Mechanical Measurement Systems
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Lecture – 18
First Order System- Frequency Response

Hello, I welcome you all in this course on Mechanical Measurement Systems. And today we will continue to discuss the First Order System with Frequency Response.

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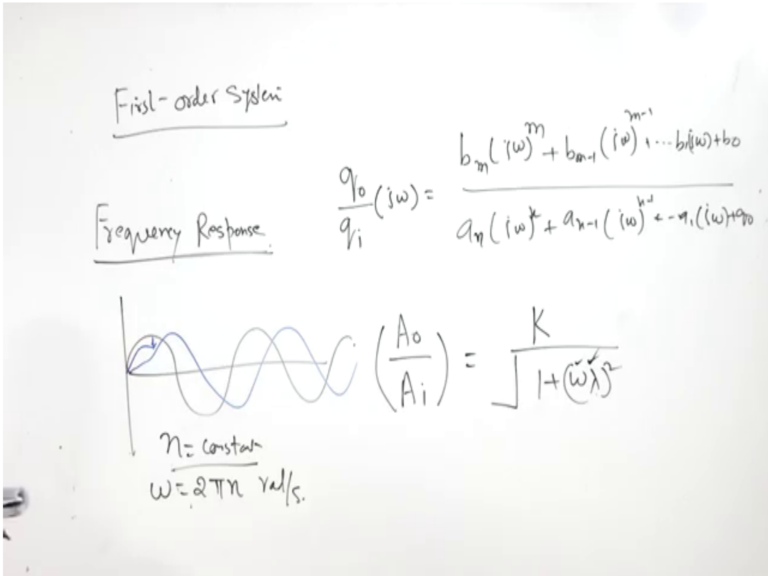

So, first order system for frequency response. When we were discussing the fundamentals of dynamic response of measuring instruments, then we have we have driven a generalized equation for or we frequency input between output and input as b o sorry this is b m; b m minus 1 I omega m minus 1 plus b 1 i omega plus b o, and this is about a o i omega raised to power n plus a n minus 1, i omega raised to power n minus 1 plus a 1 i omega plus a o this is a n.

Now, we are going to discuss earlier we were discussing the step response or the ramp response or the impulse response of measuring first order instruments now this is frequency response. Suppose there is a sign in a certain input, how the system is going to behave shall, the measuring system will also have output like this or shall it have this different type of output. Now once we have sine input, then output amplitude is normally not more than input amplitude I mean if reflects the energy of the wave. So, output amplitude is always less than input amplitude that is one thing. Second thing is phase output and input are they are going to remain the same phase or output is going to be something like this it is out of the phase right.

So, there are two things in the frequency response, that is first is amplitude ratio. So, amplitude ratio it is output amplitude divided by input amplitude. So, if I want to draw the output response I need two things frequency is going to remain same. If input frequency is 50 hertz the output in frequency will also be 50 hertz frequency will not change now frequency is constant now what is going to change? First is amplitude and the amplitude ratio is expressed by K divided by 1 by 1 plus omega lambda whole square, omega is frequency 2 pi f omega is 2 pi n radians per second right n is a frequency.

Suppose frequency is 50 hertz. So, 100 pi is going to be the omega and lambda is time constant. The time constant will reflect whether the response is sluggish or it is very quick in response. For large value of time constant definitely the system response is sluggish. Once system response is sluggish this is reflection of system response and system response we shall also be divided by the decided by the frequency. If frequency is high right you will find that there is a greater phase difference between input and output or when this lambda is high, greater difference between phase of inlet and output. So, the phase or we can phase is expressed by phi.

