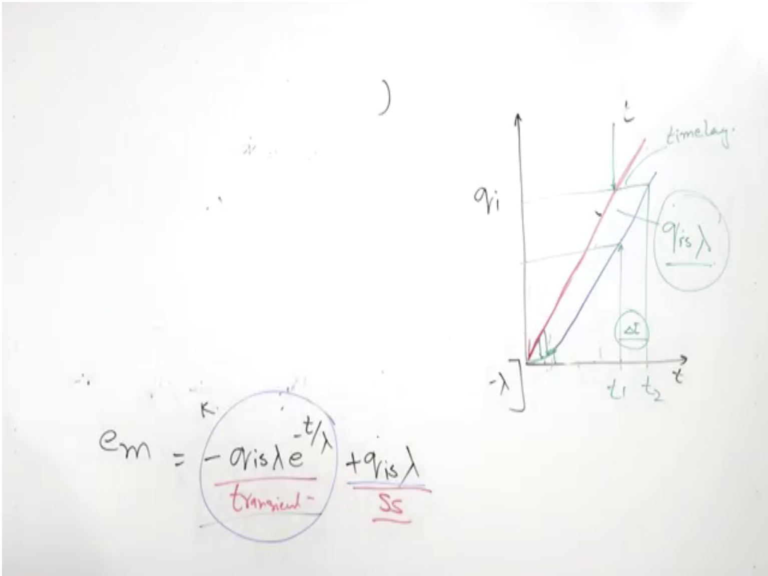

It is going to be like this. So, this clean in the because this is straight line this is these are not parallel to each other right. So, we are getting this due to presence of this factor and after certain time interval this factor diminishes and we get only q i s lambda and that is this one.

Now, error in measurement; so, at a particular time t at a particular time t now, at a particular time t here this is the error in the measurement, this is the response of the instrument. So, instrument is showing this much of input and actual input is this one. So, this is the steady state error and this can be expressed as q i s multiplied by lambda right and for example, this reading this reading we are getting at t 1, but actually this is going to happen for the instrument it is going to be a t 2.

So, this time interval is known as time lag. So, there are two thing steady state error at a particular time actual input and output of the instrument this difference is steady state error and it is shown by q i s lambda and time lag means actual input is this and the instrument will be showing actual input after time interval of delta t and this is known as time lag there are two things and this measurement during this time interval has to be avoided because it will crimp in because we will I mean we will it will because the variation is not linear. So, what time we are making the measurement it is important here, but once this transient part is diminished anytime we can have the measurement and we are going to have a constant error that is q dot i s that is rate of loading multiplied by time constant.

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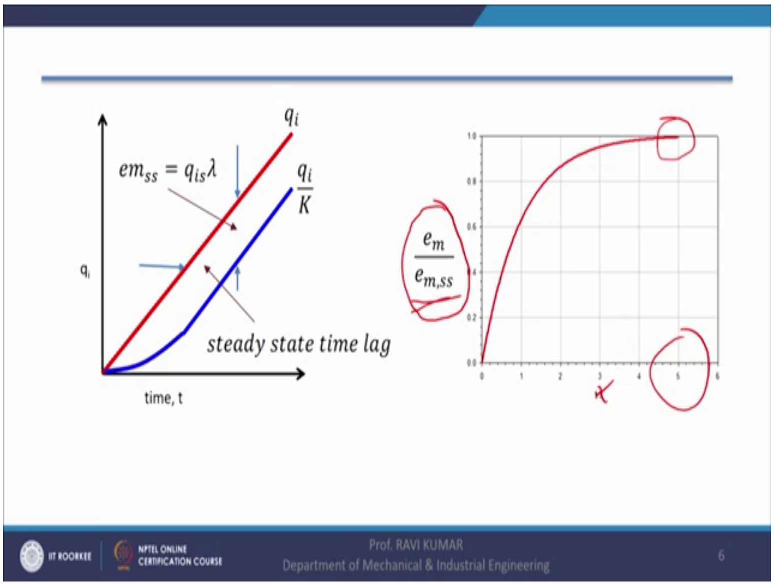

Now, regarding this error in measurement in this graph the same thing is shown here in this graph. This is error in measurement divided error in measurement at steady state and this is increasing and it is the ratio is attaining at the value of one at approximately t by lambda is 0.5 is now it is not t by lambda it is it is t only. So, at after 5 seconds right.

