Mechanical Measurement Systems
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Lecture - 20
Second Order System- Step Response (2)

Hello, I welcome you all in this course on Mechanical Measurement Systems. Today we will continue to discuss the step response of second order system. So, today we will be discussing second order system step response and we will solve certain examples.

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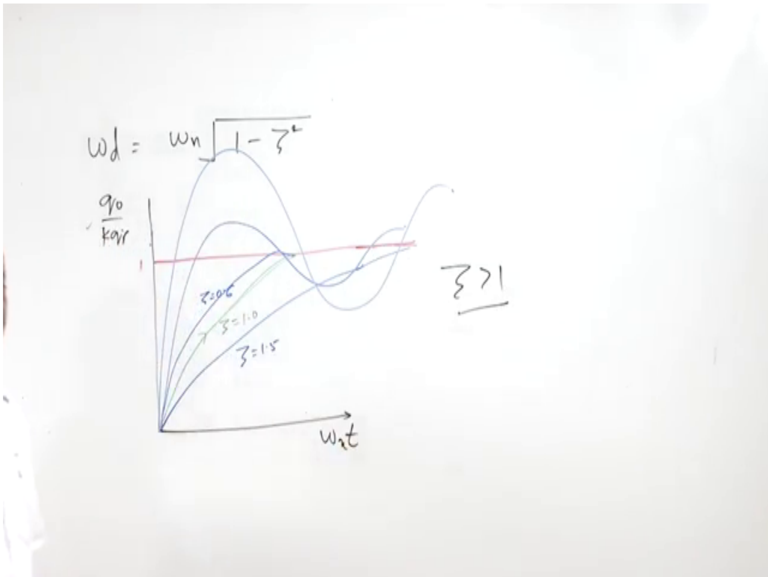

So, last time we finished at the point where I wrote the omega d of frequency of damped vibration as omega m minus 1 minus zeta square for under ramped system. Now, how this is ok, differential equations are ok, but how these systems are going to be have in actual practice.

Now, in actual practise if we draw curves using these equations response curve of different systems on y axis we can take K by q is sorry not K by qo by Kq is right and on the x axis time we can or omega t omega nt can be taken because all the expression they omega and t are appearing together, right.

So, we are taking omega nt on x axis. So, for this qo by K qi is equal to 1 suppose the line is like this where qo by K qi is equal to 1. Now, we will see the response of we will start with a response of over damped system. Over damped system means when zeta is greater than 1, now over damped system will be something like this the behaviour of over damped system will be will be something like this right very sluggishly it will respond and after certain value of omega nt it this it will come closer to this, but very sluggishly it will response. So, if you have a second order voltmeter I mean the this is over damped. So, the moment you give the input very slowly the needle will move and it will indicate the input to the instrument and this is let us say is curve for zeta is equal to 1.5.

Now, the second is critically damped. Now, the critically damped system will respond to the input and earlier it will it will occupy the value of or it will get the value of qo by Kq is, but there will not be any over shoot in this system in the response of the system. So, this is zeta is equal to 1.0.

Now, critically damped system, not critically this is under damped system. In the under damped system the system will behave let us zeta is equal to 0.6. Now, the 0.6 will go like this if it is this is let us say zeta is equal to 0.6 suppose zeta is or the damping ratio is 0.2 right. So, if it is 0.2 then it can go like this or 0.1 for the over shooting right.

So, the when the damping ratio is reducing the oscillations in the response of the system will be developed and when zeta is equal to 0 there is a situation where zeta is equal to 0 when zeta is equal to 0 there is no damping the system will continue to oscillate. So, we should choose a system while choosing a system we have to be careful.

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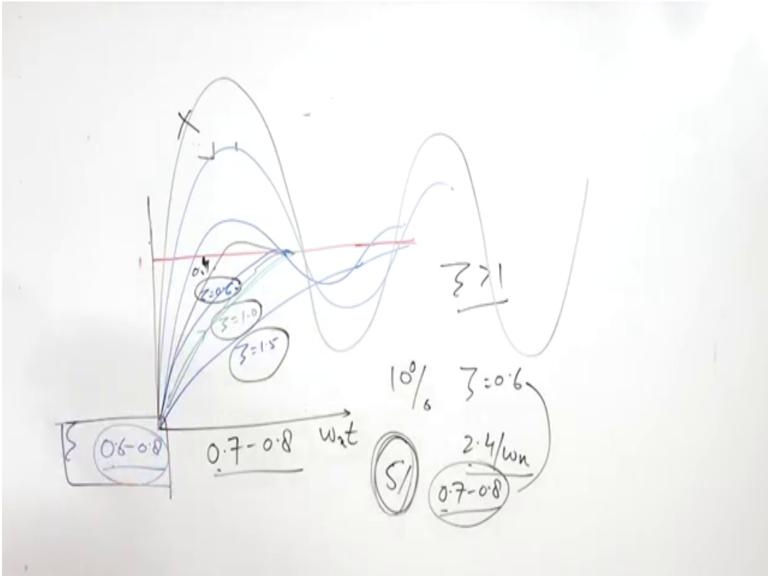