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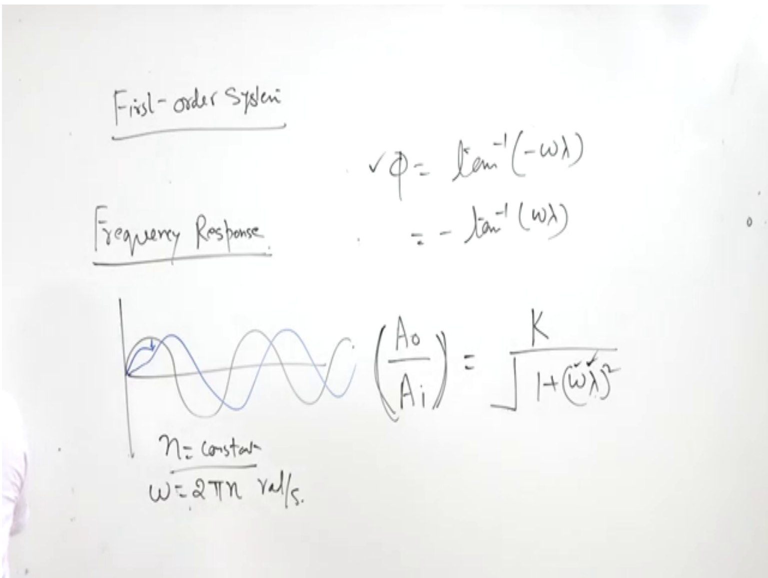

So, phase is phase of output signal is phi is equal to tan inverse minus omega lambda. So, it will always be lagging or phase lag can be written as minus tan inverse omega lambda. So, in first order system if you want to do the analysis of the first order system there are only 2 things, one is amplitude ratio and another is phase lag or phase right and we will take some numerical problems one by one and that will give you the clear inside how the system response is taking account for a frequency input to the instrument.

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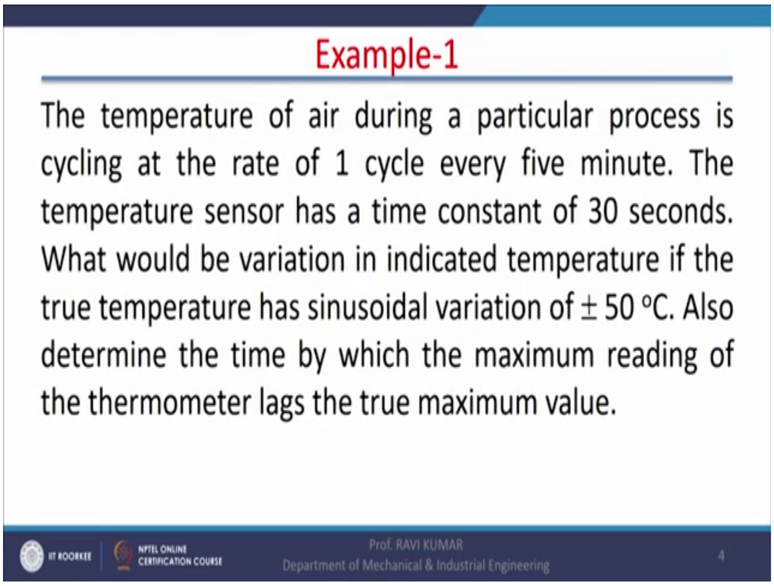

Now, first example is the temperature of air during a particular process is cycling at the rate of one cycle every 5 minute. So, there is a air in the room and temperature is cycling, one cycle for 5 minutes suppose air in this room it is cycling with the rate of 1 cycle per 5 minutes let us say how the temperature is fluctuating the temperature sensor has a time constant of 30 seconds.

So, temperatures there is a temperature sensor in the room which has time constant of 30 seconds fine. What would be the variation in indicator temperature if the true temperature has sinusoidal variation of plus minus 50 degree, plus minus 50 degree in a room it is quite large. So, it is not happening in a in a building it is happening somewhere else, where air temperature is varying between plus 50 degrees to minus 50 degrees centigrade right and the cycle is 1 cycle every 5 minutes right and the time constant of measuring instrument is 30 seconds.

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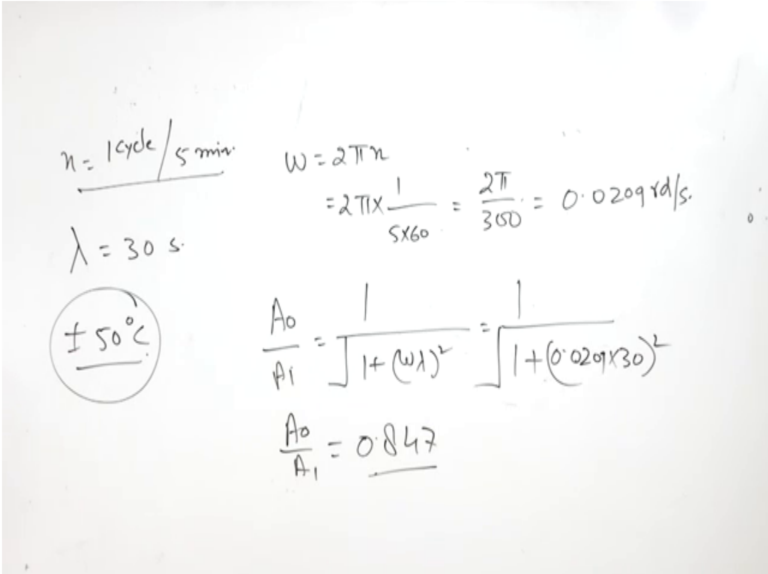

