Mechanical Measurement Systems
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Lecture – 17
First Order System - Impulse Response

Hello, I welcome you all in this course on mechanical measurement systems. Today, we will discuss first order system impulse response.

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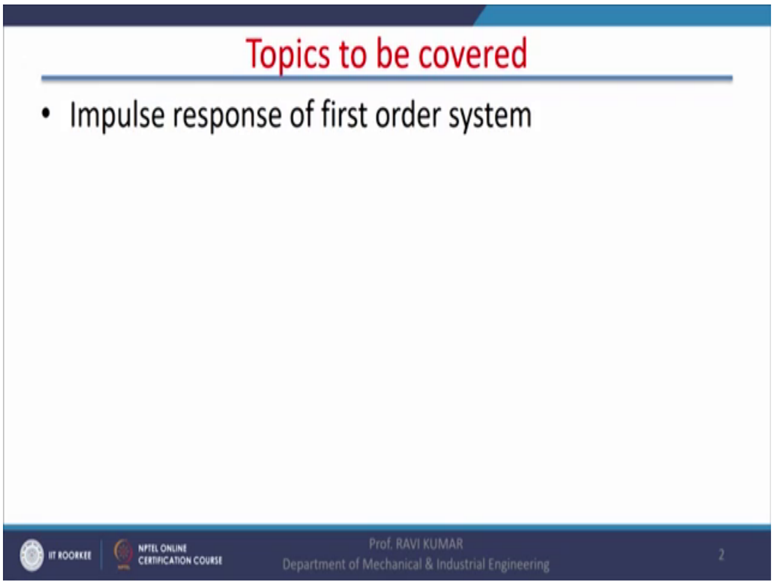

First order system.

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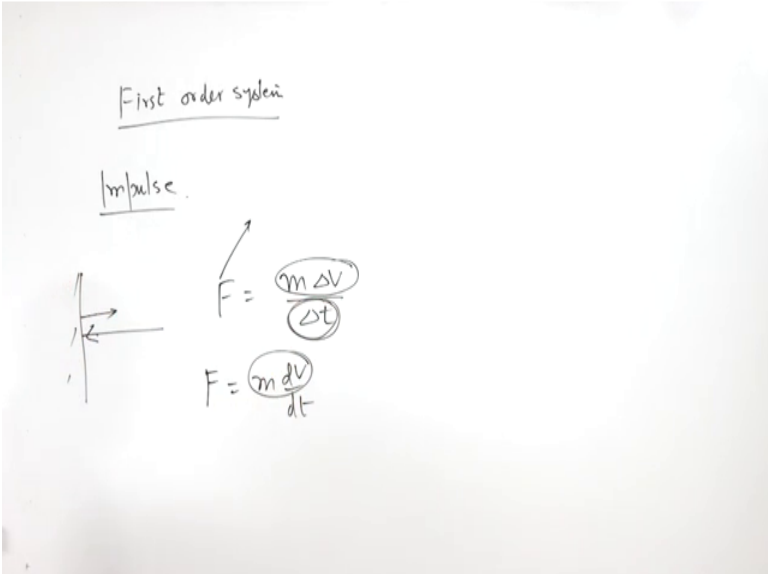

First order system impulse response as all of us all of you know the impulse is exerted when a large amount of force is put on a body or exerted on a body for a very short duration of time. By Newton's second law of motion force is defined as rate of change of momentum or it is can be defined as m delta V by delta t, fine. When this rate of change of momentum is same and delta t is very small we are going to feel or the object is going to feel very high order of force because force is again m dV by dt. So, this change in momentum is for very short time.

Suppose, for example, there is a wall and we are striking a ball on the wall when the ball strikes the wall then there is a within no time there is a change in the direction when there is a change in direction m V is suppose the momentum is m V the change in momentum becomes 2 mV right, because there is a change in the direction of the velocity and for very short duration of time and this is how the impact loading is done on the wall through a ball, but in electrical signals impulse loading can be understood as a pulse.

Suppose, there is a pulse we are measuring again voltage with the help of a voltmeter all of a sudden there is a pulse and the pulse remain for a very short time or delta t or dt for pulse tends to 0. Now, how the system will respond how our measuring instrument which is of the first order will respond to this pulse. So, for this we will do one analysis.

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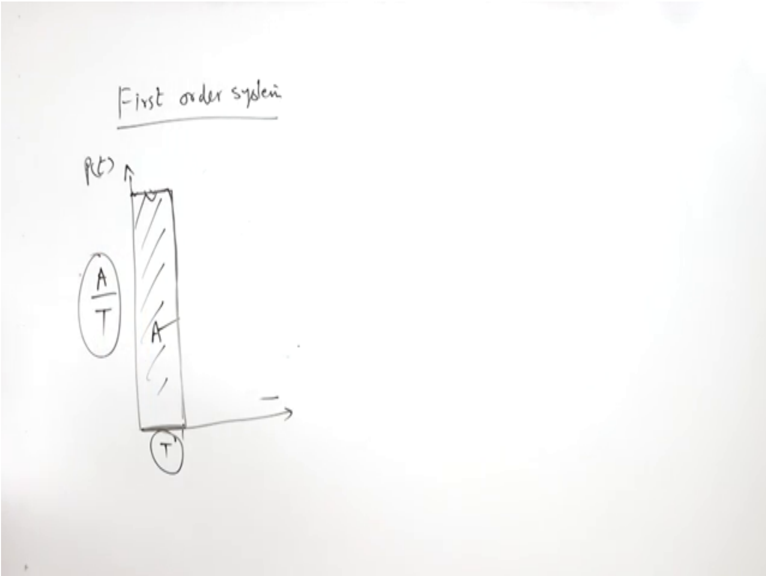