Now, if we have to be careful then we should not go for a system where zeta is equal to 0, at the same time we should not go for a system which has zeta is equal to 1.5. Even critically damped is not recommended though it appears to be from the name itself the critically damped is the most suitable instrument where it is not the most suitable instrument, because it will also take sufficient time to get settled.

The best value of zeta is between 0.6 to 0.8. So, it depends upon the, I mean the how quick the response of the instrument you want, but here there is only a little over shoot this is 0.8. So, there is a little over shoot and vibrations are not there. So, it is simply suppose you have given 220 volt it will go up to 225 and coming immediately come back to 220 it is something like this.

So, this zeta value has to be between 0.6 to 0.8 depending upon the how quick response you want from the instrument suppose. So, it is not 0.8 it is kind of a 0.4, 0.8 will be below zeta here 0.8 will be below zeta, but there will be a little over shoot in the point data also. So, the instrument which we are going for should have zeta between 0.6 to 0.8, this is about say above 0.6 it is going to be point 4 not 0.8. So, in that case suppose I want that the instrument is settled within ah 10 percent range with within 10 percent range and that for 10 percent range settlement for zeta is equal to 0.6 the settling time is 2.4 by omega n.

So, if I want to have settlement I mean if I want to get instrument settle within 5 percent of error then the value of zeta has to be in a range of 0.7 and 0.8. So, normally it is recommended that the value of zeta between 0.6 and 0.8 is fairly good or if you want quicker response then we should go for 0.7 to 0.8 right and further in this way figure a correction has to be made because this is 0.4, this is not 0.6 sorry 0.8. So, now, after this we can have one more step response for omega n is equal to 10, this is one way or generalised way of representing the response of system.

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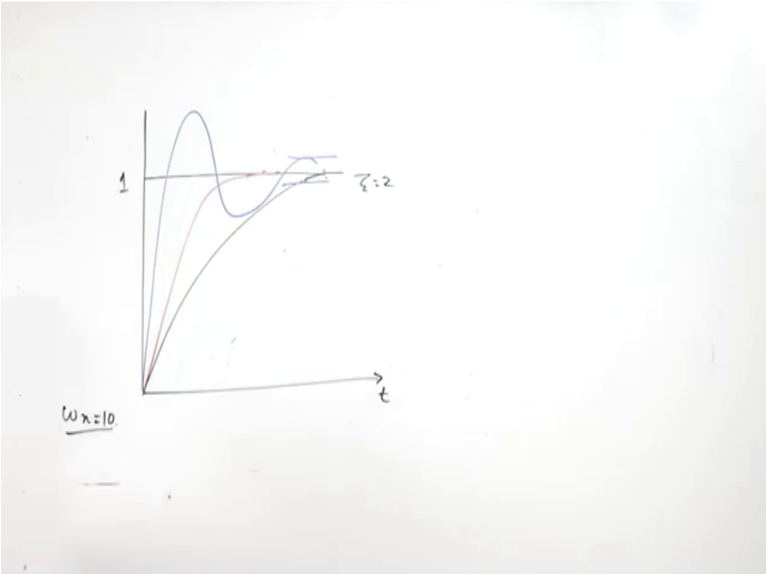

Now, the response of the system you can generate through MATLAB also. So, in MATLAB if you take omega n is equal to 10 right and then this is qo by qi 1 this is time t. Now, here we are not getting taking omega t, we are only taking t and when the damping ratio is 2 the response is like this, that is zeta is equal to 2. When we take zeta is equal to 1 the response is going to be like this, but when zeta is 0.3 in the case of 0.3 the response is they are going to be the vibrations in the output signals. So, there is going to be over shoot this is known as over shoot right and then after certain time interval it will get settled in a particular error band may be 5 percent or 10 percent.

Now, suppose in a second order system zeta is equal to 0 sorry, we will start with this error in measurement dynamic error in measurement by second order systems. If you want to have dynamic error in the measurement that is equal to qi qi minus qo by K that is error in measurement.