So, here we will we will have to find that what is the variation in indicated temperature. This is the actual variation, but what is the variation in indicated temperature by temperature sensing device.

So, in order to find indicator temperature, first of all calculate the value of omega. So, omega is 2 pi m, 2 pi multiplied by n is 1 cycle every 5 minutes 5 into 60 right and this is going to be equal to 2 pi by 300 and 0.0209 radians per second. This is frequency or omega the frequency is 1 cycle per minute and the value of omega is 0.0209. Now response of temperature sensors; so temperature sensor will have also have sinusoidal response and the cycle time is again 1 cycle every 5 minutes this is going to remain same, but amplitude will change.

Now, amplitude of output divided by amplitude of input is equal to under root 1 by 1 by under root 1 plus omega square lambda whole square is equal to 1 by 1 plus 0.029 multiplied by lambda is 30 seconds, multiplied by 30 whole square and under root fine. Now in this case the amplitude ratio is coming as A o by A i as 0.029 multiplied by 30 square plus 1 this is not 2 0.0209 209. So, 0.0209 into 30 square plus 1.5 0.847 it is 0.847. So, amplitude of the response amplitude of the frequency response is only 84.7. Let us say percent of input amplitudes. So, amplitude of output will reduce now what about the phase angle? The phase angle is.

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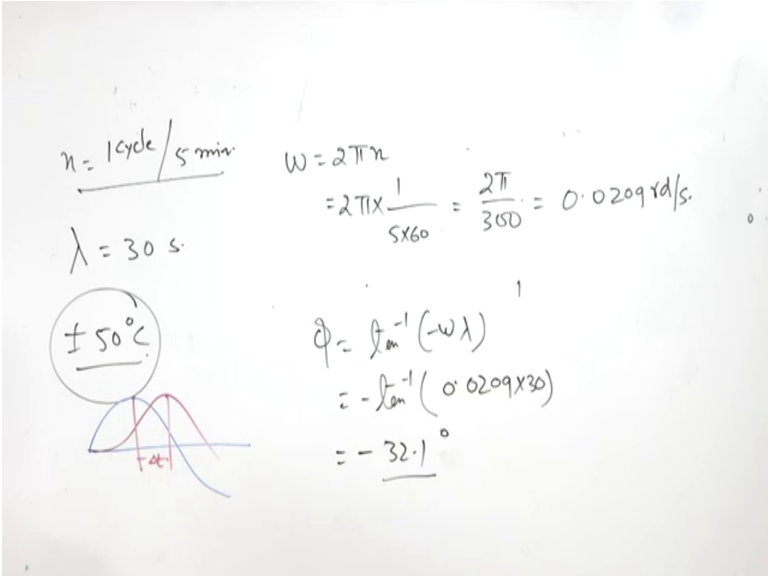

Phase angle is tan inverse minus omega lambda is equal to minus tan inverse omega is 0.0209 and lambda is 30 and if we take minus of the tan inverse 30, it is going to be equal to minus 32.1 degree not centigrade degree.

So, this is phase lagging so minus. So, it is phase lag now in this problem it is being asked determine the time by which the maximum reading of the thermometer lags the true maximum value. So, in a sine input suppose this is the maximum value, but in response will get a different maximum value or maybe somewhere here. So, there is going to be a time lag and this time lag can be calculated once we have the phase lag. Now this phase lag first of all we will convert this into the radiance. So, this phase lag 32.1 is converted into the radiance.

