Mechanical Measurement Systems

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Lecture – 21

Second Order System- Ramp Response

Hello, I welcome you all in this course on mechanical measurement systems. Today, we will discuss second order system, the ramp response and we will focus on ramp response only second order system.

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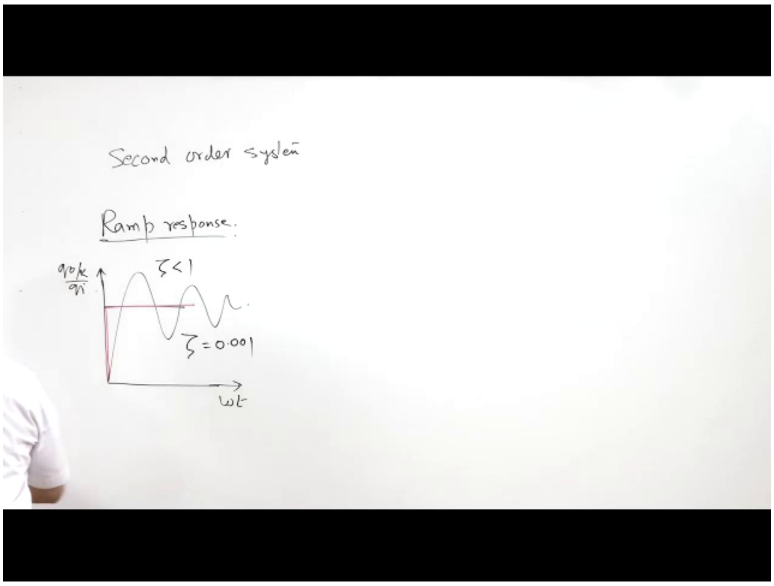

And we will focus on ramp response, it means when there is a ramp input in a second order system, how system is going to behave.

Now, before we start the ramp response, I like to mention here that in case of step response of second order system, recently we have discussed step response of second order system where input is instantaneous where input is instantaneous and remains constant.

Now in this ramp response if I say the system is under damped when the system is under damped, they are going to be the vibrations and after a certain time period, this is omega t and this is qo qik sorry qo by K divided by qi qo by K divided by qi, right.

Then we will have vibrations. Now, suppose this is for under damped system it means zeta is less than 1. Now, suppose zeta is equal to 0.001 in that case, it is obvious that in that case we have to will we are going to have vibrations of larger amplitude and they will die after a long time very long time right.

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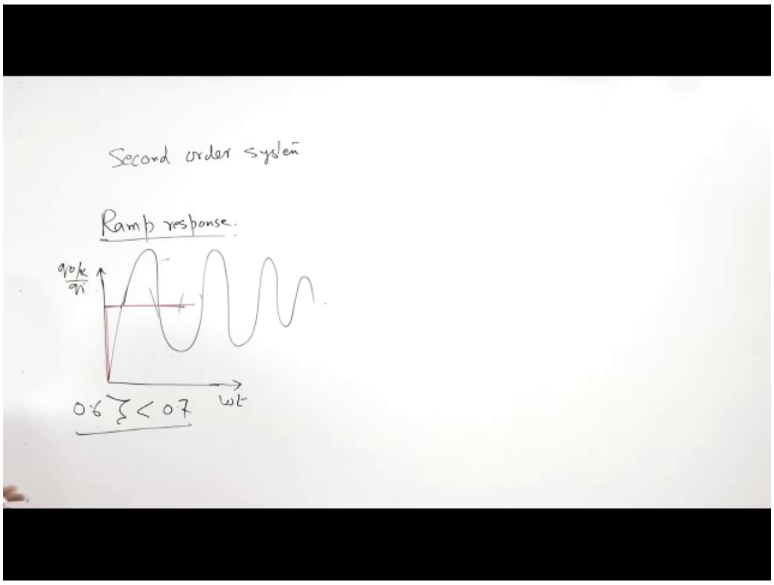

And this type of input or this pite of response for a measuring system I mean it is not appreciated a measuring system should response should stabilize as early as possible. So, that is why last time I told you that the damping ratio has to be in a range of 0.6 to 0.7 now can go up to 0.8.

So, there we strike a very good compromise between the oscillation and the sluggishness of the response of the system right.

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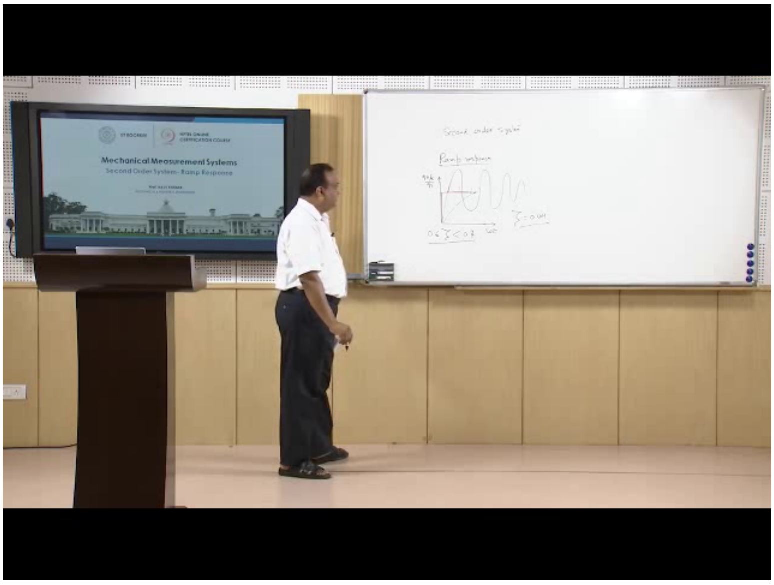

Now we have like piezoelectric transducers. So, piezoelectric transducers have a quality when a force is applied against the opposite faces EMF is generated or by applying EMF against the opposite faces the deformation will take place in the crystal. Now in piezoelectric crystal, it has zeta or the damping ratio of the order of 0.001 right.