Now, in this analysis we will assume that the pulse is for the finite duration finite duration of let us say T and in the pulse the energy is important because it is it lasts for a very short duration of time and suppose, this is area it is depicted by this area shape of pulse is not that important because either it will have a flat this or this or this it will not have much bearing on the A, right. So, shape of the pulse is not very important because it lasts for a very short duration of time and this is P t.

Now, it is a limiting process and it has a functional strength of a right and magnitude in this direction the y direction we can assume always assumes as A by T right and when the A is unit when A is equal to 1 then it is it becomes unit impulse function and unit impulse function can be always be expressed by del T or any impulse function can be represented by A del T.

Now, we will not focus on this, we will focus on system response. System response because it is I reiterate that because it is for a very short duration of time, shape of the pulse is not that important it allows only for let us say one microsecond right, but we assume it to be we assume it to represent by a rectangle having an area A and it lasts for each duration of time T. So, magnitude in this direction pt direction is going to be A by T, fine. Now, here I will redraw this diagram.

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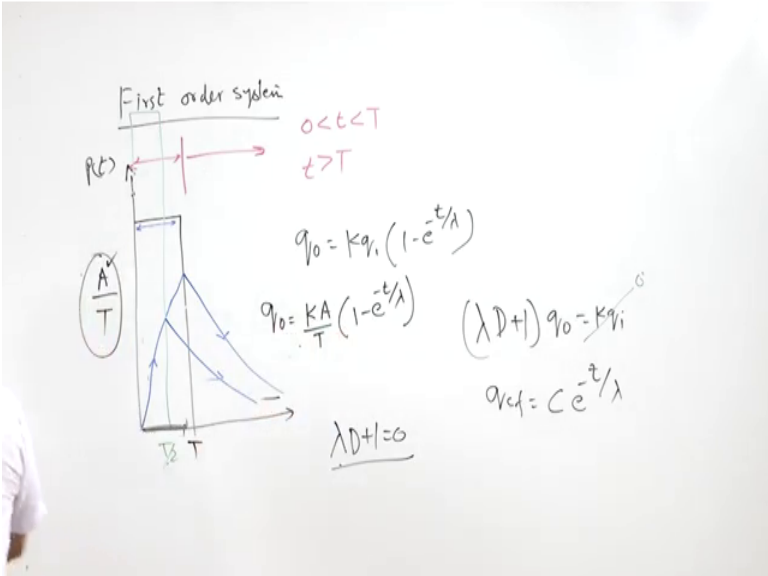

Now, here it will act as a terminated step response we can divide this in two regions; one is this region where input signal is there and another region where there is no input signal or they are two boundary conditions one is T is less than T and when is T is greater than the regarding the response of the system because all of a sudden there is an a step input, fine.

So, but the system cannot be respond this manner right for a step input it will rise the response of the system, the output will rise until before it attains the final value because this T is much smaller than the time constant. So, before it attains the final value the input is removed and the system will again respond like this not go below this. So, it will rise and then again it will fall.

Now, if we make T to T by 2 now, T is reduced to T by 2 then this amplitude will be doubled and the system response will again be like this right. Now, we will we will now mathematically we will express the system response in this case to begin with we will take this part of the impulse this part of the impulse will start with q o is equal to K q i 1 minus e raised to power minus t by lambda and this is limiting up to it here now input q i. So, here the q i we should take as A by T. So, q is going to be equal to K A by T 1 minus e raised to power minus T by lambda, right.

Now, for this part this part we do not have any standard equation right, but since it is a first order system we will assume that or we will take that lambda d plus one q o is equal to K q i this is the I mean generalized equation for first order response of the first order system. Now, in this part input is 0. So, this is this part is 0 when this is 0 then complementary function q complementary function is going to be equal to C e raise to power minus t by lambda because lambda D plus 1, we can say lambda D plus 1 is equal to 0 and for this equation will have only one root and for one root complimentary function is going to be like this. Now, particular integral will not be there without this side is 0.

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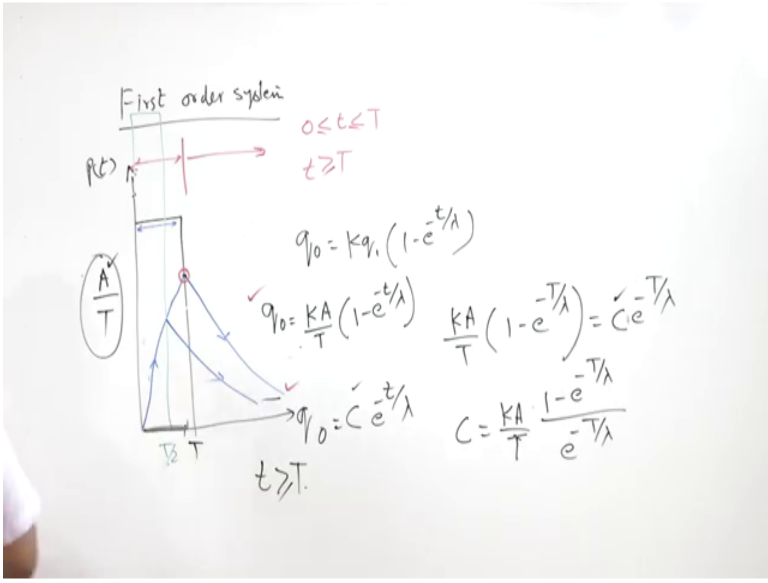

So, the solution for this equation is q o is equal to C. E raise to power minus t by lambda provided t is greater than T or equal to T right and here also will take greater than equal to and this is less than or equal to less than or equal to.