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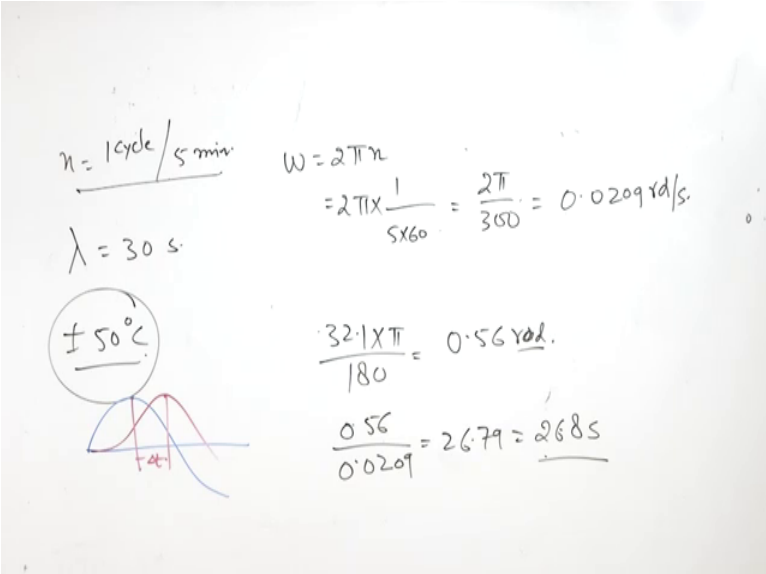

So, 32.1 multiplied by a pi divided by 180 will give 0.56 radians and we have omega radians per second right. So, this 56 0.56 divided by omega 0.0209 and this will give 26.79 or 26.8 seconds. So, peak of input signal will be reflected in the measuring instrument after a lag of 26.8 seconds right. So, in a frequency input there are two things one is amplitude ratio and another is the phase lag.

Now regarding this we will take another example, there is a first order instrument which is required to measure signals with frequency response of 100 hertz. Now the frequency is quite high 100 hertz. So, n is equal to 100 cycles per second or 100 hertz 100 hertz.

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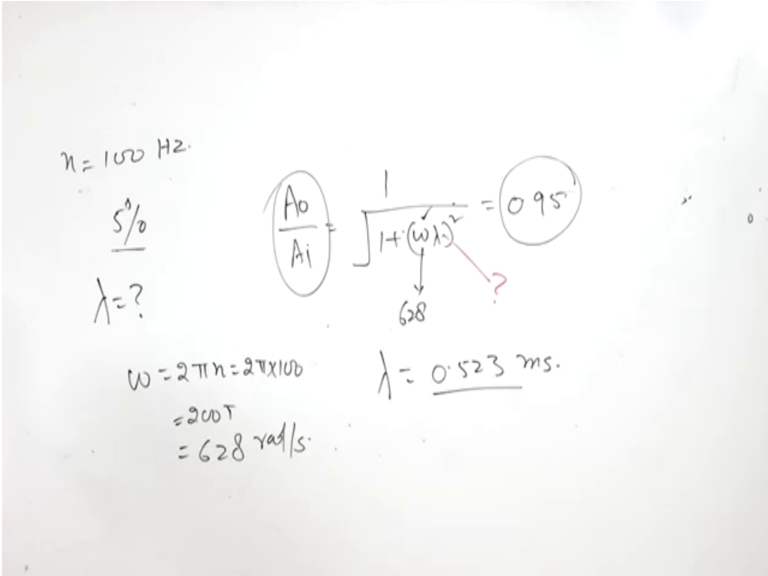

If allowable in accuracy measurement of amplitude is 5 percent, allowable inaccuracy in is 5 percent, then measure the time constant, then measure the time constant what is the time constant allowable accuracy means allowable accuracy in amplitude ratio. So, A o by A i is equal to as we know is equal to under root 1 plus omega lambda whole square. Now allowable accuracy is 5 because in first order system this the system will not overshoot. So, for input 1 we are not going to get going to get 1.05, for input 1 for level accuracy here it will be only 0.95, omega is 2 pi n is equal to 2 pi multiplied by 100 is equal to 200 pi and that is 628 radians per second.

Now, we will be putting the value omega here 628 this A o by A i is known to us the only unknown remaining here is lambda. Just with the help of these values we will calculate the value of lambda and lambda is 0.523 milliseconds. Now this lambda is 0.523 milliseconds here now phase angle. Now here the when phase shift is required when frequency is 75 hertz as per the statement of the problem.