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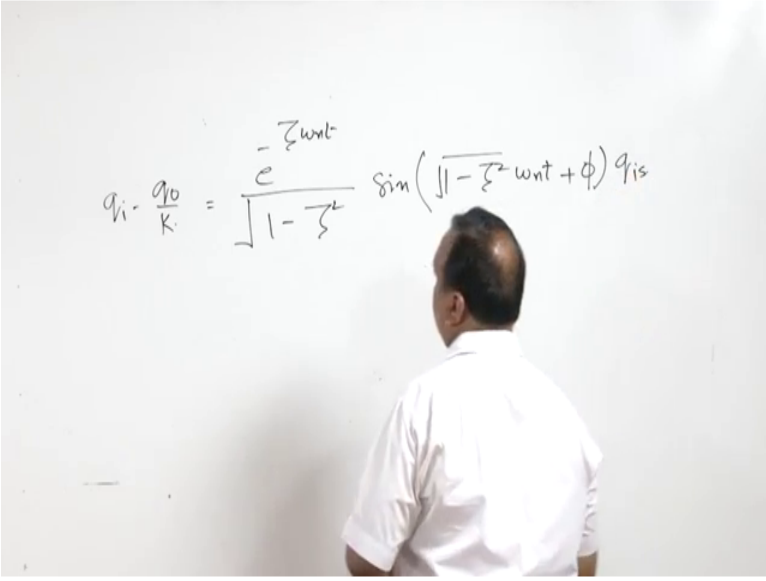

And that is going to be equal to if you look at the equation for under damped vibrations then it is going to be e raised to power minus zeta omega nt divided by under root 1 minus zeta square sine 1 minus zeta square omega nt plus phi q is. So, this is the order of error in under damped instrument which has input of q i and getting q is output. So, if you look at this under steady state when t is very high, this is tending to be towards 0.

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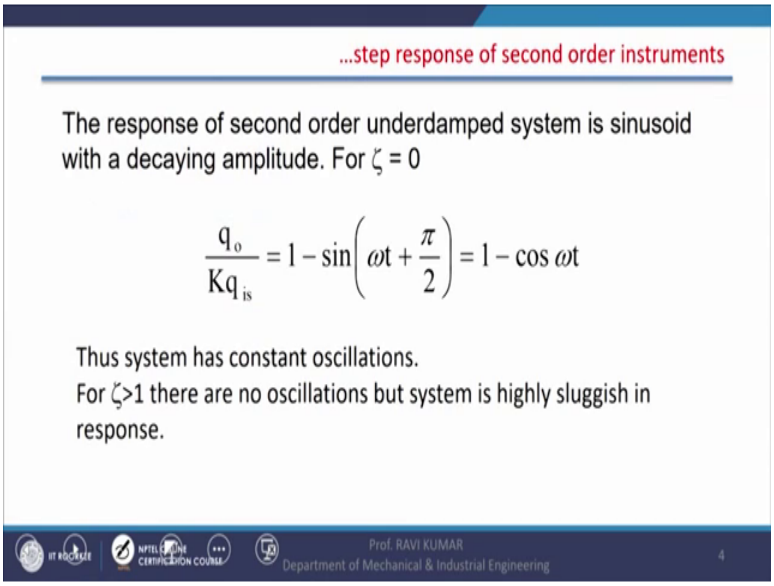

Now, response of second order under damped instrument under damped system for zeta is equal to 0.

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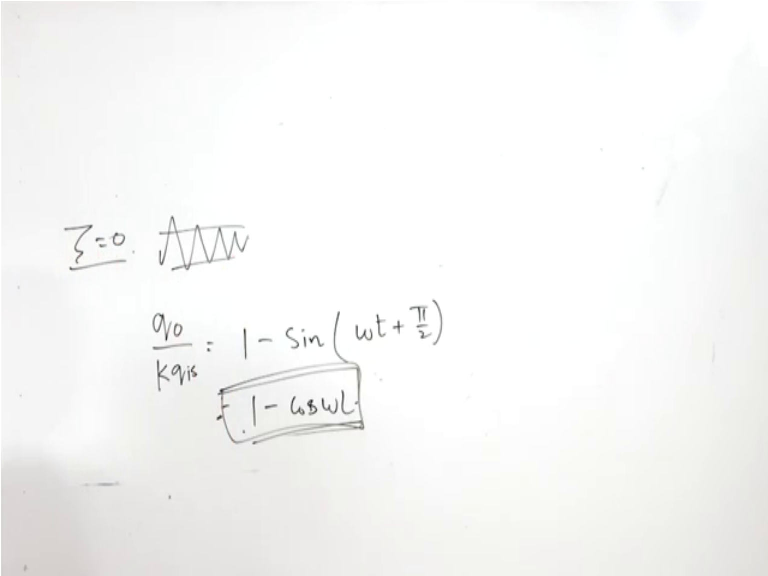

When the zeta is 0 then under damped instrument gets the response as qo by Kq is equal to 1 minus sine omega t plus pi by 2, when zeta is equal to 0 and this is going to be equal to nothing but one minus sorry cos omega t sine omega t plus pi by 2 is cos omega t. And this is the this can be considered as the response of, dynamic response of and under damped system when the value of damping ratio is 0 when this is 0.