Now, we have to find the value of C which is unknown and we do not have earlier we used to have boundary conditions, but here we have boundary conditions at infinite time this output is going to be 0, but it will not be used here. So, in order to find C what we can do at this point this particular point both of these equations are valid. So, we will what we will do here we will put t is equal to T and we equate these two equations once we are putting t is equal to T capital T T is time. So, it means K A by T 1 minus e raise to power minus T by lambda capital T by lambda is going to be equal to C e raise to power minus capital T by lambda.

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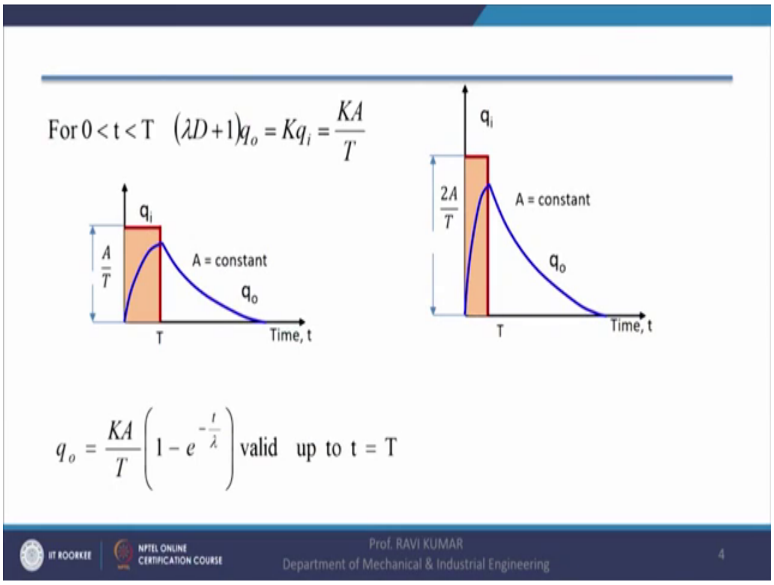

Now, in this equation now, in this equation the unknown is only C. So, C is K A by T 1 minus e raised to power minus T by lambda divided by e raise to power minus capital T by lambda, the T is known to us.

So, we can say then impulse of a short duration short duration maybe 1 microsecond or 1 millisecond we can go to 1 millisecond also so, 1 microsecond 1 milliseconds. So, for a for short for that short duration the value the normal value T is going to have now just put this equation C here.

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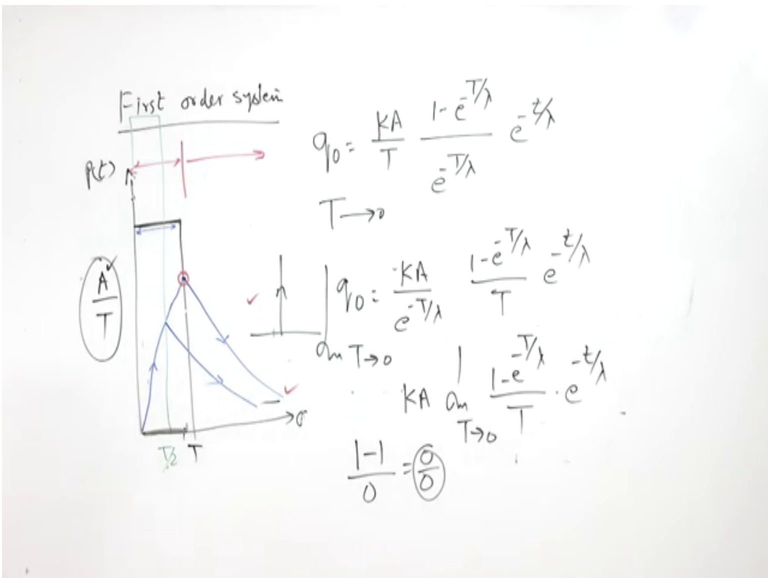

When you are putting this value of C here then we are getting q o is equal to K A by T, 1 minus e raise to power minus capital T by lambda divided by e raise to power minus capital T by lambda e raise to power minus a small t by lambda, this is the response of the instrument. So, there is a volt to suppose in electrical system there is a voltmeter and there is a pulse in the in the voltage. So, voltage suppose shoots so, normal voltage is 200 volts all of a all of a sudden it shoots to pulse maybe of let us say 1000 volts or 2000 volts and if this pulse remains for a very short duration of time. The how the system will behave how the system will behave? This is after the pulse this is going to be the governing equation, right.