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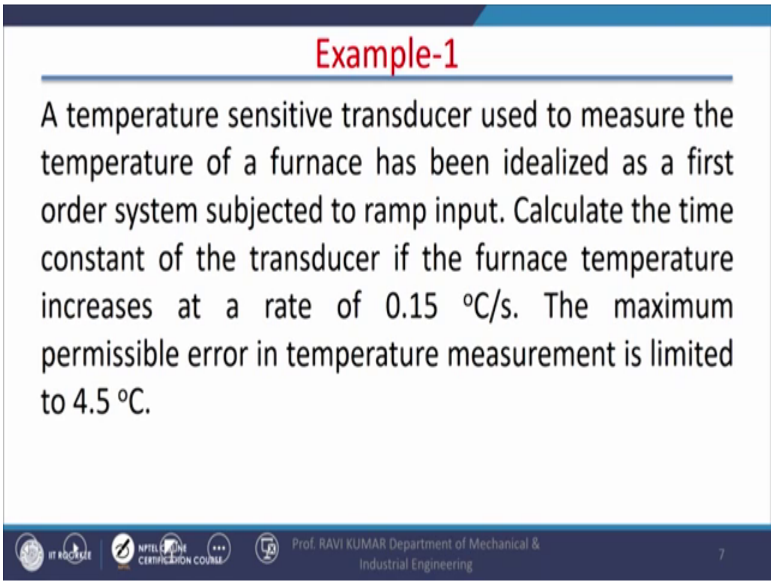

After this we can take one example now this example states that temperature the temperature sensitive transducer used to measure is used to measure the temperature of a furnace and it has been idealised as first order system subject to ramp input. So, a temperature sensitive transducer is used to measure a temperature of a surface there is a furnace and it is idealised as or it is considered as first order system subjected to ramp input.

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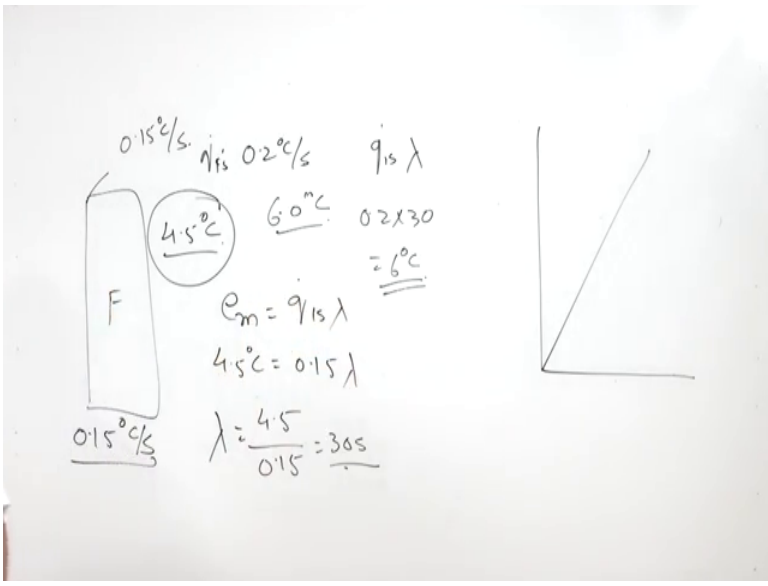

So, first order system subjected to ramp input right. Calculate the constant of transducer if the furnace time constant of transducer if the furnace temperature increases at the rate of 0.15 degree centigrade per second. So, furnace there is a furnace the the temperature of the furnace is rising and temperature of the furnace is rising with a rate of 0.15 degree centigrade per second and the maximum permissible error is 4.5 degree centigrade and we have to find time constant.