So, the system is going to have constant oscillations as I showed you earlier also and the wave form you can be determined using this equation.

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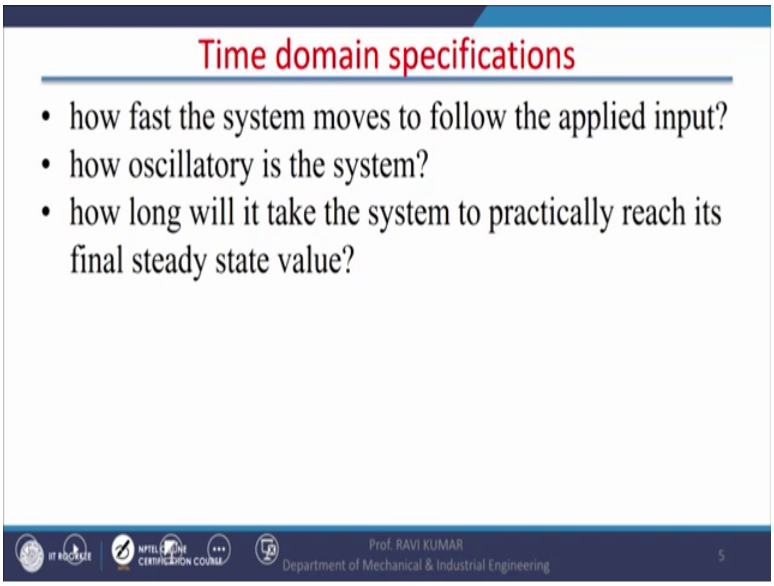

Now, time domain specifications if you go for the time domain specifications while taking for the instruments, first of all the first question we ask how fast the system moves to follow the applied input because for any purpose of measurement this is the first question which is which is which is obvious that how fast the system is responding, whether the system is oscillatory or not because it may be quick in response with a lower value of this. But if the system output is oscillating then it is of no use until and unless it stabilises.

[FL] there are three things first is how fats is system whether the system oscillates if it oscillates then how much time it takes to get settled right.

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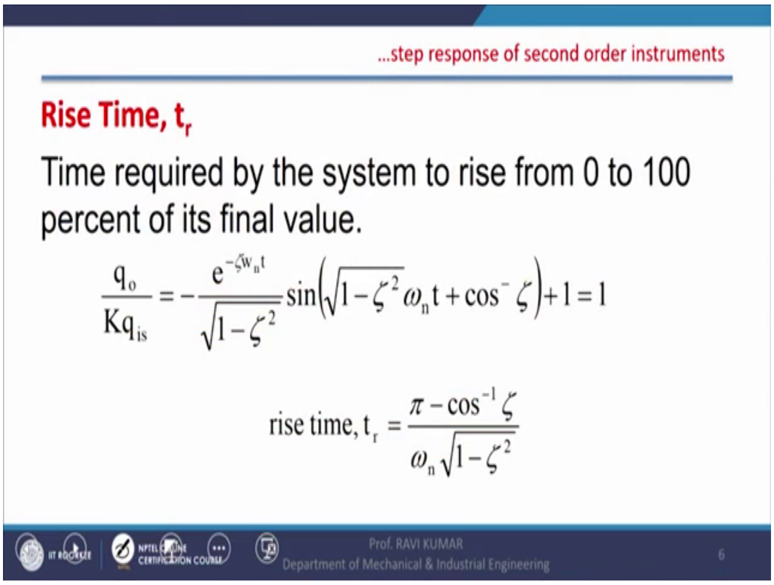

So, in order to find this we will calculate certain parameters and the first parameter is rise time.