Now, in this governing equation we have assumed T as some infinitesimal time, but in ideal case this is this is an approximation right, ideal case what is going to happen to this T? This T should be tending to 0 T it should be tending to 0. In fact, the input to the system this has to be is only a pulse vertical line this is pulse right, because we cannot have this delta T 0 that is why we have assumed that there is a ramp response and it is a ramp, ramp input sorry not ramp input, this is a step input there is a step input for a very short duration of time and after that that short duration of time the input is terminated and then we have seen how the system is going to respond.

Now, T is tending to 0. So, here q o we can take K A by e raised to power minus T by lambda and then 1 minus e to power minus T by lambda by T e raised to power minus t by lambda I have just rearranged these values right I have just taken T here and T by lambda; the reason being the reason being and the limit T is tending to 0, because when we are putting the limits then this term will become 1. So, it is K A what about this term limit T is tending to 0, 1 minus e raised to power capital T by lambda divided by T and then again e raised to power minus t by lambda.

This term this particular term it will turn out to be 1 minus 1 divided by 0 that is 0 by 0 Now, in order to cope with this situation there is a rule and there is a known as L hospital rule.

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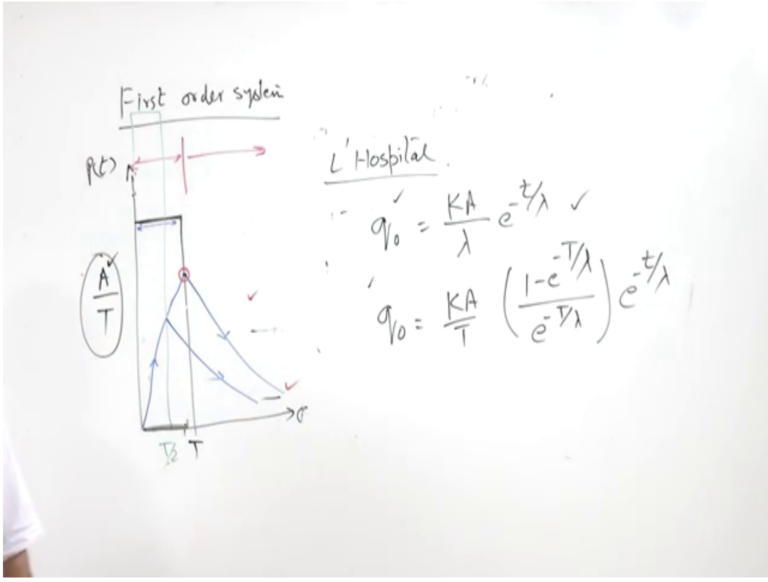

And, in L hospital rule that is L hospital both numerator and denominator both will be differentiated. So, when we differentiate both and put the value here in this equation the final expression we are going to get q is equal to K A, K is here and when we differentiate this we will get lambda e raise to power minus T by lambda. This is the final expression we are going to get here. So, this is exact this is exact solution, approximate solution is I have given you earlier for this part of for this part of the response the approximate solution is q o is equal to K A by T, 1 minus e raise to power minus T by lambda divided by e raise to power minus T by lambda e raise to power minus T by lambda. This is the approximate solution. So, this is the approximate solution in this solution we have simply taken limit tending to 0 and we have got exact solution. Now, how different is this exact solution is from an approximate solution?.

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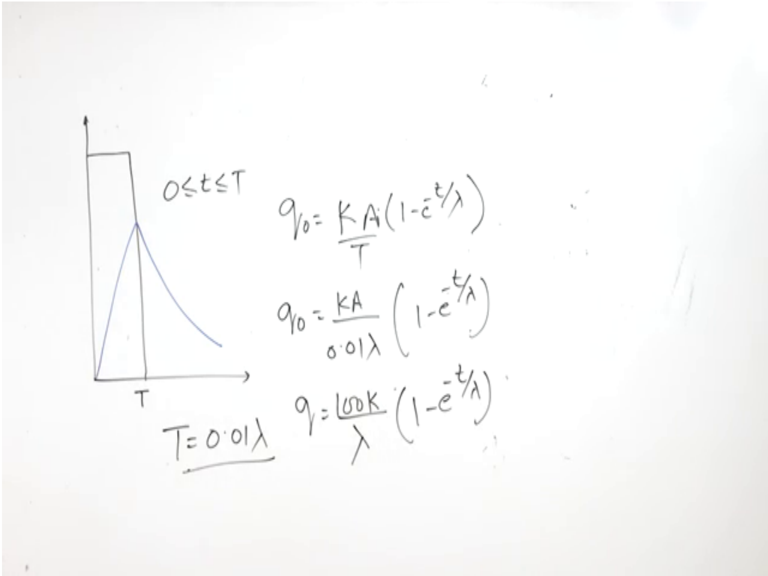

Now, we will do one analysis, now in this analysis we will study the impulse response for a particular case. Now, for particular case means there is a impulse for duration T in this impulse the response of the instrument is going to be like this for example for this reason for this reason when 0 is less than equal to t is less than equal to capital T. It will act as a first order system the first order system means q o is equal to q o is equal to K q i, 1 minus e raised to power minus T by lambda and q i we have already assumed or we have taken as A by T, right.