Then step input was given to this crystal and it was found that it is not responding like this the response is very very much acceptable, right. Now, the reason being the reason being then the analysis was done and the reason was found that first of all we should understand that no I mean model can truly represent the actual system, there has to be or there will be a deviation between a model and the working of actual system.

So, when we say there is a step input in the system it is not exactly step input the input is something like this.

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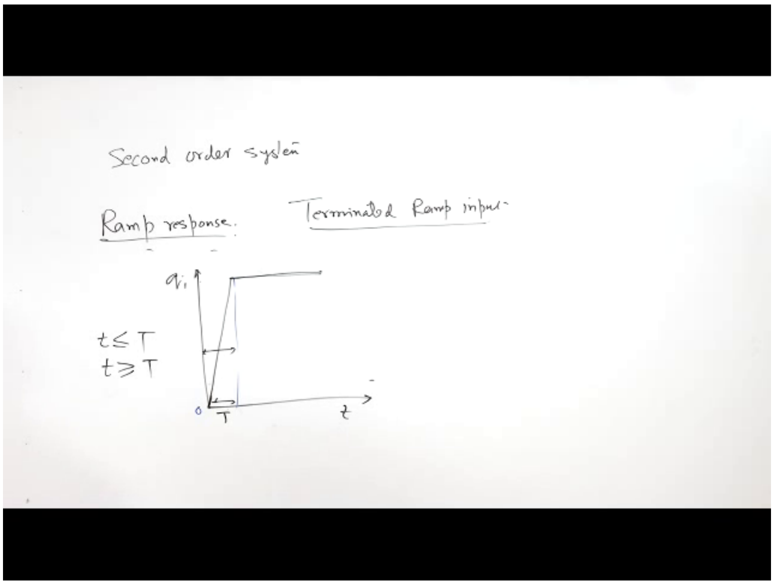

Right, this is let us say time and this is qi. So, the system will teach some infinity decimal time, it may be milliseconds, this may be in milliseconds, but there has to be some infinite decimal time for increasing the input from 0 to this value qis.

Now, this type of input shall it alter the response of the system. Now, the equation is if this type of input is given shall it respo alter the response of the of the instrument. Now, let us let we have to see that mathematically. Now, this type of input is known as terminated ramp response terminated the ramp input to the system and we help to see the terminated ramp response of the instrument.

Now for terminated ramp input the input becomes 0 here sorry change the input become 0 or slope of the ramp now you or the input becomes constant after this. So, there are two parts of this input as we have done earlier in the transient response that is for time interval t.

So, here it is divided in two zones one zone that is p is less than or equal to T and another zone when t is greater than or equal to T and it can go to up to in 3 D for this. Now again, we will start with the basics of second order system. So, first of all, we will write equation for the transient response of second order system that is D square by omega n square plus 2 zeta omega.

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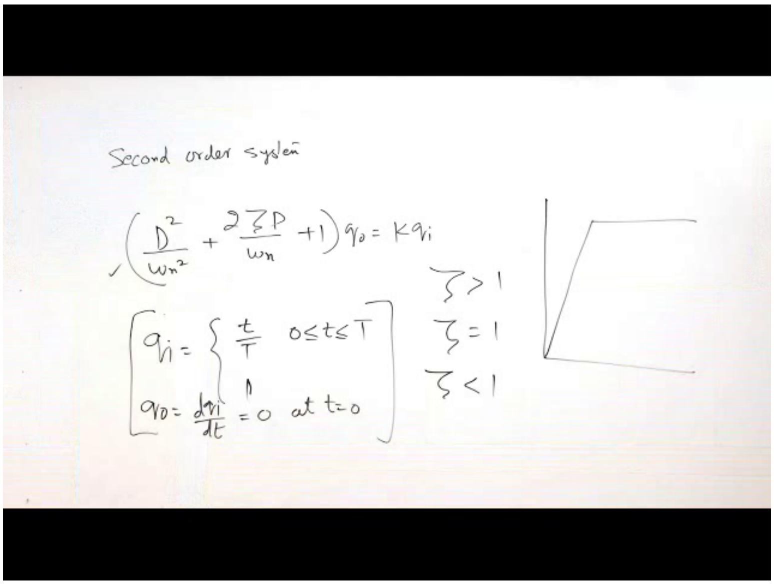

Sorry, D by omega n plus one multiplied by qo is equal to K qi and we can have higher order higher order terms also on this side input side, but because this K qi is a good enough to represent many of the engineering systems. So, we can find ourselves to kqa only.

Qi is equal to the two cases input cases one is t by T, right when 0 less than t less than capital T and then another one is one when input is one let us take input and this is the steady state value and it will continue to be one right. Now output is dqi by dt is equal to 0 at t is equal to 0 right.

Now, for these boundary conditions for these boundary conditions, we have to find solution for this and solution for this for all the cases all the cases means when the system is over damped when the system is critically damped and when the system is sorry when the system is under damped right and for both the parts for I mean in case of terminated ramp input. So, there is a variation of input and then input is constant. So, both the parts we have to find a generalized equation, and for over damped system over damped system.

So, for we will start with the cases for under damped system because most of the systems, I mean which are having very low because this terminated ramp response is only for the system where the damping ratio is 0.01.

So, when the damping ratio is very close to 0 or damping ratio let us say 0.001.