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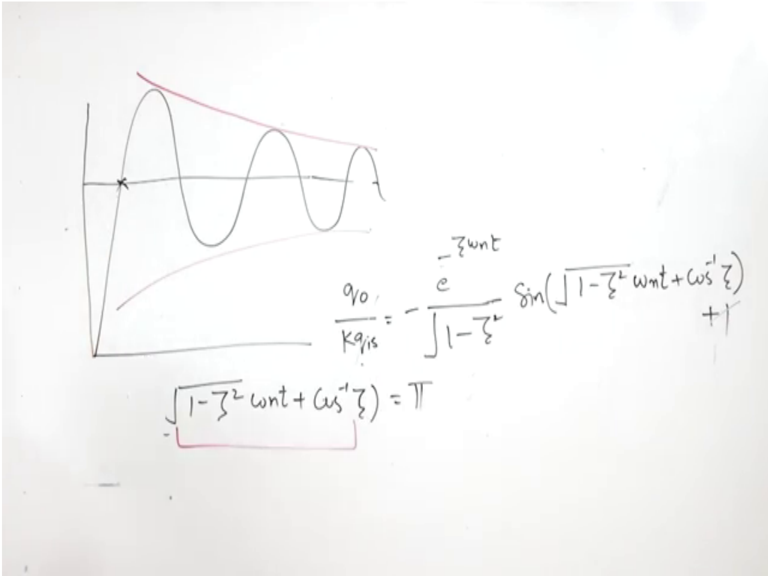

Suppose there is under damped system in under damped system this is qo by K qi and system is behaving like this for example, or amplitude of vibrations is reducing.

First thing is rise time. How much time the instrument takes to reach the 100 percent value ? And this rise time if you look at the equation for under damped vibrations that is qo by Kq is equal to minus e raised to power minus zeta omega nt divided by 1 minus zeta square sine under root 1 minus zeta square sorry; omega n t plus cos inverse zeta plus 1 that is the equation for under damped vibrations.

So, when the hundred percent value if attained this becomes one right and when it becomes 1, then this one and this one will be cancelled out. So, this expression is equal to going to be equal to 0 and when this expression is going to be equal to 0 then we can say the sine this expression is 0 or we can say that sine of under root one minus zeta square omega nt plus cos inverse zeta is equal to 0. Because this is one and one is cancelled out because output is equal to input and we are we are remained with this expression. This will naturally become 0 this is not 0. So, this has to be 0 right. So, this has to be 0 means we have written this expression like this either it is 0 or this is equal to pi because sine 0 is 0 sine pi is also 0.

So, this is equal to pi and then we can write the this is equal to pi, pi right. Now, we can further simplify this to find the t r, this is rise time and t r is pi minus cos inverse zeta, pi minus cos inverse zeta divided by omega n 1 minus zeta square this is the rise time for second order system which is under damped.

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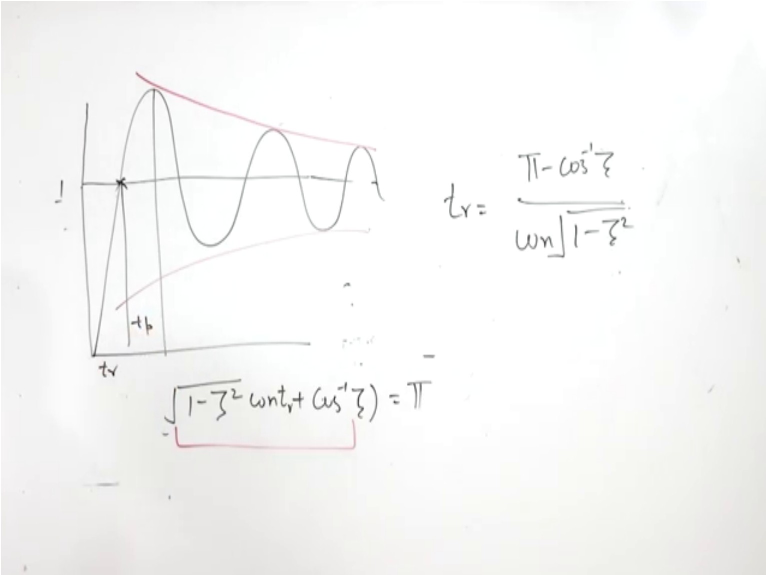

Now, after this rise time rise time is not enough because rise time gives us when the system output becomes equal to the input or 100 percent we are reaching 100 percent input, but in under damped system there is over shoot also. So, there is another term which is known as peak time t p.