Now, T we have taken as here assumed as 0.01 lambda. So, q o is going to be equal to K a by 0.01 lambda right and 1 minus e raise to power minus t by lambda or we can say that q o is 100 K, A we can take 1, right? Divided by lambda 1 minus e raised to power t by lambda, that is one case. Another case is we take approximate solution approximate solution is that is one case.

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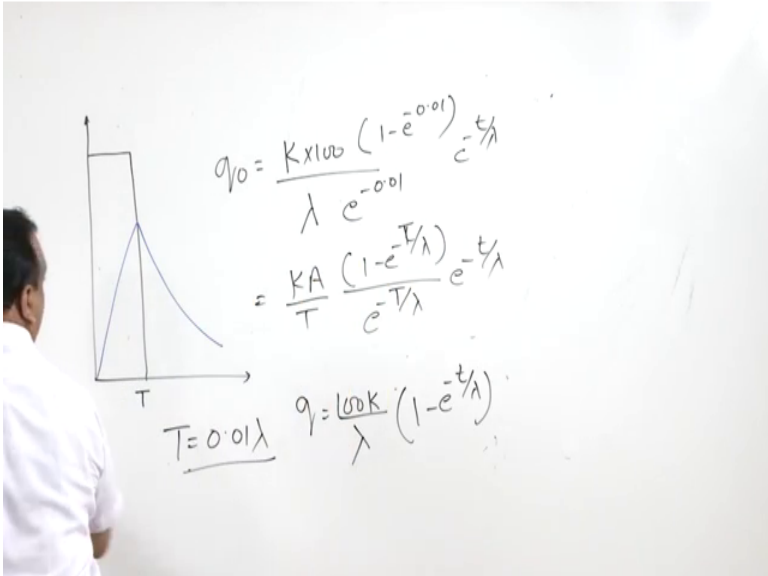

Now, approximate solution is q o is equal to K A we have already taken 1 divided by multiplied by 100 divided by lambda because it is 0.01 where T is replaced by 0.01 lambda. So, K A by T e raised to power minus 0.01 and 1 minus e raised to power minus 0.01 multiplied by e raised to power minus T by lambda this is the approximate this is out of approximate solution approximate solution is K A by T 1 minus e raised to power minus capital T by lambda divided by e raised to power minus capital T by lambda e raised to power minus a small t by lambda.

So, just we have replaced the values here because capital T by capital T by lambda is 0.01. So, 0.01 we have placed here and this is our approximate solution right. So, now, we have these two solutions.

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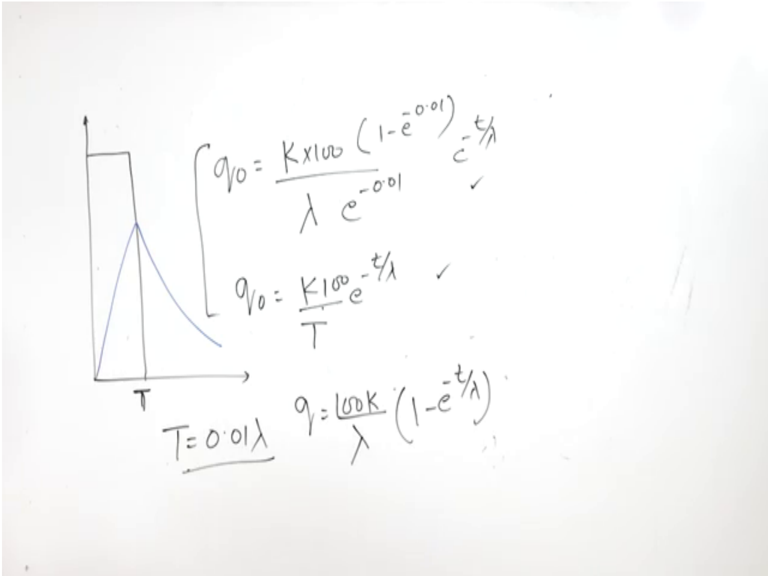

We can have exact solution also. The exact solution here is because exact solution is q o is equal to K A by lambda e raised to power minus T by lambda. Fine, now a again is 1 and lambda we have taken as T multiplied by 100 T or and this is multiplied by 100.

Now, we have these two solutions one is approximate one is exact this one there is no issue with this solution because this is well established. Now, there are issues whether this is correct or this is correct or either of this can be taken when the time is less than capital T this will operate and when the time is greater than capital T. When time is greater than capital T then we can take approximate as well as exact solution. Now, we will compare approximate solution with the exact solution.

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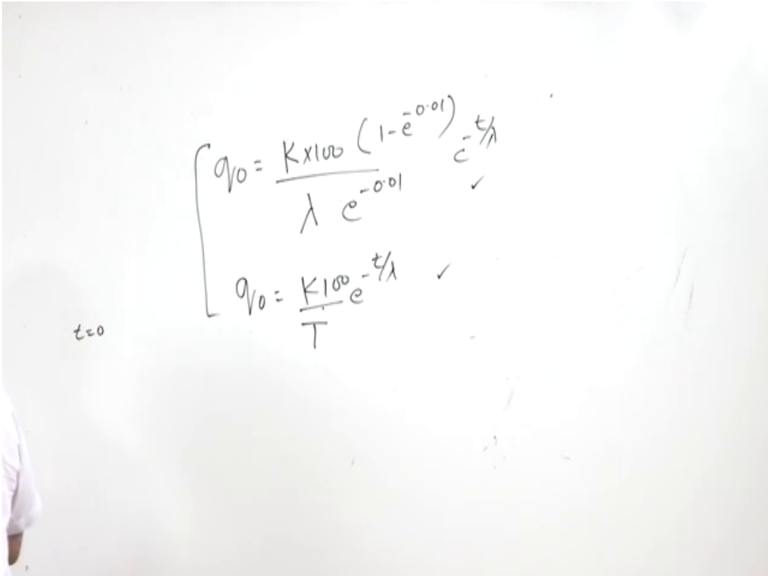

Now, when T is equal to 0, now, this let us take this as approximate or exact and approximate.