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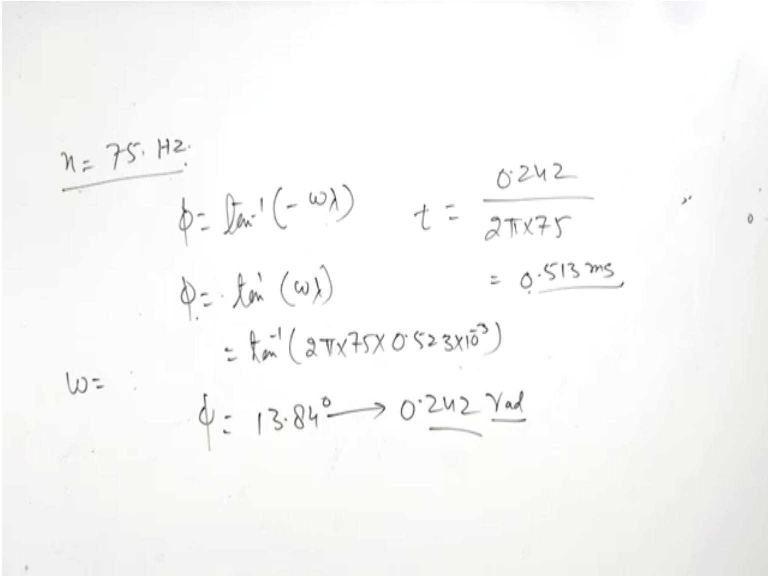

So, when the frequency input frequency 75 hertz, the phase lag or phase we can calculate phase is equal to tan inverse minus omega lambda. So, for due imposed frequency of 75 hertz this is the phase or phase lag is tan inverse omega lambda this is phase lag and omega we can calculate from here it is 2 pi f. So, here itself will calculate tan inverse 2 pi into 75 into lambda, lambda we have already calculated 0.523 into 10 to power minus 3.

Now, for this value the phase lag is coming as 13.84 degree. Now this degree will again be converted into the radians just by dividing by 180 and multiplying by pi and the radian is 0.242. Once we have converted this phase lag into the radians now omega is with us we can find the value of time, that is phase lag 0.242 divided by 2 pi into 75 and this time is going to be 0.513 milliseconds.

So, in a frequency response of a system always remember that the frequency of output and input remains same, there is a ratio of output amplitude and amplitude and output amplitude is also less than the amplitude and there is always a phase lag in the measurement. We can take another example where a thermocouple is having a time constant of 10 seconds.

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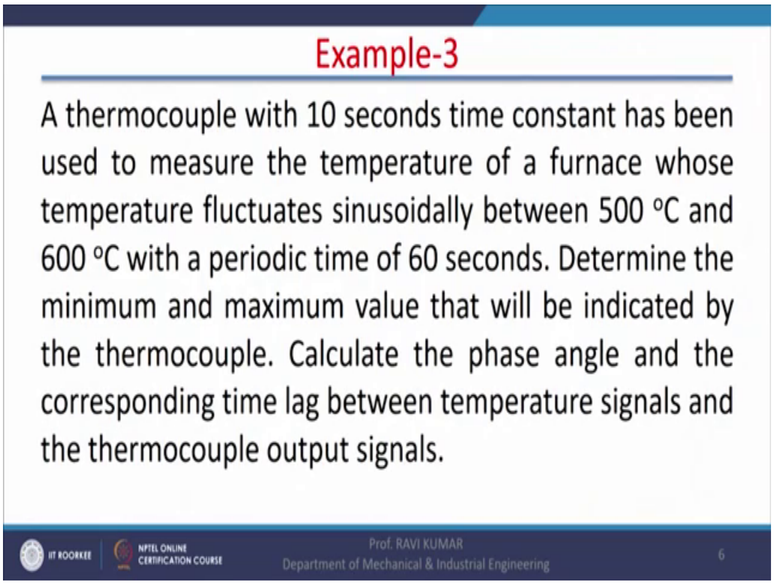

There is a thermocouple having a time constant of 10 seconds is used to measure the temperature of a furnace whose temperature fluctuates sinusoidally between. So, if there is a furnace which temperature fluctuates sinusoidally between 500 degree centigrade and 600 degree centigrade ok.

So, mean value is 550. So, around the 550 it fluctuates sinusoidally this is 550. right.