So, maximum permissible error in the measurement of the surface temperature is 4.5 degree centigrade. So, error in measurement this is steady state error is q dash i s lambda. There is a very simple formula and error is 4.5 degree centigrade rate is 0.15 lambda. So, lambda is has to be 4.5 divided by 0.15 is equal to 30 seconds. So, time constant of a thermometer has to be 30 seconds if we want to confine the error in the measurement as 4.5 degree centigrade for a temperature rise of 0.15. Suppose, the rate of temperature rise was instead of 0.15 is it is 0.45 or let us say 0.2 instead of 0.15 it was 0.2 degree centigrade per second.

Now, what is going to be the error, time constant is given 30 seconds ? So, it is going to be 6.0 degree centigrade q i s multiplied by lambda it is 0.2 into thirty 6 degree centigrade is going to be the steady state error in the measurement. So, we have changed the rate of change of temperature the error has increased. Similarly, if you reduce it if you reduce it to 0.1. Now, if you reduce it to 0.1 the error will be reduced to 0.3 degree centigrade because error in the ramp input error in the measurement is a function of this rate of input multiplied by the time constant, ok.

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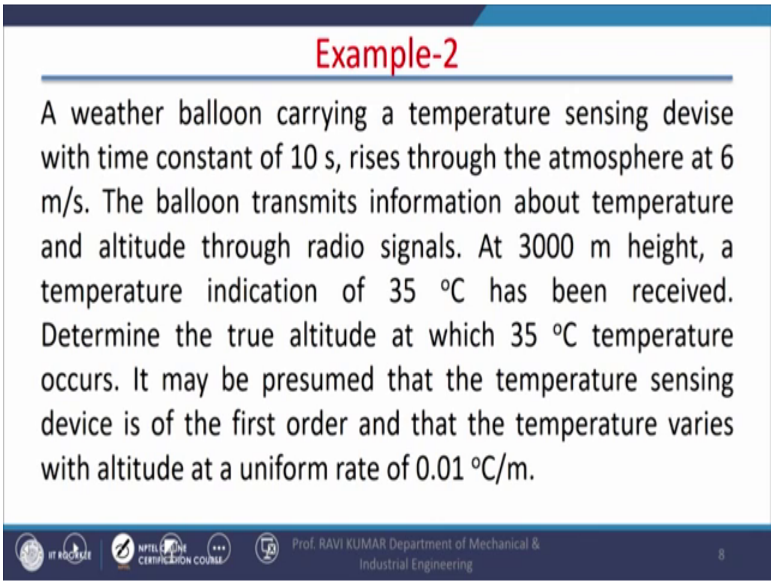

So, another example we will take one more example which is you will find more interesting. A weather balloon carrying a temperature sensing device with time constant of 10 seconds. So, they are weather balloons to find the temperatures are at high altitudes. So, there is a weather balloon which has a temperature sensor and it has time constant of 0.15, sorry what is that time constant of 10 seconds.

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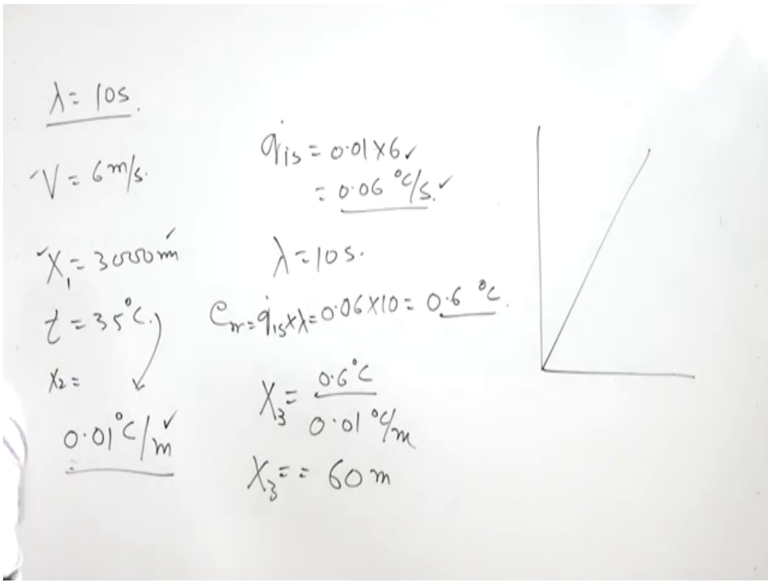