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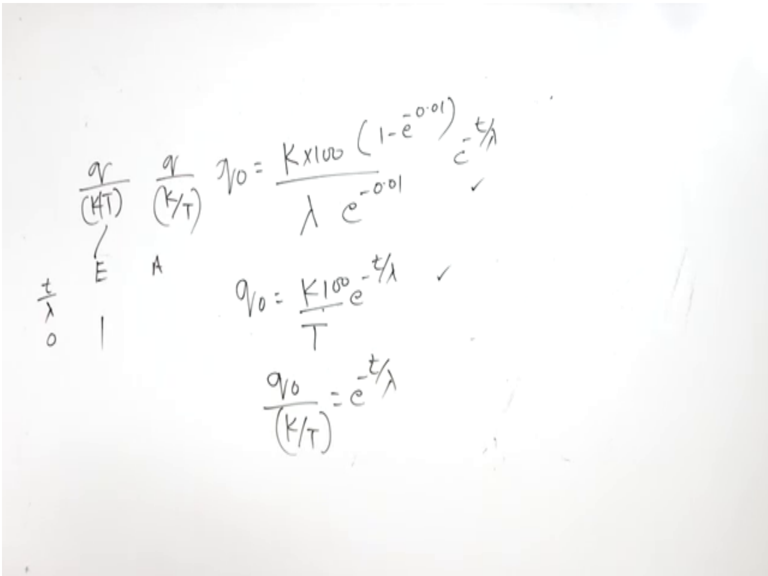

And, let us take the value of t by T t by sorry t by lambda t by lambda value of t by lambda and the value of q by K by T and this is also q divided by K by T, right. So, q divided by K by t. So, so q o divided by K by T will give e raise to power minus t by lambda. So, when T is equal to 0 or T by lambda is equal to 0 right, then in this case sorry this is approximate and exact this is exact solution. So, exact solution is going to be what? 1 1 exact solution is 1.

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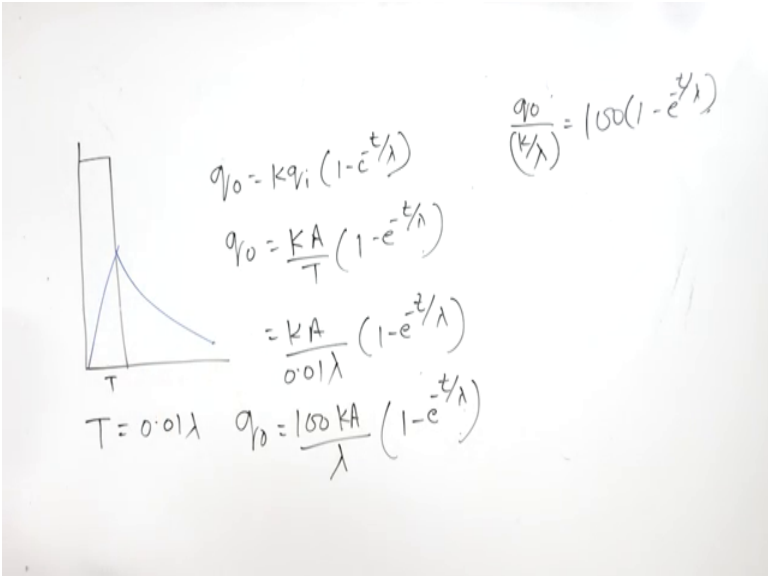

When T by T is equal to the response of the system is going to be like this now for this region for this region we can always take first order system sorry, the step response to the first order system and step strong response to the first order system means q o is equal to K q i, 1 minus e raised to power minus t by lambda. Now, we are assuming that T is equal to 0.01 lambda, it is only 1 percent of time constant of the instrument.

So, when we are assuming T is equal to 0.01 to lambda and for this we have already taken q i as K A by T, 1 minus e raise to power minus t by lambda and T is equal to 0.01 lambda. So, it is K a 0.01 lambda 1 minus e raised to power minus T by lambda and this is going to be equal to q o is equal to 100 K a divided by lambda 1 minus e raised to power minus T by lambda. Now, we will write here q o by K by lambda is equal to for a is equal to 1 suppose we take a is equal to 1 it is going to be equal to 100 1 minus e raised to power by T 1 right.

Now, the second one is beyond this beyond this we have another approximate solution.