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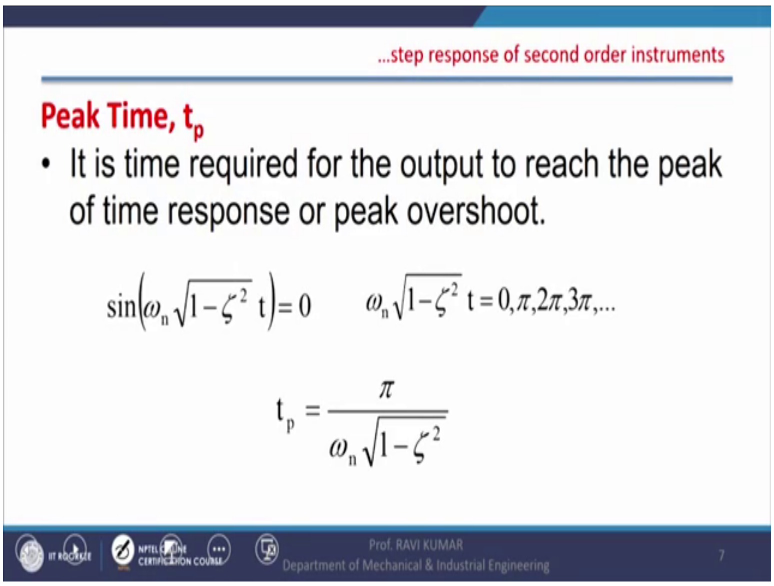

So, this is rise time this is peak time and peak time is expressed as t p is equal to pi by omega n under root 1 minus zeta square this is the peak time.

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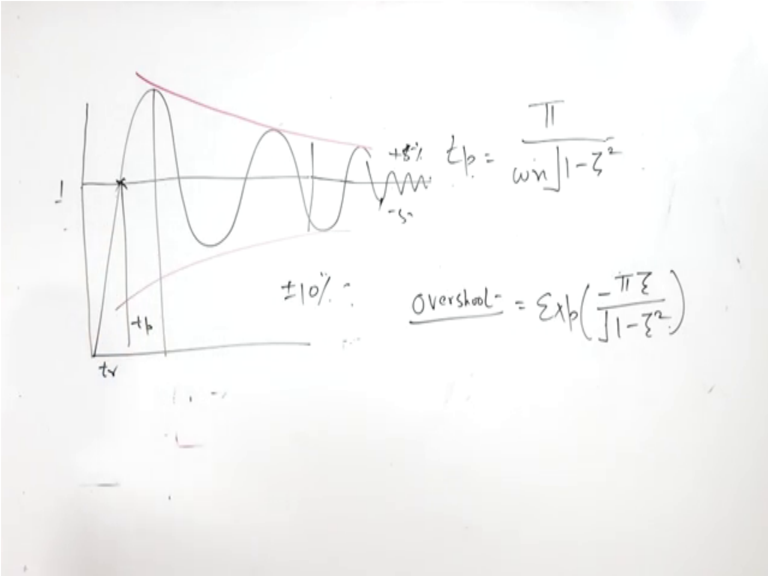

And there is one more thing which we required to know what is the amplitude of this over shoot first over shoot will have higher amplitude than the second one and it will slowly diminish that thing will take place if there is no damping the system will continue to oscillate.

So, here damping is taking place and peak overshoot is overshoot is expressed as exponential of minus pi zeta divided by under root 1 minibus zeta square right I am not going much into the mathematics of this. But these are the final expressions because rise time peak time and over shoot which are helpful in analysing the second order step response of second order system.

Another one is settling time means the time required to attain the final value in a certain error band that is settling time. So, if I say the system vibrates oscillates and after certain time interval it though it oscillates in certain plus 5 percent and minus 5 percent if oscillates in this range only. So, this time is the settling time; when the instrument the output of the instrument is settled in a particular range. So, definitely settling time for plus minus 5 percent will be more than the settling time of plus minus 10 percent.

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Sorry, yes it is more yes it is less. So, if I want settling time 2 percent the settling time will be much higher.

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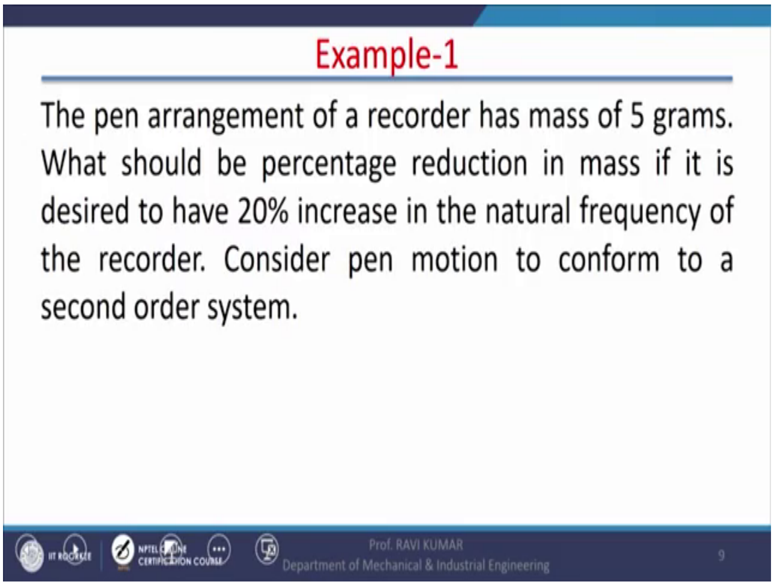