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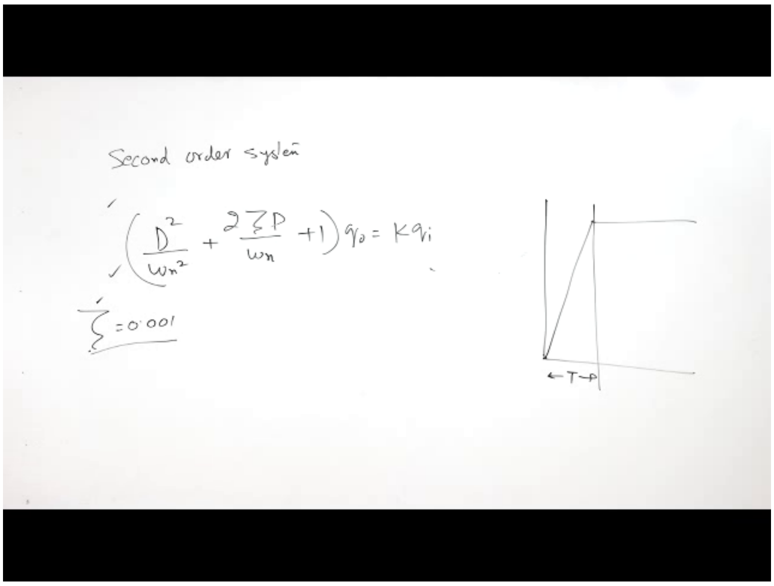

So, ideally we should have calculated for over damped critically damped ever under damped, but this terminated ramp response is useful only this analysis is useful only in the case when damping ratio is very low, right.

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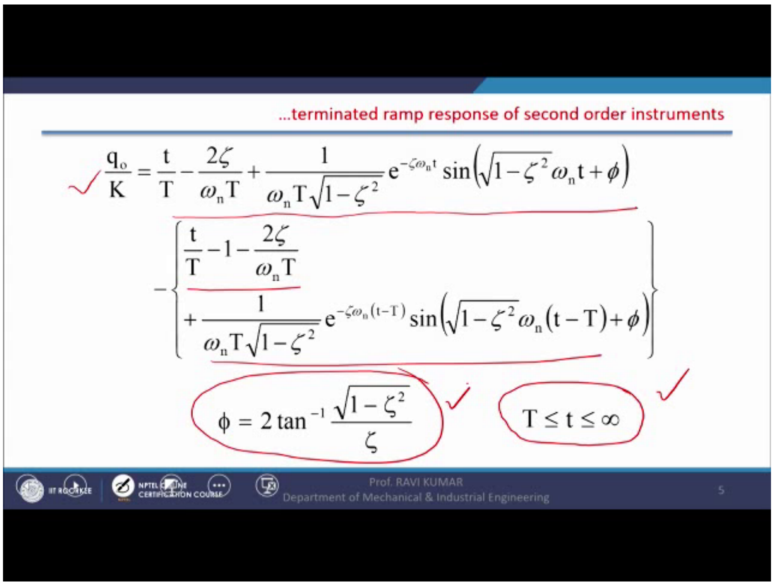

So, now here after just putting these boundary conditions here as shown here we get the output q by K is equal to t by T, then 2 zeta by omega n plus 1 by omega nT under root one minus zeta square e raise to power zeta omega nT then signed under root one minus zeta square omega nT plus phi again, the phi is the phase angle and this equation is valid for only when time is greater or is less than or equal to time for terminated ramp response because the time for terminated ramp respo ramp input is up to this only right. So, we have got one equation for this.

Now, another equation for we have this equation is valid when time is greater than T then qo by K, we have taken the previous expression as it is and then it is minus another expression and the both the expressions have certain common terms and certain different terms as well and the phase angle here in this case is 2 2 tan inverse under root zeta square by zeta, right. So, this is the case when time is greater than capital T, I mean when terminated sorry the ramp input the ramp input has stopped and it can go up to infinity.

So, this is pretty long equation and I have not gone into the details how the equation has been driven because as a mechanical engineer we are mostly concerned with the application side of the things right. So, instead of going into the details or integrase is a mathematical integrates of driving this equation I will like to discuss the implications of this equation now implications of this.

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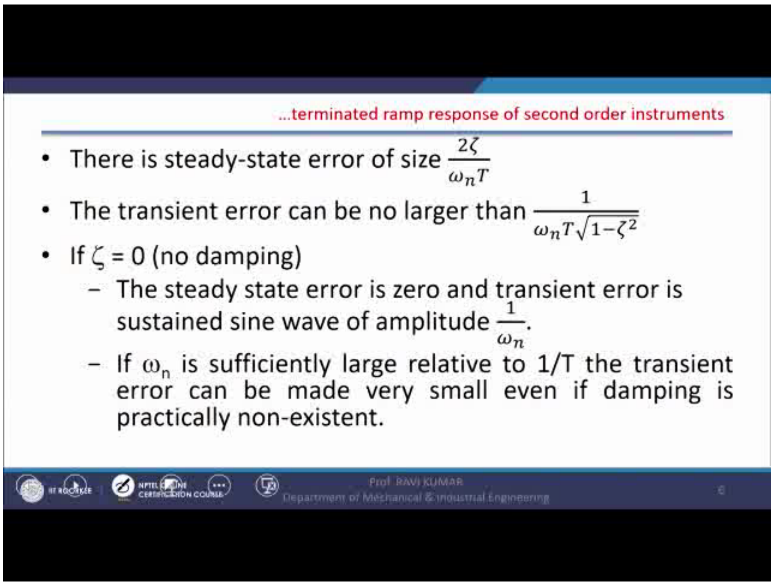

Equation is that there is a steady error of the size in this equation we will find that there is a steady state error.