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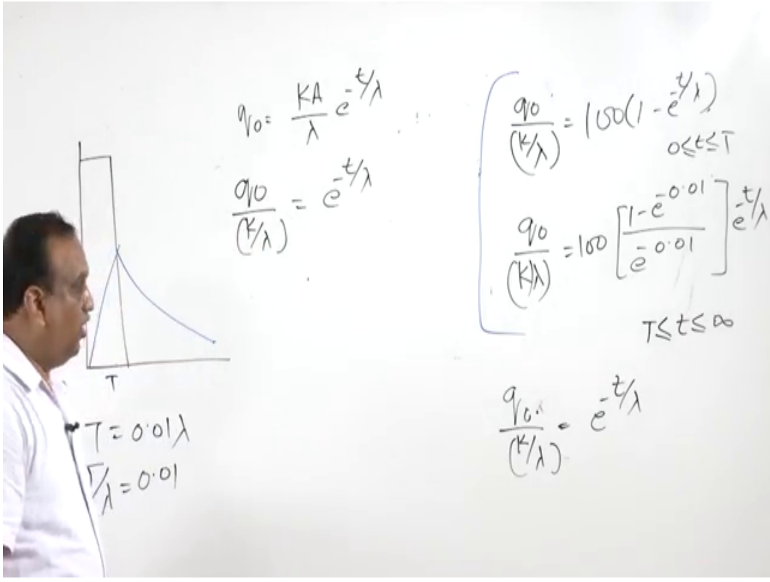

And, that approximate solution is that approximate solution is q o is equal to K A by T, K A by T 1 minus e raised to power minus T by lambda divided by e raised to power minus T by lambda. This is approximate solution for this region. So, when we take approximate solution for this region and this is multiplied by e raised to the power minus T by lambda. Now, if we take approximate and here we have already noted that T is equal to 0.01 lambda or we can say T by lambda is equal to 0.01.

So, q o is equal to again K, A we have assumed one capital T is 0.01 lambda 1 minus e raised to power minus 0.01 divided by e raised to power minus 0.01 multiplied by e raised to power by lambda or we can assume that again here we can write q o by K lambda K by lambda is equal to 100 1 minus e raised to the power 0.01 divided by e raised to power minus 0.01 and raised to multiplied by e raised to power minus t by lambda K, K has come here and K by lambda has come here, right.

So, this is valid for T is less than or equal to t is less and this is valid for there are two cases. So, we for any impulse we can find the solution using these two cases these two values. Now, before this we have derived one equation and that equation is q o is equal to that is an exact solution if you remember we have derived one equation for exact solution for impulse and that is q o is equal to K A by lambda e raised to power minus T by lambda, right and here if we take q o by K by lambda is equal to e raised to power minus t by lambda. So, either we get solution out of this or we take solution by single equation that is q o is equal to q o by K by lambda is equal to e raised to power minus t by lambda or we can use single this equation. So, this equation is valid for entire range.

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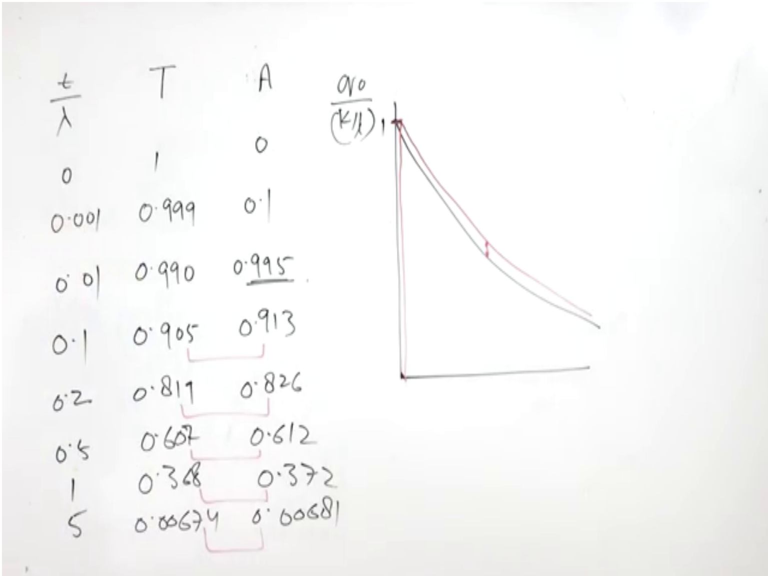

Now, we will take the different value of t by lambda and we will calculate true impulse and approximated impulse let us start with 0. So, if we take t by lambda is equal to 0, then true impulse is going to be 1 and approximated impulse will come from here and it is going to be 0.

Now, we will take t by lambda as 0.001 when we take t by lambda as the 0.001 then here we will be getting 0.999 and approximated impulse is 0.1 then we will come to 0.01 here we will be getting 0.990 and here we will be getting 0.995. Now, this is becoming closer to this approximated. Again, if we take 0.1 then for 0.1 it is 0.905, 0.913. Now, we will can continue to have 0.2, 0.819 and 0.826, right. Now, 0.5, 0.607, 0.612, 1 0.368 then 0.372 and we can go 5 then 0.00674 and 0.00681.

What I want to show you here is that initially when initially true value is one the impulse the impulse T is tending to 0. So, so if you draw the graph if you draw the graph then initially the pulse is 0 sorry, if the pulse is 1 at T is equal to 0 ideally. So, q o K by delta true value is this, 1 approximated value is 0 because it is a step response. So, initially the if we are initially we are considering a step response.