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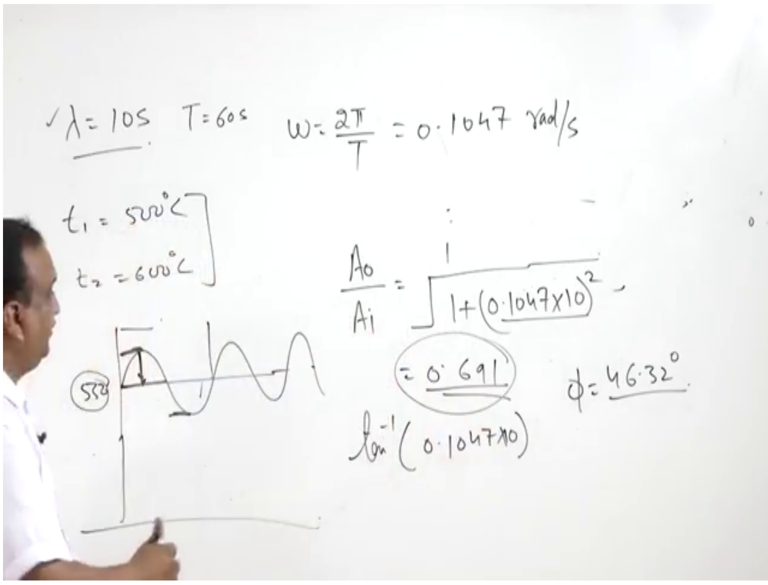

Now and the time period is 60 seconds time period is 60 seconds. So, capital T is 60 seconds for this frequency input. Determine the minimum and maximum value that will be indicated by the thermocouple again now for this minimum and maximum value of thermocouple with time constant lambda, what is going to be the minimum and maximum value for temperature measurement. So, first of all we will calculate the value of omega it is 2 pi by T, here time period is given 60 seconds. So, omega is 0.1047. T is equal to 60 we have put here 2 pi by 60 and this is the value of omega it is radians per second. Now another value we require lambda, lambda is always is given here. So, output amplitude of output and amplitude of input is 1 by under root 1 plus 0.1047 that is omega multiplied by lambda whole square.

Now, if you solve this we will be getting 0.691 that is amplitude of input and output. Now this value and this value now this is the ratio of; so actual amplitude is 50, actual amplitude is 50 now this amplitude will be reduced by 50 multiplied by this plus 550 right because 550 is the average value. So, this amplitude will be multiplied by this right and then we will add 550 here. So, that is the upper value. Similarly this amplitude 550 multiplied by this 550 minus this will give the lower value. So, temperature will fluctuate in that range. So, the range we can calculate and phase angle again calculate the phase angle and the corresponding time lag between temperature signals. So, phase angle again it is tan inverse omega lambda. So, tan inverse 0.1047 into 10 this is the phase lag and the phase lag is 46.32 degree.

Now, again this 46.32 degree will be converted into the radians and these radians will again will be divided by rotation is for the or the or this frequency input, and then we will be getting or cyclic input and then we will be getting the time this is phase lag phase lag there are 2 things one is phase angle the phase actually it is a generalized solution phase angle is a generalized solution.

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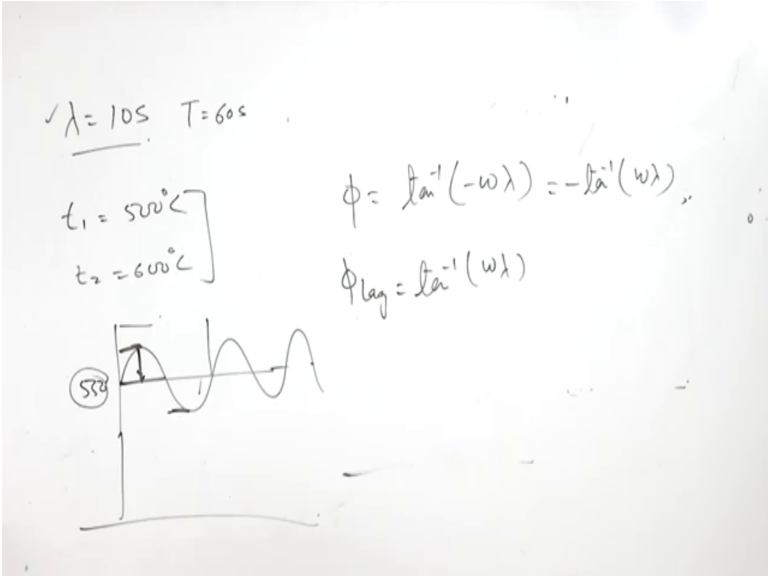

Phase angle phi is equal to tan inverse minus omega lambda, but when we say phase lag it is tan inverse omega lambda because this is always minus tan inverse omega lambda ok. Now after this there is another problem for sine input a first order system is having time constant of 0.25 seconds. So, there is a first order system and it has sine input and it is sine input is time constant is 0.25 seconds.