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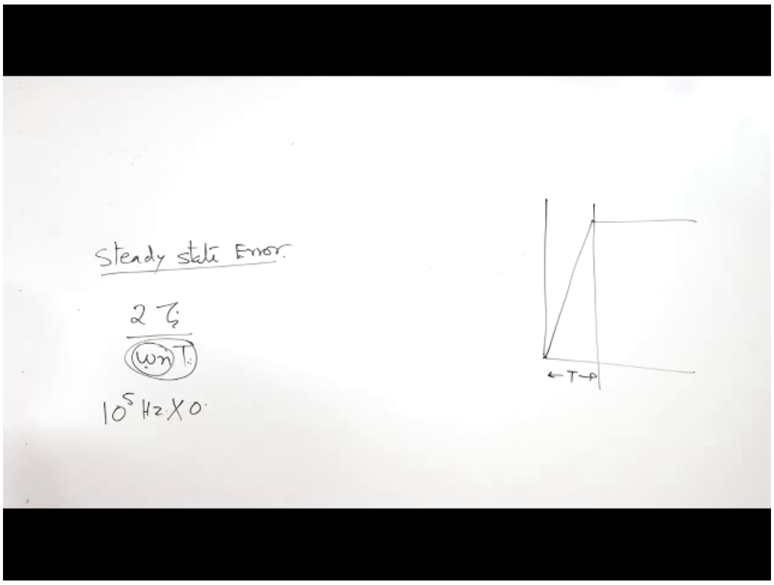

Now, steady state error sse, the steady state error is of the size of two zeta divided by omega nT. So, in terminated ramp response there is an a steady state error and steady state error can be expressed by this, Now here if the T is very small, if we make omega and very high even for that if T is small and the product has to be high higher than much higher than the zeta, then steady state error will be reduced.

So, if we take the piezoelectric crystal for piezoelectric crystal the frequency is let us say 10 to 5; 5 hertz, right, T is some milliseconds right, we will take one example for this and then if zeta is 0.001 in that case, the order of the error will be very less that we will discuss later or on.

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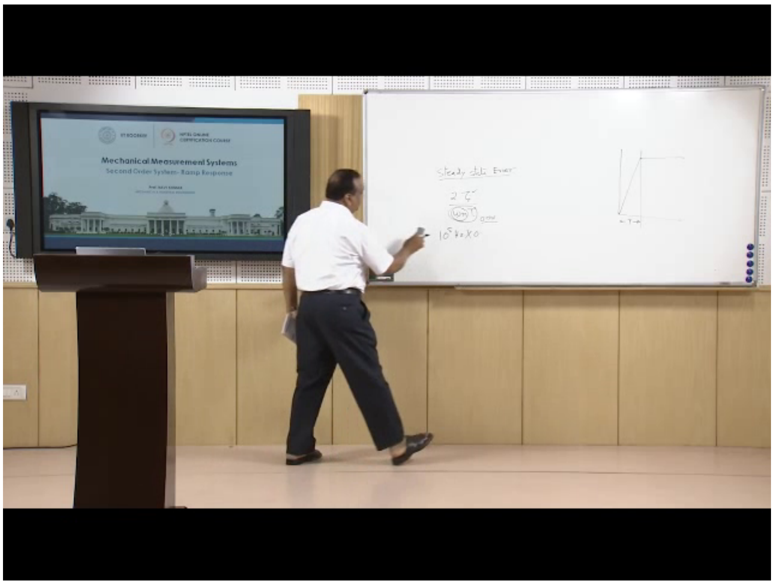

Now, first of all let us discuss about the response of the system, now how system is going to respond. Now for this using these equations using these equations some curves have been drawn like this.

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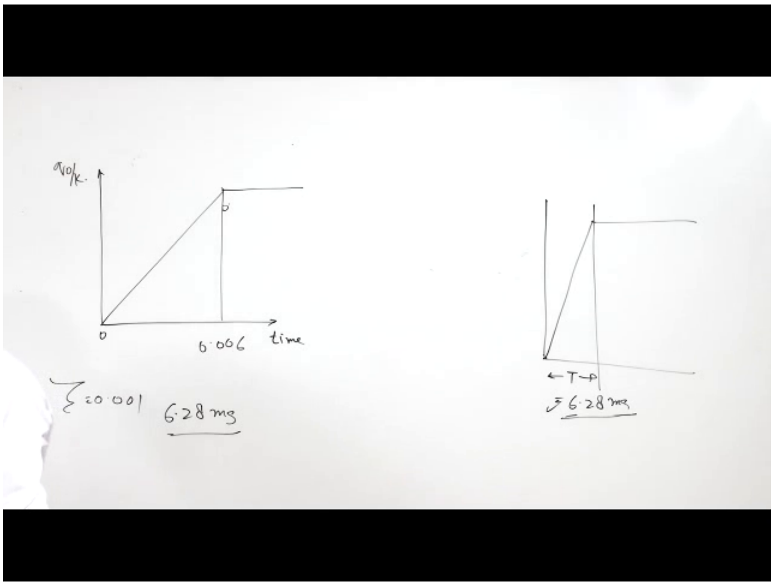

For example the first curve is about time versus qo by K and this curve starts from let us say 0.02; 0.006, I am taking example of piezoelectric crystal where zeta is 0.001 right.

and for example, the ramp response is terminated approximately 6.3 milliseconds or 6 exactly 6.28 milliseconds. So, capital T is 6.28 milliseconds for very short duration on time this cannot be 0 in ramp input, it is 0, but for actual for actual input even though, we want to have sorry for step input it has to be 0, but in actual practice we cannot have 0 to certain value in 0 for time interval there has to be some time interval and for here we have assumed the value 6.28 milliseconds right and right.

Now, we will see the output of this crystal this output characteristics; so for the output characteristics.