This is the time constant for weather balloon and the balloon is rising the speed of rising the balloon is 6 meters per second. So, there is a balloon and balloon is fitted with a temperature sensor and it is moving vertical vertically with a speed of 6 meters per second. The balloon transmits information about temperature and altitude through radio signals, at 3000 meter height a temperature indicated is 35 degree centigrade. So, 3000 meter height, X is equal to 3000 meters or 3 kilo meters from the earth surface; the temperature indicated is 35 degree centigrade, right. Determine the true altitude at which 35 degree centralise temperature occurs.

We want to have X 1 and X 2 that is true value true height at which this temperature is because the signal we are getting it has some error or it is a sludgy signal, right. So, it has some error and that correction has to be made in the measurement. So, we have to find at 3 when the balloon is at 3 3 kilo metres above the surface it has send one signal indicated the temperature 35 degree, but it is really 35 degree centigrade because temperature sensor has certain error in the measurement right and it is a steady state error. It may be presumed that at a temperature sensing devices of the first order and the temperature varies with altitude uniformly at 0.01 degree centigrade per meter temperature is changing, right.

Now, here the temperature is changing with distance this is not the rate of input in order to find the rate of input we have to multiply these 2 and then we will getting we will be getting 0.01 multiplied by 6 0.06 degree centigrade per second variation because temperature we have to find temperature variation with respect to time. Now, we have replaced velocity or distance with time. Now, we will talk in terms of time because distance is given velocity is given. So, we can always talk in terms of time and times laps in the measurement indicating the value.

So, input is q i s is 0.06 degree centigrade per every second every second this is going to be change in temperature, lambda is 10 seconds. So, steady state error here is going to be equal to how much 0.06 multiplied by 10 is equal to 0.6 degree centigrade this is going to be the steady state error. Now, once we have this steady state error we can simply find the value of X.

Now, the X is this is 0.01, ok. So, this is point 6 times q i s and q i degree centigrade. So, this is error in measurement is equal to q i s into lambda right and here because q i s we have taken 0.06 lambda is 10, it is 0.6 degree centigrade. Now, this 0.6 degree centigrade when it is divided by 0.01 degree centigrade per meter we will get the value of X or X 3 that is going to be how much 60 meter sorry 60 meter 60 meter. So, actual height actual height of the balloon when the signal was sent was 6000 minus this error 60 is equal to?

Student: 3000.

Sorry, 3000 it is the actual height when the signal was sent the actual height when the signal was sent is 3000 meter. 3000 meter minus this error minus 60 when it is going to be 2940 meters. So, the temperature indicated as thirty 5 degree centigrade is not at 3000 meter altitude it is at 2940 meter altitude. Now, we will take another example.