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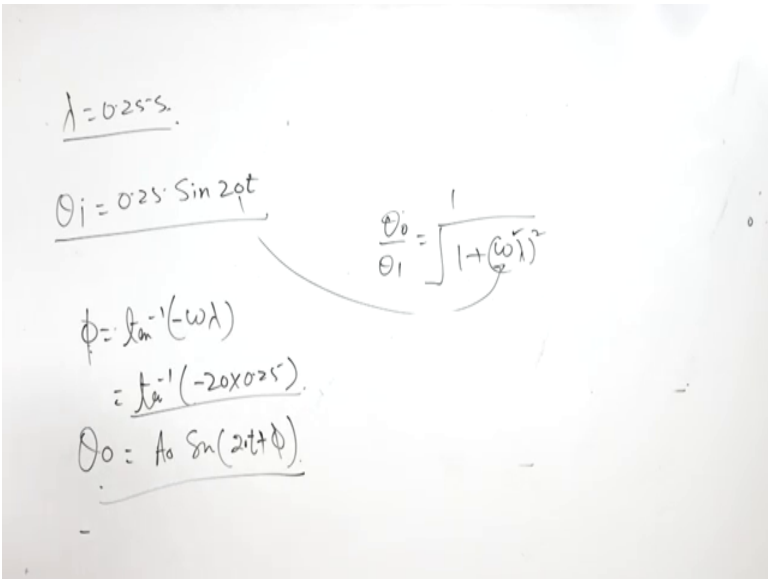

And now the first order system is having a time constant of 0.2 cells again and has been subjected to a sine input giving the relationship theta i is equal to 0.25 sine 20 t. This is the equation for input now we have to find the equation for output.

So, in order to find the output equation, we will calculate theta over theta i is equal to 1 by 1 plus omega lambda whole square lambda is here 0.25 seconds, omega is sine omega t omega is 20. So, we do not have to calculate from n. So, simply we will take from here 20 lambda will take from here and this theta o by theta i is equal to 0.196 or theta o is equal to 0.049 this is the amplitude of output.

Now second thing is phi. So, phi is tan inverse minus omega lambda right and when we take tan inverse tan inverse minus omega lambda and it is going to be equal to tan inverse minus omega is 20 lambda is 0.25 and from here we will calculate the value of phi and because this phi is negative. So, the theta o is going to be the modified amplitude A o sine 20 t 20 t minus this value phi because it is plus phi it is actually it is plus phi, because the value of phi is going to be negative. So, this sign will change to negative 2 the absolute value of this right.

Now, suppose the input to the instrument is not a simple sine wave, it is a combination of sine waves.

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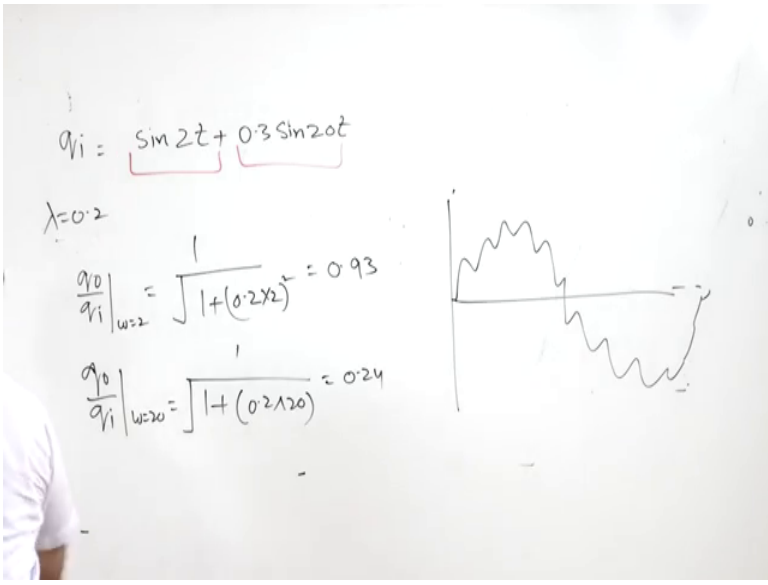