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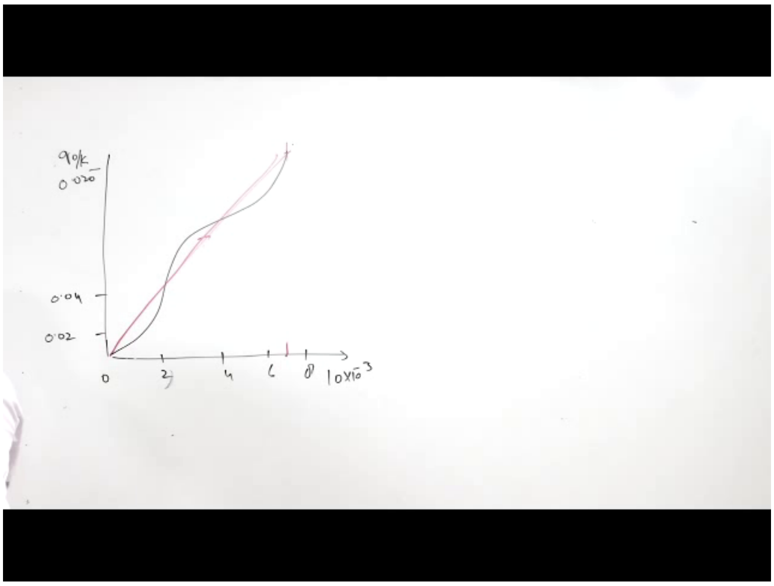

We divide in two parts one curve is for 0 to 10 into 10 to power minus 3. So, 0 2 sorry 2, 4, 6, 8 and 10 milliseconds. Now input is q by K right. So, it starting from 0.02, 0.04 and up to it goes up to 0.020 10 times right.

And then we will say this is the input it is point six two eight. So, you come somewhere here and the input is like this is the input is the ramp input. Now for this ramp input the output of the output of or the response of the system is like this if it was vertical they are remember, we could have observed the vibrations, but it is a ramp in the ramp the deviation is not much n.

Now, ramp is terminated somewhere there right. So, now, we will draw another curve in this curve we will take the type beyond 6.24 right.

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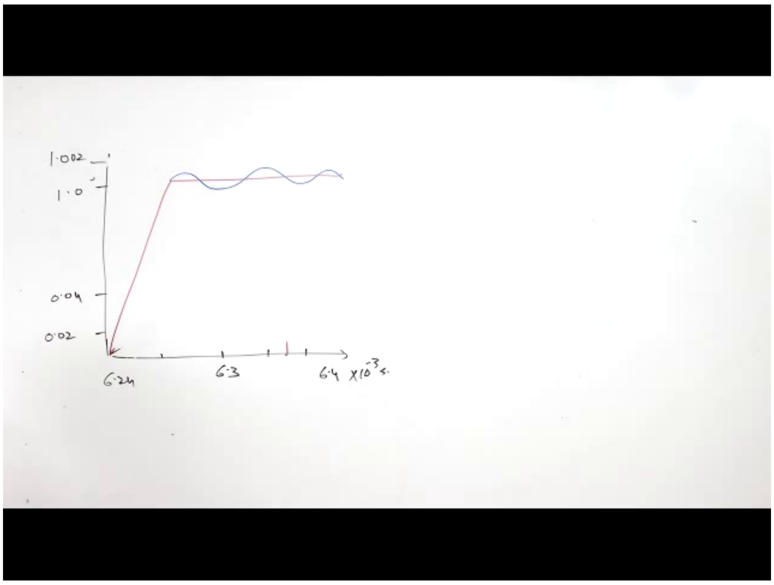

And we will come somewhere here. So, let us say 6.3 milliseconds and then let us say 6.4 milliseconds or 10 to power minus 3 seconds right. So, this will involve both response of ramp and as well as steady input.

So, now here this is sorry this is 1.0 and here is 1.002 right and there is a unit input as you said in there is note, there is a input is like this input is you have taking unit input. So, input is like this.

Now, the response of the system the response of the system is going to be like this here. So, what I mean to say for this also there is not much deviation and when the input ramp is completed in that case also the deviation is not much.

So, there is an excellent response we should say, there is an excellent response of the system for terminated ramp input right though if we give ideally if you give the step input there is going to be a lot of vibrations in the syst in the response of the instrument. So, error in this type of measurement is I said to you earlier.

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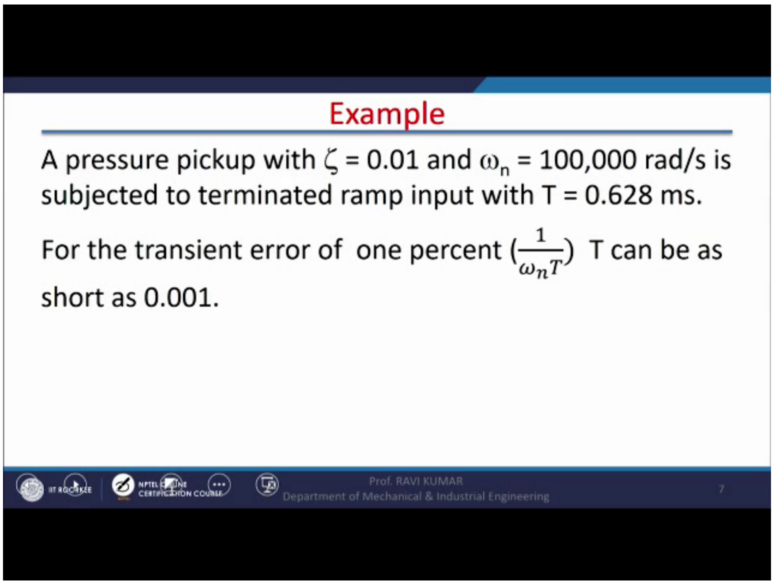

That it is 2 zeta by omega nT.

Let us take one example there is a pressure pickup which has damping ratio as 0.01. So, damping ratio is 0.01 and natural frequency is 1 lakh.