Now, I will take one example. Now, state of this problem a pen arrangement of a recorder, the pen arrangement of a recorder has mass of 5 grams. So, there is a recorder which has a pen of mass 5 grams, mass is equal to 5 grams right. What should be the percentage reduction in mass if it is desired to have 20 percent increase in natural frequency?

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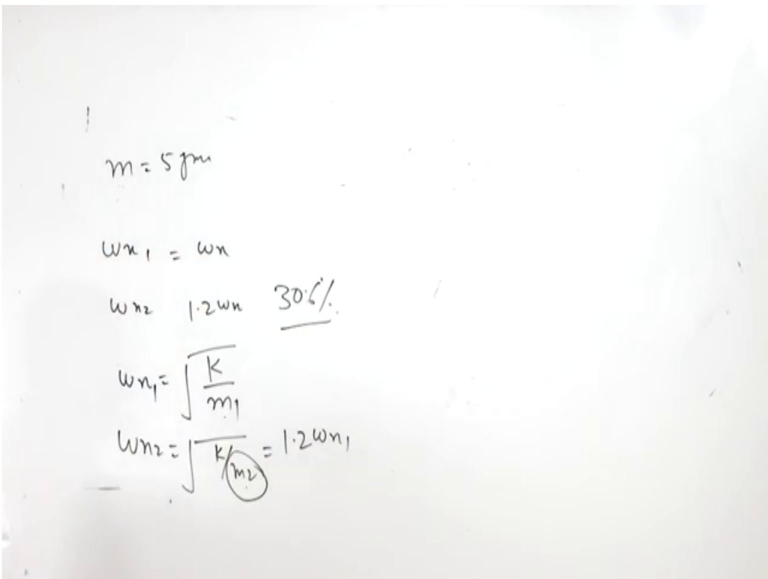

If the natural frequency is increased by 20 percent how much mass will be reduced? So, there are two cases w n 1 w n 2. So, this is w n and this is 1 point w n right. And we have to find how much mass reduction will be there and in a second order system w n is equal to under root K by m. Now, which this relation because this w n. So, w n 1 is under root K by m 1, w n 2 is equal to under root K by m 2 is equal to 1.2 w n 1.

Now, from these two equations we can always find the value of m 2 in comparison with m 1. And the answer for this is 30 point you can do it by yourself the answer is 30.6 percent.

Another example, a second order system was subjected to a step input and the measurements indicated the system the system had an overshoot of 10 percent in rise time of 0.2 seconds.

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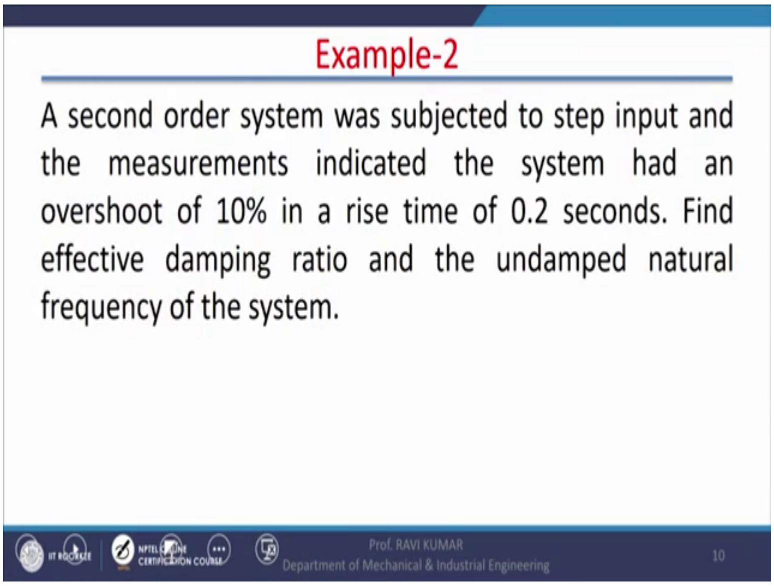

So, there is a second order system and definitely it is under damped because there is an overshoot.