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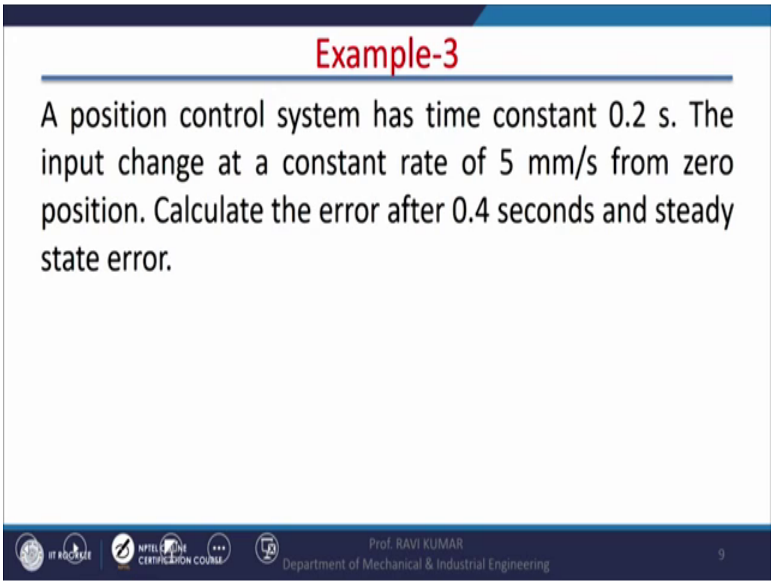

The statement of the example is a position control system has time constant 0.2 seconds. So, time constant lambda is 0.2 seconds. It is for position control system.

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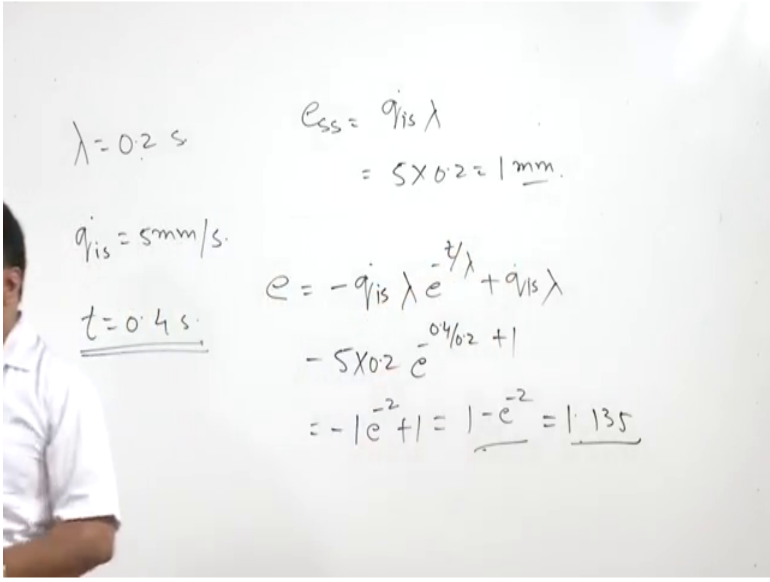

The input changes with a constant rate of 5 mm per second right from 0 position from initial position is 0. Calculate the error after 0.4 seconds and steady state error also. So, steady state errors first of all let us calculate the steady state error. So, steady state error is sorry steady state error is q i s dot lambda and q i s is 5 mm per second that is ramp slope rate of input. So, 5 mm multiplied by time constant 1 1 mm this is steady state overall. So, this instrument will give error of 1 millimetre for a ramp input 5 millimetres per second.

Now, error at t is equal to 0.4 seconds and if you look at the time constant it is 0.2. So, error at total error is transient error plus steady state error and we can always write minus q i s lambda e raised to the power minus t by lambda plus q i s dot dot lambda, this is the total error. Steady state error we have already calculated, it is 1. Now, for this transient error minus q i s if 5 mm per second right and lambda; lambda is 0.2 e raised to power minus t by lambda, t is 0.4.

So, e raised to power 0.4 divide by 0.2 and this is again minus 1, e raised to power minus 2 plus 1 or is equal to 1 minus e raise to power minus 2 and when we take 1 minus e raise to power minus 2 plus 1 and it is going to be equal to 1.135. So, after 4 seconds or 2 times of lambda the error is 1.135, right and finally, steady state error is 1 mm.

So, I think we should conclude this here itself, right and we have amply gone through the step response and the ramp response of first order system. In the next class, we will start with a impulse response of first order system.

Thank you.