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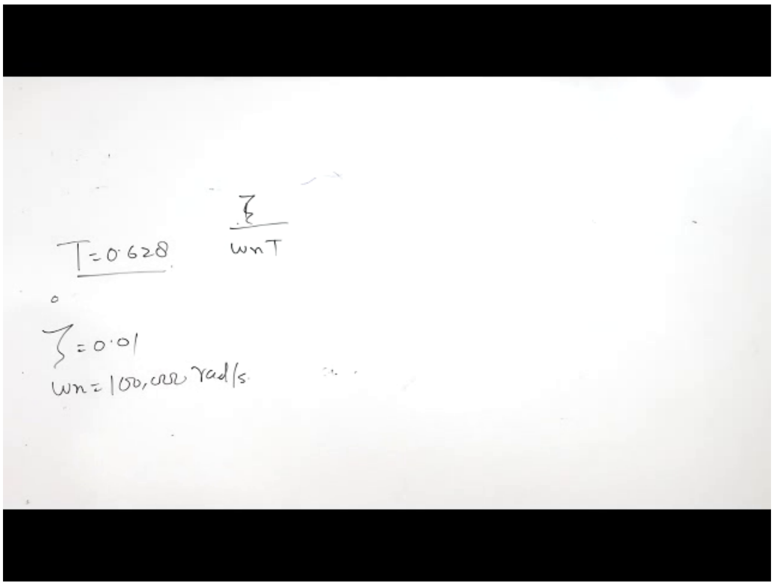

Omega n is 0 lakh cycles per second.

Now, sorry this is one is not cycle, it is radians per second it is radians per second and subjected to the ramp input with t is equal to 0.6 to 8 t is equal to 0.6 to 8, right. So, transient error in percentage is going to be equal to 2 times zeta divided by omega n or for 1 percent error it is going to be 1 by zeta divided by omega n.

So, for 1 percent error, it is going to be zeta divided by omega nT right and you can see from here if you are putting the value because this low value of T which have could have incurred very high error is compensated by very high value of natural frequency.

Now, we will take an example of pressure pickup in a pressure pick up where the piezoelectric transducer is used the damping ratio is 0.01 and natural frequency is 100,000 radians per second.

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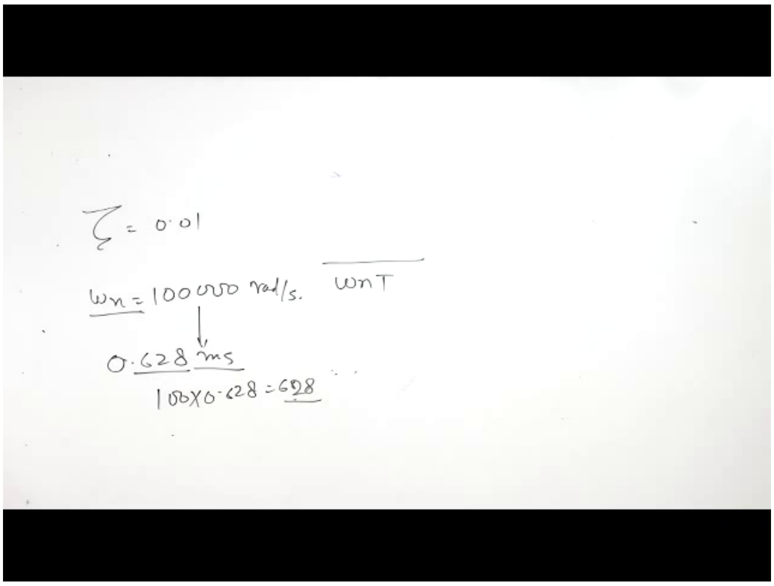

So, the damping ratio 0.01 and natural frequency is 100,000 radians per second right.

And if the terminated ramp input is point is let us say point 0.628 milliseconds. So, if you remember denominator, there is a term in the error omega nT. So, when we write this omega nT though it is for a very short duration of time, if frequencies or the wn natural frequency is substantially high as in the case of piezoelectric crystals, then the product of these two is going to be hundred multiplied by 0.628 or 628 this is multiplied by zeta n sorry, zeta this is damping coefficient this error is 2 zeta.

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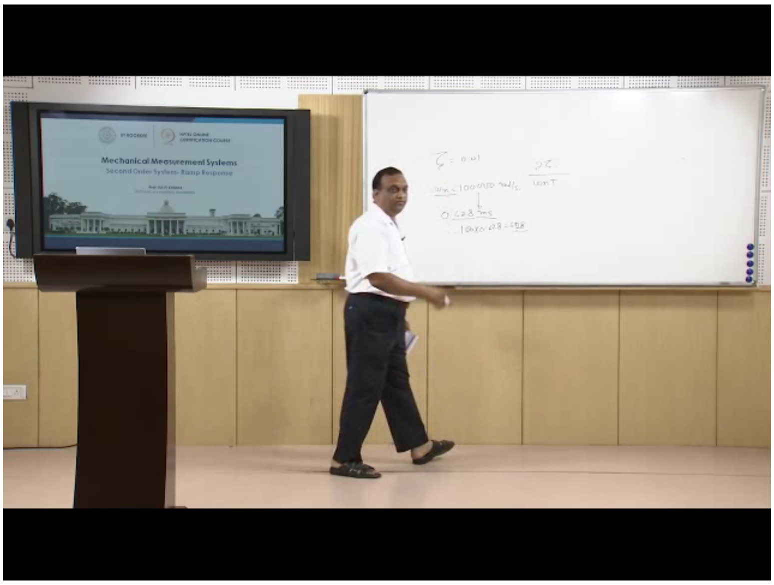

So, final error is going to be very insignificant in the response of the pickup and the response of the pickup can always be shown as for this type of.