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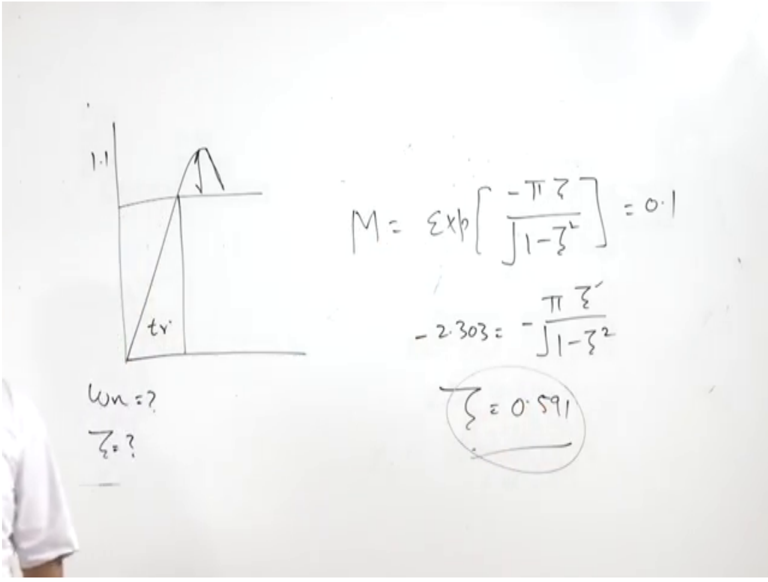

So, there is a system response overshoot of 10 percent 1.1, overshoot of 10 percent in a rise time of 0.2 seconds, rise time is given and overshoot is given 10 percent right. Find the effective damping ratio? We have to find the variety of zeta and undamped natural frequency of the system. We have to find the omega n and zeta, two quantities we have to find.

So, if we look at the expression for overshoot let us denote by m is exponential of minus pi zeta n divided by under root 1 minus zeta square and over shoot is 10 percent, so it is 0.1. So, over shoot is 10 percent. So, we can take natural log on both the sides and we can do the final manipulation. So, it is minus 2.303 is equal to minus pi zeta divided by under root 1 minus zeta square. So, here the unknown is only zeta and when we calculate the value of zeta it turns out to be 0.591, because in over shoot only this zeta is unknown and over shoot we are taking 10 percent 0.1 and by just putting the value 0.1 here then we can take natural of both sides and then only unknown remains is zeta and the zeta value of zeta is 0.591.

When we have the value of zeta then rise time, rise time is rise time is pi by damped (Refer Time: 26:07) w d, d stands for damped vibration these are damped vibration. Rise time is given rise time 0.2, 0.2 seconds.

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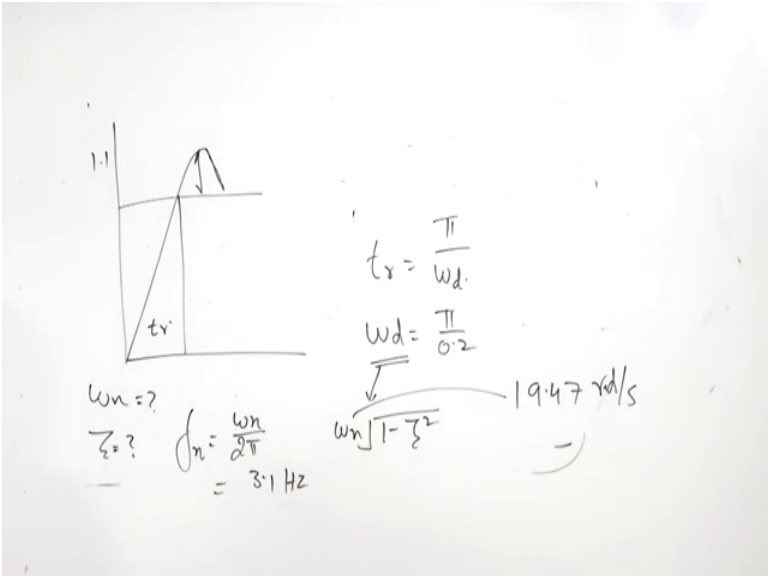

Now, here from here we can find the value of w d pi by 0.2. Now, w d is omega n under root 1 minus zeta square zeta is already with us. Now, the omega n is coming here from here the omega n is coming as 19.47 radiance per second right. Once we have calculate the omega n you can always find the value of frequency. Now, the frequency natural frequency is omega n divided by 2 pi and the answer for this is 3.5 hertz or cycles per second.

So, this is all for today. And from the next class we will start with a we will continue with the second order system and in the subsequent class we will be discussing the ramp response of the second order system.

Thank you.