Combination of sine waves means now the input is sine 2 t 2 t plus 0.3 sine 20 t. This is input to the instrument and time constraint of the instrument is let us say lambda is 0.2. Now simply what we will do in this case we will deal these 2 forms because there will be superimposed on each other. The actual wave you will find that is going to be because here this is frequency is 10 times of this. This is omega t, this is omega 2, this is omega 20 right.

So, it means we will be getting wave something like this something like this if we superimpose these 2 waves. |Now here the amplitude is 1 here amplitude is 0.3. So, there is going to be substantial variation in this amplitude also. So, the ultimate waveform out of this equation is going to be like this. Now we will deal each part of this problem as a individual problem, so for this for first one. So, we will take let us say q o by q i let us say omega is equal to 2 right. So, amplitude is going to be 1 pi under root 1 plus lambda is 0.2, omega is 2 whole square because it is 1 by 1 plus omega lambda whole square under root of.

Now, if we solve this we are going to get to 0.93. Now similarly if we do q o by q i for omega is equal to 20, then we will get 1 by under root 1 plus 0.2 into 20, and this ratio is going to be 0.24. So, 0.32 it has come to a ratio is 0.24. So, 0.3 will be multiplied by 0.24 for this output 1 will be multiplied by 0.93.

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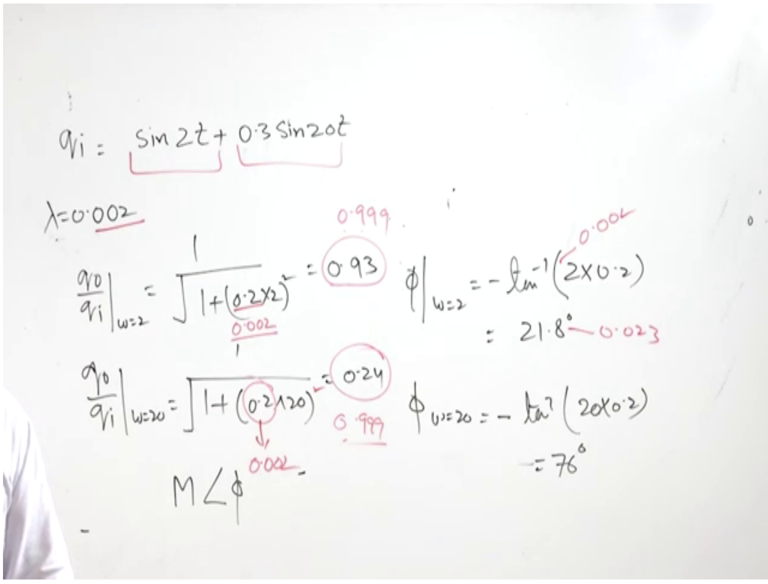

Now phi, phi omega is equal to 2 is going to be tan inverse omega lambda. So, omega is 2 into 0.2 and that is going to be equal to 21.8 degree and this phi for omega is equal to 20 is equal to minus tan inverse 20 into 0.2 and that is going to be equal to 76 degree right. Second thing is when we express this response we always express in terms of M phi. So, ratio 93 and then 21.8. 0.24 M multiplied n within angle our phase is 76. Now if

I change this to 0.02 instead of lambda is 02 it is 0.02 or let us take further 0.002 then this amplitude ratio will improve. The amplitude ratio will become 0.999 this will use another colour. Now if I take 0.022 here. So, this is replaced by 0.002.

And this will be 0.9 sorry 0.999. Now here also if I replace lambda by 0.002 it is also here also we are getting 0.999 right and regarding the phi this will be 0.002. So, it is going to be the tan inverse 0.002 into 0.2 0.023 0.023 similarly this angle will also get changed. So, we will be getting very close if a time constant is that low 0.02. So, output say the response of the instrument is going to be the exactly same as the input nature of the signal right.

That is all for today and from the next class we will start with the dynamic response of second order system.

Thank you.