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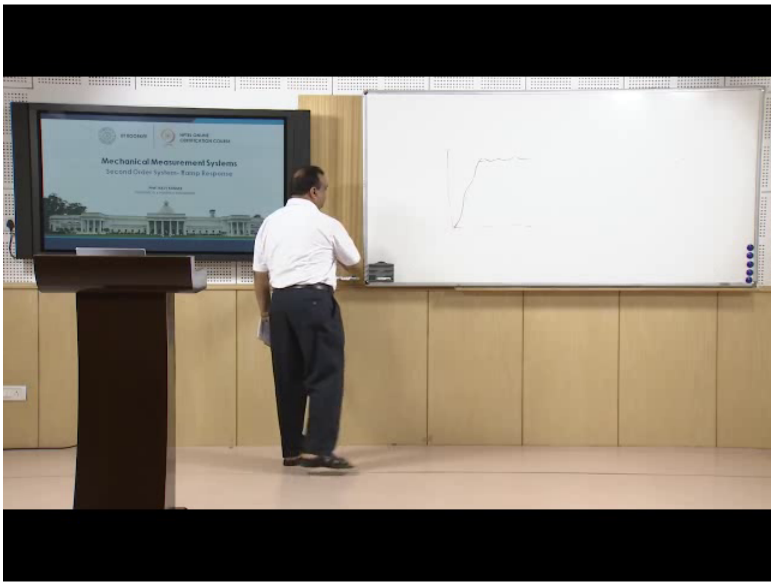

Input the response is going to be like this it will closely follow this like right.

After this we will take the ramp response of measuring systems in the ramp response of measuring systems or the second order system ramp response second order system a ramp response if you remember in the first order system.

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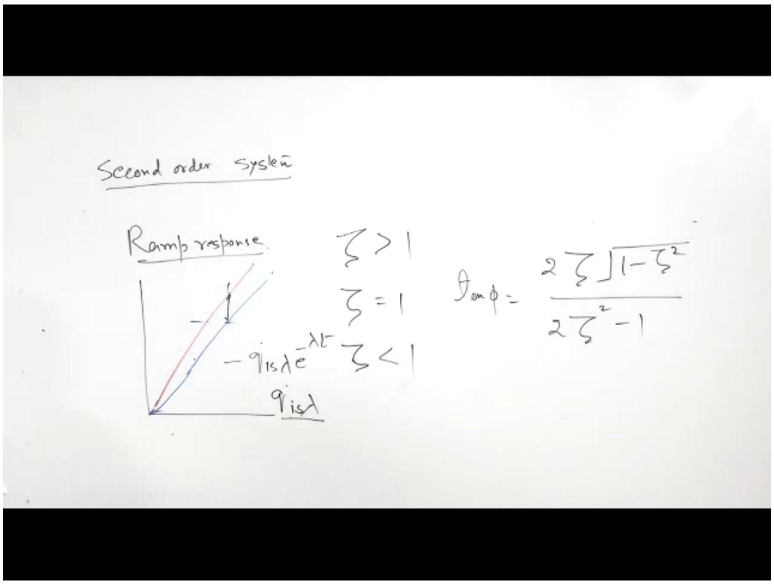

We have taken ramp response as something like this, suppose this is ramp input, then the output ramp output we have taken something like this, right and there is a steady state error in the measurement or there is a time lag and steady state error in the measurement.

In the first order system and it was a like transient error was minus q dot is lambda e to power minus lambda t that was a transient error and the steady stake error was steady state error was q dot is lambda. So, this is the steady state error with the measurement.

Now here also we have for ramp response for the same initial conditions as in the case of step response there is a final equation this is for the damped case when zeta is equal to here in this case now we are going for the full fledge ramp response we will drive equations for both all three cases when damping coefficient is greater than one when system is critically damped and when the damping coefficient is less than 1.

So, when the system is over damped in that case the output.

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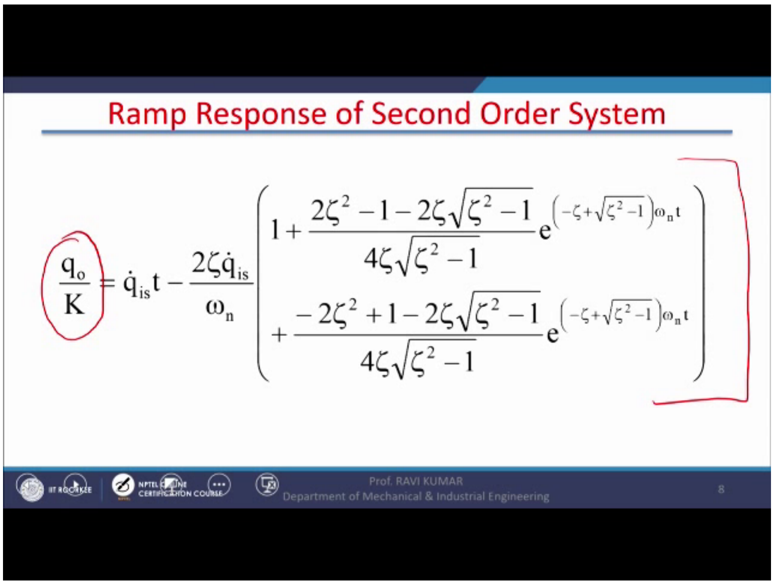

Or the response of the system is expressed by this equation this appears to be a very complicated equation and normally because and normally our systems have the de damping ration in a in a range of 0.6 to 0.7.