So, step response at T is equal to 0 the output is 0 then 0.001, right? Then it is coming 0.1 then 0.01 they almost become equal to each other. So, the response the true response of the instrument is like this, but if you look at the approximated response of the instrument it will be something like this because in the rest of the terms you see the difference is not much. So, approximate solution is also as good as true solution, right. So, error is very little in fact, an infinitesimal right. So, this is this is how the system behaves when it comes under impulse input and initially it is it has to be infinite, ideally it has to be infinite, right. So, we have taken q o by K by lambda is 1, it has not infinite it has to have unit value and this will start from 0 and within very short duration of time this will reach the 0.995.

Now, in addition to this when the pulse is for very short duration the shape of the pulse does not matter, right. It is only energy that is imparted to the instrument and we can take one that we can mathematically also we can ensure this. Let us take first order system it is lambda D plus 1 q o is equal to q i, K q i or lambda dq o by dt plus q o is equal to K q i.

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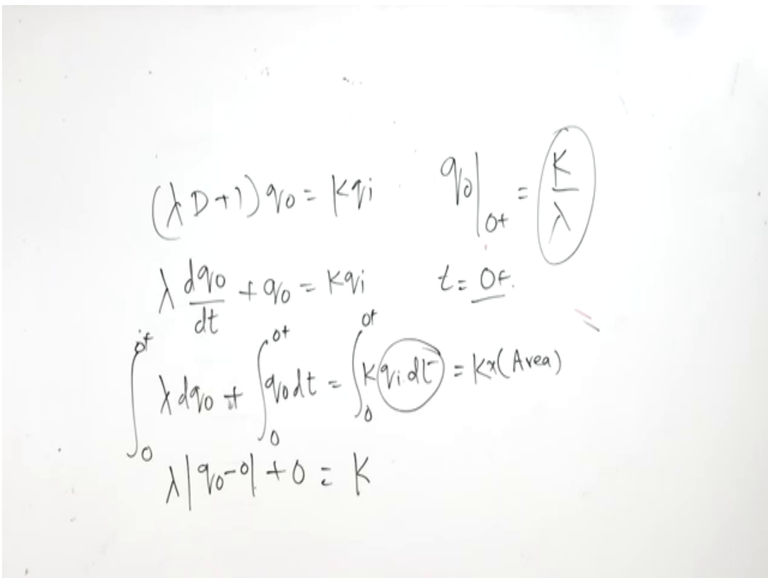

Or lambda dq o plus q o dt is equal to K q i dt, fine. Now, this q i dt this area this is K multiplied by area. So, it is K multiplied by area. Now, we will integrate this. Now, we will integrate this equation from 0 to 0 plus 0 to 0 plus right and this will also get integrated q i dt 0 to 0 plus. Now, we have taken this already considered this q i dt as area and when we integrate this we get a h lambda q o minus 0 plus it is q o at T is equal to 0 it is q o and T is equal to 0 plus it is q o and this is going to be 0 is equal to K multiplied by area, area we have taken unit area for the sake of analysis, right.

So, we can say that q o at 0 plus is K by lambda, right. So, when it is K by lambda right and this is the value when the time is equal to 0 plus just before the starting 0 plus means just before the starting right. Now, if there is a impulse so, impulse signal can also be represented as a differential of step input.

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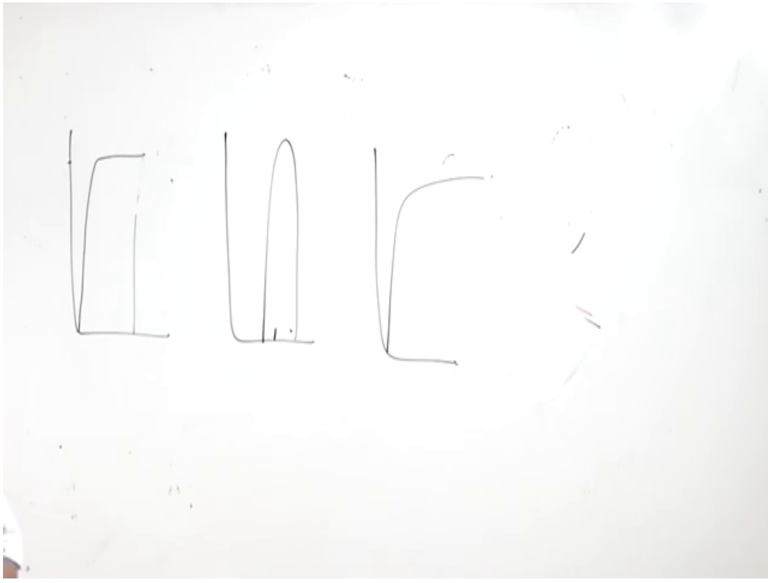

Suppose, in a in a instrument there is an step input, if I differentiate this signal I will be getting a vertical line, right or if because in actual practice in actual practice the step input is not like this. Step input is it takes it is it is something like this well it is something like this then this vertical line will become a dome, right. So, step input if you differentiate a step input you will get impulse you know perfect step input will give you differentiation of perfect step input will give you perfect this impulse signal. If you integrate this again you may get step signal, this is how this can also be represented.

And, this is all for this lecture. In the next class we will start with the frequency response of first order systems.