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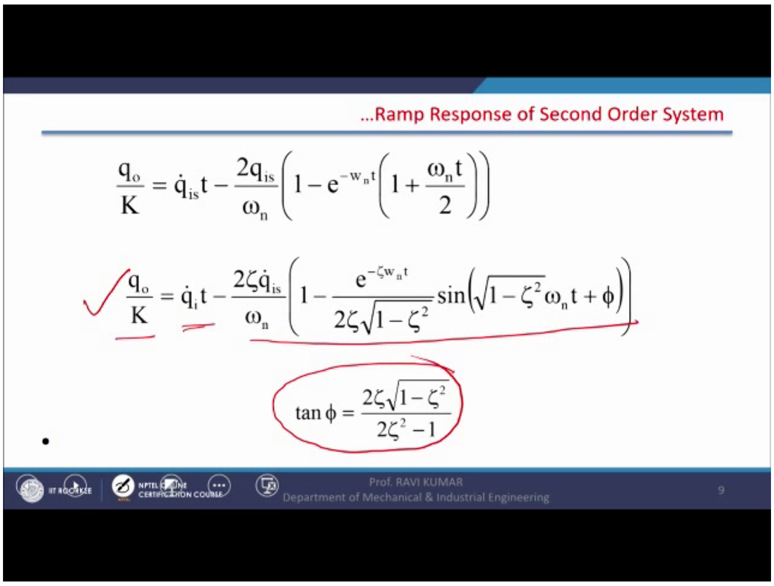

So, normally our systems second order systems are followed by the equation governing equation by under damped system and that is qo by K this is rate of input minus this expression the angle phase angle can be calculated by using this equation which is a function of damping ratio.

So, phi in under damped system the phase angle phi can be calculated by using this equation. So, it is a function of damping ratio only and if we what we show the error in the measurement when there is a ramp input, what is the error in the measurement. So, like a first order system, we can have a graph for second order system also because as a user we are more concerned with the error in the measurement right.

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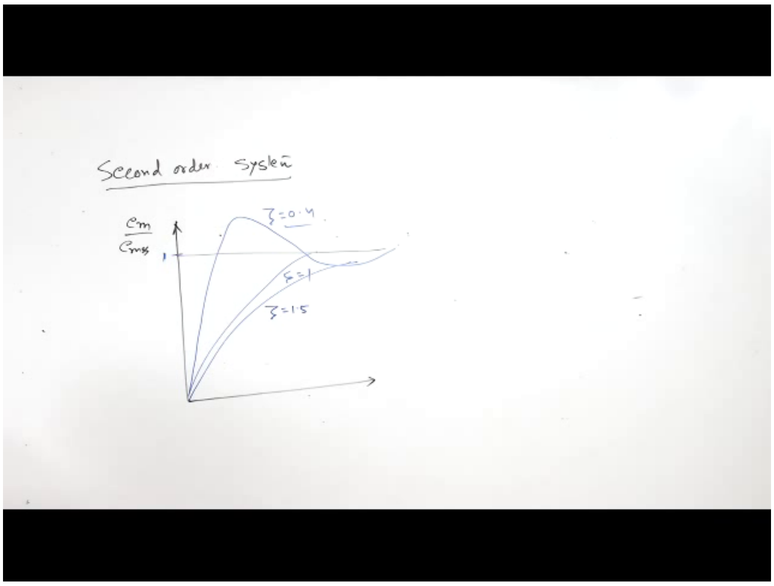

So, error in the measurement for second order system ramp input is going to be zeta is equal to 1.5, it is going to be like this for zeta is equal to 1.5, this is over damped system for a critical damped system.

So, this is let us say this is one for critically damped system is one for critically dam system it will go like this is zeta is equal to 1 for under damped system there is going to be change and a change is let us say zeta is equal to 0.4 the change is going to be like this it is some degree of vibrations and that will also incur error in the measurement and that is equal to 0.4 right.

Now, when the system is stabilized when the system is stabilized when the system is stabilized the system is going to have and a steady state error and if we draw a curve between time.

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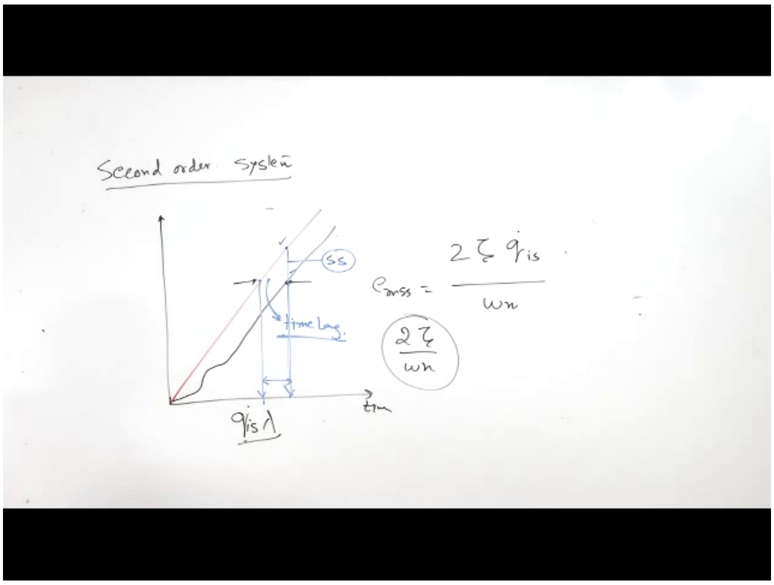

And the output and the output then suppose this is ramp input to the system this is the ramp input to the system right.

Now, error in the output eventually there is going to be some error, but after a certain time interval, there is going to be the steady state and there is going to be and steady state error and this steady state error as I mentioned earlier it is 2 zeta 2 zeta qis rate divided by omega n.

So, this is the steady state error in the ramp response of in first order system it was qis rate multiplied by lambda. Now here this damping coefficient has come into the picture and natural frequency has also come into the picture and this time constant has gone and there is a time lag also there is one steady state other it means for this input output is this much. So, this is a steady state error and this one is time lag.

So, in a ramp response there two things one is the steady state error that is at a particular instant what is actual input and what is input indicated by the instrument and there is another is time lag time lag means this input is shown at this time actual system is responding for the same showing the same input at this time. So, this is the time lag and time lag is represented by 2 zeta by omega n this is the time lag in the measurement and this is steady state error in ramp response of measuring instrument.

So, that is all for today. And, in the next lecture we will start with the impulse and frequency response of second order system.